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**San Andrés**

## Departamento de Economía Ciclo de Seminarios

*"Evaluación del Aval Bancario  
para Fondos de Pensión  
Argentinos, por el Método de  
Quasi Monte-Carlo"*

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PRICING AN INSURANCE AGAINST INVESTMENT YIELD  
DEVIATIONS FOR ARGENTINE PENSION FUNDS BY A  
QUASI MONTE CARLO METHOD

JEL classification code: G13, G18, G23

Keywords: insurance, pension funds, call spread, Quasi Monte-Carlo.

**Abstract** In this paper a Quasi Monte-Carlo method, based on Halton sequences, is used to price an insurance on profit deviations for the Argentine pension funds. The insurance is replicated as a 3D European call spread type option, and the problem of pricing this option is solved in continuous time.



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# 1 Introduction

The pension funds (PFs), managed by private firms called Administradoras de Fondos de Jubilaciones y Pensiones (AFJPs), were created in Argentina, in 1994, by the Law 24.241, which is the main legal frame that, although recent modifications, still regulates their activities.

To protect the affiliates from excessive volatility and dispersion in returns among the PFs, there are minimum return requirements relative to the weighted average performance of all PFs over a twelve months period. If the return of a PF is less than 70% of the weighted average or if it is lower than the weighted average minus two percentage points, whichever results in a greatest shortfall, the AFJP managing that fund is required to make up for the difference. The AFJPs do this by transferring cash and other assets from a reserve fund of their property to the PFs.

In November 2001, Argentina's National Administration issued a new a regulation modifying the law, namely the decree Nr. 1495/01. As a result, the AFJPs will be allowed, starting during the first half of 2002, the possibility of partially or totally constitute the required reserve funds through a bank guarantee. The same decree reduced the required value of the reserve funds from 2% of the total value of the PFs, and a minimum value of 3 million Argentine pesos (AR\$), to 1% of the PFs, and a minimum value of 1.5 AR\$. A previous attempt of introducing the bank guarantee as an alternative for funding the required reserves, was the suspended decree Nr. 1306/00, intended to be applied starting in September 2001. For an analysis of the consequences the decree Nr. 1306/00 should had brought on the AFJP's decisions concerning reserves, we refer the reader to Cané de Estrada (2001).

The bank guarantee should behave as an insurance on yield deviations, and to value its price we represent it as a 3D European call spread option. We solve the problem in continuous time, and calculate the price using a Quasi Monte-Carlo (or low discrepancy) numerical integration method, based on Halton sequences.

The remainder of the paper is organized as follows. In Section 2 we introduce the problem. In Section 3 the pricing method is outlined. Section 4 includes a brief review of low discrepancy methods, and the definition of Halton sequences. Section 5 contains the numerical results, and in Section 6 we present the conclusions.

## 2 The Valuation Problem

In the practice, the return deviation of a PF, denoted by  $D$ , is computed monthly as

$$D = R_1 - R_2,$$

where  $R_2$  is the return of a PF, and  $R_1$  is a benchmark given by

$$R_1 = \min[0.7\text{Average Return, Average Return} - 2\%].$$

The return  $R_2$  of a particular PF, calculated monthly, is defined as

$$R_2(T) = \frac{S_2(T) - S_2(T - \Delta T)}{S_2(T - \Delta T)} = \frac{S_2(T)}{S_2(T - \Delta T)} - 1, \quad (1)$$

where  $T$  coincides with the end of a month,  $S_2(T)$  is the average price of the fund's share over the month, and  $\Delta T = 1$  year.  $S_2$  is obviously a tradable. The minimum return  $R_1$  can be exactly replicated as the return of an index  $S_1$ ,

$$R_1(T) = \frac{S_1(T) - S_1(T - \Delta T)}{S_1(T - \Delta T)} = \frac{S_1(T)}{S_1(T - \Delta T)} - 1 \quad (2)$$

where  $T$  and  $\Delta T$  are the same as in (1).

In what follows we will assume that the reserves are totally integrated through the bank guarantee, which is agreed monthly, and gives the PF the right to receive, at the end of the month, a payoff equivalent to

$$\text{Payoff} = \begin{cases} \max(0, D)P, & \text{if } DP < E; \\ E & \text{if } DP \geq E, \end{cases} \quad (3)$$

where  $P$  denotes the total PF shareholders fund at the expiry date in AR\$, and  $E$  are the required reserves given, as a function of  $P$ , by

$$E = \max(\text{AR}\$1, 500, 000; 0.01P).$$

The payoff (3) can be written as the payoff of the European option

$$V(R_1, R_2, P, T) = \max[0, (R_1(T) - R_2(T)) P(T)] - \max[0, (R_1(T) - R_2(T)) P(T) - E(T)], \quad (4)$$

for the expiry time  $T = 1$  month. After replacing (1) and (2), equation (4) becomes

$$V(S_1, S_2, S_3, T) = \max \left[ 0, \left( \frac{S_1(T)}{S_1(T - \Delta T)} - \frac{S_2(T)}{S_2(T - \Delta T)} \right) S_3(T) \right] - \\ - \max \left[ 0, \left( \frac{S_1(T)}{S_1(T - \Delta T)} - \frac{S_2(T)}{S_2(T - \Delta T)} \right) S_3(T) - E(S_3(T)) \right], \quad (5)$$

where  $S_3$  denotes the random variable corresponding to the total PF shareholders fund.

We will work in a continuous time framework, and we will solve the general problem of pricing an European call spread option on 3 underlyings,  $S_i(t)$  ( $i = 1, 3$ ), with payoff given by (5).

Our assumptions are:

- i)  $S_i(t)$ ,  $i = 1, 3$ , follow lognormal diffusions

$$dS_i(t) = \mu_i S_i dt + \sigma_i(t) S_i dW_i, \quad (6)$$

where  $W_i$  ( $i = 1, 3$ ) are correlated standard Brownian motions with drifts  $\mu_i$ , volatilities  $\sigma_i$ , and correlation matrix  $\Sigma$ .

- ii) The risk-free rate  $r$  is a constant.

- iii) There is no credit-risk.

### 3 The pricing method

Some financial derivatives in  $n$  dimensions can be evaluated by computing an  $n$ -dimensional integral.

For an European non path dependent multi asset option over log-normal underlyings, we have the following integral formulation that gives the value  $V$  of the option as a function of the underlyings  $S_1, \dots, S_n$  at time  $t$  (see Barrett et al. (1992) and Wilmott (1998))

$$V(S_1, \dots, S_n, t) = \frac{e^{-r(T-t)} (2\pi(T-t))^{-n/2}}{\sigma_1 \dots \sigma_n (\text{Det}\Sigma)^{\frac{1}{2}}} \int_0^\infty \dots \int_0^\infty \frac{G(S'_1 \dots S'_n)}{S'_1 \dots S'_n} e^{-\frac{1}{2}\alpha^T \Sigma^{-1} \alpha} dS'_1 \dots dS'_n, \quad (7)$$

where

$$\alpha_i = \frac{\log \frac{S'_i}{S_i} - (r - \frac{\sigma_i^2}{2})(T - t)}{\sigma_i(T - t)^{\frac{1}{2}}}, \quad (i = 1, \dots, n),$$

$G(S_1, S_2, \dots, S_n)$  is the payoff,  $r$  is a constant interest rate,  $\sigma_1, \dots, \sigma_n$  are the volatilities, and  $\Sigma$  is the correlation matrix for the  $n$  assets.

Since the random variables  $S_1, \dots, S_n$  are log-normally distributed, the value of the option given by (7) can be rewritten as

$$V(S_1, \dots, S_n, t) = e^{-r(T-t)} \int_{-\infty}^{\infty} \dots \int_{-\infty}^{\infty} G(S'_1, \dots, S'_n) p(\phi_1, \dots, \phi_n) d\phi_1 \dots d\phi_n, \quad (8)$$

where  $p(\phi_1, \dots, \phi_n)$  is the probability density function for the  $n$  correlated Normal variables  $\phi_1, \dots, \phi_n$ , with zero mean and unit variance (cfr. Wilmott (1998)). In most of the cases, these integrals have no analytic solution and must be computed numerically.

Theoretical and numerical results in Spanier et al. (1994), Paskov (1994), Paskov et al. (1995), and Bruno et al (1999), allow to affirm that approximations to these integrals using deterministic sequences are more accurate and faster to compute than the basic Monte-Carlo method.

The price of an European call spread option with a payoff given by (5) may be written as the integral (8) for  $n = 3$ , and  $G(S_1, S_2, S_3)$  replaced by (5), and the problem of approximating this integral can be efficiently solved by using a Quasi Monte-Carlo method.

## 4 Quasi Monte-Carlo Methods

The Monte-Carlo technique to calculate integrals uses random points, and in its application it is crucial to generate adequate random samples. Its success is related to the quality of the random samples, and here *quality* means how well these samples exhibit a real randomness. Due their important role, the generation of random numbers or random vectors has become a fundamental issue in the development of these techniques.

The Monte-Carlo integration methods have several advantages when compared to classic formulae: (1) they may be applied to a very general class of functions; (2) the error in the speed of convergence is essentially independent of the dimension of the problem; (3) the error may be easily estimated "a posteriori".

However, serious drawbacks are also associated with them. Despite the fact that the speed of convergence does not depend on the dimension, it is too slow. Moreover, there are practical problems in the construction of random samples. In the practice, we use the so called *pseudo-random* numbers or vectors, generated by a computer through a completely deterministic algorithm, and when pseudo-random numbers substitute random samples it is not obvious that the statistical analysis of the error holds.

From these considerations, the statistical analysis of the error was dismissed, and, instead, the emphasis was placed on rigorous bounds to the absolute integration error. Therefore, what becomes relevant is that the samples were distributed as uniformly as possible, not their real randomness. The role played by sequences distributed in this way is to assure that, for an enough wide class of functions, the integration error may be made as small as required. The idea of using selected deterministic nodes is the starting point of the *Quasi Monte-Carlo* integration methods.

A Quasi Monte-Carlo method (cfr. Niederreiter (1992) and Spanier et al. (1994)) may be described, in a simple way, as a deterministic version of a Monte-Carlo method, in the sense that the random samples are replaced by selected deterministic points. The selection criterium depends on the particular problem under study, and the main goal is to choose points for which the deterministic error bound is less than the Monte-Carlo probabilistic error.

To estimate the magnitude of the quadrature error as a function of the sample size it is necessary to introduce the *discrepancy* (a concept that arises in number theory), a quantitative refinement of the concept of a uniform distribution. It measures the deviation from uniformity of an  $n$ -dimensional set of numbers, and its usefulness consists in providing, for enough regular integrands, an "a priori" higher bound for the integration error. Although the problem of finding the  $n$ -dimensional set of numbers that has the lower discrepancy is still open, several low discrepancy sets are already known, e.g. Halton, Sobol, and Faure sequences, amongst others (cfr. Paskov et al. (1995), Faure (1982), Tezuka (1994), and Tezuka (1991)).

#### 4.1 The Halton Sequences

For integers  $n \geq 0$  and a prime number  $b \geq 2$ , let us expand each integer  $n$  in terms of the base  $b$ , with  $m = \lceil \log_b n \rceil$ ,

$$n = a_1(n) + a_2(n)b + \cdots + a_{m+1}(n)b^m = \sum_{i=1}^{m+1} a_i(n)b^{i-1},$$

where  $0 \leq a_i < b$  and  $a_{m+1}(n) \neq 0$ .

The  $N$ -point Halton sequence for a given basis  $b$  in 1 dimension is

$$\Phi_b(1), \Phi_b(2), \dots, \Phi_b(N),$$

where

$$\Phi_b(n) = \sum_{j=1}^{m+1} a_j(n) b^{-j}, \quad n = 1, \dots, N.$$

The Halton sequence in  $S$  dimensions is

$$\{\Phi_{b_1}(n), \Phi_{b_2}(n), \dots, \Phi_{b_S}(n)\}$$

with  $b_1, b_2, \dots, b_S$  prime.

Figures 1 and 2 illustrate the covering of the unit square by 10,000 pairs of Halton points, in basis 2 and 3, and 10,000 pairs of a pseudo-random realization, respectively. The latter evidences a lack of uniformity.

<Insert Figs 1 & 2>

## 4.2 Discrepancy

The *Discrepancy* is defined to give a quantitative measure of the uniformity of the covering of an interval. For a sequence of  $N$  points  $X_1, X_2, \dots, X_N$  in  $I_S = [0, 1]^S$ ,  $S \geq 1$ , and subintervals

$$J = \prod_i^S [0, n_i], \quad \text{where } i = 1, \dots, S \text{ for } 0 < n_i \leq 1,$$

the *Discrepancy* is (cfr. Niederreiter (1992))

$$D_N^{(S)} = \sup_J \left| \frac{A(J; N)}{N} - V(J) \right|,$$

where  $A(J, N)$  is the number  $n$ ,  $0 \leq n < N$ , with  $X_n \in J$ , i.e. the number of points in the sequence  $\{X_n\}$  that lie in  $J$ ,  $V(J)$  is the volume of  $J$ , and the supremum is taken over all the subintervals  $J$ .

A *Low Discrepancy Sequence* (Halton, Sobol, Faure, etc.) is a sequence  $X_1, X_2, \dots, X_N$  in  $[0, 1]^S$  such that, for all  $N > 1$ , its discrepancy satisfies



$$D_N^{(S)} \leq C_S \frac{(\log N)^S}{N},$$

where the constant  $C_S$  only depends on the dimension  $S$ .

The *Discrepancy* for the first  $N$  points of a Halton sequence is given by (cfr. Tezuka (1994))

$$D_N^{(S)} = C(b_1, b_2, \dots, b_S) \frac{(\log N)^S}{N},$$

where

$$C(b_1, b_2, \dots, b_S) = \prod_i^S \frac{b_i}{\log b_i}.$$

## 5 Numerical Results

The approximated numerical solution is calculated by computing the integral (8) for  $n = 3$ , and the payoff given by (5).

The integration points in  $(-\infty, \infty)$  are generated from Halton sequences in basis 2, 3, 5 and 7 by the Box-Muller method (see Knuth (1981)), and a Cholevsky factorization.

To estimate the log-volatilities  $\sigma_i$  ( $i = 1, 3$ ), and the correlation matrix  $\Sigma$ , we used seven years of historical monthly data of the share for each PF, and of the index corresponding to the minimum return. The log-volatilities were estimated by the Exponentially Weighted Moving Average (EWMA) method. Furthermore, we have used a time step  $\Delta t = 1$  month, and an annual risk-free rate  $r = 4.5\%$ .

The convergence of the method, with the number  $N$  of integration points varying between 10,000 and 1,000,000, may be observed in Figures 4 and 5, that show the results obtained for Consolidar and Nación, two PFs that in December 2001 exhibited good and bad performances, respectively.

<Insert Figs 3 & 4 >

The prices of the insurance, calculated with  $N = 400,000$ , for eleven PFs, and the nineteen last months for which data were available, are summarized in Tables 1 and 2. For each PF, the second row corresponds to the price written as a percentage of the required reserves at the last day of the previous month.

< Insert Tables 1 & 2 >

The fundamental characteristic that we found on the monthly cost of the bank guarantee is its very high volatility. Meaning that in many cases its cost increases by a factor of one hundred in a span of two or three months (see, for example, Orígenes between Sep-00 and Nov-00).

For asserting the convenience of choosing or not the bank guarantee as a replacement of the assets in the reserve funds, we need to compare its cost with the opportunity cost of funding the reserves in the old fashioned way. The average cost of opportunity on an annual basis for these firms (the AFJPs) lies within a range that goes from 15% to 20% of the reserve funds.

As a result of these figures, Tables 1 and 2 show that the alternative provided by the bank guarantee would be clearly convenient for Arauca, Consolidar, Máxima and Previsol. On the other hand, while the bank guarantee results inconvenient for Profesión, Siembra and Orígenes, it becomes extremely expensive for Nación, Futura, Unidos and, in the last months, for Prorenta.

Another peculiarity of the bank guarantee is that, during some period of the observations, its cost descends to almost nothing for all PFs. This fact is associated with the formula for calculating the minimum required return, which becomes increasingly lower than the average return after the latter surpasses 6,67%. Note that the period referred to (from Jul-00 to Oct-00), corresponds to high positive average annual returns, while most of 2001 exhibits negative or poor average annual returns.

## 6 Conclusions

In this paper we applied a low discrepancy method, based on Halton sequences, to estimate the cost of insurance on profit deviations for the Argentine Pension Funds, during a nineteen months period, and found that the result of this exercise provides some interesting insights:

- We found that this new alternative provided by Argentine pension fund's legislation would be profitable for some Pension Funds. Compared to the cost of funding the reserves through the standard mechanism, the bank guarantee resulted cheaper for four out of eleven pension funds studied.
- We found a very high volatility on insurance prices, which introduces a potential solvency risk after replacing asset reserves with the bank guarantee. The volatility in the monthly cost of the insurance, creates a new risk of bankruptcy for AFJPs. The danger of

becoming exposed to this bankruptcy risk is particularly high during long periods of high average returns, when the AFJPs may delude themselves on the cheapness of the insurance and be unprepared for large payments in the future.

- **The very high level of sensitivity of the insurance premium to the relationship between a fund's performance and the endogenous benchmark magnifies the prevailing incentives within the industry.** The behavior commonly referred to as "flock effect", derived from the existence of a minimum return linked to an endogenous benchmark, is amplified. With the bank insurance, AFJPs are not only penalized when they fall below the minimum return, but also when they approach it (even still being far) through the rise in the cost of the insurance.

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Table 1. Prices in AR\$, calculated with  $N = 400,000$ , for Arauca, Consolidar, Futura, Máxima, and Nación

	Arauca	Consolidar	Futura	Máxima	Nación
June-00	49,091 0.45%	332 0.00%	352,270 12.34%	511,620 1.85%	292,560 2.32%
July-00	38,365 0.33%	105 0.00%	175,400 5.94%	382,310 1.33%	637,570 4.88%
Aug-00	22 0.00%	0 0.00%	482 0.02%	606 0.002%	971 0.007%
September-00	110 0.001%	0 0.00%	1,257 0.04%	1,807 0.006%	84 0.001%
October-00	1,575 0.01%	13 0.00%	15,328 0.50%	23,960 0.08%	1,022 0.01%
November-00	30,711 0.24%	5,925 0.02%	315,430 10.53%	330,940 1.11%	35,289 0.26%
December-00	65,496 0.51 %	23,629 0.06 %	406,670 13.72 %	354,870 1.19 %	273,310 1.98 %
January-01	65,938 0.49%	43,811 0.11%	278,460 9.17%	200,740 0.65%	525,230 3.69%
February-01	82,979 0.57%	168,790 0.39%	345,520 10.64%	241,380 0.73%	428,180 2.78%
March-01	76,935 0.54%	131,720 0.31%	362,160 11.52%	243,650 0.75%	386,540 2.57%
April-01	63,459 0.44%	71,900 0.17%	281,340 9.02%	268,240 0.83%	834,200 5.58%
May-01	44,081 0.30%	117,670 0.28%	307,340 9.82%	364,430 1.12%	2,551,100 16.99%

Table 1. (continuation)

	Arauca	Consolidar	Futura	Máxima	Nación
June-01	115,630 0.77%	189,590 0.44%	266,060 8.39%	426,890 1.29%	2,832,300 18.55%
July-01	81,199 0.53%	116,120 0.26%	276,940 8.75%	459,570 1.37%	2,050,100 13.33%
August-01	115,340 0.79%	48,180 0.12%	299,350 10.17%	262,480 0.83%	4,601,800 31.98%
September-01	126,210 0.83%	31,439 0.07%	374,830 12.38%	250,210 0.76%	4,879,000 32.60%
October-01	183,920 1.22%	15,497 0.04%	471,400 16.05%	259,590 0.81%	4,519,500 30.85%
November-01	280,230 1.87%	9,480 0.02%	563,950 19.60%	194,900 0.61%	3,616,200 24.97%
December-01	294,250 2.03%	5,361 0.01%	504,690 18.36%	129,770 0.43%	1,982,600 14.19%

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**Table 2. Prices in AR\$, calculated with  $N = 400,000$ , for Orígenes, Previsol, Profesión, Prorenta, Siembra, and Unidos**

	Orígenes	Previsol	Profesión	Prorenta	Siembra	Unidos
June-00	718,560 2.39%	1,112 0.03%	54,452 3.63%	391 0.01%	80,086 0.30%	807,990 53.87%
July-00	658,430 2.11%	996 0.02%	65,244 4.35%	8 0.00%	63,751 0.23%	315,390 21.03%
Aug-00	1822 0.01%	0 0.00%	2631 0.18%	0 0.00%	82 0.00%	2,267 0.15%
September-00	12,676 0.04%	5 0.00%	4,149 0.28%	0 0.00%	615 0.00%	5,226 0.35%
October-00	139,530 0.43%	274 0.01%	15,489 1.03%	0 0.00%	9,173 0.03%	58,803 3.92%
November-00	1,252,400 3.85%	12,648 0.29%	101,180 6.75%	5 0.00%	275,520 0.95%	384,280 25.62%
December-00	1,557,200 4.83%	34,782 0.81%	124,960 8.33%	218 0.005%	668,750 2.32%	254,920 16.99%
January-01	1,460,500 4.40%	31,427 0.70%	95,061 6.34%	283 0.01%	801,760 2.69%	140,680 9.38%
February-01	2,774,100 5.22%	34,932 0.72%	27,476 1.83%	1,942 0.04%	1,149,200 3.57%	137,900 9.19%
March-01	1,989,800 3.85%	31,057 0.65%	27,905 1.86%	6,209 0.12%	1,047,900 3.33%	127,580 8.51%
April-01	1,214,200 2.35%	24,374 0.51%	16,917 1.13%	15,163 0.30%	1,099,000 3.48%	141,850 9.46%
May-01	985,800 1.91%	21,274 0.44%	10,896 0.73%	51,002 1.01%	1,131,800 3.56%	132,090 8.81%

Table 2. (continuation)

	Orígenes	Previsol	Profesión	Prorenta	Siembra	Unidos
June-01	1,057,400 2.02%	18,107 0.37%	23,684 1.58%	97,021 1.89%	2,180,600 6.72%	44,081 2.94%
July-01	1,347,400 2.56%	21,431 0.43%	45,676 3.03%	230,590 4.50%	1,830,600 5.62%	91,986 6.13%
August-01	2,185,200 4.44%	22,770 0.49%	61,305 4.09%	282,510 5.88%	2,317,300 7.58%	37,410 2.49%
September-01	1,990,500 3.93%	14,300 0.30%	37,222 2.48%	510,710 10.36%	2,434,500 7.70%	58,694 3.91%
October-01	1,967,600 3.98%	14,688 0.31%	74,005 4.93%	1,252,900 26.00%	2,122,700 6.84%	258,060 17.20%
November-01	2,358,900 4.83%	18,835 0.40%	66,089 4.41%	909,170 19.10%	2,855,900 7.22%	292,330 19.49%
December-01	3,233,600 6.92%	29,056 0.64%	79,858 5.32%	281,810 6.13%	3,303,500 8.85%	338,090 22.54%

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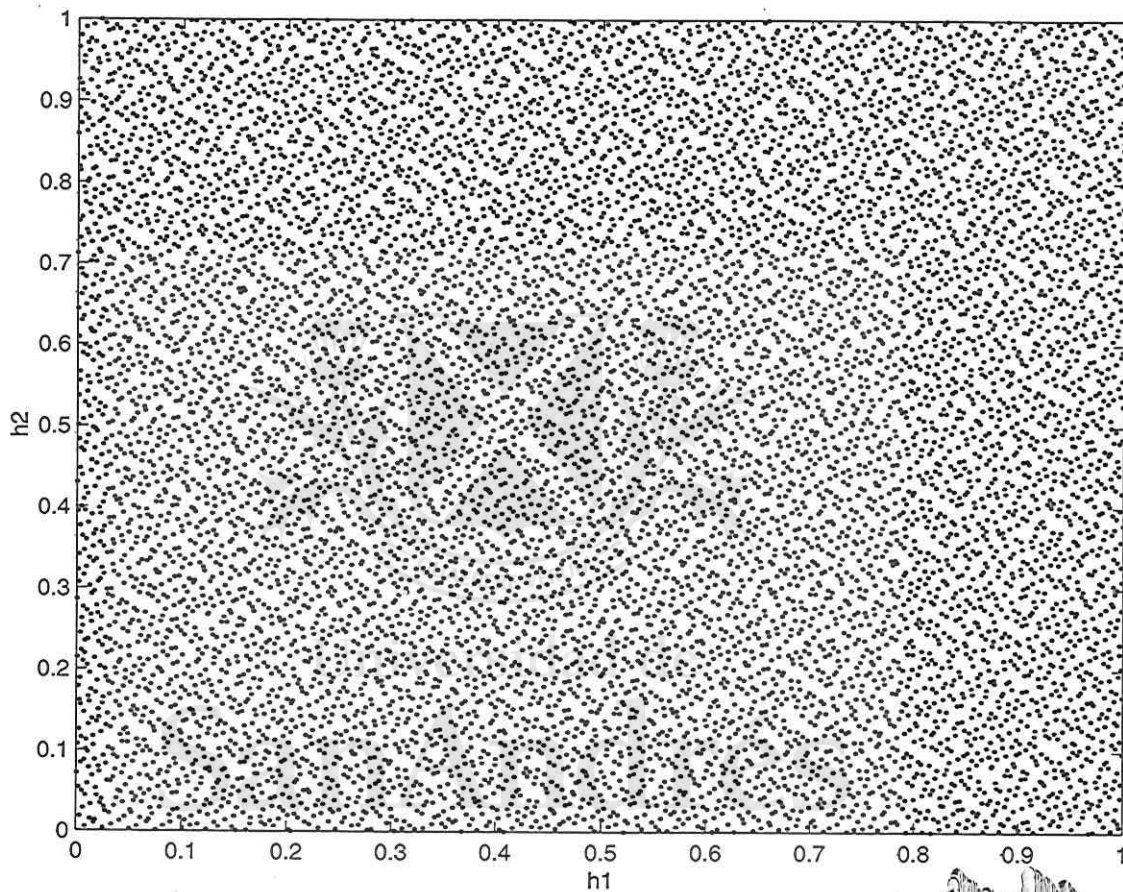
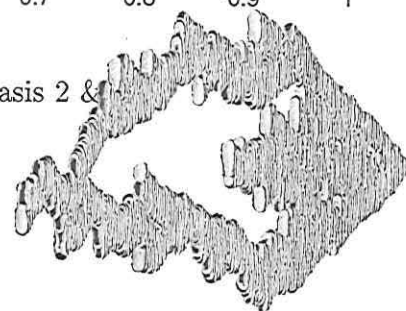


Figure 1: Distribution of 10,000 Halton pairs- basis 2 &



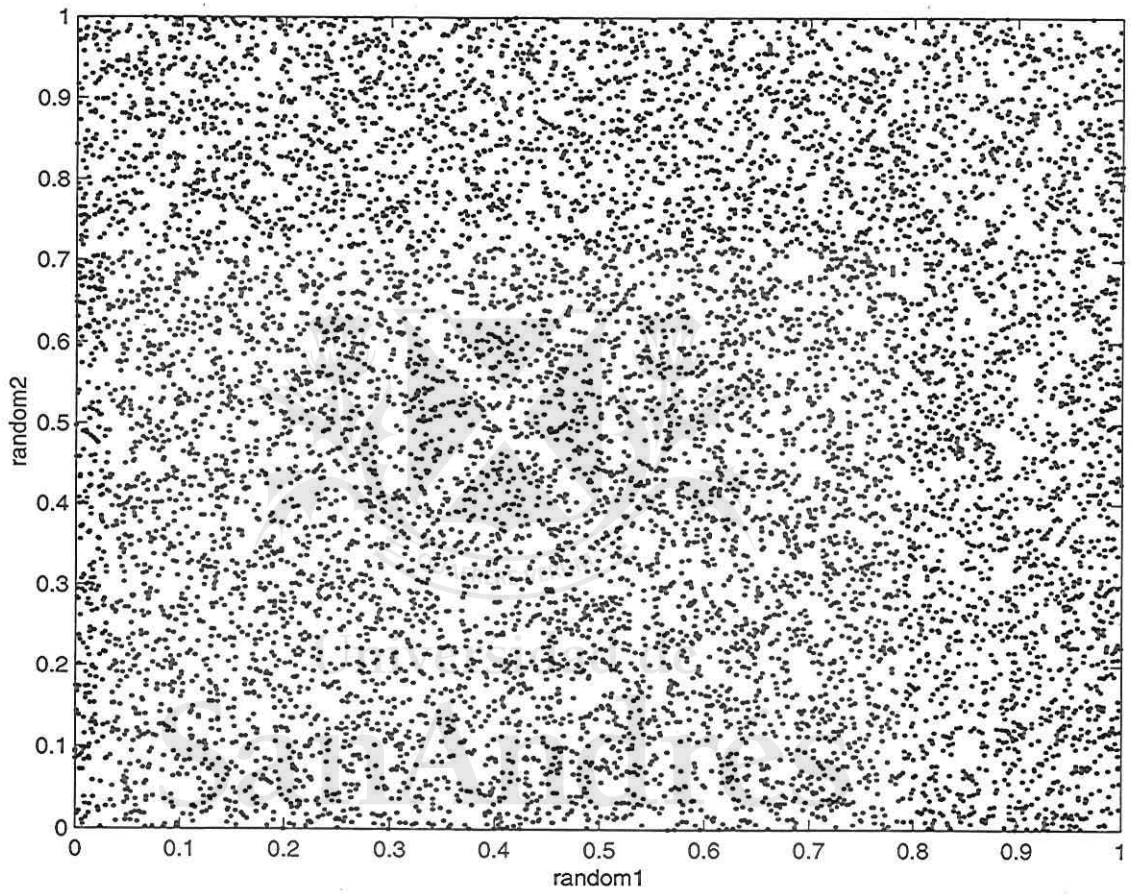


Figure 2: Distribution of 10,000 pseudo-random pairs

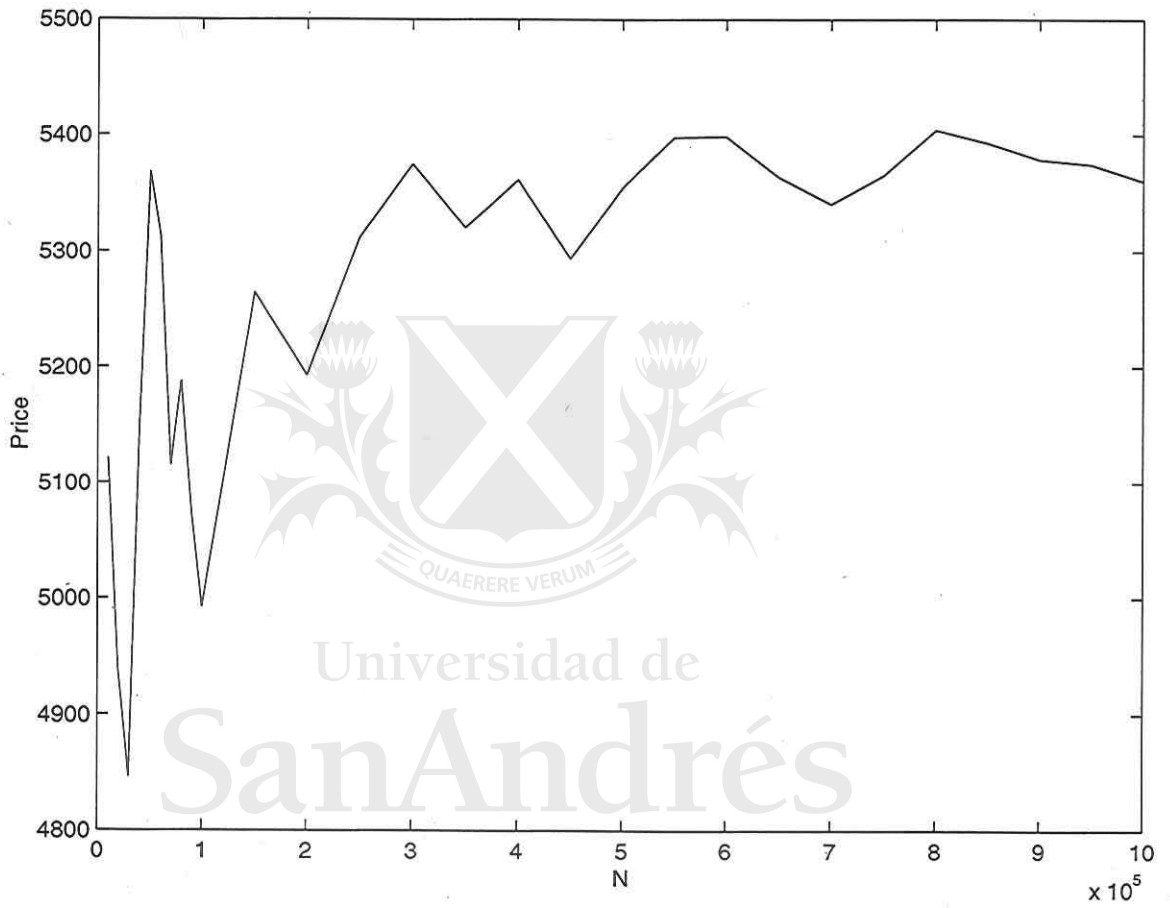


Figure 3: Convergence for the PF Consolidar, Dec-01

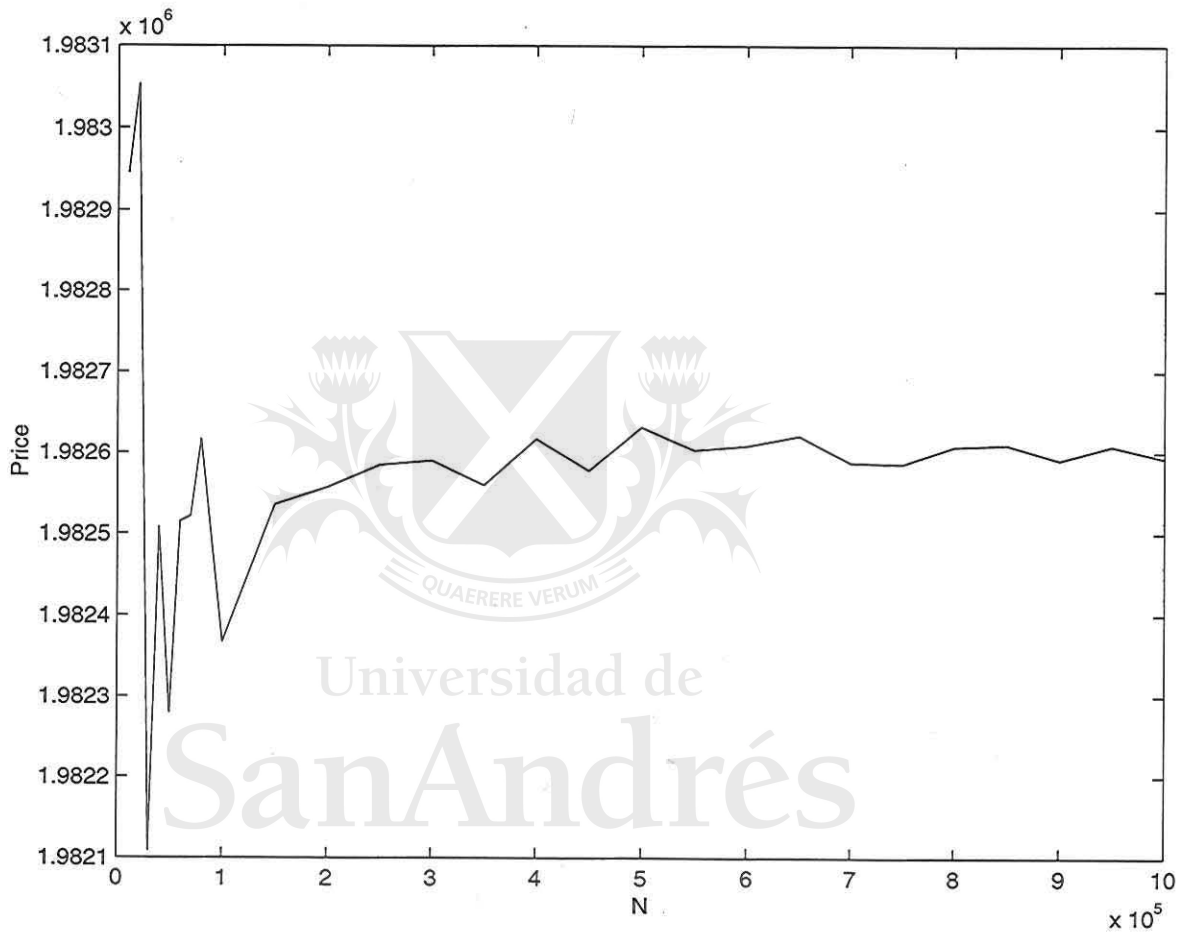


Figure 4: Convergence for the PF Nación, Dec-01