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Diamond- Dybvig Banks in two-good, two-currencies, small open economies with cash-in-advance constraints

Enrique Kawamura (Universidad de San Andrés)

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Departamento de Economía

Director
Mariano Tommasi
Ph.D. in Economics, University of Chicago

Coordinador del Ciclo de Seminarios
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Enrique Kawamura
Universidad de San Andrés
Vito Dumas 284
Victoria, Buenos Aires (1644)
Argentina
kawa@udesa.edu.ar

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Abstract

This paper analyzes a two-good version of the Diamond and Dybvig model in a small open economy. This structure is used to analyze the interaction between banks as liquidity insurers, real exchange rates and monetary policies. With fixed exchange rates and local lender of last resort, non-tradeables price deflation is necessary for existence as well as for implementation. Conditions for currency crises are reduced to the standard international illiquidity condition of Chang and Velasco (1998). The paper also discusses flexible exchange rates with peso-denominated deposits as well as dollarized banking systems.

1 Introduction

A vast recent theoretical as well as empirical literature has recently emphasized the role of the banking sector fragility to explain recent currency crisis observed in Asia and Russia. The main contributions are found in successive papers by Chang and Velasco (1998, 2000a, 2000b and 2001). Specially,

*VERY PRELIMINARY VERSION, NOT TO BE QUOTED.

Chang and Velasco (2000b) emphasize the role of exchange rates in determining the possibility of currency crisis due to a so-called *international illiquidity* condition (see [1998] also for a definition). One of the main criticisms of this work is the way money (specially, local currency) is modelled. They use a simple version of a money - in - the - utility function model to address questions on exchange rates, monetary policy, optimal (liquidity) risk sharing and bank (or currency) runs.

However, this type of preferences may be misleading. A special assumption taken in the Chang and Velasco (2000b) model is the fact that the amount of pesos must be carried over between periods 1 and 2 by the patient consumers. That local currency stock implies some level of utility. What that assumption really means is unclear. On the other hand, even though the authors emphasize that it is possible to introduce local currency holdings in the utility function of the impatient agents, this amount of pesos must also be carried over between periods 1 and 2. A question is then why an impatient person would be interested in carrying over pesos between these two dates, given that she cares only about consumption at date 1.

This and other criticisms may imply that a more serious role for money is needed in the banking model. It is well known that the two main roles for money are store of value and mean of payment. In the first case, there exists already a series of papers with overlapping generation models where money has positive value embedded in the Diamond - Dybvig structure[1]. This structure allows to take money seriously as a store of value within the mentioned framework. However, it is hard to find a model with the Diamond - Dybvig structure where money is used as a mean of exchange.

The model presented here combines the Diamond and Dybvig structure with the fact that currency is used as a mean of exchange. To my knowledge, this is the first attempt to introduce this fact. This economy assumes a tradeable as well as a non - tradeable good. There are also two currencies, the dollar (foreign) and the peso (local). This model assumes then that some currency *must* be exchanged for these goods and viceversa. Hence some type of cash - in - advance constraint is assumed to hold in the economy. Given that this is a finite horizon economy, this type of constraint forces to introduce frictions in the transaction technology. The story used here is similar to the traditional shopper - seller division of the household as in, for example, Lucas (1980, 1982). In some cases a Central Exchange institution centralizing exchange is introduced as in Magill and Quinzii (1992, 1996). The main objective for this is to study what its consequences are

[1] See, for example, Betts and Smith (1997), Champ, Smith and Williamson (1996), Schreft and Smith (1997) and Williamson (1998).

about the optimality and fragility of different exchange rate and monetary arrangements as analyzed in Chang and Velasco (2000b).

The main results are the following. With fixed exchange rates and a local lender of last resort, the optimal allocation can be implemented by a Diamond and Dybvig banking system with suitable monetary policies. The main difference with the literature is the fact that local credit in pesos by the Central Bank is only needed within a period, not between periods. The main use of this *liquidity* in local currency is to allow the exchange of non - tradeables. There is no other role for the Central Bank in this case. It is also shown that the contract also allows for an equilibrium currency run. With logarithmic preferences conditions for a run are simpler.

Within the same banking system, it is also shown that an exchange rate policy contingent on the proportion of consumers withdrawing in the intermediate period prevents runs. This is the result in Chang and Velasco (2000b), showing that the main factor that threats a run is the so called *international illiquidity*.

A dollarized banking system with no use for pesos is also presented here. This in fact can also be interpreted as a banking system within a currency board regime. First, implementation of the optimal allocation is at most a non - generic property. In the logarithmic preferences case, implementation could hold if the share of tradeables and non - tradeables are equalized (which implies a non - generic property). This system may also be subject to runs. In the logarithmic utility function case, the degree of illiquidity needed to generate a run is more demanding than with fixed exchange rates and a lender of last resort. In this (loose) sense, it can be said that with logarithmic preferences a dollarized banking system is less *likely* to suffer a crisis than a fixed exchange rate regime with a local lender of last resort. This conclusion of course cannot be generalized to other preferences.

This paper is organized as follows. In section 2 I present the general environment to be used throughout the paper. Section 3 shows the characterization of the optimal allocation. Sections 4 and 5 discuss the implementation through a banking system within a fixed exchange rate regime in which the Central Bank acts as a lender of last resort. Section 6 studies the currency run equilibrium. Section 7 presents a contingent exchange rate policy and the impossibility of runs. Section 8 shows a dollarized banking system, discusses its optimality and also its fragility. Section 9 shows concluding remarks and extensions while section 10 contains all the proofs of lemmas and propositions.

2 The Environment

The economy lasts for three periods, $t = 0, 1, 2$. There are two consumption goods in periods 1 and 2. One good is called *tradable* and the other, *non−tradable*. In period 0 there are only tradeables. Non-tradeables are produced with a constant-returns-to-scale technology. For every period of maturity, the per - unit gross return is $A > 1$ non tradeables per unit of tradable invested at date 0. In this same period there is a continuum of ex-ante identical consumers, with names in the unit interval. As it is standard, at the beginning of date 1 there exists a preference shock that determines the ex-post type of each consumer. With probability π the consumer becomes *impatient* and with the remaining probability she becomes *patient*. An impatient agent has preferences represented by the utility function $u(c_{1T}) + v(c_{1N})$, where c_{1l} is the consumption by an impatient consumer (in period 1) of good $l = T, N$, where T stands for *tradables* and N for *non − tradables*. A patient consumer has utility function $u(c_{2T}) + v(c_{2T})$, where c_{2l} stands for the consumption at date 2 by the impatient agent of good l. Hence, the ex-ante utility function is

$$\pi [u(c_{1T}) + v(c_{1N})] + (1 - \pi) [u(c_{2T}) + v(c_{2T})]$$

There exist two other investment technologies. There is a long term investment project that gives $R > 1$ units of tradable goods at date 2 per unit of the T good invested in period 0. As in the literature, assume that if liquidated at date 1 the gross return in terms of tradeables is $r < 1$. The other investment corresponds to the fact that the tradable good is assumed to be storable with net return equal to 0. There is no endowment of none of the goods in this economy.

3 A planner's problem with limited international credit.

This section characterizes the planner's problem assuming the existence of credit at date 0 that allows the planner to borrow tradable goods directly in the first period, with a net interest rate equal to 0. This planner allocates this amount in the different available technologies. Let d be the amount of tradeables borrowed by the planner at date 0, let x be the amount of tradeables invested in the long term project, y be the amount of tradeables stored between periods 0 and 1. Let z be the amount of tradeables invested in 0 to produce non-tradeables. The planner's problem can be written as follows.

$$\max \quad \pi\left[u\left(c_{1T}\right)+v\left(c_{1N}\right)\right]+(1-\pi)\left[u\left(c_{2T}\right)+v\left(c_{2T}\right)\right]$$

subject to

$$\begin{aligned}
x+y+z_1+z_2 &\leqq d \\
\pi c_{1T} &\leqq y \\
(1-\pi)\, c_{2T} &\leqq Rx-d \\
\rho_t c_{tN} &\leqq A^t z_t
\end{aligned}$$

where $\rho_1=\pi$ and $\rho_2=1-\pi$. The problem can be written as

$$\begin{aligned}
\max_{x,z_1,z_2} \quad & \pi\left[u\left(\frac{d-x-z}{\pi}\right)+v\left(\frac{Az_1}{\pi}\right)\right] \\
& +(1-\pi)\left[u\left(\frac{Rx-d}{1-\pi}\right)+v\left(\frac{A^2 z_2}{1-\pi}\right)\right]
\end{aligned}$$

The first order conditions for an interior solution are

$$\begin{aligned}
u'\left(c_{1T}\right) &= Ru'\left(c_{2T}\right) \\
u'\left(c_{1T}\right) &= Av'\left(c_{1N}\right) \\
u'\left(c_{1T}\right) &= A^2 v'\left(c_{2N}\right)
\end{aligned}$$

Since $R>1$ and $A>1$, it is easy to show that $c_{1T}<c_{2T}$ and $c_{1N}<c_{2N}$. Therefore impatient consumers obtain always a strictly less ex - post utility than patient consumers. The next result characterizes the optimal allocation more sharply. (All the proofs are in the appendix).

Lemma 1 *The optimal amount of investment liquidated at date t in the non-tradeables technology is equal to $z_t^*=\rho_t z$, where z is the total amount invested in the non-tradeable technology at date 0.*

This will be useful when discussing the conditions for runs in a banking system. For illustrative purposes consider the following example.

Example 2 *Assume $u\left(c_{tT}\right)=\theta \ln c_{tT}$, $v\left(c_{tN}\right)=(1-\theta)\ln c_{tN}$. It can be shown that the optimal allocation satisfies the linear system*

$$\begin{aligned}
Rx+(1-\pi)Rz_1+(1-\pi)Rz_2 &= d\left(R(1-\pi)+\pi\right) \\
(1-\theta)x+z_1+(1-\theta)z_2 &= d(1-\theta) \\
(1-\theta)(1-\pi)x+(1-\theta)(1-\pi)z_1+\left(\pi\theta+(1-\pi)(1-\theta)\right)z_2 &= (1-\theta)(1-\pi)d
\end{aligned}$$

whose solution is

$$x^* = \frac{d\left(R\theta\left(1-\pi\right)+\left(1-\theta\right)+\theta\pi\right)}{R}$$

$$z_1^* = \frac{d\pi\left(\left(R-1\right)\left(1-\theta\right)\right)}{R}$$

$$z_2^* = \frac{d\left(1-\pi\right)\left[\left(R-1\right)\left(1-\theta\right)\right]}{R}$$

and so, consumption allocations are

$$c_{1T}^* = \frac{d\theta\left(R-1\right)}{R}$$
$$c_{2T}^* = d\theta\left(R-1\right)$$
$$c_{1N}^* = \frac{\left(1-\theta\right)Ad\left(R-1\right)}{R}$$
$$c_{2N}^* = \frac{\left(1-\theta\right)A^2d\left(R-1\right)}{R}$$

This example will be kept for future reference.

4 Implementation through a banking system with peso - denominated deposits. Preliminaries.

This section presents a banking system similar to that in Chang and Velasco (2000) embedded in a monetary system with two currencies, the *dollar* and the *peso* (foreign and local currency respectively). Assume that the traded good is the numeraire in terms of dollars. This implies immediately that the price of the T - good is equal to 1 dollar for every period. There exists a large number of commercial banks that compete to get customers (alternatively, we can assume that these banks act directly on behalf of consumers). There is also a Central Bank that borrows dollars from abroad and lends to commercial banks. Throughout the rest of the paper (for simplicity) I will assume that the Central Bank is the only creditor to commercial banks.

Following the cash-in-advance literature [2] I assume that each consumer is in fact a household constituted by two parts, the shopper and the entrepreneur. How these two parts interact will be discussed below, and it will

[2] See, for example, Lucas (1980, 1982) or Lucas and Stokey (1987).

depend on how trasanctions take place in this economy. Also, to simplify the interpretation I will assume that the two technologies producing tradeables (the liquid short run technology and the long term, illiquid project) give dollars instead of physical tradable goods. In this regard, $R > 1$ is the dollar-gross return on the long term project if liquidated at date 2, while the dollar net return on the storage technologies is equal to zero.

The basic sequence of actions by the households and banks is described here. However, the exchange rate policy and the organization of transactions will be discussed later on because of all the different varieties to be considered. The Central Bank borrows d dollars in the first period, through a two-period loan. The Central Bank lends a portion of this amount to banks that invest in the two technologies for tradable goods. The remaining dollars are lent (directly or through commercial banks) to consumers who produce non-tradable goods. Banks give a contract to each shopper that specifies the amount of *pesos* to be withdrawn in each period. The contract gives the right to withdraw at either date 1 or date 2, but not both.

In period 1, each household learns its type. Then impatient shopper and impatient entrepreneur separate from each other until the end of date 1. The impatient entrepreneur sells the produced non-tradable goods in exchange for pesos (this will be clarified below). Entrepreneurs return these pesos to their creditors. On the other hand, shoppers withdraw the corresponding pesos from commercial banks. Shoppers sell these pesos in exchange for non-tradable goods and dollars (to buy tradeables). At the end of period 1 shopper and entrepreneur gets together again and consume what is left for them. Commercial banks sell the dollars received from the storage technology to the Central Bank in exchange for pesos. Note that in period 1 all patient households do nothing (in the absence of runs).

In period 2 a similar sequence of actions is observed. All banks receive the amount of dollars from the long term investment. A portion of these dollars are returned to the Central Bank to repay the period 0 debt. The remaining dollars are purchased by the Central Bank. This institution then uses a portion of all dollars to repay the outstanding foreign debt. On the other hand, at the beginning of date 2 patient entrepreneurs and patient shoppers separate from each other. Then patient entrepreneurs sell the non-tradable goods in exchange for pesos, which are returned to their creditors. At the same time, patient shoppers withdraw pesos from commercial banks, which are sold afterwards to buy non-tradeables and dollars (to buy then tradeables). At the end of this last period entrepreneur and shopper get together and consume what is left.

The next sections specify who may the entrepreneurs' s creditors be and how the exchange of non-tradeables may take place.

5 Peso-denominated loans for Commercial banks and Central Exchanges with fixed exchange rates with Local Lenders of Last Resort.

This section assumes that the Central Bank fixes the exchange rate to one for all periods and may become a local lender of last resort for commercial banks (although this is not a necessary condition for implementation, see below). At the same time, assume that the Central Bank issues a certain amount of pesos at date 0, which is lent to commercial banks, which lend these pesos to the entrepreneurs at date 0. This amount of pesos must be equal to the amount of dollars needed to start production of non-tradeables, so that the equilibrium exchange rates is indeed equal to one. Since entrepreneurs sell pesos to get dollars, the net supply of pesos at the end of date 0 is also null.

The problem of the commercial bank is then

$$\max \quad \pi\left[u\left(c_{1T}\right)+v\left(c_{1N}\right)\right]+(1-\pi)\left[u\left(c_{2T}\right)+v\left(c_{2T}\right)\right]$$

subject to

$$x+y \leqq \delta \tag{1}$$

$$z_1+z_2 \leqq m_0^{CB} \tag{2}$$

$$\pi w_1 \leqq p_1 A z_1+y+h_1 \tag{3}$$

$$\pi c_{1T} \leqq y \tag{4}$$

$$(1-\pi)\, w_2 \leqq p_2 A^2 z_2+Rx-d-h_1 \tag{5}$$

$$(1-\pi)\, c_{2T} \leqq Rx-d \tag{6}$$

$$c_{tT}+p_t c_{tN} \leqq w_t \tag{7}$$

with $t=1,2$, and where $d=\delta+m_0^{CB}$ (and all these three variables are taken as exogenous by the commercial bank). Equation (1) shows the constraint faced by the bank when investing in the short and long run dollar - denominated investment technologies. Equation (2) shows the constraint in terms of pesos lent to households for non-tradable production. Equation (3) is the constraint faced by the commercial bank in period 1, whereas equation (4) states that tradable consumption by impatient households must be entirely financed by dollars. Equations (5) and (6) are the corresponding counterparts in period 2. Finally, equation (7) shows that purchasing dollars and non-tradeables must be done using the pesos withdrawn from the commercial banks.

The next result shows a preliminary characterization of the solution to the banking problem.

Proposition 3 *The solution to the banking problem satisfies the incentive - compatibility constraint*

$$u(c_{1T}) + v(c_{1N}) \leqq u(c_{2T}) + v(c_{2T})$$

A necessary condition to get a solution is that the gross rate of growth of non-tradable prices must be less than or equal to $\frac{1}{A} < 1$. Therefore there must be a deflation in non-tradeables. The policy that implements the optimal allocation implies that δ maximizes $V(\delta, d - \delta)$, where V is the indirect utility function of the banking problem (written as a function of δ and $m_0^{CB} = d - \delta$). In this optimal allocation the credit policy in pesos pursued by the Central Bank in period 1 is irrelevant (although a currency board is not allowed for implementation).

This result characterizes the fact that non-tradeables must become cheaper through time. This will have implications on the monetary policy that is consistent with equilibrium. The next question is whether this equilibrium is able to implement the first best. The following gives an answer for the logarithmic utility case.

Example 4 *Assume that preferences are of the log type as before. The first order conditions in this case can be written as*

$$\pi R x - \pi d = (1-\pi) R\delta - ((1-\pi) R) x$$

and so

$$x^e = (1-\pi)\delta + \frac{\pi d}{R}$$

and therefore

$$c_{2T}^e = \frac{Rx^e - d}{1-\pi} = \frac{R(1-\pi)\delta + \pi d - d}{1-\pi} = R\delta - d$$

$$c_{1T}^e = \frac{\delta - x^e}{\pi} = \frac{1}{\pi}\left(\delta - (1-\pi)\delta - \frac{\pi d}{R}\right) = \frac{1}{\pi}\left(\pi\delta - \frac{\pi d}{R}\right) = \delta - \frac{d}{R}$$

Thus

$$c_{1N} = \frac{w_1 + \frac{d}{R} - \delta}{p_1}$$
$$c_{2N} = \frac{w_2 - R\delta + d}{p_2}$$

From the first order conditions we obtain

$$c_{2N} = Ac_{1N}$$

so

$$p_1\left(w_2 - R\delta + d\right) = Ap_2\left(w_1 + \frac{d}{R} - \delta\right)$$

and from the constraints

$$\pi w_1 = p_1 A\left(m_0^{CB} - z_2\right) + \left(\pi\delta - \frac{\pi d}{R}\right) + h_1$$

$$\left(1-\pi\right)w_2 = p_2 A^2 z_2 + R\left(1-\pi\right)\delta - \left(1-\pi\right)d - h_1$$

On the other hand, when $h_1 > 0$ first order conditions imply that

$$w_1 + \frac{d}{R} - \delta = w_2 - R\delta + d$$

which implies then that $p_1 = Ap_2$ in order to get a solution. If this is the case, one of the four variables is undetermined (see the appendix for a more general proof). Then we set h_1 exogenously and we then solve the system

$$w_1 + \frac{d}{R} - \delta = w_2 - R\delta + d$$
$$\pi w_1 = p_1 A\left(m_0^{CB} - z_2\right) + \left(\pi\delta - \frac{\pi d}{R}\right) + h_1$$
$$\left(1-\pi\right)w_2 = p_1 A z_2 + R\left(1-\pi\right)\delta - \left(1-\pi\right)d - h_1$$

The solution being

$$z_2^e = \left(1-\pi\right)m_0^{CB} + \frac{h_1}{Ap_1}$$

$$w_1^e = p_1 A m_0^{CB} + \left(\delta - \frac{d}{R}\right)$$

$$w_2^e = p_1 A m_0^{CB} + R\delta - d$$

which implies

$$c_{1N}^e = A m_0^{CB}$$

$$c_{2N}^e = A^2 m_0^{CB}$$

In order to implement the optimal allocation, we must first have that

$$c_{1T}^* = \frac{d\theta\left(R-1\right)}{R} = c_{1T}^e = \delta - \frac{d}{R}$$

which gives

$$\delta^{imp} = \frac{d\left(\theta\left(R-1\right)+1\right)}{R}$$

This is the amount of dollars that the Central Bank should lend to the commercial banks at date 0. Therefore the amount of dollars converted to pesos that will be used to produce non-tradeables should be

$$m_0^{CB,imp} = d - \delta^{imp} = \frac{d\left(1-\theta\right)\left(R-1\right)}{R}$$

Replacing these in c_{tN}^e *we obtain*

$$c_{1N}^e = \frac{Ad\left(1-\theta\right)\left(R-1\right)}{R}$$

$$c_{2N}^e = \frac{A^2 d\left(1-\theta\right)\left(R-1\right)}{R}$$

Therefore, $c_{tN}^* = c_{tN}^e$*. Note that the date 1 price of non-tradeables is not yet specified until the monetary policy is specified.*

The second case is such that $h_1 = 0$*. This implies that* $p_1 \geq Ap_2$*. On the other hand, we have that the relevant system of equations must be*

$$p_1\left(w_2 - R\delta + d\right) = Ap_2\left(w_1 + \frac{d}{R} - \delta\right)$$

$$\pi w_1 = p_1 A\left(m_0^{CB} - z_2\right) + \left(\pi\delta - \frac{\pi d}{R}\right)$$
$$\left(1-\pi\right) w_2 = p_2 A^2 z_2 + R\left(1-\pi\right)\delta - \left(1-\pi\right) d$$

The solution to this system is

$$z_2^{e'} = \frac{(1-\pi)\, p_1 m_0^{CB}}{(1-\pi)\, p_1 + \pi A p_2}$$

and

$$w_1^e = A p_1 \left(\frac{A p_2 m_0^{CB}}{(1-\pi)\, p_1 + \pi A p_2} \right) + \delta - \frac{d}{R}$$

$$w_2^e = A^2 p_2 \left(\frac{p_1 m_0^{CB}}{(1-\pi)\, p_1 + \pi A p_2} \right) + R\delta - d$$

As a consequence:

$$c_{1N}^e = \frac{w_1^e + \frac{d}{R} - \delta}{p_1} = \frac{A^2 p_2 m_0^{CB}}{(1-\pi)\, p_1 + \pi A p_2}$$

$$c_{2N}^e = \frac{w_2^e - R\delta + d}{p_2} = \frac{A^2 p_1 m_0^{CB}}{(1-\pi)\, p_1 + \pi A p_2}$$

Since it is still true that $m_0^{CB} = \frac{d(1-\theta)(R-1)}{R}$, *therefore*

$$c_{1N}^e = \frac{A^2 p_2 \left(\frac{d(1-\theta)(R-1)}{R} \right)}{(1-\pi)\, p_1 + \pi A p_2}$$

$$c_{2N}^e = \frac{A^2 p_1 \frac{d(1-\theta)(R-1)}{R}}{(1-\pi)\, p_1 + \pi A p_2}$$

But we want to have that $c_{tN}^e = c_{tN}^*$ *for* $t = 1, 2$ *then it must still be true that* $p_1 = A p_2$. *Under this price, it is possible to implement the first best allocation when* $h_1 = 0$.

The next step is to shed light on the monetary policy that the Central Bank must follow to allow implementation. To get this, we need to specify the equations that must satisfy the allocations. Let M_1 be the (per - capita) amount of pesos issued by the Central Bank. Hence, a portion of this is destined to buy the non-tradeables to impatient entrepreneurs, while the rest is dedicated to either buy dollars from commercial banks or to lend pesos to the financial intermediaries. The date 1 constraint for feasibility in the market for pesos is as follows.

$$M_1 = p_1 A z_1 + y + h_1$$

Given that the right hand side depends on p_1, and possibly on p_2, this equation determines a relationship between those prices and M_1. In period 2, let M_2 be the amount of pesos issued by the Central Bank, used to purchase the non-tradable goods sold by patient entrepreneurs and to buy dollars from the commercial banks. This amount is net of the pesos returned by commercial banks. Thus

$$M_2 = p_2 A^2 z_2 + Rx - d - h_1$$

It is then clear that M_t must be equal to $\widehat{\pi}_t w_t$, where $\widehat{\pi}_1 = \pi$ and $\widehat{\pi}_2 = 1 - \pi$. This gives conditions on the implementability of the optimal allocation. Consider this in the logarithmic case.

Example 5 *Continuing with our example, if $h_1 > 0$ we had that*

$$M_1 = \pi w_1^e = p_1 A \frac{\pi d (1-\theta)(R-1)}{R} + \frac{\pi d (\theta (R-1))}{R}$$

Therefore

$$p_1^e = \frac{RM_1 - \pi d (\theta (R-1))}{A \pi d (1-\theta)(R-1)} \tag{8}$$

which is positive if and only if

$$M_1 > \frac{\pi d (\theta (R-1))}{R}$$

For period 2 the analysis is similar

$$\begin{aligned} M_2 = (1-\pi) w_2^e &= (1-\pi) p_1 A m_0^{CB} + (1-\pi) R\delta - (1-\pi) d \\ &= (1-\pi) p_1 A \frac{\pi d (1-\theta)(R-1)}{R} + (1-\pi)(d\theta (R-1)) \end{aligned}$$

By replacing equation (8) in here we get

$$\begin{aligned} M_2 &= \frac{(1-\pi)}{R} (RM_1 - \pi d (\theta (R-1))) + (1-\pi)(d\theta (R-1)) \\ &= (1-\pi) M_1 + d\theta (R-1) \left[1 - \pi - \frac{\pi}{R} \right] \end{aligned}$$

Note that if $p_1^e > 0$, then $M_2 > 0$. This last equation gives a link between money supply in periods 1 and 2 to support the optimal allocation as an equilibrium. Given the link between the dates 1 and 2 's non-tradable prices, this forces to get a sequence of supply of pesos consistent with that condition.

If $h_1 = 0$ *then*

$$M_1' = \pi w_1^{e'} = \pi \left[A p_1 \left(\frac{d(1-\theta)(R-1)}{R} \right) + \frac{d(\theta(R-1))}{R} \right]$$

Therefore

$$p_1^{e'} = \frac{\left(\frac{M_1}{\pi}\right) - \frac{d(\theta(R-1))}{R}}{\left(\frac{d(1-\theta)(R-1)A}{R}\right)}$$

Again, this is positive if $M_1 > \pi\left(\frac{d(\theta(R-1))}{R}\right)$ *(the same condition as before). It can be shown then that* M_2' *must also satisfy the equation* $M_2' = (1-\pi) M_1' + d\theta (R-1)\left[1 - \pi - \frac{\pi}{R}\right]$.

This monetary rule is of course specialized to the logarithmic preferences case. In general, money supply need to be adjusted to get the prices that allow implementation. The technique is still the same as in the example. If we replace in the expression $M_1 = p_1 A z_1 + y$ by z_1^* and y^* it is clear that the price of non - tradeables must be equal to $p_1^{eq} = \frac{M_1 - y^*}{A z_1^*}$. To ensure positivity of the price we demand that $M_1 > y^*$. Replacing this in $M_2 = p_2 A^2 z_2^* + R x^* - d$ $= p_1 A z_2^* + R x^* - d$ we get $M_2 = \left(\frac{M_1 - y^*}{A z_1^*}\right) A z_2^* + R x^* - d$. This is the generalization of the formulae presented in the logarithmic case.

What about the real exchange rate? In our case it is just $1/p_t^e$ (the price of tradeables in terms of non - tradeables). Clearly $\frac{1}{p_2} = \frac{A}{p_1} > \frac{1}{p_1}$, so the equilibrium that implements the optimal allocation implies a real appreciation of the local currency between periods 1 and 2. It is clear that the price of non - tradeables satisfies the simplest quantitative theory of the money demand. This is the direct consequence of assuming cash - in - advance constraints for all goods. Therefore the real exchange rate in this model is determined entirely by the stock of pesos in period 1 (ceteris paribus).

6 Financial and Currency crisis

This section explores the conditions under which this banking system induces some type of run or crisis on the financial system. As Chang and Velasco (2000b) state, it is obvious that the only type of run that could arise in this economy with fixed exchange rates and a local lender of last resort is a *currency* crisis. Commercial banks can always get enough liquidity in pesos to satisfy any withdrawal pattern. The problem will be faced by the Central Bank, when trying to sell dollars in exchange for pesos. This arises when the long term, dollar-denominated asset is illiquid enough. Assuming that the banking system and the Central Bank have perfect commitment to repay all foreign debt, the next proposition shows the conditions for a crisis to happen.

Proposition 6 *In a banking system with the exchange rate regime described in section 4 there is a currency crisis equilibrium if the following conditions hold*

$$c_{1T}^{*} > y^{*} + r\left(x^{*} - \frac{d}{R}\right)$$

$$u\left(c_{1T}^{*}\right) + v\left(c_{1N}^{*}\right) > u\left(0\right) + v\left(c_{2N}^{*}\right)$$

If any of the two conditions fail to hold, then there is no equilibrium with a currency crisis.

In the logarithmic case, the assumption $r < 1$ is enough to get an equilibrium currency crisis.

Example 7 *With logarithmic preferences, the first order condition of the consumer gives the period 1 demand function for non-tradeables:*

$$\frac{\left(\delta - \frac{d}{R}\right) + p_1 A m_0^{CB}}{2p_1} = c_N\left(p_1\right)$$

So the price that equilibrates the date 1 non-tradeable market is

$$p_1^e = \frac{d\theta\left(R-1\right)}{AR}$$

Then the amount left to buy dollars at a one-to-one rate is just $c_{1T}^e = \frac{d\theta(R-1)}{R}$. *However, the total amount of dollars available at the Central Bank is*

$$\begin{aligned} y^e + r\left(x^e - \frac{d}{R}\right) &= \frac{\pi d\theta\left(R-1\right)}{R} + r\left(\frac{\left(1-\pi\right)d\theta\left(R-1\right)}{R}\right) \\ &= \frac{d\theta\left(R-1\right)}{R}\left(\pi + r\left(1-\pi\right)\right) \end{aligned}$$

Since $r < 1$, *then* $y^e + r\left(x^e - \frac{d}{R}\right) < \frac{d\theta(R-1)}{R} = c_T^e$. *This implies that the Central Bank runs out of dollars but also it fails to satisfy (at a fixed exchange rate) the total demand for dollars. Therefore, it is rational for any household to withdraw early since otherwise the Central Bank closes. (In this case, it is never optimal to wait in the event of a run given the fact that* $\ln\left(0\right)$ *is minus infinity. However this result depends on the form of the functions* u *and* v*, as the proposition states).*

This result generalizes the result in Chang and Velasco (2000b). It states that not only an international illiquidity condition is required for a crisis to occur. It also suggests that preferences may matter. The second inequality of proposition (6) shows this suggestion.

The question about the behavior of the date-1 real exchange rate in period 1 has an obvious answer here. When the crisis occurs, that is, after the last consumer is able to sell pesos for dollars at the one - to - one rate the (implicit) nominal exchange rate *jumps to infinity*, which makes the real exchange rate also *jump to infinity*. Since this is not interesting, I delay a more complete discussion for the subsequent work in the following sections.

7 Devaluation contingent rules with peso - denominated deposits and the elimination of runs.

Chang and Velasco (2000b, section 6) demonstrated that flexible exchange rates eliminates equilibrium currency crisis when coupled with suitable monetary policies. This section extends this result to the case of two goods. I start by considering peso-denominated deposits, as in the previous section. Subsequent sections will analyze the same issues in partially dollarized banking systems.

Consider the following exchange rate policy. In period 0 the exchange rate is equal to one. At the beginning of date 1 the Central Bank buys all dollars sold by commercial banks also at an exchange rate equal to unity. When shoppers go to the Central Bank to sell their pesos for dollars, the exchange rate may not be equal to one. Instead, the Central Bank may devalue the peso whenever the proportion of consumers selling pesos at that stage is strictly larger than π. More precisely, the exchange rate set by the monetary authority is equal to $\max\left\{1, \frac{\widehat{\pi}}{\pi}\right\}$, where $\widehat{\pi}$ is the observed proportion of shoppers selling pesos for dollars at date 1. As before, the Central Bank lends to commercial banks any amount of pesos to satisfy any withdrawal pattern. In period 2 the exchange rate is reset to one at all times.

This policy is enough to ensure that no currency run equilibrium exists under this policy. The following proposition states this with precision.

Proposition 8 *Under the exchange rate and monetary policy, there is no currency crisis equilibrium. Moreover, the only possible equilibrium corresponds to the optimal allocation.*

This result is not surprising at all. The key point here is as in Chang and Velasco (2000), that is, the absence of a serious sequential service constraint at the Central Bank. What really matters is the fact that the currency run here is prevented when the patient shoppers foresee that any run will be handled by the Central Bank by depreciating the local currency transitorily (i.e., only in period 1). By anticipating this, all patient households know that long - term investments of the commercial bank are preserved (not early liquidated) so that by waiting until period 2 all the payments promised by the contract will be fulfilled.

This then shows that the result in the literature does not rely on preferences at all. The important fact here is that this depreciation policy preserves the international liquidity of the system. However, this model does not say anything about what happens when the sequential service constraint at the Central Bank is reintroduced. This remains to be answered in subsequent research.

8 Dollarized banking systems.

When discussing a dollarized version of the former banking system, it is obvious that the only way to get exactly the same outcome is when the Central Bank as a local lender of last resort is replaced by an international lender of last resort. This institution is assumed to be willing to lend any dollar at the beginning of period 1 to be returned at the end of period 1 at zero net interest rate. Also, the Central Bank as the institution that centralizes the non - tradeable goods exchanges is replaced by a Central Exchange, as in Magill and Quinzii (1992 and 1996). With these new assumptions, it is very easy to show that the same allocation is implementable when only dollars are traded. The next proposition shows this

Proposition 9 *When there exists an international lender of last resort, willing to lend dollars at zero net interest rate in intra - period loans, then there exists a version of the former banking system fully dollarized that implements the optimal allocation as an equilibrium. Moreover, if the same international lender of last resort also gives zero - net - interest - rate loans to local banks between periods 1 and 2, then there is no bank run equilibrium.*

However, in practice the US Federal Reserve is usually reluctant to act as a lender of last resort in foreign countries (mainly due to moral hazard considerations not modelled here). The rest of this section assumes a different dollarized banking system without such a lender [3]. The following analysis suggests that the discussion on the relationship between banking, exchange rates and optimality can be very sensitive to changes in the process of exchange of goods and assets.

Suppose that in period 0 commercial banks borrow d dollars from abroad. These banks decide the amount to be invested in the liquid (short run) and illiquid (long run) dollar denominated investments. The rest is assigned to purchase the amount of tradable inputs to be given to the entrepreneurs in order to produce non - tradeables. This means that the commercial bank directly purchases the inputs on behalf of entrepreneurs.

Assume that at the beginning of date 1 impatient entrepreneurs and shoppers split as before. Commercial banks obtain y^* dollars by liquidating the storage technology. The dollars are paid to the impatient shoppers, who purchase non-tradable goods from other entrepreneurs at a price $\bar{p}_1$. After this first session shoppers and entrepreneurs get together. Shoppers bring non-tradeables to be consumed at the end of the period and entrepreneurs bring y^* dollars. Then consumers use these dollars to purchase tradable goods abroad. Then consumption takes place and period 1 ends.

In period 2 actions are similar. At the beginning of this period patient entrepreneurs and shoppers split. Then commercial banks obtain the results of the long term project (in dollars), and repay to the foreign creditors the outstanding debt. The remaining dollars are paid to patient shoppers who use them to purchase non - tradeables at a price equal to $\bar{p}_2$. After this, shoppers and entrepreneurs meet again. They use the remaining dollars to buy tradable goods. The period ends with the consumption of tradeables and non-tradeables.

8.1 Suboptimality of the dollarized banking system.

This subsection analyzes whether this banking system can decentralize the optimal allocation studied in section 3. The problem of the bank can be formalized as follows.

$$\max \quad \pi \left[u\left(c_{1T}\right)+v\left(c_{1N}\right)\right]+(1-\pi)\left[u\left(c_{2T}\right)+v\left(c_{2T}\right)\right]$$

subject to

$$x+y+z \leqq d$$

[3] Alternatively, it can also be interpreted as a two - currency banking system where the Central Bank becomes a currency board at a one - to - one rate.

$$
\begin{aligned}
\pi w_1 &\leqq y \\
p_1 c_{1N} &\leqq w_1 \\
\pi c_{1T} &\leqq p_1 A(z - z_2) + w_1 - p_1 c_{1N}
\end{aligned}
$$

$$
\begin{aligned}
(1-\pi) w_2 &\leqq Rx - d \\
p_2 c_{2N} &\leqq w_2 \\
(1-\pi) c_{2T} &\leqq p_2 A^2 z_2 + w_2 - p_2 c_{2N}
\end{aligned}
$$

The next proposition shows the characterization of the equilibrium allocation.

Proposition 10 *Under the stated conditions, the equilibrium allocation is characterized by the constraints above and the equations*

$$\frac{v'(c_{1N})}{p_1} = R\frac{v'(c_{2N})}{p_2}$$

$$v'(c_{1N}) = u'(c_{1T})\, p_1^2 A$$

$$u'(c_{1T})\, p_1 = A p_2 u'(c_{2T})$$

Then, except for a Lebesgue - measure zero of parameters, the optimal allocation cannot be decentralized in this banking system.

This result shows formally the intuition that only in special cases a dollarized banking system (in a dollarized economy) can be optimal ex-ante. For illustrative purposes, I present the case of logarithmic preferences

Example 11 *In the case where utility functions are logarithmic the first order conditions imply*

$$p_2 c_{2N} = R p_1 c_{1N}$$

$$(1-\theta)\, c_{1T} = p_1^2 A \theta c_{1N}$$

$$p_1 c_{2T} = A p_2 c_{1T}$$

Replacing by the constraints (assuming that they hold with equality)

$$\pi (Rx - d) = (1-\pi) R (d - x - z)$$

$$(1-\theta)(z-z_2)=\theta(d-x-z)$$

$$\pi z_2=(1-\pi)(z-z_2)$$

so

$$\begin{aligned} z_2 &= (1-\pi)z \\ z_1 &= \pi z \end{aligned}$$

Then we must solve for (x,z) from the following linear system:

$$Rx+(1-\pi)Rz=[R(1-\pi)+\pi]d$$

$$\theta x+[(1-\theta)\pi+\theta]z=\theta d$$

The solution to the system is

$$x^*=(R(1-\pi)(1-\theta)+(1-\theta)\pi+\theta)\left(\frac{d}{R}\right)$$

$$z^*=\frac{[R-1]\theta d}{R}$$

so

$$\begin{aligned} y^* &= d-x^*-z^* \\ &= \frac{d}{R}[R-(R(1-\pi)(1-\theta)+(1-\theta)\pi+\theta)-[R-1]\theta] \\ &= \frac{d}{R}[R-1][(1-\theta)\pi] \end{aligned}$$

Note that this coincides with the optimal z if and only if $\theta=1-\theta$, or when $\theta=\frac{1}{2}$. The equilibrium demand functions are

$$c_{1N}=\frac{\frac{d}{R}[R-1][(1-\theta)\pi]}{\pi p_1}$$

$$c_{1T}=p_1Az^*=p_1A\frac{[R-1]\theta d}{R}$$

$$\begin{aligned} c_{2N} &= \frac{\left(R\left(1-\pi\right)\left(1-\theta\right)+\left(1-\theta\right)\pi+\theta-1\right)d}{\left(1-\pi\right)p_2} \\ &= \frac{\left(R-1\right)\left(1-\pi\right)\left(1-\theta\right)d}{p_2\left(1-\pi\right)} \end{aligned}$$

$$c_{2T} = p_2 A^2 z^* = p_2 A^2 \frac{\left[R-1\right]\theta d}{R}$$

In equilibrium, $\pi c_{1N} = A\pi z^*$ *and* $\left(1-\pi\right)c_{2N} = A^2\left(1-\pi\right)z^*$. *From the first equation we solve for the equilibrium value of* p_1^* :

$$\frac{\left(1-\theta\right)}{p_1} = A\theta$$

or

$$p_1^* = \frac{1-\theta}{A\theta}$$

and from the second equation we get p_2^* :

$$\frac{\left(1-\theta\right)}{p_2} = A^2\frac{\theta}{R}$$

Hence

$$p_2^* = \frac{\left(1-\theta\right)R}{\theta A^2}$$

Hence the equilibrium consumption allocations are

$$c_{1N}^* = \frac{d}{R}\left[R-1\right]A\theta$$

$$c_{1T}^* = \left(1-\theta\right)\frac{\left[R-1\right]d}{R}$$

$$c_{2N}^* = \left(R-1\right)d\left(\frac{\theta A^2}{R}\right)$$

$$c_{2T}^* = \left(1-\theta\right)\left[R-1\right]d$$

and so

$$w_1^* = \frac{\frac{d}{R}\left[R-1\right]\left[\left(1-\theta\right)\pi\right]}{\pi} = \left(\frac{d}{R}\right)\left(R-1\right)\left(1-\theta\right)$$

Note that when the banking system is dollarized the price system depends entirely on real fundamentals. This is obvious since the relevant money supply is not controlled by any local agent. In the logarithmic case, the inflation in terms of non - tradeables is $p_2^* \ / \ p_1^* = \frac{A}{R}$. Then the change in the real exchange rate between periods 1 and 2 depends entirely on the ratio between the marginal productivity of the non - tradeable production technology and the marginal productivity of the long - term tradeable - generating investment technology.

8.2 Banking system fragility

The second point in this section is whether this contract implies a form of bank runs. It is obvious to see that now we are referring to bank runs and not to currency crisis. The firs obvious result is the following.

Proposition 12 *The necessary and sufficient condition for an equilibrium run to occur in a dollarized banking system is* $w_1^* > y^* + rx^*$.

The proof of this is standard and left to the reader. To illustrate this condition I present the logarithmic example.

Example 13 *If preferences are logarithmic, the condition is replaced by*

$$\left(\frac{d}{R}\right)(R-1)(1-\theta)(1-\pi) > r\left((R(1-\pi)(1-\theta)+(1-\theta)\pi+\theta)\left(\frac{d}{R}\right)\right)$$

or

$$(R-1)(1-\theta)(1-\pi) > r(R(1-\pi)(1-\theta)+(1-\theta)\pi+\theta)$$

or

$$r < \frac{(R-1)(1-\theta)(1-\pi)}{R(1-\pi)(1-\theta)+(1-\theta)\pi+\theta} = \bar{r}^{dol}$$

Note that, $\bar{r}^{dol}$ *is strictly less than the upper bound for* r *in the case of fixed exchange rates with a local lender of last resort (which is equal to one). This fact could be interpreted loosely stating that it is less likely the run in this dollarized system than with a fixed exchange rate and local lender of last resort, although the optimal allocation may not be implementable. Therefore, if we parameterize the economy through* r*, then we can say that the probability, measured as the Lebesgue measure of* $[0, \bar{r}]$*, is lower the closer is* R *to* 1.

In the logarithmic case it is then possible to state that, even though the fixed exchange rate regime with a Central Bank printing unbacked pesos to provide liquidity in transactions decentralize the optimal allocation, this system seems *more fragile* (in the sense that the Lebesgue measure of the interval $[0, \bar{r})$ is larger) than a system where the Central Bank cannot do that, although this system does not typically decentralize the optimal allocation.

Does this hold in the traditional Chang and Velasco framework? I answer this in the following example using also logarithmic preferences.

Example 14 *Take the Chang and Velasco (2000b) framework with the following preferences:* $g(z) = \ln z$ *and* $\chi(m) = -\frac{1}{2}m^2 + \alpha m + \beta$, $\alpha > R - 1 > 0$, $\beta \geq 0$. *Consider the case of the Currency board studied in section 3: in this example, according to the authors, we have*

$$\widetilde{M} = \alpha - (R - 1)$$

and the x and y must satisfy equations 3.7, 3.8 and 3.9, or:

$$Rx = \alpha\widetilde{M} + \beta - \frac{1}{2}\widetilde{M}^2 + y$$

$$R\lambda x + (1 - \lambda)\, y = eR - (R - 1)(1 - \lambda)\widetilde{M}$$

From here we get

$$\widetilde{x} = e + \frac{[\alpha - (R - 1)]^2 (1 - \lambda)}{2R} + \frac{\beta(1 - \lambda)}{R}$$

$$\widetilde{y} = eR + \left(\frac{2 - \lambda}{2}\right)[\alpha - (R - 1)]^2$$

and then condition 3.10 in CV implies

$$r < \frac{eR + [\alpha - (R - 1)]^2 \left(\frac{1-\lambda}{2}\right) + \beta(1 - \lambda)}{eR + \left(\frac{2-\lambda}{2}\right)[\alpha - (R - 1)]^2 - \beta\lambda - (1 + \alpha)[\alpha - (R - 1)]}$$

Take now the banking system with Fixed Exchange Rates and a local lender of last resort (section 4). Then equations (4.4), (4.5), (4.6) *must hold. In our example they imply:*

$$\widetilde{M} = \alpha$$

$$R\lambda x + (1 - \lambda)\, y = eR$$

$$y + \beta + \frac{\alpha^2}{2} = Rx$$

Hence

$$\bar{x} = e + \frac{\alpha^2 (1 - \lambda)}{2R} + \frac{\beta (1 - \lambda)}{R}$$

$$\bar{y} = eR - \lambda \left[\frac{\alpha^2}{2} + \beta\right]$$

Therefore condition (4.11) *is equivalent in this case to*

$$r < \frac{e + \left[\frac{\alpha^2}{2} + \beta\right]\left(\frac{1-\lambda}{R}\right)}{e - \left[\frac{\alpha^2}{2} + \beta\right]\left(\frac{\lambda}{R}\right)}$$

Suppose now these numerical values for the parameters $(e, R, \alpha, \beta, \lambda)$:

e	R	α	β	λ
1	1.01	0.5	0	0.35

Then the upper bound for r under a currency board is equal to 2.2998, while under fixed rates with a local lender of last resort is equal to 1.1294. Hence, by applying the arguments above, it is less likely under a fixed exchange rate with a Central Bank acting as a local lender than under a currency board regime.

This then shows that the result obtained in this paper may depend upon the role that currencies have in transactions, and how these are arranged. Clearly, the transaction arrangements differ between the fixed exchange rate regime with a local lender with the dollarized (or currency board) regime (without an international lender of last resort). This may be driving part of the result. However, as stated above, it is impossible to have an equilibrium with a dollarized system with the arrangements as described in the fixed rate with local lender case, since in the latter case some type of currency stock is needed for purchasing and selling non - tradeables before withdrawing from banks, while in the former this is not necessary. This result then suggests that cash - in - advance constraints limit the validity of the results since they depend on the *timing* of transactions.

9 Concluding Remarks and Extensions

This paper presents an extension of the Chang and Velasco small open economy version of the Diamond and Dybvig model with cash - in - advance constraints, in order to give money a more serious role for consumers. In this way the results in Chang and Velasco (2000b) about the interaction between banks, exchange rate and monetary policies can be restated and analyzed more properly. The main conclusion is that the way exchanges in the non - tradeables sector are organized has a very strong influence on the properties of each policy and the banking system that can be constructed. When agents trade in non - tradeables before withdrawing from banks, it is clear that dollarized systems without an international lender of last resort cannot work since there is simply no stock of currency to generate exchanges in non - tradeables. In this situation fixed exchange rates with the Central Bank providing unbacked pesos (local lender of last resort) does implement the optimal allocation. However this system is vulnerable to currency runs. A devaluation threat as the one in Chang and Velasco (2000b) eliminates this risk. However, this relies heavily on some form of absence of sequential service constraint at the Central Bank, as in the original work.

If the exchange in non - tradeables must be done with money withdrawn from banks, then a dollarized banking system can be now considered. However, almost never is the optimal allocation implemented under this assumption, that is, implementation is at best non - generic. In the special case of logarithmic preferences, however, runs under dollarized systems are (in an informal sense) less likely than in the banking system with fixed exchange rates and the local lender of last resort. This is different from the Chang and Velasco (2000b) model, for which there are parameter values where the reverse is true.

The assumptions used throughout the paper may be subject to various criticisms. Perhaps one of the most important problems is the introduction of ad - hoc cash - in - advance constraints. It is crucial to clarify that this paper does not want to lead with the question of why money is used for transactions. The model takes this assumption as given (as in most of the cash - in - advance literature) to address other issues on the relationship between banks and monetary and exchange rate policies. Even though this is not completely satisfactory, the cash - in - advance approach seems to add at least a more explicit role for money, which was absent in the original framework.

Still, this framework can be extended to consider other questions that seem relevant. For example, what happens when there is some international aggregate (locally non - diversifiable) shock in the different arrangements?

Given the randomness in the international price of commodities, this may affect the optimal design of the banking contract as well as the properties of different monetary and exchange rate policies. This framework has also a natural two - country extension. This is interesting to study the interaction between international banks, monetary and exchange rate policies and the possibility of contagion, as studied in the real version of the Diamond and Dybvig framework in Allen and Gale (2000).

A deeper question is the fact that money in this economy cannot considered as essential, at least in the sense of Wallace (2000). The ad - hoc assumption may not be satisfactory if we want to study the interaction between monetary and exchange rate policies, banking stability and currency substitution, for example. This calls for endogeneizing the use of money for transactions, as studied by the already broad literature on search models starting from Kiyotaki and Wright (1989). However this implies a banking system with possibly infinitely lived households, issue that to my knowledge was not addressed yet. This last extension seems much more complicated to construct.

10 Appendix: Proofs

10.1 Proof of Lemma 1

If z is fixed then the problem in terms of how much of z is liquidated early is the solution to

$$\max_{z_2 \in [0,z]} \quad \pi v\left(\frac{A\left(z-z_2\right)}{\pi}\right)+\left(1-\pi\right) v\left(\frac{A^2 z_2}{1-\pi}\right)$$

whose FOC is $v'\left(\frac{A(z-z_2)}{\pi}\right) = Av'\left(\frac{A^2 z_2}{1-\pi}\right)$. Strict concavity of v ensures that the solution to this problem is unique. But then, if $z_2 = (1-\pi)\, z$ clearly this equality is satisfied. Hence we can state that $z_1 = \pi z$ and $z_2 = (1-\pi)\, z$.

10.2 Proof of Proposition 3

Note that the problem is a standard concave programming. Hence we know that the first order conditions are necessary as well as sufficient to characterize the optimum. Let the Lagrangian be

$$\begin{aligned}
\mathcal{L} &= \pi\left[u\left(c_{1T}\right)+v\left(c_{1N}\right)\right]+(1-\pi)\left[u\left(c_{2T}\right)+v\left(c_{2T}\right)\right]+ \\
&+\phi\left[\delta-x-y\right]+\mu\left[m_0^{CB}-z_1-z_2\right]+\phi_1\left[p_1Az_1+y+h_1-\pi w_1\right] \\
&+\lambda_1\left[y-\pi c_{1T}\right]+\phi_2\left[p_2A^2z_2+Rx-d-h_1-(1-\pi)w_2\right] \\
&+\lambda_2\left[Rx-d-(1-\pi)c_{2T}\right]+\sum_{t=1}^{2}\theta_t\left[w_t-c_{tT}-p_tc_{tN}\right]
\end{aligned}$$

The decision variables are $(c_{1T}, c_{2T}, c_{1N}, c_{2N}, x, y, z_1, z_2, w_1, w_2, h_1)$. The FOC are

$$\begin{aligned}
\pi u'\left(c_{1T}\right) &= \pi\lambda_1+\theta_1 \\
(1-\pi)u'\left(c_{2T}\right) &= (1-\pi)\lambda_2+\theta_2 \\
\pi v'\left(c_{1N}\right) &= \theta_1 p_1 \\
(1-\pi)v'\left(v_{2N}\right) &= \theta_2 p_2 \\
\phi &= R\left(\phi_2+\lambda_2\right) \\
\phi &= \phi_1+\lambda_1 \\
\mu &= \phi_1 A p_1 \\
\mu &= \phi_2 A^2 p_2 \\
\pi\phi_1 &= \theta_1 \\
(1-\pi)\phi_2 &= \theta_2 \\
\phi_1 &\leqq \phi_2
\end{aligned}$$

Therefore we obtain $\phi_1+\lambda_1 = R(\phi_2+\lambda_2)$, but since also $u'(c_{tT}) = \lambda_t+\phi_t$ we then obtain

$$u'\left(c_{1T}\right) = Ru'\left(c_{2T}\right)$$

Also we do have $v'(c_{tN}) = \phi_t p_t$ for $t = 1, 2$. This of course implies that

$$v'\left(c_{1N}\right) = Av'\left(c_{2N}\right)$$

The two equalities, together with $R > 1$ and $A > 1$, imply that the equilibrium allocation satisfies the incentive - compatibility constraint. On the other hand, since $\phi_1 \leqq \phi_2$ (with equality whenever $h_1 > 0$) then it is clear that we also should have

$$\frac{v'\left(c_{1N}\right)}{p_1} \leqq \frac{v'\left(c_{2N}\right)}{p_2}$$

Both expressions then give the inequality $p_1 \geq Ap_2$. Hence, the gross rate of growth of non-tradeables prices must satisfy $\frac{p_2}{p_1} \leqq \frac{1}{A} < 1$.

Note that if $\phi = \mu$ then we also obtain

$$u'(c_{1T}) = Av'(c_{1N})$$

Therefore, when $\phi = \mu$, and under suitable conditions on δ and m_0^{CB} we obtain the same first order conditions as those corresponding to the optimal allocation. The equality $\phi = \mu$ is the same as stating that the partial derivative of the indirect utility function (call this V) with respect to δ must be equal to the partial derivative of V with respect to m_0^{CB}. However, we know that $m_0^{CB} = d - \delta$. Hence, if we write the indirect utility function V as a function of (δ, m_0^{CB}), then we can rewrite $V(\delta, d-\delta)$. Hence $\phi = \mu$ is equivalent to the (interior) solution of the problem $\max_{\delta \in [0,d]} V(\delta, d-\delta)$. Therefore, the Central Bank (if aiming to implement the optimal allocation) must set δ as the solution to this problem. Note that h_1 may be either strictly positive or zero. This shows the last result.

10.3 Proof of proposition 6

The proof is standard. Suppose that the condition holds. From lemma (1) each household producing non-tradeables in period 1 gets Am_0^{CB}, while in period 2 is $A^2 m_0^{CB}$. Each shopper who withdraw early must get a total amount of pesos equal to $w_1 = w_1^e = p_1 Am_0^{CB} + c_{1T}^e$. This is because the contract specifies that every shopper withdrawing at date 1 has the right to get w_1^e pesos. Commercial banks then do not fail. Each consumer maximizes $u(c_T) + v(c_N)$ subject to $c_T + p_1 c_N \leqq w_1^e$. The first order condition is then $u'(c_T)p_1 = v'(c_N)$. Next, find the price p_1^* such that $c_N = Am_0^{CB}$. This exists because of the following (standard) argument. The first order condition can be rewritten as

$$u'\left(p_1 Am_0^{CB} + c_{1T}^e - p_1 c_N\right) p_1 = v'(c_N)$$

From here, it is possible to get $c_N(p_1, w_1^e)$ using the Intermediate Value Theorem. By applying then the Implicit Function Theorem, we can compute the partial derivative $\frac{\partial c_N}{\partial p_1}$:

$$\frac{\partial c_N}{\partial p_1} = \frac{u'(c_{1T}) + \left(Am_0^{CB} - c_N\right) u''(c_{1T})}{p_1^2 u''(c_{1T}) + v''(c_N)}$$

It is obvious that $\frac{\partial c_N}{\partial p_1} > 0$ if and only if $u'(c_{1T}) - c_N u''(c_N) > -Am_0^{CB} u''(c_{1T})$. Also, note that

$$\lim_{p_1 \to 0} c_N = +\infty$$

Why? If this does not hold, then $\lim_{p_1 \to 0} c_N < +\infty$. But then

$$\lim_{p_1 \to 0} \left[v'(c_N) - p_1 u'(w_1^e - p_1 c_N) \right] > 0$$

So, for p_1 sufficiently small, the first order condition is not true. This contradiction implies that $\lim_{p_1 \to 0} c_N = +\infty$. An analogous argument shows that when p_1 is sufficiently large, we must have c_N going to zero. Otherwise the first order condition is violated. By the intermediate value theorem, there must be a p_1^* that implies $c_N = Am_0^{CB}$. Note that when this happens, in its neighborhood we get $\frac{\partial c_N}{\partial p_1} > 0$. Therefore this price is (at least locally) unique. This condition just states that the demand for non-tradeables must equal its supply. Replacing this in the consumer's budget constraint implies that exactly c_T pesos are left to purchase dollars at a one-to-one exchange rate. However, the Central Bank only has $y^e + r\left(x^e - \frac{d}{R}\right)$ dollars (sold by commercial banks after liquidating the long term project). Hence, if the first inequality in the statement holds, the Central Bank fails. If the second inequality also holds, then it is individually rational to withdraw early. Clearly, if the second inequality fails to hold, then it is not individually rational to run to the bank (for a patient). The failure of the second condition implies that even when the Central Bank fails in period 1, any patient gets higher utility by waiting until period 2 than by running.

Suppose that only the first inequality in the statement is not true, that is, $c_{1T}^* \leqq y^* + r\left(x^e - \frac{d}{R}\right)$. I show that there is no run equilibrium, by using similar arguments as in Chang and Velasco (2000). Suppose without loss of generality that a proportion $\widehat{\pi} > \pi$ withdraws early (we will keep $\widehat{\pi} < 1$ but as close to one as desired). Let $\widehat{l} < x^e - \frac{d}{R}$ be the amount of the long term, dollar-denominated asset liquidated in $t = 1$. Again, the arguments in the previous paragraph can be applied to say that every early withdrawer is left with an amount of pesos to buy c_{1T}^* dollars. The amount of dollars left at the Central Bank in period 1 is $y^* + r\widehat{l}$. Hence, let us set $\widehat{l}$ equal to $\frac{\widehat{\pi} c_{1T}^* - y^*}{r}$. We know that this is feasible because $c_{1T}^* \leqq y^* + r\left(x^e - \frac{d}{R}\right)$. In period 2 the amount of dollars available for patient consumers (waiting until the last date) is equal to $R\left(x^e - \widehat{l}\right) - d$. Since $\widehat{l} < x^e - \frac{d}{R}$, then it is clear that, since $\widehat{\pi} < 1$ then $\widehat{l} < \left(\frac{\widehat{\pi} - \pi}{1 - \pi}\right)\left(\frac{Rx^e - d}{R}\right)$, and so $(1 - \pi) R\widehat{l} < (\widehat{\pi} - \pi)(Rx^e - d)$, which implies $(Rx^e - d)(1 - \widehat{\pi}) < (1 - \pi)\left(Rx^e - d - R\widehat{l}\right)$, and so $\frac{(Rx^e - d)}{1 - \pi} < \frac{\left(Rx^e - d - R\widehat{l}\right)}{(1 - \widehat{\pi})}$. Therefore, $c_{2T}^* < \frac{\left(Rx^e - d - R\widehat{l}\right)}{(1 - \widehat{\pi})}$, or $(1 - \widehat{\pi}) c_{2T}^* < \left(Rx^e - d - R\widehat{l}\right)$. This means that the amount of dollars available to each patient consumer who does not withdraw early strictly greater than c_{2T}^*. Hence it is rational to wait. This eliminates the run equilibrium.

10.4 Proof of proposition 8

Suppose that a proportion $\widehat{\pi} > \pi$ of the population behave as impatient consumers in period 1. This means that a proportion $\widehat{\pi}$ of entrepreneurs liquidate the non - tradable technology to produce each Az units of N. They sell these non-tradeable goods to the Central Bank, and repay their debt to commercial banks. These banks only liquidate $y^e = \pi c_{1T}^e$ dollars from the storage technology in order to purchase pesos from the Central Bank. Then a proportion $\widehat{\pi}$ of shoppers withdraw w_1 pesos. These are used to purchase non - tradeables as well as dollars for tradeables. The exchange rate at which each shopper must pay every dollar is equal to $\frac{\widehat{\pi}}{\pi} > 1$. Then the budget constraint faced by the shopper is

$$\left(\frac{\widehat{\pi}}{\pi}\right) c_{1T} + p_1 c_{1N} \leqq w_1$$

Following similar arguments as in proposition 6 the first order condition is

$$u'\left(\left(w_1^e - p_1 c_N\right)\left(\frac{\pi}{\widehat{\pi}}\right)\right)\left(\frac{\pi}{\widehat{\pi}}\right) p_1 = v'\left(c_N\right)$$

By the same arguments in the proof of proposition 6 there exists some$\left(\frac{\pi}{\widehat{\pi}}\right) p_1$ that makes c_N equal to Am_0^{CB}. For this price, the amount available of pesos to purchase dollars is again equal to $w_1^e - p_1 Am_0^{CB} = c_{1T}^e$. But the amount of dollars that can be purchased is equal to $c_{1T}^e\left(\frac{\pi}{\widehat{\pi}}\right)$. Hence, the total amount of dollars demanded by the shoppers is equal to $\widehat{\pi}\left[c_{1T}^e\left(\frac{\pi}{\widehat{\pi}}\right)\right] = \pi c_{1T}^e$. But this is equal to the amount of dollars to be sold by the Central Bank in period 1, $y^e = \pi c_{1T}^e$. Hence the Central Bank does not fail. The banking system did not need to liquidate early any of the long term project. Hence all remaining patient households who are active in period 2 can get strictly higher consumption than the corresponding optimal allocation. Notice then that each patient household who withdraw early obtains $c_{1N} = c_{1N}^* < c_{2N}^*$ and $c_{1T} = c_{1T}^*\left(\frac{\pi}{\widehat{\pi}}\right) < c_{1T}^* < c_{2T}^*$. Hence it is not individually rational for an impatient household to run against the banking system in period 1. This shows that the only equilibrium in this banking system, under this exchange rate and monetary policy, is that corresponding to the optimal allocation.

10.5 Proof of Proposition 9

In the dollarized banking system, all debt is now denominated in dollars. At the beginning of period 1, there is a Central Exchange that borrows from the international lender an amount M_1 dollars. Impatient entrepreneurs sell the non - tradeable production to this Central Exchange at a price equal to p_1^e

(now this price is set in dollars per unit of good N). Impatient entrepreneurs must use all these dollars to return the debt to commercial banks. Then the financial intermediaries liquidate the storage technology. Hence banks have a per-capita amount of $M_1 + y^* = p_1^e A z_1^e + y^*$ dollars. These are paid to impatient shoppers, who spend them in buying non-tradeables and tradeables. Non - tradeables are bought to the Central Exchange in exchange for M_1 dollars, who are used to repay the debt with the international lender of last resort. In period 2 the timing is also equivalent to that in the fixed exchange rate regime. At the beginning of period 2, the Central Exchange borrows M_2 dollars. These are used to purchase $A^2 z_2^*$ units of good N at the price p_2^e. Patient entrepreneurs receive M_2 dollars, which are used to repay their debt against commercial banks. The financial intermediaries receive these dollars and also liquidate the long - term investment technology. They set aside d dollars to be returned to the institutions who lent these dollars in period 0. The rest is paid to patient shoppers, who spend this money in purchasing goods N and T. Non - tradeables are again bought to the Central Exchange at the price p_2^e, so the amount of dollars paid is equal to M_2 (in equilibrium). These dollars are returned to the international lender of last resort. The reader can easily verify that the equations involved in this system are exactly the same as in the banking system considered above. Hence the allocations must coincide

For the second part, if the international lender of last resort is willing to lend at zero net interest rates between periods 1 and 2, the run equilibrium is eliminated. To show this, suppose that a fraction $\widehat{\pi} > \pi$ of the population behaves as impatient agents in period 1. If this is so, the total period - 1 supply of non - tradeables is equal to $A\widehat{\pi}m_0^{CB}$. The Central Exchange borrows M_1 dollars to be used to purchase the total period - 1 supply of non - tradeables at a price $\widehat{p}_1 = \frac{M_1}{A\widehat{\pi}m_0^{CB}}$. Those dollars are returned by the active entrepreneurs to the commercial banks. When facing a proportion $\widehat{\pi}$ of the population withdrawing, the commercial bank asks for a one-period, zero net interest rate loan with size equal to $(\widehat{\pi} - \pi)\, c_{1T}^*$. Hence, the total amount of dollars to be paid to shoppers withdrawing at date 1 is equal to $M_1 + y + (\widehat{\pi} - \pi)\, c_{1T}^* = \widehat{p}_1 A\widehat{\pi}m_0^{CB} + y + (\widehat{\pi} - \pi)\, c_{1T}^*$. This must be equal to $\widehat{\pi}w_1 = \widehat{\pi}\left[p_1^* A\pi m_0^{CB} + y\right]$, since every shopper showing up at date 1 has the right to withdraw w_1 dollars. Then each shopper purchases $c_{1N}^* = Am_0^{CB}$ units of N at the price $\widehat{p}_1$. The remaining amount of dollars are used to purchase c_{1T}^* units of T goods. This is feasible because $\widehat{\pi}\left(w_1 - \widehat{p}_1 c_{1N}^*\right) = y + (\widehat{\pi} - \pi)\, c_{1T}^* = \widehat{\pi}c_{1T}^*$. In period 2, banks have in net terms an amount of $Rx - d - (\widehat{\pi} - \pi)\, c_{1T}^*$ of dollars to be paid to the remaining fraction $(1 - \widehat{\pi})$ of the population (all of them of the patient type). Actions take place as before. The amount w_2

is spent in purchasing c_{2N}^* units of good N at the price $\widehat{p}_2 = \frac{M_2}{(1-\widehat{\pi}) A^2 m_0^{CB}}$. The remaining amount of dollars is equal to $\frac{Rx-d-(\widehat{\pi}-\pi)c_{1T}^*}{1-\widehat{\pi}}$. Since $Rx - d = (1-\pi)\,c_{2T}^*$, $(\widehat{\pi}-\pi) > 0$ and $c_{2T}^* > c_{1T}^*$ then $(\widehat{\pi}-\pi)\,c_{2T}^* > (\widehat{\pi}-\pi)\,c_{1T}^*$, and so $(1-\pi)\,c_{2T}^* - (1-\widehat{\pi})\,c_{2T}^* > (\widehat{\pi}-\pi)\,c_{1T}^*$, therefore $(1-\pi)\,c_{2T}^* - (\widehat{\pi}-\pi)\,c_{1T}^* > (1-\widehat{\pi})\,c_{2T}^*$. Thus, $\frac{Rx-d-(\widehat{\pi}-\pi)c_{1T}^*}{1-\widehat{\pi}} > c_{2T}^*$. Each patient shopper and household who waits until date 2 gets a higher consumption in tradeables than what the optimal allocation implies. Hence there is no incentive for any patient household to misbehave. This shows that an international lender willing to provide funds at zero net interest rate between periods 1 and 2 eliminates the run equilibrium.

10.6 Proof of proposition 10

The problem is

$$\max \quad \pi\left[u\left(c_{1T}\right)+v\left(c_{1N}\right)\right]+(1-\pi)\left[u\left(c_{2N}\right)+v\left(c_{2N}\right)\right]$$

subject to

$$x+y+z \Box d$$

$$\begin{aligned}
\pi w_1 &= y \\
p_1 c_{1N} &\Box w_1 \\
\pi c_{1T} &\Box p_1 A\left(z-z_2\right)+\pi\left(w_1-p_1 c_{1N}\right)
\end{aligned}$$

$$\begin{aligned}
(1-\pi)\, w_2 &= Rx-d \\
p_2 c_{2N} &= w_2 \\
(1-\pi)\, c_{2T} &= p_2 A z_2+(1-\pi)\left(w_2-p_2 c_{2N}\right)
\end{aligned}$$

The Lagrangian then is the following expression,

$$\begin{aligned}
L \;=\; & \pi\left[u\left(c_{1T}\right)+v\left(c_{1N}\right)\right]+(1-\pi)\left[u\left(c_{2N}\right)+v\left(c_{2N}\right)\right]+\phi_0\left\{d-x-y-z\right\} \\
& +\phi_1\left[y-\pi w_1\right]+\eta_1\left[w_1-p_1 c_{1N}\right]+\psi_1\left[p_1 A\left(z-z_2\right)+\pi\left[w_1-p_1 c_{1N}\right]-\pi c_{1T}\right] \\
& +\phi_2\left[Rx-d-(1-\pi)\, w_2\right]+\eta_2\left[w_2-p_2 c_{2N}\right] \\
& +\psi_2\left[p_2 A^2 z_2+(1-\pi)\left[w_2-p_2 c_{2N}\right]-(1-\pi)\, c_{2T}\right]
\end{aligned}$$

and the first order conditions (necessary and sufficient conditions due to our assumptions) are:

$$\begin{aligned}
L_{tT} &= \rho_t u'(c_{tT}) - \rho_t \psi_t = 0 \\
L_{tN} &= \rho_t v'(c_{tN}) - (\eta_t + \rho_t \psi_t) p_t = 0 \\
L_x &= -\phi_0 + R\phi_2 = 0 \\
L_y &= -\phi_0 + \phi_1 = 0 \\
L_z &= -\phi_0 + \psi_1 p_1 A = 0 \\
L_{z_2} &= -\psi_1 p_1 A + \psi_2 A^2 p_2 = 0 \\
L_{w_1} &= -\pi\phi_1 + \eta_1 + \psi_1 \pi = 0 \\
L_{w_2} &= -(1-\pi)\phi_2 + \eta_2 + (1-\pi)\psi_2 = 0
\end{aligned}$$

Then we need to have

$$\phi_1 = R\phi_2 = \psi_1 p_1 A$$

$$\begin{aligned}
u'(c_{tT}) &= \psi_t \\
v'(c_{tN}) &= \phi_t p_t
\end{aligned}$$

so

$$\phi_t = \frac{v'(c_{tN})}{p_t}$$

therefore

$$\frac{v'(c_{1N})}{p_1} = R\frac{v'(c_{2N})}{p_2} \tag{9}$$

and also

$$v'(c_{1N}) = u'(c_{1T}) p_1^2 A \tag{10}$$

$$u'(c_{1T}) p_1 = A p_2 u'(c_{2T}) \tag{11}$$

which shows the first part of the proposition.

From the constraints (assuming that all of them hold with equality) we get

$$\pi p_1 c_{1N} = d - x - z$$

$$\pi c_{1T} = p_1 A (z - z_2)$$

$$\begin{aligned} (1-\pi) p_2 c_{2N} &= Rx - d \\ (1-\pi) c_{2T} &= p_2 A^2 z_2 \end{aligned}$$

If we first include the optimal consumption allocation $\left\{ (c_{tl}^*)_{t=1}^2 \right\}_{l=T}^N$ on the left hand side in each equation, then we could solve for (x, z, z_2), but we have four equations! Then it is possible to solve this if some of the prices are also considered as endogenous (in which case, the system becomes non-linear).

Suppose we get $(\bar{x}, \bar{y}, \bar{z}, \bar{p}_1)$ as a function of the parameters and $\left\{ (c_{tl}^*)_{t=1}^2 \right\}_{l=T}^N$. On the other hand, if the optimal allocation has to decentralized here then they must satisfy the three FOC before (equations (9), (10) and (11), where the only unknown would be p_2. By Theorem 11.3 in Magill and Quinzii (1996, section 11) the set of parameters in which there exists a p_2 solving this has a Lebesgue measure zero. Therefore, in a generic sense, it is not possible to implement the optimal allocation.

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