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"A Model of TFP""

RICARDO LAGOS
New York University

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Ricardo Lagos*
New York University

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Abstract

This paper proposes an aggregative model of Total Factor Productivity (TFP) in the spirit of Houthakker (1955-1956). It first considers a simple general equilibrium competitive model where labor is a fully flexible and mobile factor while capital is relatively fixed and firms' productivity is subject to idiosyncratic shocks. Within this context, an aggregate production function is derived by aggregating across production units in equilibrium. In this simple model individual decisions affect aggregate inputs but the level of TFP is a constant that depends on the underlying assumed heterogeneity. The model is then extended to allow for a frictional labor market where jobs are created and destroyed as in Mortensen and Pissarides (1994). Within this context, the level of TFP is shown to depend on all the characteristics of the labor market as summarized by the job-destruction decision.

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1 Introduction

Hall and Jones (1999) and Parente and Prescott (2000) have established that differences in Total Factor Productivity (TFP) account for a large fraction of the variation in output per worker across countries. Hall and Jones (1999) use data on output, labor input, average educational attainment and physical capital to decompose the differences in output per worker into differences in capital intensity, human capital per worker and TFP. Their levels accounting exercise implies that differences in physical capital and educational attainment explain only a small amount of the differences in output per worker.[1] Parente and Prescott (2000) show that the standard growth model without TFP differences is not consistent with the observed income differences even when augmented to include a human capital sector. [2] So in order to understand income differences across countries, one first needs to understand what determines the level of TFP.

Hall and Jones (1999) conjecture that differences in observed TFP are driven by differences in the institutions and government policies they collec-

[1] As a way of illustration, consider the following fact reported by Hall and Jones (1999). In 1988, output per worker in the five richest countries was on average 31.7 times that of the five poorest. Differences in capital intensities and educational attainments contributed factors of 1.8 and 2.2, respectively, to this difference. The remaining difference, a factor of 8.3 was accounted for by the TFP differential. Without this productivity difference, the average output per worker of the five richest countries would have only been about four times that of the five poorest.

[2] Specifically, they find that reasonable differences in saving rates cannot account for observed differences in steady-state income levels; and that the small diminishing returns to individuals investing in human capital that are needed to fit the empirical income differences imply that the time allocated to schooling is implausibly high. They also show that the factor difference in TFP needed to account for the income differences between the world's richest and poorest countries is between 2 and 3, not unreasonably high.

tively refer to as "social infrastructure". Corrupt government officials, severe impediments to trade, poor contract enforcement and government interference in production are their examples of bad "social infrastructures" leading to low levels of TFP.[3]

Parente and Prescott (1994) propose that some countries have lower TFP than others because their process of technology adoption at the micro level is constrained by "barriers to riches". These barriers are essentially any institution or government policy that increases the cost of technology adoption. Parente and Prescott (1999) show that monopoly rights can be a "barrier to riches".

In this paper I focus on the theory underlying the aggregate production function and show how labor-market policies affect this function in general, and the level of measured TFP in particular. Specifically, I construct an aggregative model of TFP in the spirit of Houthakker (1955-1956): the basic idea is to derive an aggregate production function by aggregating across active production units. In equilibrium, the levels of output, inputs and TFP as well as the shape of the aggregate relationship between them depend on individual production decisions –such as which production units remain active in the face of idiosyncratic shocks– and these decisions are in turn

[3] Examples aside, the institutions and policies that Hall and Jones (1999) refer to as "social infrastructure" are defined by the two variables they use to proxy it in their regressions. The first is a measure of openness to trade; and the second, an index of government "anti-diversion" policies measuring (i) law and order, bureaucratic quality, (iii) corruption, (iv) risk of expropriation, and (v) government repudiation of contracts. In their empirical investigation, Acemoglu, Johnson and Robinson (2001) also highlight the role of institutions in determining different income levels across countries. Their focus is on the role property rights and checks against government power, and their definition of "institutions" is a risk-of-expropriation index.

affected by policies. So the model can be used to study the precise interaction between all these variables explicitly.

In the model proposed here, policy affects TFP because the latter is related to the average productivity of the units which are active, and policy induces changes in the productivity composition of active units. By distorting the way in which individual production units react to the economic environment, labor-market policies can make an economy exhibit a low level of TFP. As a result, an economy may have a higher level of aggregate measured TFP than another even when production units in both operate the same technologies. In this sense the determinants of TFP levels I focus on here are different from the barriers to technology adoption of Parente and Prescott (1999, 2000).[4]

At a theoretical level, the paper also shows that under some conditions, a standard search model of the labor market –with its underlying meeting frictions and simple fixed-proportions micro-level production technologies– can generate an aggregate production function just like the one implied by

[4]Although they focus on monopoly rights in their formal modelling, Parente and Prescott (2000) mention a few labor-market policies as examples of "barriers to riches":

"In India, for example, firms with more than 100 workers must obtain the government's permission to terminate any worker, and firms of all sizes are subject to state certification of changes in the tasks associated with a job." (pp. 107-108). "Another way the state protects the monopoly rights is by requiring large severance payments to laid-off workers." "Also in India, regulations require certain firms to award workers with lifetime employment and require firms with more that twenty-five workers to use official labor exchanges to fill any vacancy." (p. 108). "In Bangladesh, for example, private buyers of the state-owned jute mills were prohibited for one year from laying off any of the workforce they inherited. After one year, a worker could be laid off but not without a large severance payment."

Parente and Prescott (2000) use these as instances of policies that can lower TFP by making technology adoption costly. But as we show below, these policies can also have a direct impact on level of TFP.

the textbook neoclassical model of growth in which firms have access to a standard constant-returns Cobb-Douglas production technology. So in this sense, from the perspective of aggregate output, inputs and productivity, the neoclassical and the search paradigms can seem quite close.

The rest of the paper is organized as follows. Section 2 considers a simple general equilibrium competitive model where labor is a fully flexible and mobile factor while capital is relatively fixed and firms' productivity is subject to idiosyncratic shocks. Within this context, an aggregate production function is derived by aggregating across production units in equilibrium. In this simple model individual decisions affect aggregate inputs but the level of TFP is a constant that depends on the underlying assumed heterogeneity. In Section 3 the model is extended to allow for a frictional labor market where jobs are created and destroyed as in Mortensen and Pissarides (1994). The equilibrium is characterized in Section 3.1, and the classical aggregation result of Houthakker (1955-1956) is extended to the dynamic general equilibrium search setup in Section 3.3. When aggregate inputs are correctly measured, the level of TFP is shown to depend on all the characteristics of the labor market as summarized by the job-destruction decision. Section 4 introduces four policies: employment and hiring subsidies, firing taxes and unemployment benefits, and studies their effects on TFP. Section 5 extends the basic model to the more realistic case of serially-correlated shocks and elaborates on how the observed level of TFP is affected by the different ways of measuring aggregate inputs that can be found in the literature. Section 6 concludes. All propositions are proved in the Appendix.

2 The Competitive Model

Time is discrete and continues forever. The economy is populated by a continuum of identical and infinitely-lived individuals of size L. There is also a large number of firms determined by free entry.

2.1 Preferences and Technology

Individual preferences are represented by

$$U = \sum_{t=0}^{\infty} \beta^t u\left(C_t\right)$$

where $\beta \in (0,1)$ is the discount factor and C_t is consumption in period t. The instantaneous utility function u is strictly concave, twice continuously differentiable and satisfies Inada conditions. For now I assume no utility from leisure and normalize the time endowment to 1.

Firms have access to a production technology that combines capital and labor to produce the only consumption good in the economy:

$$f\left(z, n_i, k_i\right) = \min\left(z n_i, k_i\right)$$

where n_i and k_i denote the levels of labor and capital inputs chosen by firm i. Output is linear in labor but is bounded above by the stock of capital the firm is operating with. Labor productivity is stochastic at the firm level and indexed by a random variable z with cumulative density $G(z)$. This idiosyncratic productivity shock is assumed to be *iid* across firms. The timing convention is that firm i has to choose k_{it+1} –its "scale of operation" for period $t+1$– at the end of period t, before observing the realization of the idiosyncratic shock. This timing captures the idea that labor is a fully flexible factor while capital is relatively fixed in the short run (in this case within the period). But more important in what follows, is that this timing makes the notion of a job well defined. A standard neoclassical firm effectively has an unlimited number of jobs; in contrast this formulation formalizes the idea that in any period a firm only has a fixed number of jobs to fill.

2.2 Competitive Equilibrium

Individuals choose a sequence of consumption in order to maximize U subject to

$$K_{t+1} = (r_t + 1 - \delta) K_t + w_t - C_t, \tag{1}$$

where K_{t+1} is the amount of capital each chooses to save at time t and $\delta \in (0,1)$ is the discount factor. The rental and wage rates are denoted r_t and w_t respectively.

Every period firms must decide how much labor to hire given the current realization of the idiosyncratic shock and the capital it put in place at the end of the previous period. In other words, each solves[5]

$$\max_n f(z, n, k) - wn - rk.$$

The demand for capital was specified at the end of the previous period and its rental must be paid regardless of the firm's production decision. Then

[5]Notice that we are regarding k as independent of z. As we show below, this is indeed the case when shocks are *iid* through time. In a later section we explore the case where shocks are serially correlated.

the firm's demand for labor is

$$n(z) = \begin{cases} 0 & \text{if } z < w \\ \frac{k}{z} & \text{if } w \leq z, \end{cases} \tag{2}$$

output is $zn(z)$ and profits are

$$\pi(z) = \begin{cases} -rk & \text{if } z < w \\ \left(1 - \frac{w}{z} - r\right)k & \text{if } w \leq z. \end{cases}$$

If the shock renders the marginal product of labor smaller than the wage, the firm hires no workers and loses the rental on capital. On the other hand, if the marginal product exceeds the wage, then the firm will hire workers to produce at capacity. Profits are positive iff $z > w/(1-r)$.

At the end of every period the firm chooses the capital stock it will employ in production in the following period. I begin by analyzing the case in which shocks are *iid* through time. The amount of capital is chosen before observing the realization of next period's shock, so at the end of period t the firm solves

$$\max_{k_{t+1}} \left[\int_{w_{t+1}} \left(1 - z^{-1} w_{t+1}\right) dG(z) - r_{t+1} \right] k_{t+1}. \tag{3}$$

The *iid* assumption keeps the firm's problem a series of static, one-period maximizations as in the standard neoclassical setup.

A competitive equilibrium is a sequence of allocations $\{C_t, K_{t+1}\}_{t=0}^{\infty}$ together with a sequence of prices $\{w_t, r_t\}_{t=0}^{\infty}$ such that the former solves the individual's maximization problem given the latter, firms maximize expected profit and markets clear. Given an initial capital stock K_0, the solution to the agent's maximization problem is characterized by the Euler equation

$$u'(C_t) = \beta(r_{t+1} + 1 - \delta) u'(C_{t+1}),$$

the budget constraint (1) and the transversality condition $\lim_{t=\infty}\beta^t u'(C_t) K_{t+1} = 0$. As usual, r_t and w_t are obtained by imposing market clearing in the factor markets. From (3):

$$1 - G(w) - w\int_w z^{-1} dG(z) - r = 0 \tag{4}$$

must be the case in equilibrium.[6] Hence any feasible k_i solves firm i's capacity problem: as for the standard neoclassical firm, individual firm-size is indeterminate in equilibrium. Market clearing requires that the sum of the k_i's across firms equals the aggregate capital stock K. Condition (4) ensures that a firm's expected profit is zero and can be seen to deliver the equilibrium interest rate r given the wage rate w.

I now characterize the equilibrium wage rate. The individual firm's inverse labor demand in (2) can be used to derive the aggregate (inverse) demand for labor:

$$N = K\int_w z^{-1} dG(z). \tag{5}$$

Similarly, aggregate output is

$$Y(K, w) = [1 - G(w)]K. \tag{6}$$

Following Houthakker (1955-1956), one could imagine solving (5) for the aggregate labor demand $w(K, N)$ and then plugging it in (6) to obtain

[6] The firm's problem when choosing capacity has a solution iff

$$1 - G(w) - w\int_w z^{-1} dG(z) - r \leq 0.$$

If the condtion held with strict inequality then each firm i would choose $k_i = 0$ and the capital market wouldn't clear.

$Y[K, w(K, N)]$. Hereafter I impose $N = L$ (the labor market clears) and will use $F(K, L)$ to denote $Y[K, w(K, L)]$ in order to simplify notation and to stress the fact that it is the economy's aggregate production function. From (5) one sees that $-w_2(K, L) g[w(K, L)] K = w(K, L)$ and from (6) that $F_2(K, L) = -w_2(K, L) g[w(K, L)] K$. Hence $F_2(K, L) = w(K, L)$.[7] In equilibrium labor is paid its marginal product in the aggregate production function. Taking another look at (4), one sees that using (5) and (6) it can be rewritten as $rK = F(K, L) - wL$. So if the aggregate production function exhibits constant returns to scale (CRS), then $r = F_1(K, L)$ and capital is also paid its (aggregate) marginal productivity. And indeed, the aggregate production function is CRS. To see this notice from (5) that $w(K, L)$ is homogeneous of degree zero and hence (6) indicates that for any $\xi \geq 0$, we have $F(\xi K, \xi L) = \xi F(K, L)$.

To conclude this section I show that the aggregate technology is consistent a balanced growth path, namely a growth path along which all endogenous variables grow at constant (albeit possibly different) rates. For this purpose, assume the firm-level technology is $f(z, x_t n_i, k_i)$, where $x_{t+1}/x_t = \gamma_x$. That is, x_t represents labor-augmenting (exogenous) technical progress, and it is aggregate in the sense that it affects all firms symmetrically. The aggregate production function corresponding to this micro structure is $F(K, x_t L)$. To see this notice that the firm-level (inverse) labor demand is

$$n(z) = \begin{cases} 0 & \text{if } x_t z < w \\ \frac{k}{x_t z} & \text{if } w \leq x_t z \end{cases}$$

[7] I owe this argument to Erzo G. J. Luttmer.

and firm-level output is $x_t z n(z)$. Hence aggregate labor demand and output are

$$N = (K/x_t) \int_{w/x_t} z^{-1} dG(z), \text{ and } Y(K, w/x_t) = K \int_{w/x_t} dG(z).$$

The former defines $w/x_t = w(K, x_t N)$ and plugging this into the latter yields $Y[K, w(K, x_t N)]$, which is $F(K, x_t N)$. So with labor-augmenting technical change at the micro level, the model will be consistent with balanced growth provided the instantaneous utility function u is.[8]

2.3 An Example

Now suppose the idiosyncratic shock follows a Pareto distribution; that is

$$g(z) = \begin{cases} 0 & \text{if } z < \varepsilon \\ \frac{\alpha\varepsilon^\alpha}{z^{1+\alpha}} & \text{if } \varepsilon \leq z. \end{cases}$$

For this case (5) and (6) indicate that aggregate inverse demand and aggregate output specialize to

$$\begin{aligned} N &= \frac{\alpha}{1+\alpha}\varepsilon^\alpha w^{-1-\alpha} K, \text{ and} \\ Y(K, w) &= \varepsilon^\alpha w^{-\alpha} K \end{aligned}$$

respectively. Inverting the former to get the aggregate labor demand

$$w(K, N) = \left(\frac{\alpha}{1+\alpha}\varepsilon^\alpha \frac{K}{N}\right)^{\frac{1}{1+\alpha}}$$

[8]With the firm-level technology adopted, the aggregation process in the spirit of Houthakker preserves the type of technical change prevailing at the micro level. Above we showed that labor-augmenting technical change at the micro level implies labor-augmenting technical change at the macro level. Similarly, it can be shown that $f(z, n, x_t k)$ and $x_t f(z, n, k)$ aggregate to $F(x_t K, L)$ and $x_t F(K, L)$ respectively.

and substituting this in the latter we arrive at

$$F(K,N) = AK^{\sigma}N^{1-\sigma}$$

where $A \equiv \left(\frac{1+\alpha}{\alpha}\varepsilon\right)^{\frac{\alpha}{1+\alpha}}$ and $\sigma \equiv \frac{1}{1+\alpha}$. This is a modified version of the classic aggregation result of Houthakker (1955-1956).[9] Notice that A is what economists normally think of as TFP. In this case it only depends on the parameters of the distribution of productivity shocks. Below I extend the model by incorporating a frictional labor market and show how labor-market institutions affect the level of TFP.

3 Model with a Frictional Labor Market

I model the labor market as in Mortensen and Pissarides (1994). Time is continuous and the horizon infinite. There is a continuum of infinitely lived agents of two types: workers and firms. The size of the labor force is normalized to unity while the number of firms is determined endogenously by free entry. Both types are risk-neutral. Workers derive utility only from consumption. Each firm has a single job that can either be filled or vacant and searching. Similarly, workers can either be employed by a firm or unemployed and searching. No new offers arrive while an agent is in a relationship

[9] Houthakker performs the aggregation over production units that employ two variable factors and face capacity constraints due to a fixed (unmodelled) factor. Here we have assumed each production unit employs a single variable factor (labor) as well as capital. Capital is chosen a period ahead and hence plays the role of the fixed factor constraining output at the time employment and production decisions are made. This formulation delivers an aggregate production function with constant returns to scale. In contrast, the setup used by Houthakker generates a function of the variable inputs only and hence it exhibits diminishing returns to scale.

(i.e. there is no on-the-job search). I abstract from capital accumulation and assume labor-market participants take aggregate stock of capital, K, as given.[10]

Assume meeting frictions can be represented by a function $m(u, v)$ that determines the instantaneous number of meetings as a function of the numbers of searchers on each side of the market; namely unemployed workers u and vacancies v. Suppose m exhibits constant returns to scale and is increasing in both arguments. Let $q(\theta)$ denote the (Poisson)-rate with which a vacancy contacts an unemployed worker, where $\theta = v/u$.[11]

Each firm has access to a technology $f(x, n, k)$ that combines hours n and capital k to produce a homogeneous consumption good. The match-specific level of technology is indexed by x. I assume that

$$f(x, n, k) = x \min(n, k) \tag{7}$$

and interpret k as the firm's "capacity". So output is linear in hours but is bounded above by the stock of capital the firm is operating with. The convention is that firm i has to choose and put in place k_i –its "scale of operation"– in order to engage in search and that this choice is irreversible.[12]

[10] I abstract from saving and accumulation because the focus here is on isolating the effects of labor-market policies on the level of TFP. But even when trying to explain income differences, Parente and Prescott (2000) forcefully argue that one cannot rely on policies that cause differences in saving rates, as they do not vary systematically with countries' incomes.

[11] Note that $q(\theta) = m(1/\theta, 1)$ and hence $q' < 0$. The probability a worker contacts a vacancy in a small time interval is $\theta q(\theta)$ and is increasing in θ. See Lagos (2000) for an environment in which a constant-returns matching function is explicitly derived from first principles.

[12] The idea is that in order to search, the firm must have borrowed some capital (e.g.

This captures the idea that hours are a fully flexible factor while capital is relatively fixed. Firms rent capital from a competitive market at flow cost c.

Match productivity is stochastic and indexed by the random variable x. For an active match, the process that changes the productivity is Poisson with finite arrival rate λ. When a match of productivity x suffers a change, the new value x' is a draw from the fixed distribution $G(z)$. So the productivity process is persistent (since $\lambda < \infty$) but –conditional on change– it is independent of the firm's previous state.[13] The Poisson process and the productivity draws are *iid* across firms and there is no aggregate uncertainty. The focus will be on steady state outcomes.

Below I will show that there is a productivity level R such that active matches dissolve if productivity ever falls below R and new matches form only if their initial productivity is at least R.[14] Let $H_t(x)$ denote the cross-sectional distribution of productivities among active matches. That is, $H_t(x)$ is the fraction of matches producing at productivities x or lower at time t. The time path of $(1-u_t)\,H_t(x)$, namely of the number of matches producing

to set up a plant). The firm is initially free to pick any size of plant k_i, but this choice is irreversible in the sense that once put in place, k_i cannot be changed. In a similar vein, technologies are assumed fixed and irreversible in Gilchrist and Williams (2000) and in Mortensen and Pissarides (1994).

[13] This is the process used by Mortensen and Pissarides (1994). For reasons that will be clear below, we later generalize the model by specifying that when a match of productivity x suffers a change, the new value x' is a draw from the fixed distribution $G(z|x)$. If $G(z|x_1) < G(z|x_0)$ when $x_0 < x_1$, then apart from persistent, idiosyncratic shocks are also positively correlated through time.

[14] Mortensen and Pissarides (1994) work with a bounded support and assume new matches start off with the highest productivity. We relax these assumptions and treat active and new matches symmetrically. In the model we consider, the initial productivity of a match is a non-degenerate random variable drawn from the same distribution as the innovations to active matches.

at productivities x or lower at time t is given by[15]

$$
\begin{aligned}
\frac{d}{dt}\left[(1-u_t)\, H_t(x)\right] &= \lambda (1-u_t)\left[1-H_t(x)\right]\left[G(x)-G(R_t)\right] \\
&\quad +\theta q(\theta)\, u_t \left[G(x)-G(R_t)\right] \\
&\quad -\lambda (1-u_t)\, H_t(x)\, G(R_t) \\
&\quad -\delta (1-u_t)\, H_t(x) \\
&\quad -\lambda (1-u_t)\, H_t(x)\left[1-G(x)\right].
\end{aligned}
$$

The first term accounts for the matches with productivities above x that get innovations below x. The newly-formed matches that start off with productivities no larger than x are in the second term. The third term is the number of matches in the interval $[R_t, x]$ that get shocks below R_t and are destroyed. Let δ denote the parameter of an independent Poisson process that causes separations for unmodelled reasons. Then the fourth term accounts for matches in the interval $[R_t, x]$ that are destroyed for exogenous reasons. The last term accounts for the number of matches in the same interval that "move up" by virtue of having drawn productivities larger than x. Imposing steady states we arrive at:

$$
H(x) = \left[\frac{\lambda}{\delta+\lambda} + \frac{\theta q(\theta)\, u}{(\delta+\lambda)(1-u)}\right]\left[G(x)-G(R)\right].
$$

In addition, the steady-state unemployment rate is

$$
u = \frac{\delta + \lambda G(R)}{\delta + \lambda G(R) + \theta q(\theta)\left[1-G(R)\right]}. \tag{8}
$$

[15]The fact that active matches will form and continue only for productivities at least as large as R_t means that $H_t(R_t) = 0$. So in the derivation below we only focus on $x \geq R_t$.

Using this expression, the steady-state cross-sectional productivity distribution can be rewritten as

$$H(x) = \frac{G(x) - G(R)}{1 - G(R)}. \tag{9}$$

Firms can be either vacant and searching or filled. The problem of a searching firm is summarized by

$$rV = \max_k \left[-ck + q(\theta) \int \max\left[J(z) - V, 0\right] dG(z) \right], \tag{10}$$

where V is the asset value of a vacancy, $J(x)$ is the asset value of a filled job and r is the discount factor. I assume there is entry of firms until all rents are exhausted, so $rV = 0$ in equilibrium. Letting $\pi(x)$ denote flow profit,

$$rJ(x) = \pi(x) + \lambda \int \max\left[J(z), V\right] dG(z) - \lambda J(x) - \delta\left[J(x) - V\right], \tag{11}$$

where

$$\pi(x) = \max_n \left[x \min(n, k) - \phi n - ck - C(x, \phi) k - w(x)\right].$$

Instantaneous profit is the residual output after the wage $w(x)$ and all other costs of production have been paid out. There are three such costs in this formulation: a fixed one, ck, which is the cost of capital; a variable cost, ϕn, that can be managed by varying hours; and a "maintenance cost" $C(x, \phi)$ per unit of capital, which is independent of n. Our assumptions[16] imply that w is independent of n, so the profit-maximizing choice of hours is

$$n(x) = \begin{cases} k & \text{if } \phi < x \\ 0 & \text{if } x \leq \phi \end{cases} \tag{12}$$

[16] In particular the fact that workers derive no utility from leisure.

and hence flow profit is $\pi(x) = [\max(x-\phi,0) - c - C(x,\phi)]\,k - w(x)$.

One can think of ϕ as the cost of electricity, for instance, with electricity usage being proportional to hours worked. This variable cost is introduced to allow for the possibility of "labor hoarding". Specifically, for some parametrizations, it is possible that at low productivity realizations the firm may keep the worker employed despite requiring that she supplies zero hours. Below I show that this extreme variety of labor hoarding has interesting aggregate implications when it occurs in equilibrium. The maintenance cost is introduced in this section as a simple device to avoid a "flat spot" in flow profit which would otherwise carry over to the value functions. Since it is perhaps the only non-standard element of the model, I redo the whole analysis without this device in a later section. So for now, I use a convenient specification for the maintenance cost, namely $C(x,\phi) = \max(\phi - x, 0)$.[17] With this specification, instantaneous profit is just $\pi(x) = (x - \phi - c)\,k - w(x)$ for any x.

The value of employment and unemployment to a worker are denoted

[17] One way to interpret this formulation is that machines require no maintainance if they are being operated by workers; so if $x \geq \phi$, the maintainance cost is zero and flow profit is just $(x - \phi - c)\,k - w(x)$. But when they stand idle, machines need to be run, even without a worker, in order to keep them operational. The time they need to be run depends on productivity. If $x = 0$, say, then the machine needs to be run at the cost of a full shift, ϕ, but if $x > 0$, then each machine needs to be run for less time, at cost $\phi - x$ per machine. So for $x < \phi$, output is zero, and flow profit is $-[(\phi - x + c)\,k + w(x)]$; the firm loses the maintainance cost, the cost of capital, and the wage payment to labor.

$W(x)$ and U respectively and solve

$$rU = b + \theta q(\theta) \int \max[W(z) - U, 0]\, dG(z) \tag{13}$$

$$rW(x) = w(x) + \lambda \int \max[W(z) - U, 0]\, dG(z) - (\delta + \lambda)[W(x) - U], \tag{14}$$

where $b \geq 0$ is a worker's flow income while unemployed.[18]

3.1 Equilibrium

I follow the bulk of the unemployment search literature by letting $\beta \in [0,1)$ and assuming the instantaneous wage $w(x)$ continuously solves

$$\max_{w(x)} [W(x) - U]^{\beta} [J(x) - V]^{1-\beta}$$

and therefore it satisfies

$$(1-\beta)[W(x) - U] = \beta J(x) \tag{15}$$

at all times. Letting $S(x) = J(x) + W(x) - U$ denote the surplus from a match, notice that (15) implies $J(x) = (1-\beta) S(x)$ and $W(x) - U = \beta S(x)$. These together with (11), (13), and (14) imply

$$(r + \delta + \lambda) S(x) = (x - \phi - c) k - rU + \lambda \int \max[S(z), 0]\, dG(z),$$

where

$$rU = b + \frac{\beta}{1-\beta} kc\theta. \tag{16}$$

[18] Below we will model b as a fraction of the average market wage and interpret it as unemployment insurance income.

Since $S'(x) = \frac{k}{r+\delta+\lambda} > 0$, there exists a unique R such that $S(R) > 0$ iff $x > R$. Hence matches separate whenever productivity falls below R.[19] Using this reservation strategy the surplus can be written as

$$(r+\delta+\lambda) S(x) = (x-\phi-c) k - rU + \lambda \int_R S(z) dG(z). \qquad (17)$$

For completeness, (15) and the value functions can be manipulated to obtain expressions for instantaneous wages and profit:

$$w(x) = \beta(x-\phi-c) k + (1-\beta) rU \qquad (18)$$

$$\pi(x) = (1-\beta)[(x-\phi-c) k - rU]. \qquad (19)$$

Intuitively, the wage is a weighted average of output (net of the rental on capital and the variable and maintenance costs) and the worker's reservation wage.

I now do some analysis to characterize the job-creation and destruction decisions as summarized by θ and R respectively. Evaluating (17) at $x = R$ we see that

$$\lambda \int_R S(z) dG(z) = rU - (R-\phi-c) k.$$

Notice that since the expected capital gain on the left-hand-side is positive, at $x = R$ net output is smaller than the worker's reservation wage. From (18) and (19) we see that this implies that $w(R) < rU$ and $\pi(R) < 0$: workers and firms sometimes tolerate instantaneous payoffs below those they could

[19] Notice that separations are privately efficient. Moreover, they are also consensual in the sense that by (15), $J(x) > 0$ iff $W(x) - U > 0$; so the firm wants to destroy the match iff the worker wants to quit.

get by separating, in anticipation of a future productivity improvement.[20] Substituting this simpler expression for the expected capital gain term into (17) gives

$$S(x) = \frac{x - R}{r + \delta + \lambda} k. \tag{20}$$

Evaluating (17) at $x = R$ and using (20) to substitute $S(\cdot)$ we arrive at the job-destruction condition:

$$R - \phi - c - \left(\frac{b}{k} + \frac{\beta}{1 - \beta} c\theta \right) + \frac{\lambda}{r + \delta + \lambda} \int_R (x - R)\, dG(x) = 0. \tag{21}$$

Notice that as is standard, the destruction decision is independent of scale if b is. The natural interpretation of b is that it is unemployment insurance income. Along these lines, if we assume $b = \rho E_x [w(x) | x \geq R]$, where $\rho \in [0, 1)$ is the replacement rate, then $b = \hat{b}k$, with

$$\hat{b} = \frac{\rho\beta [\tilde{x}(R) - \phi - c + c\theta]}{1 - (1 - \beta)\rho}$$

and $\tilde{x}(R) \equiv E[x | x \geq R] = [1 - G(R)]^{-1} \int_R x dG(x)$. Under this specification, b is linear in k and hence (21) is independent of k. For future reference, in this case (21) becomes:

$$R - \frac{\rho\beta\tilde{x}(R)}{1-(1-\beta)\rho} - \frac{(1-\rho)(\phi+c)}{1-(1-\beta)\rho} - \frac{\beta c\theta}{(1-\beta)[1-(1-\beta)\rho]} + \frac{\lambda}{r+\delta+\lambda} \int_R (x - R)\, dG(x) = 0.$$

In what follows I will always abstract from scale effects caused by unemployment income b by assuming it is a fraction of the average going wage. At times I may even resort to the especially tractable case with $\rho = b = 0$.

[20]This feature of the model is a consequence of the costly and time-consuming meeting process, as noted by Mortensen and Pissarides (1994).

Equation (10) together with $rV = 0$ imply that at the optimal k, we have

$$(1-\beta)\int_R S(x)\,dG(x) = \frac{ck}{q(\theta)},$$

namely that the expected profit from a filled job equals the expected hiring cost in an equilibrium with free entry. Using (20) to substitute $S(\cdot)$ out of this expression we arrive at the job-creation condition:

$$\int_R (x-R)\,dG(x) = \frac{(r+\delta+\lambda)\,c}{(1-\beta)\,q(\theta)}. \tag{22}$$

The job-creation and destruction conditions jointly determine R and θ, and under our maintained assumptions they are independent of the choice of scale, k.[21] For given c and ϕ, an equilibrium is a vector $[\theta, R, H, U, w, u, k]$ such that (θ, R) jointly solve (21) and (22); and given (θ, R), H satisfies (9); U is given by (16); w by (18); and u by (8). In addition, the market for capital should clear, so k must satisfy $[1-(1-\theta)\,u]\,k = K$, where K is the aggregate supply of capital, which labor-market participants take as given.[22]

[21] For the case with $\rho = 0$, for instance, it is easy to show that there is a unique pair (θ, R) that satisfies (21) and (22). To see this notice that the slopes (in θ-R space) of the job-creation and destruction conditions are

$$\frac{-(r+\delta+\lambda)\,c\eta(\theta)}{(1-\beta)\,\theta q(\theta)\,[1-G(R)]} < 0 \text{ and } \frac{\beta c}{(1-\beta)\left\{1-\frac{\lambda[1-G(R)]}{r+\delta+\lambda}\right\}} > 0$$

respectively, with $\eta(\theta) \equiv \frac{-\theta q'(\theta)}{q(\theta)}$.

[22] Notice that using (20), (10) can be written as

$$rV = \max_k \left[-c + \frac{(1-\beta)\,q(\theta)}{r+\delta+\lambda}\int_R (x-R)\,dG(x)\right]k.$$

Since in equilibrium θ and R are independent of k, the objective is linear and the problem has a solution iff

$$\int_R (x-R)\,dG(x) \leq \frac{(r+\delta+\lambda)\,c}{(1-\beta)\,q(\theta)}.$$

Note that if $R < \phi$, then the capital and workers in matches with realizations in $[R, \phi)$ remain employed but are not engaged in production. The firms in these states have excess capacity and hoard labor. In the following section I provide a sharper characterization of aggregate outcomes for a particular distribution of idiosyncratic shocks.

3.2 Aggregation

Let K_e denote the demand for capital from all firms with filled jobs. Since firms irreversibly choose the same amount of capital k upon entering the market, $K_e = \frac{1-u}{1-(1-\theta)u}K$. In general, aggregate output Y and the aggregate number of hours worked, N, are given by $Y = (1-u)\int_{\mu} f[x, n(x), k]\, dH(x)$ and $N = (1-u)\int_{\mu} n(x)\, dH(x)$ respectively, with $\mu \equiv \max(R, \phi)$. Using (7) and (12), these expressions become

$$N = [1 - H(\mu)]\, K_e \tag{23}$$

$$Y(K_e, \mu) = [1 - H(\mu)]\, K_e E(x|x \geq \mu), \tag{24}$$

where $E(x|x \geq \mu) = [1 - H(\mu)]^{-1} \int_{\mu} x dH(x)$. Intuitively, since every firm-worker pair is setting hours either to zero or to full capacity k, the aggregate number of hours worked is just equal to the fraction of firm-worker pairs who engage in production times the total capital stock in filled jobs. Similarly, aggregate output equals the number of active units of capital, $[1 - H(\mu)]\, K_e$,

But if the inequality is strict, then each firm i will choose $k_i = 0$ and the market is inactive. So a nontrivial equilibrium requires (22) to hold. Then any feasible k_i solves firm i's capacity problem: as for the standard neoclassical firm, individual size is indeterminate in equilibrium.

times their average productivity.[23] Following Houthakker (1955-1956), one could imagine solving (23) for the aggregate "labor demand" by active firms $\mu(K_e, N)$ and then plugging it in (24) to obtain $Y[K_e, \mu(K_e, N)]$. Hereafter, I use $F(K_e, N)$ to denote $Y[K_e, \mu(K_e, N)]$ in order to simplify notation and stress the fact that it is the economy's aggregate production function. Even for an arbitrary H, the aggregate production function is CRS. To see this, notice that $\mu(K_e, N)$ is homogeneous of degree zero and hence (24) indicates that for any $\zeta > 0$, we have $F(\zeta K_e, \zeta N) = \zeta F(K_e, N)$. Also, from (23) one sees that $-\mu_2(K_e, N) K_e dH(\mu) = 1$ and from (24) that $F_2(K_e, N) = -\mu_2(K_e, N) K_e \mu dH(\mu)$. Thus $F_2(K_e, N) = \mu$. So the marginal product of labor in the aggregate production function is equal to the marginal product of the least efficient unit of labor employed in production.[24]

Now suppose idiosyncratic shocks are draws from a Pareto distribution with parameters ε and α, namely

[23] As mentioned previously, Mortensen and Pissarides (1994) assume that G has support $[0, 1]$ and that all new matches start off with productivity 1. So, using our notation but setting $\delta = 0$, aggregate output in their model evolves according to

$$\dot{Y} = k\theta q(\theta) u - \lambda Y + \lambda(1-u) k \int_{\mu}^{1} x dG(x).$$

Replacing $(1-u)k$ with K_e, steady state output is

$$Y = \frac{\theta q(\theta) uk}{\lambda} + [1 - H(\mu)] K_e E(x|x \geq \mu),$$

which is essentially (24) except for the first term. Our assumption that the initial productivity of a new match is a random draw from G –just as the innovations to the productivity of ongoing matches– allows us to consider a density G with unbounded support. In addition, this alternative assumption smoothes aggregate output by getting rid of the "spike" $\theta q(\theta) uk\lambda^{-1}$. This will turn out to be key in the aggregation procedure that follows.

[24] The marginal product of capital in the aggregate production function is $[1 - H(\mu)][E(x|x \geq \mu) - \mu]$.

$$G(x) = \begin{cases} 0 & \text{if } x < \varepsilon \\ 1 - \left(\frac{\varepsilon}{x}\right)^{\alpha} & \text{if } \varepsilon \le x \end{cases} \tag{25}$$

where $\varepsilon > 0$ and $\alpha > 2$. [25] Then, provided $R \geq \varepsilon$, $1 - G(R) = \left(\frac{\varepsilon}{R}\right)^{\alpha}$; and for any $x \geq R$,

$$G(x) - G(R) = \left(\frac{\varepsilon}{R}\right)^{\alpha}\left[1 - \left(\frac{R}{x}\right)^{\alpha}\right].$$

Plugging these expressions in (9) one sees that the steady state productivity distribution of active matches is

$$H(x) = \begin{cases} 0 & \text{if } x < R \\ 1 - \left(\frac{R}{x}\right)^{\alpha} & \text{if } R \le x. \end{cases} \tag{26}$$

This is the *cdf* of a Pareto distribution with parameters R and α. Using (26), $1 - H(\mu) = \left(\frac{R}{\mu}\right)^{\alpha}$ and $E(x|x \geq \mu) = \frac{\alpha}{\alpha-1}\mu$, so (23) and (24) specialize to

$$N = \left(\frac{R}{\mu}\right)^{\alpha} K_e \tag{27}$$

$$Y(K_e, \mu) = \frac{\alpha}{\alpha - 1} R^{\alpha}\mu^{1-\alpha}K_e. \tag{28}$$

Inverting the former to get the aggregate labor demand

$$\mu(K_e, N) = \left(\frac{K_e}{N}\right)^{1/\alpha} R,$$

and substituting it in the latter we arrive at

$$F(K_e, N) = AK_e^{\gamma}N^{1-\gamma} \tag{29}$$

where

$$A = \frac{R}{1-\gamma} \tag{30}$$

[25] This distribution has mean $\bar{x} = \frac{\alpha}{\alpha-1}\varepsilon$ and variance $\sigma^2 = \frac{\bar{x}}{(\alpha-2)(\alpha-1)}$. We assume $\alpha > 2$ so that both are finite.

and $\gamma \equiv 1/\alpha$. This is a modified version of the classic aggregation result of Houthakker (1955-1956).[26] The factor A is what macroeconomists normally refer to as TFP. Its level depends on α, a parameter of the primitive distribution of productivity shocks, as well as on all the characteristics of the labor market as summarized by the destruction decision R. Notice that F expresses output as a function of the aggregate number of hours worked, N, and the total amount of capital hired by firms with filled jobs, K_e. One can also express output as a function of the aggregate capital stock, K, simply by substituting

$$K_e = \frac{1-u}{1-(1-\theta)\,u}K \tag{31}$$

in (29) to get $\hat{F}(K,N) = \hat{A}K^{\gamma}N^{1-\gamma}$, with $\hat{A} \equiv \left[\frac{1-u}{1-(1-\theta)u}\right]^{\gamma} A$.

The aggregate production function is Cobb-Douglas despite fixed proportions in the micro-level technologies. This results when only a fraction of the capital stock included as an argument in the aggregate production function is actually being used in production. To see this, notice that if there is no hoarding in equilibrium (i.e. if $\mu = R$) then $N = K_e$ and $F(K_e, N) = AK_e$.[27]

[26] Houthakker performed the aggregation over production units that employ two variable factors and face capacity constraints due to a fixed (unmodelled) factor. Here we have assumed each production unit employs a single variable factor (labor) as well as capital. Capital is chosen before engaging in search and then remains fixed, hence playing the role of the fixed factor constraining output at the time employment and production decisions are made. This formulation delivers an aggregate production function with constant returns to scale. In contrast, the setup used by Houthakker generates a function of the variable inputs only and hence it exhibits diminshing returns to scale. Another difference is that the shift parameter in Houthakker's production function is solely a function of the parameters in the primitive productivity distribution. But here, decisions can shift the aggregate production function.

[27] Here is another way to see that hoarding (with imperfect measurement of utilization) is necessary for the aggregate to be Cobb-Douglas in capital and hours. Let K_p denote the capital stock being used in production, that is $K_p = [1 - H(\mu)]\,K_e$. Then it is clear

Since having firm-worker pairs that sometimes choose to be inactive affects the shape of the aggregates, I now establish under what conditions the equilibrium exhibits this property. For the remainder of the section I assume $\rho = 0$ to ease the algebra.

With G given by (25), (21) and (22) specialize to:

$$R - \phi - c - \frac{\beta}{1-\beta} c\theta + \frac{\lambda}{r+\delta+\lambda} \frac{\varepsilon^{\alpha} R^{1-\alpha}}{\alpha - 1} = 0 \quad (32)$$

$$\frac{\varepsilon^{\alpha} R^{1-\alpha}}{\alpha - 1} - \frac{(r+\delta+\lambda) c}{q(\theta)(1-\beta)} = 0. \quad (33)$$

By totally differentiating, we find that

$$\frac{\partial R}{\partial \phi} = \frac{(r+\delta+\lambda)\eta(\theta)}{\beta\theta q(\theta)[1-G(R)] + (r+\delta+\lambda)\eta(\theta)\left\{1 - \frac{\lambda[1-G(R)]}{r+\delta+\lambda}\right\}} > 0 \quad (34)$$

and

$$\frac{\partial \theta}{\partial \phi} = \frac{-(1-\beta)\theta q(\theta)[1-G(R)]}{(r+\delta+\lambda)\eta(\theta) c} \frac{\partial R}{\partial \phi} < 0,$$

where $1 - G(R) = (\varepsilon/R)^{\alpha}$ and $\eta(\theta) \equiv -\theta q'(\theta)/q(\theta)$. An increase in ϕ has no direct effect on the job-creation condition, and it shifts the job-destruction condition up in θ-R space. This increases the equilibrium value of R and decreases the equilibrium value of θ. Combining (32) and (33), one sees that the sign of $\phi - R$ is the sign of

$$\frac{\lambda}{q(\theta)} - [1 - (1-\theta)\beta].$$

So at low productivity realizations, the firm is more likely to hoard labor than to break the match when λ is large (and hence the option value of keeping a

from (24) that aggregate output is again linear in the relevant capital stock: $Y = \bar{A} K_p$, with $\bar{A} \equiv E(x|x \geq \mu)$. We are now in a position to explain why the model was extended to include the variable cost ϕ. If $\phi = 0$ then we always have $\mu = R$ in equilibrium, and the model always aggregates to a production function of the AK type.

match is large), and when q is small (and hence the expected cost of hiring a new worker is high). Market tightness θ enters the expression with an ambiguous sign because on the one hand a large theta makes hoarding more likely by increasing the expected recruiting cost; but on the other, through its effect on the worker's reservation wage, it also increases the value of her threat point in the wage bargain, which makes keeping an unproductive worker employed more costly and hoarding less likely. In fact, the latter effect disappears if the worker has no power in the wage bargain (i.e. if $\beta = 0$). Next, I provide a sufficient condition for $R < \phi$ to be possible in equilibrium under some parametrizations.

Let θ_ε^* be defined by $q(\theta_\varepsilon^*) = \frac{(\alpha-1)(r+\delta+\lambda)c}{(1-\beta)\varepsilon}$ and $\phi_\varepsilon = \left[1 + \frac{\lambda}{(\alpha-1)(r+\delta+\lambda)}\right]\varepsilon - \left(1 + \frac{\beta}{1-\beta}\theta_\varepsilon^*\right)c$. Then if $\phi = \phi_\varepsilon$, (32) and (33) are solved by $\theta(\phi_\varepsilon) = \theta_\varepsilon^*$ and $R(\phi_\varepsilon) = \varepsilon$. Notice that if $R(\phi_\varepsilon) < \phi_\varepsilon$, then there is a nondegenerate interval $[\phi_\varepsilon, \phi_R)$ such that $R(\phi) < \phi$ iff $\phi \in [\phi_\varepsilon, \phi_R)$, where ϕ_R is defined by $R(\phi_R) = \phi_R$.[28] The function $R(\phi)$ is illustrated in Figure 1.

So a sufficient condition for hoarding to occur in equilibrium is that $\phi_\varepsilon - \varepsilon > 0$, or equivalently, that $T(\lambda, \zeta) > 0$, where

$$T(\lambda,\zeta) \equiv \frac{\lambda\varepsilon}{(\alpha-1)(r+\delta+\lambda)} - \left(1 + \frac{\beta}{1-\beta}\theta_\varepsilon^*\right)c.$$

The parameter ζ summarizes the efficiency of matching, with the property

[28] Letting $\theta_R^* = q^{-1}\left[\frac{(\alpha-1)(r+\delta+\lambda)c}{(1-\beta)\varepsilon^\alpha\phi_R^{1-\alpha}}\right]$, it is a matter of algebra to verify that

$$\phi_R = \left[\frac{\lambda\varepsilon^\alpha}{(\alpha-1)(r+\delta+\lambda)\left(1+\frac{\beta}{1-\beta}\theta_\varepsilon^*\right)c}\right]^{\frac{1}{\alpha-1}}.$$

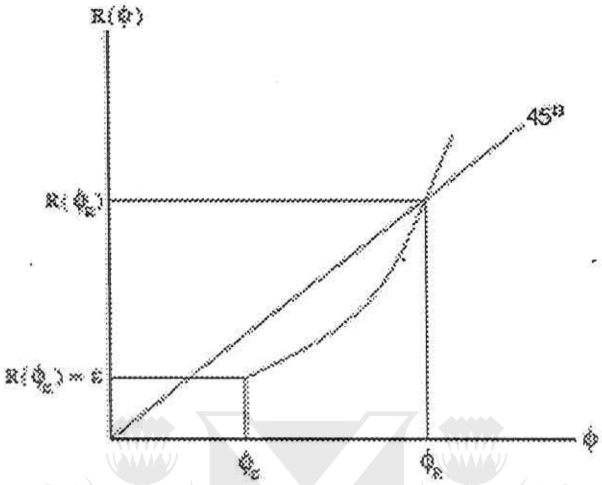

Figure 1: Destruction decision as a function of the variable cost.

that $\partial m(u,v)/\partial\zeta > 0$ and hence that $\partial q(\theta)/\partial\zeta > 0$ for all θ. Figure 2 plots the boundary $T(\lambda,\zeta) = 0$ in λ-ζ space. The condition $\phi_\varepsilon - \varepsilon > 0$ is satisfied for the values of the parameters λ and ζ that lie below boundary.[29] Intuitively, the parameter restriction that makes hoarding possible holds for relatively large λ (i.e. when bad shocks are very transitory) and relatively low ζ (i.e. when the search process needed to replace the worker is very costly).

Having characterized the main properties of the equilibrium, I now look

[29] Note that θ_ε^* goes to zero as ζ goes to zero. So $T(\lambda, 0) = 0$ iff $\lambda = \lambda_0$, where $\lambda_0 \equiv \frac{c(\alpha-1)(r+\delta)}{\varepsilon - c(\alpha-1)}$ is the point at which the boundary intercepts the horizontal axis in Figure 2. Formally, this boundary is upward-sloping because

$$\frac{\partial T}{\partial\zeta} = -\frac{\beta\theta_\varepsilon^*}{1-\beta}\frac{\partial\theta_\varepsilon^*}{\partial\zeta} < 0 \text{ and } \frac{\partial T}{\partial\lambda} = \frac{\varepsilon(r+\delta)}{(\alpha-1)(r+\delta+\lambda)^2} - \frac{bc^2(\alpha-1)}{(1-\beta)c(1-\beta)q'(\theta_\varepsilon^*)} > 0.$$

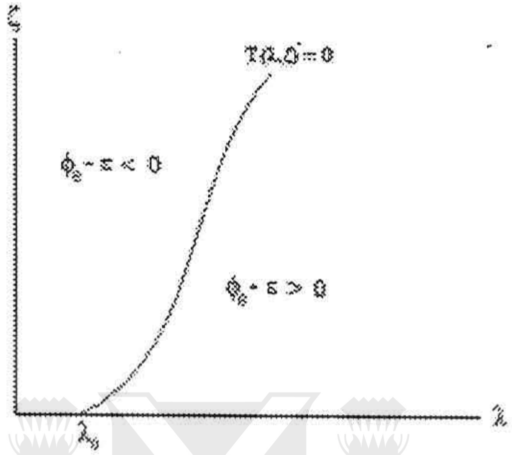

Figure 2: Range of parameters for which there is hoarding.

at the effects that labor-market policies have on the level of TFP.

4 Labor-Market Policies and the Level of TFP

In this section I consider the effects of four policies: employment and hiring subsidies, firing taxes and unemployment benefits. I follow Pissarides (2000) and model the subsidies as transfers from the government to the firm and the firing tax as a payment from the firm to the government.[30] For reasons that will become clear below, I will assume that the subsidies and the tax

[30] We assume that upon separation the firm must pay the firing tax the government because in the present setup, firing taxes would be completely neutral in the alternative scheme were the firm compensates the fired worker directly. (The effects of such a policy would be completely undone by the wage bargain.) To keep the analysis simple, we will ignore financing constraints. A natural extension would be requiring the government to run a balanced budget. An example of a scheme which is self-financing in the steady state is $\tau_f = \tau_h$ and $\tau_e = \rho = 0$.

are proportional to the firm's size, as measured by k.[31] The value function $W(x)$ is still given by (14), while (10), (11) and (13) generalize to

$$\begin{aligned}
rV &= \max_k \left[-ck + q(\theta) \int \max\left[J_o(z) + \tau_h k - V, 0\right] dG(z)\right], \\
rJ(x) &= \pi(x) + \tau_e k + \lambda \int \max\left[J(z), V - \tau_f k\right] dG(z) - \lambda J(x) \\
&\quad -\delta\left[J(x) + \tau_f k - V\right], \\
rU &= b + \theta q(\theta) \int \max\left[W_o(z) - U, 0\right] dG(z).
\end{aligned}$$

The policy variables are τ_h (hiring subsidy), τ_e (employment subsidy), τ_f (firing tax) and b (unemployment benefit). There are two reasons why the bargaining situation faced by a firm and worker when they first meet and are still considering whether to form a match is different from the one they face every instant after having agreed to form a match. The first is that in the initial bargain there is a one-time hiring subsidy at stake. The second, is that at that point the firm is not yet "locked in" by the firing tax. I use $w_o(x)$ to denote the wage that solves the initial bargain and $w(x)$ to denote the subsequent one.[32] So $W_o(x) - W(x) = w_o(x) - w(x)$, $J_o(x) - J(x) = w(x) - w_o(x)$, and hence

$$J_o(x) + W_o(x) = J(x) + W(x). \tag{35}$$

[31] Essentially, this assumption is convenient because it implies that the policies introduce no "scale effects" into the job-creation and destruction decisions.

[32] Pissarides (2000) calls the initial wage the "outsider wage" and the subsequent one the "insider wage".

The wages $w_o(x)$ and $w(x)$ are respectively characterized by

$$\begin{aligned} \beta\left[J_o(x) + \tau_h k\right] &= (1-\beta)\left[W_o(x) - U\right] \\ \beta\left[J(x) + \tau_f k\right] &= (1-\beta)\left[W(x) - U\right]. \end{aligned}$$

Letting $S_o(x) = J_o(x) + W_o(x) + \tau_h k - U$ and $S(x) = J(x) + W(x) + \tau_f k - U$ be the initial and the subsequent surplus respectively, the first-order conditions imply that $W_o(x) - U = \beta S_o(x)$, $W(x) - U = \beta S(x)$, $J_o(x) + \tau_h k = (1-\beta) S_o(x)$ and $J(x) + \tau_f k = (1-\beta) S(x)$. Combining these with the value functions we arrive at

$$(r + \delta + \lambda) S(x) = (x - \phi - c) k + \tau_e k + r\tau_f k - rU + \lambda \int \max\left[S(z), 0\right] dG(z),$$

with rU as in (16). Since $S'(x) > 0$, there is exists a unique R such that $S(x) \geq 0$ iff $x \geq R$. Using this reservation property, the surplus of an ongoing match can be written as

$$(r + \delta + \lambda) S(x) = (x - \phi - c) k + \tau_e k + r\tau_f k - rU + \lambda \int_R S(z) dG(z), \quad (36)$$

a natural generalization of (17). One can work with the value functions and the first order conditions of the Nash problem to derive expressions for wages and profit. The key observations are that $w_o(x)$ is decreasing in the firing tax but increasing in the hiring and employment subsidies, while $w(x)$ is increasing in the employment subsidy and the firing tax and independent of the hiring subsidy.[33] Evaluating (36) at $x = R$,

$$\lambda \int_R S(z)\, dG(z) = rU - [(R - \phi - c)\, k + \tau_e k + r\tau_f k],$$

and substituting this back into (36) yields (20). Using (20) to substitute $S(z)$ out of (36), evaluating at $x = R$ and using (16) we arrive at the job-destruction condition that generalizes (21):

$$R - \phi - c + \tau_e + r\tau_f - \left(\rho + \frac{\beta}{1-\beta} c\theta\right) + \frac{\lambda}{r+\delta+\lambda} \int_R (x - R)\, dG(x) = 0.$$

For simplicity, I have specified $b = \rho k$ where $\rho \in [0, 1)$ is akin to a replacement rate.[34] Increases in the employment subsidy and the firing tax reduce R for given θ. In other words, an increase in τ_e or τ_f shifts the job-destruction condition down in θ-R space. Conversely, an increase in ρ raises the worker's outside option and hence increases R for given θ.

[33] The wages and profit agreed upon in an ongoing match are:

$$\begin{aligned} w(x) &= \beta[(x - \phi - c)\, k + \tau_e k + r\tau_f k] + (1-\beta)\, rU \\ \pi(x) &= (1-\beta)[(x - \phi - c)\, k - rU] - \beta[\tau_e k + r\tau_f k], \end{aligned}$$

while those in an initial match are:

$$\begin{aligned} w_o(x) &= \beta[(x - \phi - c)\, k + \tau_e k + (r+\delta+\lambda)\tau_h k - (\delta+\lambda)\tau_f k] + (1-\beta)\, rU \\ \pi_o(x) &= (1-\beta)[(x - \phi - c)\, k - rU] - \beta[\tau_e k + (r+\delta+\lambda)\tau_h k - (\delta+\lambda)\tau_f k]. \end{aligned}$$

Finally, notice that

$$w(x) - w_o(x) = \pi_o(x) - \pi(x) = \beta(r+\delta+\lambda)(\tau_f - \tau_h)\, k.$$

[34] This formulation of the unemployment compensation is a clean way to ensure the job-destruction equation is independent k. Another –perhaps more realistic– way to obtain the same result would be to adopt the specification outlined before, where $b = \rho E_x[w(x)\,|x \geq R]$. The formulation in the text yields the same qualitative results, but it is simpler because b remains independent of R.

By free entry, $rV = 0$, and

$$(1-\beta)\int_R S_o(x)\,dG(x) = \frac{ck}{q(\theta)}. \tag{37}$$

Finally, using (35) we see that $S_o(x) = S(x) + (\tau_h - \tau_f)\,k$, which combined with (20), can be used to substitute $S_o(x)$ from (37) to obtain the job-creation condition:

$$\frac{1}{r+\delta+\lambda}\int_R (x-R)\,dG(x) + [1-G(R)](\tau_h-\tau_f) = \frac{c}{(1-\beta)\,q(\theta)}.$$

For given R, the hiring subsidy increases and the firing tax decreases job-creation. The other policy instruments have no direct effect on the entry decision.

Assuming G is as in (25), then

$$\begin{aligned} R-\phi-c+\tau_e+r\tau_f-\left(\rho+\frac{\beta}{1-\beta}c\theta\right)+\frac{\lambda\varepsilon^{\alpha}R^{1-\alpha}}{(\alpha-1)(r+\delta+\lambda)} &= 0 \quad (38)\\ \frac{\varepsilon^{\alpha}R^{1-\alpha}}{(\alpha-1)(r+\delta+\lambda)}+\left(\frac{\varepsilon}{R}\right)^{\alpha}(\tau_h-\tau_f)-\frac{c}{q(\theta)(1-\beta)} &= 0 \quad (39)\end{aligned}$$

The main properties of the equilibrium are summarized in the following proposition.

Proposition 1. Let θ_ϵ^* be defined by

$$q(\theta_\epsilon^*) = \frac{(\alpha-1)(r+\delta+\lambda)c}{(1-\beta)\left[\varepsilon+(\alpha-1)(r+\delta+\lambda)(\tau_h-\tau_f)\right]}, \text{ and let}$$

$$\phi_\epsilon = \left[1+\frac{\lambda}{(\alpha-1)(r+\delta+\lambda)}\right]\varepsilon-\left(1+\frac{\beta}{1-\beta}\theta_\epsilon^*\right)c+\tau_h+r\tau_f-\rho > 0.$$

If $\varepsilon+(\alpha-1)(r+\delta+\lambda)(\tau_h-\tau_f) > 0$, then for any $\phi > \phi_\epsilon$: (a) there exists a unique equilibrium; (b) $R > \varepsilon$; (c) $\partial R/\partial\phi > 0$ and (d) $\partial\theta/\partial\phi < 0$. If in addition, $\phi_\varepsilon - \varepsilon > 0$, then: (e) there is a nondegenerate interval $(\phi_\varepsilon, \widetilde{\phi})$ such that $R(\phi) < \phi$ for all $\phi \in (\phi_\varepsilon, \widetilde{\phi})$.

Proof. See the Appendix.

Aggregate output is still given by (29); the aggregate stock of capital demanded by filled jobs, K_e, is still given by (31); and the aggregate number of hours worked, N, is still as in (27). In addition, if the measure of capital used to construct aggregate output is K_e, then the level of TFP is still given by (30). The following proposition, which holds under the assumptions stated in Proposition 1, summarizes the effects that labor market policies have on A, the level of observed TFP.

Proposition 2. Employment subsidies and firing restrictions reduce A. Hiring subsidies and unemployment benefits increase A.

Proof. See the Appendix.

Since A is proportional to R, policy instruments have the same qualitative effect on TFP as on the destruction rate. Proposition 2 is illustrated in Figure 3. Employment subsidies make firms more tolerant of low productivity realizations, and hence lower the average productivity of active firms. Firing taxes have a similar qualitative effect on job-destruction, but that mechanism is reinforced by a decrease in job-creation (which reduces the reservation wage and hence makes firms even more tolerant to low productivity realizations). Hiring subsidies have no direct effect on the destruction decision, but they stimulate job-creation. This increases market tightness which in turn increases the workers' outside option and raises R. Unemployment benefits also cause R to rise through an increase in the worker's reservation wage.

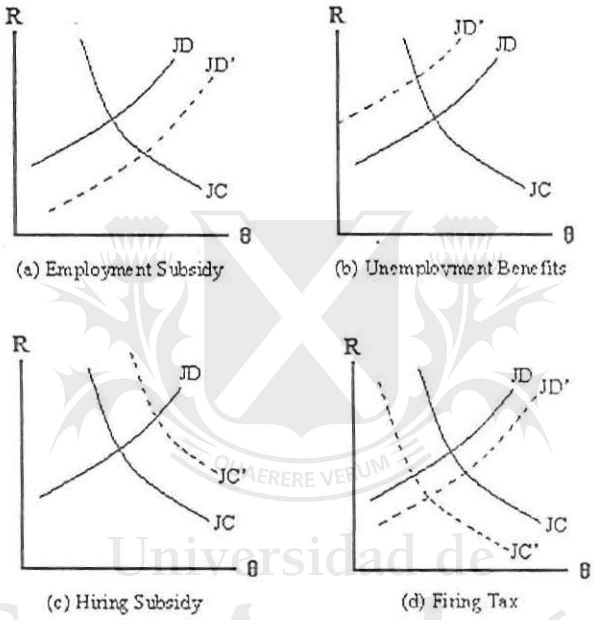

Figure 3: Equilibrium effects of various policies.

5 Extensions

The maintenance cost $C(x, \phi)$ was introduced as a simple device to avoid "flat spots" in the value functions.[35] Here I show that by extending the model

[35] If $\pi(x) = [\max(x - \phi, 0) - c]\, k - w(x)$, then $\pi(x)$ is flat up to ϕ and then rises with slope k. It is easy to show that in this case $J(x)$ is also flat up to ϕ and then rises with slope $\frac{k}{r+\delta+\lambda}$. Note that since R is defined by $J(R) = 0$, this implies that generically the

in a natural way, one can drop the maintenance cost without affecting the main results. To this end, I generalize the productivity process by allowing for serially correlated shocks: when match of productivity x suffers a change, the new value x' is a draw from the fixed distribution $G(x'|x)$. Assuming $G(x|x_1) < G(x|x_0)$ if $x_0 < x_1$, allows idiosyncratic shocks to be positively correlated through time. For this case, the cross-section of productivities evolves according to

$$\begin{aligned}
\frac{d}{dt}\left[(1-u_t) H_t(x)\right] &= \lambda(1-u_t)\int_x^{\infty}\left[G(x|s)-G(R_t|s)\right]dH_t(s) \\
&\quad +\theta q(\theta)u_t\int_{-\infty}^{\infty}\left[G(x|s)-G(R_t|s)\right]dH_t(s) \\
&\quad -\lambda(1-u_t)\int_{-\infty}^{x}G(R_t|s)\,dH_t(s) \\
&\quad -\int_{-\infty}^{x}\left[1-G(x|s)\right]dH_t(s) \\
&\quad -\delta(1-u_t)H_t(x)\lambda(1-u_t).
\end{aligned}$$

The first term accounts for the matches with productivities above x that get innovations below x. The newly-formed matches that start off with productivities no larger than x are in the second term. Notice our assumption that upon contact, the worker and firm draw their productivity level from the density corresponding to the average productivity among active matches.[36] The

equilibrium will have $\varphi < R$ (except for the knife-edge case in which R is indeterminate). We want to avoid this type of flat spots in J to allow for the possibility that $R < \varphi$ in equilibrium.

[36] If shocks were *iid*, we could just specify that new matches draw z from $G(z)$ just as active matches do when forced to update their shock. However, with correlated shocks active matches with state z draw the new shock z' from $G(z'|z)$. Since vacancies and unemployed workers have no productivity attached to them, we assume their initial draw z' is from the average density $\int G(z'|z)\,dH(z)$. As a way of motivating this, imagine –as

third term is the number of matches in the interval $[R, x]$ that get shocks below R and are destroyed. The fourth term accounts for the number of matches in the same interval that "move up" by virtue of having drawn productivities larger than x. The last term accounts for matches in the interval $[R_t, x]$ that are destroyed for exogenous reasons. Imposing steady states and re-arranging, we arrive at

$$H(x) = \left[\frac{\lambda}{\delta + \lambda} + \frac{\theta q(\theta) u}{(\delta + \lambda)(1 - u)}\right] \int [G(x|s) - G(R|s)]\, dH(s).$$

The steady-state unemployment rate is

$$u = \frac{\delta + \lambda \int G(R|s)\, dH(s)}{\delta + \lambda \int G(R|s)\, dH(s) + \theta q(\theta) \int [1 - G(R|s)]\, dH(s)}. \tag{40}$$

Using this expression, the steady-state cross-sectional productivity distribution can be rewritten as

$$H(x) = \frac{\int [G(x|s) - G(R|s)]\, dH(s)}{\int [1 - G(R|s)]\, dH(s)}, \tag{41}$$

which is a natural generalization of (9).

The firm's problem upon entering the market is now summarized by

$$rV = \max_k \left[-ck + q(\theta) \int \int \max[J(z) - V, 0]\, dG(z|x)\, dH(x)\right]. \tag{42}$$

As usual, I assume there is entry of firms until all rents are exhausted, so $rV = 0$ in equilibrium. The value of a filled job with productivity x is

$$rJ(x) = \pi(x) + \lambda \int \max[J(z), V]\, dG(z|x) - \lambda J(x) - \delta [J(x) - V], \tag{43}$$

Mortensen and Pissarides (1994) do– that firms must irreversibly adopt a "technology" to engage in production. Our specification then means that they pick their "technology" at random from all those active at the time the match is created.

where $\pi(x) = \max_n [x \min(n,k) - \phi n - ck - w(x)]$. Flow profit $\pi(x)$ is the residual remaining after the wage $w(x)$ and all other costs of production have been paid out. There are only two such costs in this formulation: the fixed cost, ck, and the variable one, ϕn. Our assumptions still imply that w is independent of n, so the profit-maximizing choice of hours is still given by (12), and hence $\pi(x) = y(x) - w(x)$, where $y(x) \equiv [\max(x-\phi, 0) - c] k$ is output net of the variable cost and the rental on capital.

The value of unemployment and employment and to a worker are

$$\begin{aligned} rU &= b + \theta q(\theta) \int\int \max[W(z) - U, 0]\, dG(z|x)\, dH(x) && (44) \\ rW(x) &= w(x) + \lambda \int \max[W(z) - U, 0]\, dG(z|x) && (45) \\ &\quad - (\delta + \lambda)[W(x) - U]. \end{aligned}$$

I still assume the wage solves the Nash bargaining problem and hence it is still characterized by (15). Letting $S(x) = J(x) + W(x) - U$ denote the surplus from a match, notice that (15) implies that $J(x) = (1-\beta) S(x)$ and $W(x) - U = \beta S(x)$. These together with (43), (44), and (45) imply

$$(r + \delta + \lambda) S(x) = y(x) - rU + \lambda \int \max[S(z), 0]\, dG(z|x)$$

where rU is given by (16). The fact that $S'(x) > 0$ implies that there exists a unique R such that $S(R) > 0$ iff $x > R$. Hence matches separate whenever productivity falls below R. For completeness, (15) and the value functions can be manipulated to obtain expressions for instantaneous wages and profit:

$$\begin{aligned} w(x) &= \beta y(x) + (1-\beta) rU && (46) \\ \pi(x) &= (1-\beta)[y(x) - rU]. && (47) \end{aligned}$$

Intuitively, the wage is a weighted average of net output and the worker's reservation wage.

It turns out that one can get a much sharper characterization of the equilibrium by putting some structure on the conditional distribution $G(s|x)$. In what follows, I assume that $dG(s|x) = \xi(x)\hat{g}(s)\,ds$ where $\xi'(x) > 0$.[37] Note that this allows us to rewrite the surplus from a match x as

$$(r+\delta+\lambda)\,S(x) = y(x) - rU + \lambda\xi(x)\int_R S(z)\,\hat{g}(z)\,dz, \qquad (48)$$

and that evaluating it at $x = R$ yields

$$\lambda\int_R S(z)\,\hat{g}(z)\,dz = \frac{rU - y(R)}{\xi(R)}.$$

Since the expected capital gain on the left-hand-side is positive, at $x = R$ net output is smaller than the worker's reservation wage. From (46) and (47) we again verify that $w(R) < rU$ and $\pi(R) < 0$. Substituting the simpler expression for the expected capital gain term into (48) we obtain[38]

$$(r+\delta+\lambda)\,S(x) = y(x) - rU + \frac{\xi(x)}{\xi(R)}\left[rU - y(R)\right]. \qquad (49)$$

[37] As an example, the Pareto distribution that we adopt below to derive the main aggregation result satisfies this condition.

[38] A word of caution is in order here. In general, the lower bound of the support of the density $\xi(x)\,\hat{g}(s)\,ds$ could be a function of x itself. For instance assume the support is $[\varepsilon(x), \infty)$. Then, formally, the capital gain term in (48) should be written as $\lambda\xi(x)\int_{\max[R,\varepsilon(x)]} S(z)\,\hat{g}(z)\,dz$, and then evaluating the surplus at $x = R$ would yield

$$\lambda\int_{\max[R,\varepsilon(R)]} S(z)\,\hat{g}(z)\,dz = \frac{rU - y(R)}{\xi(R)}.$$

Thus

$$\lambda\int_{\max[R,\varepsilon(x)]} S(z)\,\hat{g}(z)\,dz = \frac{rU - y(R)}{\xi(R)}$$

and (49) follows iff $R > \varepsilon(x)$ for all x. A restriction that we assume –and later verify– to be satisfied in equilibrium.

Note that $y'(x) \geq 0$, and that the expected capital gain from the next draw (the second term) is increasing in current productivity because $\xi'(x) > 0$ (i.e. a higher shock today means the next innovation will be drawn from a better distribution). Thus $S'(x) = y'(x) + [\xi'(x)/\xi(R)][rU - y(R)] > 0$. Just as before, in equilibrium we can have $\phi < R$ or $R < \phi$. We now use (48) and (49) to derive the job-creation and destruction conditions. Evaluating (48) at $x = R$ and using (49) to substitute $S(z)$ we arrive at the job-destruction condition:

$$y(R) - rU + \frac{\lambda}{r+\delta+\lambda}\int_R \left\{ y(z) - rU + \frac{\xi(z)}{\xi(R)}[rU - y(R)] \right\} dG(z|R) = 0.$$

Equation (42) together with $rV = 0$ imply that at the optimal k, we have

$$(1-\beta)\int\int_R S(z)\, dG(z|x)\, dH(x) = \frac{ck}{q(\theta)},$$

namely the expected profit from a filled job equals the expected recruiting cost. Using (49) to substitute $S(z)$ out of this expression we arrive at the job-creation condition:

$$\int\int_R \left\{ y(z) - rU + \frac{\xi(z)}{\xi(R)}[rU - y(R)] \right\} dG(z|x)\, dH(x) = \frac{(r+\delta+\lambda)\, ck}{(1-\beta)\, q(\theta)}.$$

After some manipulations, the job-destruction and creation conditions respectively simplify to:

$$\tfrac{-[rU-y(R)]}{k} + \tfrac{\lambda}{r+\delta+\lambda}\left\{ \varphi(\mu|R) + \tfrac{rU-y(R)}{\xi(R)k}\int_R [\xi(x) - \xi(R)]\, dG(x|R) \right\} = 0$$

$$\int \varphi(\mu|z)\, dH(z) + \tfrac{rU-y(R)}{\xi(R)k}\int\int_R [\xi(x) - \xi(R)]\, dG(x|z)\, dH(z) = \tfrac{(r+\delta+\lambda)c}{(1-\beta)q(\theta)},$$

where $\varphi(\mu|R) \equiv \int_{\mu} [1 - G(x|R)]\, dx$ and $\mu \equiv \max(\phi, R)$. Given the cross-sectional productivity distribution H, the job-creation and destruction conditions jointly determine R and θ. Observe that these conditions are analogous to those in Mortensen and Pissarides (1994) when $\phi = 0$ and $\xi(x) = \xi$ for all x. More formally, for given c and ϕ, an equilibrium is a list $[R, \theta, H, U, w, u, k]$ such that R, θ and H jointly solve (41) and the job-creation and the job-destruction conditions; rU is given by (16); w by (46); and u satisfies (40). In addition, the market for capital should clear, so k must satisfy $[1 - (1 - \theta) u] k = K$, where K is the aggregate supply of capital, which labor-market participants take as given. I now turn to the case when the distribution of idiosyncratic shocks is Pareto.

Suppose idiosyncratic shocks are draws from

$$G(x|s) = \begin{cases} 0 & \text{if } x < \varepsilon(s) \\ 1 - \left[\frac{\varepsilon(s)}{x}\right]^{\alpha} & \text{if } \varepsilon(s) \leq x \end{cases}$$

where $\varepsilon(\cdot)$ is a continuously differentiable function and $\alpha > 2$. I introduce positively correlated shocks by assuming that $\varepsilon' > 0$. The special case of *iid* shocks corresponds to $\varepsilon' = 0$. In addition, I assume there is an $\underline{\varepsilon} > 0$ such that $\varepsilon(\underline{\varepsilon}) = \underline{\varepsilon}$ and $\varepsilon(s) = 0$ if $s < \underline{\varepsilon}$, and that $\lim_{s \to \infty} \varepsilon(s) = 1 + \underline{\varepsilon} \equiv \overline{\varepsilon}$.[39]

Then if $R \geq \varepsilon(s)$, $1 - G(R|s) = \left[\frac{\varepsilon(s)}{R}\right]^{\alpha}$ and for any $x \geq R$,

$$G(x|s) - G(R|s) = \left[\frac{\varepsilon(s)}{R}\right]^{\alpha} \left[1 - \left(\frac{R}{x}\right)^{\alpha}\right].$$

Substituting these expressions in (41) we see that the steady state produc-

[39]This distribution has mean $\mu = \frac{\alpha}{\alpha-1}\varepsilon(s)$ and variance $\sigma^2 = \frac{\mu}{(\alpha-2)(\alpha-1)}$. We assume $\alpha > 2$ so that both are finite. An example of an $\varepsilon(\cdot)$ satisfying all these conditions is $\varepsilon(s) = 1 + \underline{\varepsilon} - e^{\underline{\varepsilon}-s}$, for any $\underline{\varepsilon} > 0$.

tivity cross-section is still given by (26). So for this case, the job-creation and destruction conditions, respectively, specialize to

$$\frac{\mu^{1-\alpha}R^{\alpha}}{\alpha-1}-\frac{\alpha[y(R)-rU]}{k}\left[\frac{R}{\varepsilon(R)}\right]^{\alpha}\int_{R}\frac{\varepsilon(x)^{\alpha}-\varepsilon(R)^{\alpha}}{x^{1+\alpha}}dx=\frac{(r+\delta+\lambda)c}{(1-\beta)q(\theta)}\left[\alpha\int_{R}\frac{\varepsilon(x)^{\alpha}}{x^{1+\alpha}}dx\right]^{-1}$$

$$\left[1-\frac{\lambda\alpha}{r+\delta+\lambda}\int_{R}\frac{\varepsilon(x)^{\alpha}-\varepsilon(R)^{\alpha}}{x^{1+\alpha}}dx\right]\frac{y(R)-rU}{k}+\frac{\lambda}{r+\delta+\lambda}\frac{\varepsilon(R)^{\alpha}\mu^{1-\alpha}}{\alpha-1}=0$$

Under relatively mild conditions, it can be shown that the job-creation condition slopes down and the destruction condition up in θ-R space, implying a unique (θ, R) pair. A parameter restriction analogous to the one depicted in Figure 2 guaranteeing that there is a range of values for ϕ such that $R < \phi$ can still be derived. In addition, one should always make sure that the equilibrium satisfies $R > \overline{\varepsilon}$.[40] Following the same procedure used for the simpler model, it is easy to verify that output still aggregates to (29).

I conclude this section by showing how the observed level of TFP is affected by the different ways of measuring aggregate inputs that can be found in the literature. The measure of capital input used by Hall and Jones (1999) did not adjust for utilization. This means that K instead of K_e was used in the production function, which would imply $\hat{F}(K,N) = \hat{A}K^{\gamma}N^{1-\gamma}$, with $\hat{A} = \left[\frac{1-u}{1-(1-\theta)u}\right]^{\gamma} A$, as mentioned in Section 3.3. But in addition, Hall and

[40] Recall that the derivation of (49) implicitly assumes that $R > \varepsilon(s)$ for all s. This condition is satisfied if $R > \overline{\varepsilon}$. Showing that equilibria with $R < \phi$ are possible for some parametrizations is now rather tedious, so we just outline the idea here. Let $\phi_{\overline{\varepsilon}}$ be the value of ϕ such that $\theta^{*}_{\overline{\varepsilon}}$ and $R(\phi_{\overline{\varepsilon}}) = \overline{\varepsilon}$ solve the job-creation and destruction conditions. Then if $\phi_{\overline{\varepsilon}} - \overline{\varepsilon} > 0$, there will be an interval $(\phi_{\overline{\varepsilon}}, \widehat{\phi})$ such that $R(\phi) < \phi$ iff $\phi \in (\phi_{\overline{\varepsilon}}, \widehat{\varphi})$. If, in addition, we guarantee that $\partial R(\phi)/\partial\phi > 0$, then $\phi_{\overline{\varepsilon}} - \overline{\varepsilon} > 0$ also implies $\overline{\varepsilon} < R(\varphi)$ for all $\varphi \in (\phi_{\overline{\varepsilon}}, \widehat{\phi})$. Finally, notice that $R > \overline{\varepsilon}$ also implies that every match faces a positive probability of being destroyed for endogenous reasons. To see why, suppose $R = \varpi < \overline{\varepsilon}$; then any match that reaches a state $s > \varepsilon^{-1}(\varpi)$ will never be destroyed endogenously.

Jones (1999) report they did not have data on hours per worker, so they used the number of employed workers instead of hours worked to measure labor input. Letting $E = 1 - u$ denote employment and using (27) and (31), the number of hours worked is $N = (R/\mu)^{1/\gamma} \frac{KE}{1-(1-\theta)u}$, so their measurements of inputs imply that the aggregate relationship between inputs, output and TFP that they observed was $\tilde{F}(K, E) = \tilde{A}K^{\gamma}E^{1-\gamma}$, with $\tilde{A} = \left[\frac{(R/\mu)^{1/\gamma}K}{1-(1-\theta)u}\right]^{1-\gamma} \hat{A}$.

6 Concluding Remarks

I have presented a theory of aggregate TFP differences based on the interaction between institutions and the microeconomics underlying the aggregate production function. I focused on a precise type of institutions, namely labor-market policies as measured by the magnitudes of hiring and employment subsidies, unemployment benefits and firing restrictions. In the model, firm-level technologies are subject to idiosyncratic shocks which induce a cross-sectional distribution of productivities. Through their effect on the job-creation and destruction rates, labor market policies affect the distribution of productivities among active firms.

Policies that make firing difficult make firms less willing to give up relatively unproductive opportunities to search for better ones, lowering the average productivity among active matches, and aggregate TFP. Employment subsidies also make firms more tolerant of bad productivities and hence they also decrease TFP. Unemployment benefits have the opposite effect. Hiring subsidies stimulate job creation and cause more competition among firms. As a result, firms become more selective and only pursue very productive ventures. The cross-sectional distribution of productivities shifts to the right, and aggregate TFP rises.

The model could be used as a guide to understand aggregate productivity data. It could be calibrated to find out how large the differences in the mix and magnitude of labor-market policies have to be in order to explain the differences in TFP levels among a relevant set of countries. It may also prove to be a helpful tool to the econometrician interested in measuring aggregate productivity.

A Appendix

Proof of Proposition 1.

Let $\theta(\phi)$, and $R(\phi)$ denote the solution to (38) and (39) when it exists; and define $\tau(R) = R + \alpha(r+\delta+\lambda)(\tau_h - \tau_f)$. By totally differentiating (38) and (39):

$$
\begin{aligned}
\frac{\partial R(\phi)}{\partial \phi} &= \frac{(r+\delta+\lambda)\,\eta(\theta)}{[\beta\theta q(\theta)/R]\,(\varepsilon/R)^{\alpha}\,\tau(R) + (r+\delta+\lambda)\,\eta(\theta)\left[1-\frac{\lambda(\varepsilon/R)^{\alpha}}{r+\delta+\lambda}\right]} \\
\frac{\partial \theta(\phi)}{\partial \phi} &= \frac{-(1-\beta)\,\theta q(\theta)\,(1/R)\,(\varepsilon/R)^{\alpha}\,\tau(R)}{(r+\delta+\lambda)\,\eta(\theta)\,c}\frac{\partial R}{\partial \phi}.
\end{aligned}
$$

So $\tau(R) > 0$ is sufficient for $\partial R/\partial\phi > 0$. If $\phi = \phi_{\varepsilon}$, then (38) and (39) have a unique solution, namely $\theta(\phi_{\varepsilon}) = \theta^{*}_{\varepsilon}$ and $R(\phi_{\varepsilon}) = \varepsilon$. But $\tau(\varepsilon) > 0$ by assumption, so $\partial R(\phi_{\varepsilon})/\partial\phi > 0$. This and the continuity of $R(\phi)$ implies that $R(\phi) > \varepsilon$ for all $\phi > \phi_{\varepsilon}$. Since $\tau' > 0$, for any $\phi > \phi_{\varepsilon}$ we know that $\tau(R) > 0$ and therefore $\partial R(\phi)/\partial\phi > 0$ and $\partial\theta(\phi)/\partial\phi < 0$. This establishes parts (b), (c) and (d). In θ-R space, the slopes of the job-destruction and creation conditions are

$$
\frac{\beta c}{1-\beta\left[1-\frac{\lambda(\varepsilon/R)^{\alpha}}{r+\delta+\lambda}\right]} > 0, \text{ and } \frac{-c\eta(\theta)\,(r+\delta+\lambda)\,R}{(1-\beta)\,\theta q(\theta)\,(\varepsilon/R)^{\alpha}\,\tau(R)} < 0
$$

respectively, which establishes (a). Finally, $\phi_{\varepsilon} - \varepsilon > 0$ is equivalent to $\phi_{\varepsilon} - R(\phi_{\varepsilon}) > 0$, which implies (e). ■

Proof of Proposition 2.

Define

$$
\Delta = \frac{(\varepsilon/R)^{\alpha}\,\tau(R)}{(r+\delta+\lambda)\,R} + \frac{\eta(\theta)}{\beta\theta q(\theta)}\left[1-\frac{\lambda(\varepsilon/R)^{\alpha}}{r+\delta+\lambda}\right].
$$

Since $\tau(R) > 0$ by Proposition 1, we have $\Delta > 0$ in any equilibrium. By totally differentiating (38) and (39) we get

$$\begin{aligned}
\frac{\partial R}{\partial \tau_e} &= \frac{-\eta(\theta)}{\beta\theta q(\theta)\Delta} < 0,\ \frac{\partial R}{\partial \tau_f} = -(1/\Delta)\left[(\varepsilon/R)^{\alpha} + \frac{r\eta(\theta)}{\beta\theta q(\theta)}\right] < 0, \\
\frac{\partial R}{\partial \tau_h} &= (1/\Delta)(\varepsilon/R)^{\alpha} > 0,\ \frac{\partial R}{\partial \rho} = -\frac{\partial R}{\partial \tau_e} > 0,
\end{aligned}$$

and this concludes the proof. ■

For completeness, here I report the effects of all policies on market tightness.

$$\begin{aligned}
\frac{\partial \theta}{\partial \tau_e} &= \frac{-(1-\beta)\theta q(\theta)(\varepsilon/R)^{\alpha}\tau(R)}{c\eta(\theta)(r+\delta+\lambda)R}\frac{\partial R}{\partial \tau_e} > 0, \\
\frac{\partial \theta}{\partial \tau_h} &= \frac{1-\beta}{\beta c}\left[1 - \frac{\lambda(\varepsilon/R)^{\alpha}}{r+\delta+\lambda}\right]\frac{\partial R}{\partial \tau_h} > 0, \\
\frac{\partial \theta}{\partial \rho} &= \frac{-(1-\beta)\theta q(\theta)(\varepsilon/R)^{\alpha}\tau(R)}{c\eta(\theta)(r+\delta+\lambda)R}\frac{\partial R}{\partial \rho} < 0, \\
\frac{\partial \theta}{\partial \tau_f} &= \frac{1-\beta}{\beta c}\left\{r + \left[1 - \frac{\lambda(\varepsilon/R)^{\alpha}}{r+\delta+\lambda}\right]\frac{\partial R}{\partial \tau_f}\right\}.
\end{aligned}$$

Without additional restrictions the sign of $\partial\theta/\partial\tau_f$ is ambiguous. It is negative in any equilibrium with $\phi > \phi_{\varepsilon}$ if $\delta > r(1-\varepsilon)/\varepsilon$.

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