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"On the timing of Balance-of-Payments Crises: Asymmetric Information and Interest Rate Policy"

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On the Timing of Balance-of-Payments Crises: Asymmetric Information and Interest Rate Policy

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Abstract

This paper provides a simple dynamic framework for understanding the timing of balance-of-payments crises. It presents an asymmetric information model that focuses on investors' learning process and its crucial interaction with interest rate policy. The model incorporates two basic ingredients: (i) investors have private information; and (ii) investors interact in a dynamic setting, weighing the high returns they receive while holding domestic assets against the incentives to pull out before crises take place. The model shows that the presence of private information delays the onset of BOP crises, giving rise to large drops in asset prices when crises finally take place. It also shows that, even though there is a positive relationship between domestic interest rates and the speed at which investors can learn from each other, high interest rates are an effective defense against speculative attacks. The effect of interest rates on the timing of crises increases with the degree of private information. Finally, I characterize the optimal interest rate policy for the monetary authority: the optimal policy is to raise interest rates sharply as fundamentals become very weak. However, this policy is time inconsistent, suggesting a role for commitment devices such as currency boards or IMF pressure.

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1 Introduction

During the 1990's the world witnessed a large number of balance-of-payments (BOP) crises, including the EMS crisis in 1992, the Mexican crisis in early 1995, the Asian crisis in 1997, and the recent crises in Russia and Brazil. The large, and rapidly growing, literature on BOP crises has provided many insights into the causes behind these crises. A consensus now exists about the importance of institutions (e.g. bank supervision, corporate governance), debt management, and consistency in the setting of monetary and fiscal policy. Despite this progress, however, economists still have a limited understanding of the dynamics and timing of crises.

The main difficulty in studying the timing of BOP crises stems from the need to account for two seemingly contradictory characteristics. On the one hand, BOP crises are usually "large," in that they involve massive asset reallocations, wild swings in asset prices, and heavy output losses. On the other hand, BOP crises are often triggered by shocks that seem too small to account for these effects. This paper proposes a simple dynamic framework for studying the timing of BOP crises that accounts for these two characteristics. It emphasizes the dynamics of investors' learning process and its crucial interaction with interest rate policy.

This paper models BOP crises as the equilibrium outcome of a game between a monetary authority, which attempts to keep a fixed exchange rate, and a set of investors that at each point in time decide how much of their capital to invest in the country. The model relies on two basic ingredients: (i) investors have private information; and (ii) investors interact in a dynamic setting, weighing the high returns they receive while holding domestic assets against the incentives to pull out before the crisis takes place. The crisis is triggered by some investors selling their domestic assets and starting a run on the central bank's reserves, with other investors following suit until reserves are exhausted. Investors have private information regarding the level of the exchange rate in case the peg is abandoned. The run up to the crisis is characterized by a slow learning process, in which the high returns on domestic assets more than compensate for the risk of capital losses due to devaluation. During the crisis most of the remaining uncertainty regarding investors' private information is resolved. Furthermore, when the peg is abandoned the exchange rate experiences a discrete devaluation. As a result, investors' strategies incorporate an incentive to take their capital out before the crisis takes place.[1] Although the timing of the crisis is unpredictable based on public information, the model has a unique equilibrium, in which the timing depends on investors' private information.

Two versions of the model are presented. The first, in which the return on domestic assets is taken as exogenous, emphasizes investors' learning process and its implications for the timing of BOP crises. This version provides insights into the behavior of asset prices during crises, as well as into the effects of interest rates and asymmetric information on the timing of crises. First, it shows that even in a model with a single equilibrium, "large shocks" are not necessary in order for crises to involve large drops in asset prices. Large movements in asset prices at the time the peg is abandoned are possible as a result of the large amount of private information that is revealed during crises. Furthermore, an asymmetry exists in that revaluations never take place.

Second, it shows that the presence of private information delays the crisis, in the sense that the peg lasts longer for all realizations of investors' private information. This result follows from two features of BOP crises that are captured by the model. As the crisis progresses, investors become more informed because they can infer the private information of the investors who take their capital out. The investors who would leave last then know the value of the new exchange rate and, thus, would have an incentive to wait if they expected a revaluation. As a result, arbitrage can rule out negative but not positive devaluations.[2] In addition, the high returns on domestic assets in episodes of BOP crises create an incentive to wait past the point when the expected devaluation is zero. Without private information, however, investors cannot "coordinate" into staying past this point and leave when the size of the devaluation is zero. The delay of BOP crises in the presence of private information can account for the observation that crises often occur long after problems in the affected countries are recognized. It also implies that when the peg is finally abandoned the currency always depreciates.

Third, the fixed exchange rate lasts longer when domestic interest rates

[1] This contrasts with so-called first generation models of BOP crises, in which the timing of crises is determined by the condition that the exchange rate be continuous. In fact, in the model presented in this paper crises would also involve a continuous exchange rate if private information were not present.

[2] In other words, if investors start leaving "too soon," they can recognize their mistake before reserves are exhausted, which gives rise to probing attacks. If, on the other hand, they start leaving "too late" there is a devaluation.

are high. This result follows from the fact that, conditional on other investors' actions, each investor has greater incentives to leave his capital in the country when interest rates are high. In addition, an indirect channel exists due to the presence of complementarities in investors' actions: if each investor stays longer the expected losses from devaluation decrease, further increasing the incentives not to pull out. However, in a setup in which the exchange rate is continuous at the time of the crisis it is unlikely that an increase in interest rates would postpone the abandonment of the peg.[3,4] It is the existence of positive devaluations as a result of private information that allows interest rates to delay the crisis. The model thus provides a rationale for interest rate defenses.[5]

To better understand the effect of interest rate policy on the timing of BOP crises, the behavior of the monetary authority is endogenized in the second version of the model. I assume that the monetary authority controls the return on domestic assets in order to minimize a loss function, which incorporates a cost of raising interest rates and a cost of having to abandon the peg. I also assume that there is an exogenous probability of "turnaround" in which case the peg survives and no further interest rate costs need to be incurred. The model has a number of implications for interest rate policy during BOP crises.

First, the optimal interest rate policy is to raise interest rates sharply when fundamentals become very weak, rather than raising interest rates by a smaller amount for a longer period of time. This result follows from the fact that raising interest rates when fundamentals are very weak is both "more effective" and "cheaper." For any interest rate path, the effect of increasing the interest rate at some point increases the equilibrium probability that the

[3] Calvo (1995) presents such an argument. He argues that high interest rates could induce capital inflows in the run up to the crisis, but these would be compensated by a larger portfolio reallocation when the peg is abandoned. He also argues that, since the fiscal deficit (or expected future deficits) likely increases when interest rates are raised to defend a peg, it is possible that the "defense" actually hastens the end of the peg.

[4] This paper concentrates on the effect of interest rates before the peg is abandoned. Lahiri and Végh (1999) and Salant and Henderson (1978) show that, even if no private information exists, the value of interest rates *after* the peg is abandoned affects the desired portfolio reallocation and, as a result, the timing of the crisis. Drazen (1999) argues that interest rates can affect the timing of crises by providing information about the government's objectives.

[5] Stiglitz (1998) argues that only unrealistically high interest rates would be effective in defending a peg. In this paper this is not the case.

crisis will occur at that point.[6] Correspondingly, the amount of learning that takes place at that point increases, thereby "shifting back" the crisis distribution function for all previous times. As a result, high interest rates are more effective in postponing the devaluation if they are expected to take place when fundamentals are weaker. In addition, it is possible that interest rate costs will not need to be incurred if the crisis or the turnaround take place earlier.

Second, a problem of time inconsistency arises. The monetary authority would be better off if it could commit to raising interest rates as fundamentals deteriorate. This result follows from the fact that the benefits of high interest rates at a point in time are partly "sunk" when that time is reached. This time inconsistency problem suggests a role for international organizations such as the IMF, or for commitment devices such as currency boards.

Third, empirical studies on the effectiveness of interest rate defenses should be careful in interpreting episodes in which interest rates are sharply raised but the peg is abandoned. In the model presented in this paper, although interest rates are an effective instrument for defending against speculative attacks, crises are more likely while interest rates are high, even conditioning on the level of fundamentals. Finally, these results are stronger in cases of liquidity crises than in cases of solvency crises.[7]

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Related Literature

The large shifts in asset holdings during crises initially led observers to associate such episodes with investor irrationality. The so called first-generation approach to BOP crises, initiated by Salant and Henderson (1978), Krugman (1979), and Flood and Garber (1984), provided an alternative explanation. If crises mark a switch in regimes, with inflation higher after the fixed exchange rate is abandoned, the desired holdings of domestic currency should likely fall during crises. As a result, a "run" on the central banks's reserves could be interpreted as a rational portfolio reallocation. These models, though, also have the unrealistic implications that the timing of crises should be predictable and that crises should not involve large changes in asset prices. Flood and Garber (1984) and Dornbusch (1987) develop stochastic models

[6] This corresponds approximately to uncovered interest parity.

[7] In the context of this paper, a "liquidity crisis" is a crisis in which the probability that the peg survives increases when the attack is postponed.

of BOP crises that address these shortcomings by assuming the existence of large shocks.[8]

A different approach to explain the unpredictability of crises and the drops in asset prices is to assume the existence of multiple equilibria. Starting with Obstfeld (1984), second-generation models introduce the possibility that crises be self-fulfilling: if investors expect a crisis, they will act in a way such that a crisis occurs. However, these models have little to say about the *timing* of BOP crises, as a wide range of results can be obtained by assuming different expectational dynamics. Furthermore, as Morris and Shin (1998) show in a generic second-generation model, the existence of multiple equilibria might not be very robust, as adding even a small amount of noise to investors' perceptions about a country's fundamental eliminates the multiplicity of equilibria.

The model presented in this paper is complementary to a number of asset-pricing models that present alternative amplification mechanisms for the effects of shocks on asset prices (Gennotte and Leland (1990), Romer (1993), Caballero and Krishnamurthy (1999), Hong and Stein (1999), and Yuan (1999)). These models emphasize asymmetric information, liquidity, and financial constraints considerations.

There are models in the social learning literature that share many ingredients with the one presented here. (Caplin and Leahy (1994), Gul and Lundholm (1995), and Chamley (1998) present models with "informationally-driven" crises or clustering.) Although these models provide the basic intuition for why small shocks can give rise to large crises in the presence of private information, they do not provide an adequate framework for understanding the *timing* of BOP crises. The most important difference between the model in this paper and those in the social learning literature is given by the trade offs investors face in choosing their actions. In social learning models, which are usually concerned with industry dynamics or problems that give rise to similar "reduced-form" models, investors have an incentive to wait to observe other agents actions and face a cost of waiting. In episodes of BOP crises, on the other hand, investors have an incentive to move first (take their capital out of the country before the crisis takes place) and receive a flow benefit of waiting in the form of high returns on domestic assets. In addition, in episodes of BOP crises investors care about other investors' ac-

[8] Rigobon (1999) presents an alternative argument: "small shocks," if unexpected, can give rise to large reassessments about a country's fundamental.

tions not only because they reveal their private information, but also because in case of a crisis those who leave first have a higher probability of doing so before the devaluation takes place.[9] Finally, during BOP crises there is a "terminal condition," given by zero reserves at the central bank, which will play an important role in this paper but does not have a counterpart in social learning models.

Although there are no systematic studies of whether asymmetric information exists in the context of BOP crises, suggestive evidence exists. Evans and Lyons (1999) find a strong positive correlation between order flow[10] and price movements in the US$/DM exchange rate market, which is consistent with investors' trades revealing price-relevant private information. Garber (1998) argues that the existence of derivatives "obscures true risk positions and undermine the usefulness of balance-of-payments capital account categories." For example, according to IMF's International Capital Markets (1995), published 8 months after the Mexican devaluation, most of the Tesobonos outstanding at the time of the devaluation were held by foreigners (page 62). However, according to Garber, all of the US$ 16 billion worth of Tesobonos held by foreigners were involved in swaps with Mexican banks, so that all the risk was actually held by domestic banks. Furthermore, international investors do not share information on these types of trades, for they are considered proprietary. There exists an account of the events that led to the collapse in Mexico's bond market in which the crisis was triggered by investors' realization of the size of the total Tesobono swaps.[11] Johnson, Boone, Breach, and Friedman (1999) find that measures of corporate governance have a significant explanatory power for the size of devaluations and drops in local stock markets in a cross-section of countries during the Asian crisis. Under the assumption that investors have private information regarding the extent of corporate governance problems in the firms they invest, it is plausible that private information played a role in the crisis. Other "ev-

[9] In social learning models that incorporate non-informational externalities, such as Chamley (1998), complementarities give agents an incentive to move *simultaneously*. These models do not capture the incentive to pull out first.

[10] Evans and Lyons define order flow as "a measure of buying/selling pressure. It is the net of buyer-initiated orders and seller-initiated orders."

[11] Another piece of evidence that suggests that investors learned about the situation of the Mexican banking system during the crisis is given by the fact that, in January 1995, the stock prices of the banks fell much more than that of other companies, even though banks' stock prices closely followed the stock market index throughout 1994.

idence" includes the fact that, in many cases, crises are triggered when an identifiable group of investors "pulls out," such as when domestic investors refused to roll over Russia's debt in August 1997.

The paper is organized as follows. Section (2) describes the model under the assumption that the interest rate on domestic assets is exogenous. Section (3) solves and analyzes this simpler model. Section (4) focuses on interest rate policy by endogenizing the behavior of the monetary authority. Section (5) describes the robustness of the results under alternative assumptions. Section (6) concludes and suggests some speculative applications of the theory presented in this paper for contagion, asset-market bubbles, and banking crises.

2 The Model

To simplify the analysis, the model is based on a linear first-generation-type framework, with the additional assumption that investors have private information regarding the level of the exchange rate in case the peg is abandoned. Time is continuous and there are two kinds of players; a monetary authority, which attempts to keep a fixed exchange rate, and a set of investors, who at each point in time decide how much of their capital to invest in domestic assets. The state of the economy is summarized by a fundamental that deteriorates monotonically. While the peg lasts, investors receive a return on domestic assets which is higher than the international rate of return. If there is a speculative attack, the investors who are able to convert their holdings of domestic currency into foreign currency before reserves are exhausted do not suffer any capital losses. Others suffer losses equal to the size of the devaluation. The interplay between the benefit of being able to pull out before others and the high returns on domestic assets provides the main forces affecting the behavior of investors.

Monetary Authority

The monetary authority follows a simple rule: buy and sell foreign currency at the fixed exchange rate while reserves last.[12] Without loss of generality,

[12] In Section (4) the behavior of the monetary authority is endogenized, allowing it to set interest rates in order to delay, and possibly avoid, the crisis.

the exchange rate is fixed at 1. Once reserves are exhausted, the currency is floated.

Investors

Investors are risk-neutral. They initially have some capital invested in the country, for which they receive a riskless, constant, and exogenous return $r > 0$, unless a crisis occurs. At each point in time, investors decide how much of their capital to invest in the country, and how much to invest abroad. The international rate of return is 0. There are no transaction costs associated with capital movements. I also assume that investors have a maximum amount of capital (equal to their initial holdings for simplicity) and that there are no other investors who could invest in the country.[13]

Investors are heterogeneous and have private information regarding their idiosyncratic characteristics. The specific dimension of heterogeneity is not crucial for the qualitative predictions of the model but, for concreteness, I assume that investors differ in the amount of investments in domestic liquid assets. In the context of this model, liquid assets (e.g. short-term local-currency bank deposits) are assets which can be sold instantaneously and at a price which is fixed in local currency, while illiquid assets cannot be sold at any price.[14]

There are two groups of atomistic investors of mass 1 each. All investors within each group have the same amount of liquid assets or "type," denoted a_i for $i = 1, 2$.

Assumption 1 *Each investor knows his own type (and that of the rest of his group), but does not know the type of the other group. The a_i's are distributed with density function $g(\cdot)$ and support $[a_m, a_M]$. $g(\cdot)$ has no atoms and is common knowledge.*

The proportion of liquid assets invested in the country by investor $j \in [0, 1]$ of group $i \in \{1, 2\}$ at time t is denoted by $x(i, j, t)$.

[13]This assumption can be justified by assuming that investors are capital-constrained "specialists." In section (5) I will argue that if a pool of uninformed investors existed who could bring their capital to take advantage of the high returns, the results would be stronger.

[14]At least in principle, it is possible to determine the amount of foreign investment by looking at capital flows. However, information regarding the types of investments and off-balance sheet operations is much scarcer, e.g. Garber (1998).

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Environment

Time is continuous. Investors observe capital movements by all other investors.[15] The state of the economy at time t is summarized by a fundamental $f(t)$, which affects both the level of reserves and the value of the exchange rate if the government were to abandon the peg (i.e. the "shadow" exchange rate).[16]

Assumption 2 *Reserves at time t are given by*

$$R(t) = f(t) - \sum_{i=1,2} a_i \int_0^1 (1 - x(i,j,t))\, dj.$$

Assumption 3 *The shadow exchange rate at time t is given by*

$$E_s(t) = 1 + f(t) + e_0 - a_1 - a_2 \tag{1}$$

where $e_0 \in (0, a_m)$ *is a constant.*

As a result, the size of the devaluation, which is given by $a_1 + a_2 - f(t) - e_0$, is increasing in the amount of liquid assets.[17]

The fundamental $f(t)$ deteriorates monotonically at speed μ. Time is defined such that $f(0) = 2a_m - e_0$. I assume that the game starts at a time $\underline{t} < 0$ early enough such that there is an initial period when a devaluation cannot occur.

Assumption 4 *The fundamental* $f(t)$ *follows*

$$f(t) = (2a_m - e_0) - \mu t$$

In addition, $f(\underline{t}) > 2a_M - e_0$ *(i.e.* $\underline{t} < -2\frac{(a_M - a_m)}{\mu}$*).*

Since $f(t)$ falls at speed μ, the peg cannot last forever. Let $\bar{t}$ be the time at which the peg is abandoned, which is given by

[15] It does not make any difference if I assume investors only observe net flows.

[16] For example, the fundamental could be domestic credit, as in Krugman (1979) and Flood and Garber (1984).

[17] Equation (1) implies that the size of the devaluation is increasing in the amount of liquid assets that cannot be covered by existing reserves, which equals $a_1 + a_2 - f(t)$.

$$\bar{t} = \sup\left\{t : \forall\tau \in (\underline{t}, t)\, R(\tau) > 0\right\}, \tag{2}$$

i.e. when reserves at the central bank reach zero. If investors decide to pull out and reserves are not enough to cover all liquid assets, reserves are paid out according to a sequential servicing constraint. The investors who initiated the attack are able to exchange their domestic currency before others, and reserves are assigned randomly if they are not sufficient to cover a group that moves simultaneously.

A few technical assumptions are needed to rule out some forms of unrealistic behavior. Some of these assumptions will only be used in the appendix, where a formal treatment of the game is presented.

Technical Assumption 1 *The game is the limit, as $\epsilon \to 0$, of the game in which the strategies $x(i,j,t)$ can be conditioned on flows only up to time $t-\epsilon$.*

Technical Assumption 2 *Strategies must be "well-behaved." For all flow histories, $x^-(i,j,t) \equiv \lim_{\tau\to t^-} x(i,j,\tau)$ exists, $x^+(i,j,t) \equiv \lim_{\tau\to t^+} x(i,j,\tau)$ exists, and $x(i,j,t) = x^-(i,j,t)$.*

In equilibrium, investor i in group j chooses strategy $x(i,j,t)$, taking strategies $x(i',j',t)$ as given, to maximize

$$E\left[\int_{\underline{t}}^{\bar{t}} x(i,j,t)r\,dt - x^+(i,j,\bar{t})(1-E_s(\bar{t})) - (x(i,j,\bar{t}) - x^+(i,j,\bar{t}))\left(\frac{A(\bar{t})-R(\bar{t})}{A(\bar{t})}\right)(1-E_s(\bar{t}))\right]$$

where $\bar{t}$ is given by equation (2), and $A(t)$ is the amount of *desired outflows* at time t

$$A(t) = \sum_{i=1,2} a_i \int_0^1 (x(i,j,t) - x^+(i,j,t))\,dj.$$

The first term in the maximization problem accounts for the returns on liquid capital while the peg survives. The second term accounts for the devaluation losses from the capital that the investor did not attempt to take out at $\bar{t}$. The third term accounts for the devaluation losses from the capital that the investor attempted to take out, which incorporates the fact that this capital can be taken out with probability $\frac{R(\bar{t})}{A(\bar{t})}$.[18]

Technical Assumption 3 *The model is the limit of a model with transaction costs as these costs tend to zero.*

Technical Assumption 4 *Investors within each group have access to a "correlating device" that allows them to follow "mixed-like" strategies. The two groups have independent correlating signals that cannot be observed by investors in the other group (as in mixed strategies). In addition, strategies must be individual best responses since there are no commitment devices (as in correlated equilibria).*[19]

3 Analysis

As a benchmark, it is helpful to start by analyzing the model when there is no private information:

Proposition 1 *If a_1 and a_2 are common knowledge there is a unique Nash equilibrium. Investors stay in the country until time*

$$\bar{t} = -\frac{(a_1 - a_m) + (a_2 - a_m)}{\mu},$$

which satisfies $E_s(\bar{t}) = 1$. At that point they all try to leave, the peg is abandoned, and the size of the devaluation is zero.

Proof: It is trivial to show that the proposed solution is an equilibrium. To prove uniqueness note that, since $f(t)$ falls at speed μ, the peg must be abandoned, at the latest, when $f(t) = 0$. In pure strategies, investors cannot stay past $\bar{t}$ in equilibrium, since the crisis would involve a predictable depreciation. Mixed-strategy equilibria are not possible either, because they must involve randomizations over exit times up to the time when the crisis

[18] Returns on illiquid assets are not included in the maximization problem because these assets cannot be sold.

[19] Alternatively, I could assume that investors' types have infinite dimensions, with the first dimension being the liquidity of their investments, and the other dimensions being characteristics that are not pay-off relevant but on which investors can condition their actions.

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is inevitable. As a result, the "crisis hazard rate" would approach infinity at a point at which $E_s(t) < 1$, which cannot occur in equilibrium. □

This example shows that, in the model presented in this paper, the timing of crises is *independent of the interest rate r* when there is no private information.[20] The rest of this section studies the dynamics of crises and the effect of interest rates on their timing when private information is present.

For a formal analysis of the model, the reader should see the appendix. Here, I take the following proposition as a starting point, and present a more heuristic approach.

Proposition 2 *There is a unique and symmetric Nash equilibrium. The equilibrium is symmetric both between the two groups and between different investors in a single group. Symmetry within groups means that investors "move together," i.e. for all $j, j' \in [0,1]$ and $i \in \{0,1\}$, and for all histories of capital flows, $x(i,j,t) = x(i,j',t)$. In addition, investors always want to have either all their capital in the country, or all out, i.e. $x(i,j,t) \in \{0,1\}$.*

Proof: See appendix.

The analysis is greatly simplified by two features of the model. First, since investors are atomistic, they do not act strategically, i.e. they take the actions of other investors as given, as opposed to only their strategies. Together with the absence of transaction costs, this implies that the investors' maximization problem can be solved *pointwise*.

The equilibrium of the game is composed of several "stages." In the first stage, investors' types are private information and their strategies can

[20] Salant and Henderson (1978) and Lahiri and Végh (1999) show that raising interest rates can delay the timing of BOP crises by increasing the demand for domestic assets after the devaluation thereby "shifting up" the shadow exchange rate schedule. On the other hand, raising interest rates also has negative effects on the shadow exchange rate by accelerating the accumulation of government liabilities through higher debt service costs, bailouts of banks in distress, or a fall in revenue due to lower activity. Lahiri and Végh present a framework that combines these two effects. However, in these papers only *post*-devaluation interest rates have an effect on the timing of crises. As Calvo (1995) argues, raising interest rates before the devaluation likely has only negative effects on the shadow exchange rate, which prompts the question of why interest rate defenses often include raising interest rates *prior* to the devaluation. This paper provides a possible answer, focusing on the effects of interest rates on investors' learning process rather than on the shadow exchange rate. Drazen (1999) provides an alternative explanation based on the idea that interest rates can act as a signal about the government's objectives.

be summarized by a function $\bar{f}(a)$, which indicates at which value of the fundamental they would leave, conditional on their type. The first stage ends when a group of investors starts taking their capital out, thereby revealing their type. The other group then either leaves or stays, depending on their type. If the amount of their investments in liquid assets is high enough, investors in the second group leave, exhausting the government's reserves and ending the game; otherwise, they stay, the first group returns, and the second stage begins. In the second stage, the type of the group that initiated the first attack (type 2 without loss of generality) is thus known, but the type of the other group is only known to be below some value $\underline{a}$, consistent with not having pulled out. The equilibrium in this stage is characterized by a function $\bar{f}_1(a_1; a_2)$, which indicates at which value of the fundamental investors in group 1 would leave conditional on their type and that of group 2, and a hazard rate $\bar{h}(f; \underline{a}, a_2)$, which indicates the probability of group 2's leaving when the fundamental is f, conditional on their type and the maximum possible group 2's type. If group 1 leaves first, group 2 follows, reserves are depleted, and the game ends. If the attack is again initiated by group 2, investors in group 1 follow if the amount of their investments in liquid assets is high enough, or stay and stage 3 begins. Each stage thereafter is identical to stage 2, and the same functions $\bar{f}_1(\cdot)$ and $\bar{h}(\cdot)$ apply. In general, the functions $\bar{f}(\cdot)$, $\bar{f}_1(\cdot)$, and $\bar{h}(\cdot)$ would depend on the information investors acquire *during* each stage. However, it is not necessary to include this information separately because it is uniquely determined by the value of f.

Since investors' solve their maximization problem pointwise, each stage of the equilibrium can be solved independently. I start by describing the first stage, where most of the insights become clear, and briefly analyze the rest of the game later.

With some abuse of notation, let us define $a(t) = \bar{f}^{-1}(f(t))$, where $\bar{f}^{-1}$ denotes the inverse of $\bar{f}$.[21] $a(t)$ is then the "marginal type" or type that would leave exactly at time t. For t such that $f(t) > \bar{f}(a_M)$, we define $a(t) = a_M$, since $\bar{f}^{-1}(f(t))$ is not defined. Similarly, for t such that $f(t) < \bar{f}(a_m)$, we define $a(t) = a_m$. We can solve for $a(t)$ by noting that, in equilibrium, the marginal type must be indifferent between staying or leaving when the crisis hazard rate is positive.

[21] That such inverse exists follows from the fact that if $\bar{f}(a)$ where not strictly increasing, there would be a point in time with positive mass in the crisis probability distribution, which cannot occur in equilibrium since returns are only of order dt.

Proposition 3 *In the unique and symmetric Nash equilibrium of the game, the equilibrium in the first stage is characterized by the "marginal type" function $a(t)$. Investors take all their capital out of the country when $a(t)$ reaches their type. $a(t)$ satisfies the differential equation*

$$r = \frac{g(a(t))}{G(a(t))}(-\dot{a}(t))\left(\frac{2a(t) - f(t)}{a(t)}\right)(2a(t) - f(t) - e_0) \tag{3}$$

and the boundary condition

$$a(0) = a_m.$$

Proof: Even though the equilibrium is symmetric, the intuition is more clear if we start by assuming it is not. Let us define $a_1(t)$ and $a_2(t)$ as the marginal type functions for groups 1 and 2 respectively. In equilibrium, the marginal investor must be indifferent between staying or leaving. The returns outside the country are 0, while the returns inside the country consist of the sum of r and the expected losses from devaluation.

The expected losses from devaluation arise because when an investor in group 1 is in the country, there is a positive hazard rate for group 2's pulling out, in which case the investor would suffer devaluation losses with positive probability.[22] The hazard rate for group 2's pulling out is given by

$$\gamma_2(t) = \frac{g(a_2(t))}{G(a_2(t))}(-\dot{a}_2(t))$$

where $G(\cdot)$ is the cumulative distribution of $g(\cdot)$, and $\frac{g(a_2(t))}{G(a_2(t))}$ is the density of a_2 at $a_2(t)$, conditional on $a_2 \leq a_2(t)$. The probability of an investor in group 1 not being able to take his capital out conditional on group 2 pulling out is given by $\left(\frac{a_1(t)+a_2(t)-f(t)}{a_1(t)}\right)$, since after investors in group 2 take their

[22]To make this step rigorous, TA1 is needed. Otherwise, there could be other equilibria in which a group leaves even though r is higher than the expected devaluation losses due to attacks initiated by the other group. If one group left at such a time, and the reaction time were zero, the other group would follow *immediately* with positive probability. An investor would then have no incentive to deviate from this strategy, because no time elapses between the time at which he is supposed to leave and the possible crisis time. A more formal treatment of this point can be found in the appendix.

capital out only $f(t) - a_2(t)$ reserves are left. Finally, the new exchange rate would be given by equation (1). As a result, $a_1(t)$ and $a_2(t)$ must satisfy[23]

$$\begin{aligned} r &= \frac{g(a_1(t))}{G(a_1(t))}(-\dot{a}_1(t))\left(\frac{a_1(t)+a_2(t)-f(t)}{a_2(t)}\right)(a_1(t)+a_2(t)-f(t)-e_0) \\ r &= \frac{g(a_2(t))}{G(a_2(t))}(-\dot{a}_2(t))\left(\frac{a_2(t)+a_1(t)-f(t)}{a_1(t)}\right)(a_2(t)+a_1(t)-f(t)-e_0). \end{aligned}$$

Since the solution is symmetric, these equations are equivalent to equation (3), $a_1(t) = a_2(t) \equiv a(t)$, and $\gamma_1(t) = \gamma_2(t) \equiv \gamma(t)$. Finally, let τ be such that $a(\tau) = a_m$. Then, since $\gamma(t) \to \infty$ as $t \to \tau$, it must be the case that $E_s(\tau) = 1$; otherwise, some investors could suffer predictable capital losses by staying too long or miss predictable capital gains by leaving too early. This is equivalent to $a(0) = a_m$.[24] □

Figure 1 shows the marginal type $a(t)$ for different interest rates r.[25] It is clear that, for any values of a_1 and a_2, the first attack occurs later the higher r is. The intuition behind this result is that, although "learning" (which is related to $\dot{a}(t)$) can be faster when interest rates are high, the moment at which this learning starts is determined by the terminal condition. As a result, faster learning implies that more of it can take place closer to $t = 0$.

Let t_1 be the time at which the first stage ends, i.e. $a(t_1) = \max\{a_1, a_2\}$. After the initial attack either the peg is abandoned or the second stage of the game begins. Without loss of generality, let us assume $a_2 > a_1$, so group 2 is the first to leave.

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[23] Actually, if there are no transaction costs these differential equations must be satisfied only if $\dot{a}_1(t) < 0$ and $\dot{a}_2(t) < 0$. However, this problem does not arise under TA3. In the appendix I show that if $\dot{a}_i(t) = 0$ for some t, then $a_i(t') = a_M$ for all $t' \leq t$.

[24] For equation (3) to be valid, $\left(\frac{2a(t)-f(t)}{a(t)}\right) \in (0,1)$ is needed. This is satisfied at $t = 0$ iff $e_0 \in (0, a_m)$, which I assumed in A3. For earlier times, it is also satisfied if I assume $e_0 < a_m - (a_M - a_m)$. However, A3 is enough unless r is extremely high. In addition, even if the constrain were not satisfied by the solution described in the proposition, the qualitative behavior of the model would not change: the path of $a(t)$ would be less "steep" than the one proposed, but the effects of interest rates and the information structure on the timing of the crisis would be the same.

[25] Equation (3) can only be solved analytically for $r = 0$, in which case $a(t) = a_m - \frac{\mu}{2}t$. For $r > 0$, it can be shown that $a(t; r_1) < a(t; r_2)$ for all t if $r_1 < r_2$. In addition, $\dot{a}(0) = -\frac{\mu}{2} - \frac{r}{2}\frac{a_m}{e_0}$, which is useful for the numerical simulation.

Figure 1: Marginal type a(t) for different interest rates. $a_m = 0.5$, $a_M = 1.5$, $g(a)$ is uniform, $\mu = 0.1$, and $e_0 = 0.2$. Solid line: $r = 0$. Dashed line: $r = 0.05$. Dotted line: $r = 0.2$.

Proposition 4 *If $E_s(t_1) = 1 + f(t_1) + e_0 - a_1 - a(t_1) < 1$, investors in group 1 also leave and the peg is abandoned immediately. Otherwise, investors in group 2 return and the second stage begins. The equilibrium is characterized by a marginal type function $a^1(t)$, which denotes the type of investors in group 1 that would take their capital out at time t, and $h(t)$, which denotes the hazard rate of investors in group 2's pulling out. The function $a^1(t)$ satisfies the differential equation*

$$r = \frac{g(a^1(t))}{G(a^1(t))}(-\dot{a}^1(t))\left(\frac{a^1(t) + a_2 - f(t)}{a_2}\right)\left(a^1(t) + a_2 - f(t) - e_0\right) \tag{4}$$

and the boundary condition

$$a^1\left(-\frac{a_2 - a_m}{\mu}\right) = a_m. \tag{5}$$

The hazard rate $h(t)$ solves

$$r = h(t)\left(\frac{a^1(t) + a_2 - f(t)}{a^1(t)}\right)\left(a^1(t) + a_2 - f(t) - e_0\right) \tag{6}$$

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for $a^1(t) \leq f(t_1) + e_0 - a(t_1)$ *and equals zero otherwise.*

Proof: See appendix.

To understand the second stage of the game, consider the case when r is small. In this case, investors pull out in the first stage even when the expected devaluation losses are small, i.e. when t is such that $2a(t) - f(t) - e_0$ is close to zero. As a result, unless a_1 and a_2 are very similar, the first attack occurs *earlier* in the case with private information. However, there is a low probability that the group that did not initiate the attack has a type close enough to $a(t)$ so that the shadow exchange rate $E_s(t) < 1$, which implies that the currency will likely not be devalued in the first attack. Figure (2) shows this point graphically. Assuming group 2 attacks first, the solid line displays the learning process by investors in group 2, and marks the maximum possible type a_1. The dashed line is the marginal type $a(t)$ when no attack has taken place, and the dotted line is the marginal type $a^1(t)$ for group 1, conditional on group 2's type being known.[26] The marginal type $a(t)$ falls until it reaches $\max\{a_1, a_2\}$ at some time t_1. At that point, the peg is abandoned if the other type is such that the shadow exchange rate $E_s(t_1)$ is lower than 1. If the second group does not follow, the second stage begins, with an initial period of time when a crisis cannot take place. When the maximum type consistent with the peg having survived intersects the marginal type function $a^1(t)$, investors start learning again. At some point, either $a^1(t)$ reaches the type of investors that did not initiate the first attack, which triggers a successful attack, or the group that initiated the first attack pulls out again. (Not described in the figure.) A sequence of "probing" attacks can ensue, until one takes place when $E_s(t) < 1$, in which case the attack is successful. However, the size of the devaluation is small since investors do not take high risks for small r.[27]

The model provides a number of results regarding the effects of asymmetric information and interest rates on the timing of BOP crises, and the behavior of asset prices during such episodes. First, once we account for the presence of private information, interest rates have a significant effect on the timing of crises, in contrast with the case presented at the beginning of the section.

[26] The path $a^1(t)$ depends on which value a_2 takes.

[27] An empirical prediction of the model is that, when a country defends its peg very strongly, attacks should be less frequent but more likely to be successful. In addition, the devaluation should be larger.

Figure 2: Second Stage: Probing Attacks. $a_m = 0.5$, $a_M = 1.5$, $a_1 = 1.1$, $a_2 < a_1$, $g(a)$ is uniform, $r = 0.005$, $\mu = 0.1$, and $e_0 = 0.2$. Dashed line: $a(t)$. Dotted line: $a^1(t)$. Solid line: $\max\{a_1 : a_1$ is consistent with group 1 not having left$\}$.

Second, for any interest rate r, the presence of private information delays the crisis, in the sense that the peg lasts longer for all realizations of investors' private information. The presence of private information has two main effects. On the one hand, it introduces "noise," so that investors do not know precisely when the other investors are going to start leaving. As a result, investors can "coordinate" into staying in the country for a longer period of time and receiving the high returns.[28] This effect is illustrated in figure (1), as high interest rates "push back" the distribution of initial attack times.

On the other hand, private information makes investors "too optimistic" when the types are high (large amount of investments in liquid assets), and "too pessimistic" when the types are low (small amount of investments in liquid assets). The first (second) effect tends to make the first attack take place later (sooner) than under symmetric information. Although this seems

[28]The term coordination can be misleading, since investors are better off in the asymmetric information case only if μ is not large compared to r. If μ is not too large, though, one can think of the symmetric information case as a problem of coordination failure, since investors would like to commit to staying longer.

to imply that the peg should last less under asymmetric information when types are low, this is not the case because there is an asymmetric arbitrage. When one group leaves, the second group learns whether $E_s(t_1) < 1$, and would not leave unless this would cause the exchange rate to devalue. Namely, the second group can "correct" the mistake introduced by the first group's leaving too early, but it cannot correct the mistake if the first group left too late.

Third, in the context of BOP crises, private information gives rise to discontinuous drops in asset prices and, hence, complementarities in investors actions. The asymmetry in the movement of the exchange rate is due to the high returns inside the country, the fact that investors learn as the crisis progresses, and the existence of an agent (the monetary authority) which is willing to buy domestic currency even if a depreciation is expected. In addition, the model shows that a simple change in a conventional first-generation model can give rise to crises with characteristics similar to those of multiple-equilibria models.

Fourth, the model also sheds light onto the positive relationship between the rate of return and the speed at which learning occurs.[29] The reason is that the faster investors learn, the higher the crisis-hazard rate is and, thus, the higher the risk premium demanded by investors. However, in equilibrium an increase in the interest rate implies that the learning process starts *later*, as less time is needed to do the same amount of learning. As a result, it is the expectation of high interest rates in the future, with a correspondingly high learning speed, that makes investors stay now. Namely, high interest rates at a point in time push back the marginal type functions for all *earlier* times. This indicates the possibility of time inconsistency in the setting of interest rate policy.

To better understand the role of interest rates in the learning process, and in order to obtain implications for optimal interest rate policies during BOP crises, the next section endogenizes the behavior of the monetary authority. In addition, I will also show that the results presented in this section are not due to the assumption that crises are inevitable. In fact, they are strengthened if we introduce the possibility that there is a turnaround and fundamentals stop worsening.

[29] Stock (1987) found that, empirically, the business cycle evolves on an "economic time scale" rather than on a "calendar time scale." Interestingly, he also found that the most important determinant of the economic time scale is the short-term interest rate, which has an accelerating effect.

4 Interest Rate Policy

The model in the previous section revealed the existence of a significant relationship between interest rates, the speed at which investors learn from each other, and the timing of BOP crises. This section focuses on the implications of these results for optimal interest rate policy during crises. I also include the possibility that crises be avoided if pegs last long enough. As a result, the model sheds light on how the results presented in this paper depend on whether crises are "solvency" or "liquidity" crises.[30]

Interest rate policy is endogenized by introducing an objective function for the monetary authority.

Assumption 5 *The monetary authority minimizes the loss function*

$$L = E\left[1_{[\text{peg is abandoned}]}\bar{D} + 1_{[\text{deviate}]}\Phi + \int_{\underline{t}}^{\bar{t}} c(r(t))\,dt\right]$$

where $\bar{D} > 0$ is the cost associated with abandoning the peg, $\Phi > 0$ is the cost associated with deviating from a pre-announced interest rate policy, and $c(\cdot) \geq 0$ is the flow cost associated with raising interest rates. In addition, $c(0) = 0$, $c'(\cdot) > 0$, and $c''(\cdot) > 0$.

Without further changes, the equilibrium of the model would be trivial, since the crisis would take place regardless of interest rate policy. As a result, the monetary authority would set $r = 0$, and the peg would be abandoned as soon as the shadow exchange rate $E_s(t) = 1$.[31]

However, most crises have some liquidity component associated with them. For example, governments can implement policies to take a country out of an unsustainable path (such as increasing taxes or cutting government spending). In addition, the situation in international capital markets can

[30] The model presented in the previous section was one of solvency crises, since crises occurred with probability 1. In this section, crises are less likely if the monetary authority delays the learning process by increasing interest rates.

[31] Note that I assume a zero discount rate. This is not an unreasonable assumption, since episodes of BOP crises usually only last a few months. However, the monetary authority might be interested in postponing the crisis for political reasons, which might imply a higher discount rate. In any case, the "turnaround" hazard introduced below can also account for a discount rate, without affecting the equilibrium of the game.

improve and allow the country to find new sources of financing, or a positive terms-of-trade shock can take place. Usually, though, these developments take time and are not in the hands of the monetary authority. To capture such features of crises, I modify the model slightly to allow for the possibility of a turnaround in the economy.

Assumption 6 *With hazard rate ρ (or "turnaround" hazard rate), the game ends, the peg survives, and the monetary authority is spared all further interest rate costs.*

To solve the model, we first note that, although the equilibrium in the previous section was obtained for constant r, a similar equilibrium exists when r is not constant. The only difference is that r is replaced by $r(t)$ in equation (3). Let

$$t_0 \equiv \sup\{t : a(t) = a_M\}$$

be the time at which the learning process starts. It is clear that the monetary authority does not need to set $r > 0$ for $t < t_0$. In addition, the monetary authority will not announce an interest rate policy that is not credible. Then, for $t < t_0$, the loss function is given by

$$L[t, \{r(s)\}] = \int_{t_0}^{0} \left[c(r(s)) + 2\frac{g(a(s))}{G(a(s))}(-\dot{a}(s))D(s, a(s)) \right] G(a(s))^2 e^{-\rho(s-t)} ds$$

where $D(s, a)$ equals the expected losses, as of time s, conditional on the first attack being initiated at time s by a group with type a. Interest rate costs incurred *before* time s are *not* included in $D(s, a)$.[32] The other terms in the expression are the hazard rate of having an initial attack at time s, conditional on not having had a previous attack or a turnaround before time s, which equals $2\gamma(s) = 2\frac{g(a(s))}{G(a(s))}(-\dot{a}(s))$, the interest rate flow cost at time s, conditional on the same event, $c(r(s))$, and the probability that neither a turnaround nor an attack take place before time s, $G(a(s))^2 e^{-\rho(s-t)}$.[33]

[32] $D(s, a) = \bar{D}$ if the peg is abandoned at time s. If the type of the group that did not initiate the attack is low enough such that the peg is not immediately abandoned, $D(s, a) < \bar{D}$.

[33] Note that ρ enters in the loss function $L[t, \{r(s)\}]$ in the same way a discount rate would. As a result, ρ can be taken as the sum of the turnaround hazard rate and a discount rate.

In order to determine $D(s, a(s))$, one would need to solve a similar minimization problem. However, this introduces some additional difficulties because $D(s, a(s))$ is defined recursively.[34] To keep the problem simple, I assume that $r = 0$ after an initial attack.[35] The function $D(s, a(s))$ is then given by

$$\begin{aligned} D(s, a(s)) &= \bar{D}\left[\frac{G(a(s)) - G\left(f(s) + e_0 - a(s)\right)}{G(a(s))} + \right. \\ & \left. \int_s^{-\frac{a(s)-a_m}{\mu}} \mu \frac{g((f_m - \mu\tau) + e_0 - a(\tau))}{G(a(s))} e^{-\rho(\tau - s)} d\tau \right] \end{aligned}$$

The first term is the probability that the second group has a type such that the shadow exchange rate $E_s(s) < 0$, i.e. the probability that the devaluation occurs immediately. The second term takes into account the fact that, if the second group has a type such that the devaluation occurs later, the probability that the turnaround takes place is higher. Note that, since I assume $r = 0$, there are no further interest rate costs.

To simplify notation, let

$$k(s, a(s)) \equiv \frac{G(a(s))}{g(a(s))}\left(\frac{a(s)}{2a(s) - f(s)}\right)\left(\frac{1}{2a(s) - f(s) - e_0}\right)$$

Then, from proposition (3), $a(s)$ is given by

$$\begin{aligned} \dot{a}(s) &= -r(s)k(s, a(s)) \\ a(0) &= a_m \end{aligned} \qquad (7)$$

I first ignore issues of time inconsistency by assuming $\Phi = \infty$. The monetary authority's problem is then to choose t_0 and $\{r(s)\}_{t=t_0}^{t=0}$ to minimize

[34] Minimization of $D(s, a(s))$ involves an initial period with $r = 0$, followed by an increasing path for r.

[35] This assumption does not affect the qualitative results of the model because the peg is abandoned after the first attack with high probability. The reason for this is that the optimal interest rate policy involves postponing the learning process and, as a result, after the first attack takes place the type of the group that did not initiate the attack is likely large enough such that $E_s < 1$. On the other hand, the "unconstrained" $D(s, a(s))$ is steeper than the one assumed here, so the incentives to postpone the learning would be slightly lower without this assumption.

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$$
\begin{aligned}
L[t,\{r(s)\}] &= \int_{t_0}^{0}\left[c(r(s)) + 2\frac{g(a(s))}{G(a(s))}r(s)k(s,a(s))D(s,a(s))\right]\times \\
& \quad G(a(s))^2 e^{-\rho(s-t)}ds
\end{aligned}
$$

subject to equation of motion (7), $a(t_0) = a_M$, and $a(0) = a_m$.

Proposition 5 *The solution to the monetary authority's problem when $\Phi = \infty$ is characterized by*

$$
\begin{aligned}
\dot{a}(s) &= -r(s)k(s,a(s)) \\
\dot{r}(s) &= \frac{1}{c''(r(s))}\left\{[\rho D(s,a(s)) - D_t(s,a(s))]\, 2\frac{g(a(s))}{G(a(s))}k(s,a(s)) + \right. \\
& \quad c'(r(s))\left[\rho + r(s)2\frac{g(a(s))}{G(a(s))}k(s,a(s))\right] - \\
& \quad \left. c(r(s))2\frac{g(a(s))}{G(a(s))}k(s,a(s)) + c'(r(s))\frac{k_t(s,a(s))}{k(s,a(s))}\right\} \qquad (8) \\
r(t_0) &= 0 \\
a(t_0) &= a_M \\
a(0) &= a_m
\end{aligned}
$$

where a subscript t denotes the partial derivative with respect to time.

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Proof: See appendix.

The solution is more intuitive than it looks. The $\dot{a}(s)$ equation has been described above. The equation for $\dot{r}(s)$ is composed of four terms and a scaling factor. The first term captures the fact that it is desirable to postpone the expected cost D so that the peg has more chances of survival, and also if D is expected to fall. The second term is associated with the fact that the monetary authority would like to postpone raising the interest rate since there is a higher probability that the crisis or a turnaround take place earlier and the cost be saved. The third term is due to the fact that, by postponing the increase in the interest rate, and thus the crisis distribution, it is more likely that the interest rate cost in the future will be incurred. The fourth term is associated with the fact that if the "effectiveness" of raising interest rates is increasing, the monetary authority has an incentive to postpone raising them. Finally, the larger the smoothing incentives (i.e. the higher the second derivative of the cost function) the less the previous four effects matter.[36] The condition that determines t_0 comes from the desire to smooth $r(s)$.

The solid lines in figure (3) show paths for $r(s)$, $a(s)$, and the probability density of time of first attack, corresponding to the solution to the monetary authority's problem when $\Phi = \infty$.[37] The paths of $r(s)$ and $a(s)$ are conditional on not having had an attack or turnaround prior to time s. The solution is characterized by a long initial period of "tranquility," in which $r = 0$ and the probability of a crisis is zero. Towards the end, however, interest rates must be raised sharply at the same time that the probability of crisis increases. The figure clearly illustrates the results described in the previous section. First, the positive relationship between the interest rate and the speed of learning is *point by point*. Namely, along the optimal path of $r(s)$, times of high interest rates are times in which the probability of observing an attack is high. This is true for both the conditional probability (given by the slope of $a(s)$ divided by $G(a(s))$) and unconditional probability (given by the density function). In addition, it is optimal to have a sharply increasing path for the interest rate rather than keeping it at a low constant value. This is because interest rates push back the $a(s)$ schedule for all *earlier* times; as a result, it is more "efficient" to raise them late. In addition, the probability that interest rate costs are incurred decreases with the time at which they are raised since it is possible that the game ends before that time.

However, the fact that the benefits from raising interest rates at a point in time is in postponing the crisis for *earlier* times suggests that a problem of time inconsistency might exist. As a result, I next consider the case in which $\Phi = 0$. To analyze this case, I need to make an assumption regarding the point at which the monetary authority sets interest rates for times close to $t = 0$.

Technical Assumption 5 *The model is the limit, as* $\Delta t \to 0$, *of a model in which interest rates are constant within* $(-\Delta t, 0]$, $(-2\Delta t, -\Delta t]$, $(-3\Delta t, -2\Delta t]$,

[36] Note that partial time derivatives are present instead of total time derivatives. The reason for this is that the effect of $r(s)$ on the path of a exactly cancels out the term corresponding to the partial derivative with respect to a.

[37] There are two initial conditions and one final condition for system (8). To carry out the simulation, I iterated over different t_0 until $a(0) = a_m$. It should be possible to prove (I have not done it yet) that $a(0)$ is an increasing function of t_0 and, as a result, there exists only one path that satisfies all conditions.

Figure 3: Optimal path for $r(s)$, corresponding marginal type $a(s)$, and density function of times of first attack. $a_m = 0.5$, $a_M = 1.5$, $g(a)$ is uniform, $\mu = 0.1$, $e_0 = 0.2$, $\rho = 0.1$, $\bar{D} = 1$, and $c(r) = 0.5r + r^2$. Solid line: with full commitment. Dashed line: with no commitment ($r = 0$).

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and so for. In addition the interest rate for $s \in (-(n+1)\Delta t, -n\Delta t]$ is set at time $-n\Delta t$.

Proposition 6 *If $\Phi = 0$ and TA5 holds, the monetary authority cannot commit to* any *interest rate policy different from $r(s) \equiv 0$.*[38]

Proof: The proposition follows from a simple backward induction argument. Regardless of previous play, the monetary authority will set $r = 0$ for $s \in (-\Delta t, 0]$ at time 0. As a result, the peg must be abandoned at the latest at time $-\Delta t$ if $E_s(-\Delta t) < 1$. Assume that $r = 0$ for $s \in (-n\Delta t, 0]$ and that the peg must be abandoned at the latest at time $-n\Delta t$ if $E_s(-n\Delta t) < 1$.

[38] It can be shown that if $c'(0) > \frac{\rho\bar{D}}{8}\frac{a_m}{\mu e_0}$ the proposition is true even if the interest rate for $s \in (-(n+1)\Delta t, -n\Delta t]$ is set at time $-(n+1)\Delta t$.

The monetary authority then does not have any incentive to set $r > 0$ for $s \in (-(n+1)\Delta t, -n\Delta t]$ at time $-n\Delta t$. By induction, $r(s) \equiv 0$. $\square$

The dashed line in figure (3) shows paths for $r(s)$, $a(s)$, and the probability density of time of first attack, corresponding to the solution to the monetary authority's problem when $\Phi = 0$. The monetary authority cannot commit to raising interest rates and, as result, $r(s) \equiv 0$. The case of low interest rates was discussed in the previous section, and involves investors' leaving as soon as a devaluation is possible. This can be seen in the $a(s)$ schedule, which satisfies $f(s) + e_0 - 2\bar{a}(s) = 1$. The no-commitment case is then characterized by low interest rates, small devaluations, and vulnerable pegs.

In order to highlight the problem of time inconsistency, figure (4) illustrates the incentives to deviate from the optimal full-commitment interest rate policy. The solid line shows the expected *future* costs faced by the monetary authority as the crisis progresses, conditional on no previous attacks or turnaround. For early times the expected costs are an increasing function of time, since as time passes the probability that the turnaround takes place decreases. The expected costs eventually become larger than $\bar{D} = 1$, since they include both the likely devaluation and interest rate costs. As the interest rate costs become sunk, the expected future costs start decreasing. As $s \to 0$, the costs tend to $\bar{D} = 1$ since the crisis is imminent but no further interest rate costs need to be incurred. This is in sharp contrast with the behavior of the expected future costs if the monetary authority deviated from its pre-announced policy. The dashed line shows the expected costs, as of time s, assuming the monetary authority deviates at s and sets $r = 0$ thereafter.[39] The reputation cost Φ is not accounted for in the schedule. For early times, while $f(t) + e_0 - 2a_M > 1$, this path coincides with the expected costs under $\Phi = 0$, which are much higher than under full commitment. As the crisis progresses, though, the benefits from high future interest rates become sunk, and the two schedules start approaching each other. Eventually, the schedule becomes lower than that under commitment, and the monetary authority has an incentive to deviate.[40]

As a result, in order for the pre-announced full-commitment interest rate policy to be credible, Φ needs to be larger than the maximum distance be-

[39] Once the monetary authority deviates, Φ is sunk and it becomes impossible to credibly announce any policy different from $r(s) \equiv 0$.

[40] Note that the schedule with deviation can *never* be larger than $\bar{D} = 1$.

tween the two schedules in figure (4). For intermediate Φ, the monetary authority can only credibly commit to an interest rate defense which is less "aggressive."

Figure 4: Solid line: $L(t)$, as of t, conditional on reaching t before the first attack. Dashed line: Idem, but assuming $r = 0$ thereafter.

Finally, note that if investors did not have private information, the timing of the crisis would simply be given by the condition that the size of the devaluation be zero. But this is exactly the same condition that determines the time of devaluation under the no-commitment solution. As a result, as was hinted by the model in the previous section, the monetary authority is better off in the presence of asymmetric information. Note that this is not due to some form of transfer from investors, because with perfect information investors would get 0 ($r = 0$), whereas they get, in expectation, a positive return under imperfect information. In a sense, the monetary authority is willing to pay investors to stay longer in order to have a higher probability that the peg be saved. In addition, investors would like to "coordinate" into staying for longer in order to receive this payment, but they cannot do so unless there is private information.[41]

[41] I cannot make strong claims as to the welfare implications of private information, since this paper ignores important ingredients of BOP crises, such as moral hazard consider-

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Furthermore, the results presented in this section are *stronger* when crises have a higher liquidity component. The higher ρ, the larger the incentives to pre-announce a strong interest rate defense because it is less likely that their associated costs will need to be incurred. Also, it is important to note that the effect of future interest rates on the speed of learning does *not* depend on ρ. As a result, it is optimal to have a more aggressive interest rate defense when ρ is high.[42]

5 Robustness and Alternative Scenarios

Many of the ingredients of the model were introduced in reduced form. Apart from making the model more tractable, the reduced-form approach allows for a fairly general interpretation of the results. However, special attention needs to be paid to the question of robustness. This section explains which assumptions are essential for the results, and in which scenarios they are likely to be valid.

Monetary authority instruments:

In a more general setting, the government's decision to float, possibly when there are still some reserves left, could be endogenized. If the objective function of the monetary authority included a benefit from reserves left after the crisis, the problem of time inconsistency would be even more serious, as the monetary authority would have an incentive to devalue before selling its reserves.

Sources of private information:

If I assumed that investors have private information about the post-devaluation exchange rate, without assuming that there is any relationship between "types" and holdings of liquid assets, the results of the model would

ations. The results presented in this paper suggest that, although there might be good reasons for monetary authorities to require financial institutions to provide them with information regarding their activities, it might not be a good idea to make this information public. However, it is possible that monetary authorities that are better informed than investors would have even more time consistency problems than an uninformed one.

[42] An aggressive defense in the context of this paper means a commitment to raise interest rates sharply when fundamentals are very weak. When ρ is high, most times crises can be avoided by committing to raising interest rates in the few cases in which the turnaround does not take place. If ρ is low, it is not worth committing to raising interest rates because the peg will have to be abandoned with high probability.

not change. However, I prefer to assume that this information is due to some characteristic that also affects the size of the outflows because, otherwise, I would need to assume that reserves are enough to cover one group but not both. In the model presented in this paper, this constraint is satisfied without any special assumptions on initial reserves. Other sources of private information that would imply similar results include: (i) risk characteristics of bank lending, since banks have better information about their own clients than about those of other banks; (ii) liquidation value of investments; (iii) outside opportunities of investors; (iv) margin calls investors would be forced to make if a crisis occurs; and (v) investors' assessments about the prospects of the country.

Information structure:

The assumption of aggregate uncertainty is necessary for the results and, as a result, it is important that there be only two groups. However, similar results can be obtained with a unimodal distribution if two requirements are met: there exist "steep" edges or discontinuities, because if the distribution were smooth private information would be revealed slowly; in addition, if an investor with type a knows that the discontinuity in the distribution is to the left of $a + \epsilon$, he must assign a probability to the discontinuity being in $[a, a+\epsilon]$ that goes to zero as ϵ goes to zero. A distribution that satisfies these requirements is the one used in Chamley (1998): a rectangular distribution whose position is unknown, on top of a wider rectangular distribution. In addition, these two assumptions can be somewhat relaxed if one adds observation "noise." I chose to assume the existence of two groups for a number of reasons. First, this allows for the existence of "probing attacks," which are necessary to illustrate the one-sided arbitrage channel in which private information delays the crisis. Second, with the unimodal distribution I would need to make an ad-hoc assumption about the order in which investors that did not initiate the attack access foreign currency reserves, which might make the results suspect. Third, there are actually different types of investors and sometimes crises can be traced to the actions of one of them. For example, some researchers believe the behavior of hedge funds was important in the onset and spread of the Asian crisis, and versions exist about the Russian crisis being triggered by domestic investors refusing to roll over Russia's short-term debt which prompted a similar response from foreign investors.

Existence of excess returns:

The assumption that investors cannot bring in more capital is not crucial.

What is needed is that the informed investors (or specialists) be capital constrained in a way that their types are not revealed by how much more capital they bring in the run-up to the crisis. A totally inelastic supply of capital serves this purpose, but it is not necessary that the maximum amount of capital they have access to equal their initial holdings.[43] Adding uninformed investors with a more elastic supply of capital actually makes the effects presented in this paper stronger. To see this point, assume that uninformed investors bring $k(\bar{r}(t))$ capital, where $\bar{r}(t)$ are excess returns which take into account expected devaluation losses for investors who do not know a_1 or a_2. In addition, assume that when a group pulls out, the uninformed investors have the same probability of taking their capital out before the devaluation as the group of investors that did not initiate the attack. The only difference this would make to the equilibrium comes from the ratio $\left(\frac{2a(t)-f(t)}{a(t)}\right)$ in equation (3), which would be replaced by $\left(\frac{2a(t)-f(t)}{a(t)+k(\bar{r}(t))}\right)$. This would imply that learning could take place even faster, and the crisis would be postponed more than before.[44]

Different scenarios:

There are other possible scenarios that could be associated with the model presented in this paper. Most literally, one could think that investors have their capital deposited in local banks in domestic currency. Or that they have to decide whether to attack the currency by borrowing in domestic currency at the prevailing interest rate. Another possible scenario is that of a government which is trying to roll over short-term debt, with investors deciding whether the promised returns compensate for the risk of default. Even if government finances are in order, the private sector (especially domestic

[43]This simplification can be thought of as representing the fact that even though the supply of capital is not perfectly elastic, there is enough noise and uncertainty in the economy such that investors cannot infer other investors' types perfectly. In the context of herding in financial markets, Avery and Zemsky (1998) show that if there exists uncertainty in a large enough number of dimensions, it can take a long time for investors to learn even if they observe at which price other investors trade. In the context of my model, I could make the case that it is possible that investors are unable to determine other investors characteristics even if they observe capital flows.

[44]Uninformed investors are at an informational disadvantage. As a result, even if excess returns exist from the point of view of specialists, uninformed investors might still find it optimal to stay out. In addition, the existence of excess returns for informed investors does not mean that the free entry condition to become a specialist does not hold in the first place, since this decision is made earlier and might involve other costs.

banks) might face similar liquidity needs. The fundamental could then represent domestic credit, as in first-generation models, the size of government's or banks' short-term liabilities, or the size of bad loans in the financial sector. All that is needed is that investors receive high returns while the crisis does not occur, that their decisions about whether to invest and receive this return have an effect on the timing of the crisis, and that there be an incentive to be the first to "leave."

6 Concluding Remarks

This paper presents a framework for understanding the dynamics and timing of BOP crises. It shows that the presence of private information on the part of investors in a simple first-generation model can account for important features of BOP crises. First, crises can involve large drops in asset prices in the absence of large shocks even in a single equilibrium model. Second, even countries whose fundamentals are known to be weak can delay the onset of crises for long periods of time by raising interest rates.

The paper shows that the effectiveness of interest rate defenses increases with the degree of private information. When interest rates are low *or* there is no private information, pegs are abandoned at a time such that the exchange rate is continuous. When interest rates are high *and* there is private information, pegs last longer and are abandoned at a point such that the exchange rate depreciates by a large amount.

In addition, the paper shows that the optimal interest rate policy in episodes of BOP crises is to sharply raise interest rates if fundamentals deteriorate, rather than raising interest rates by a smaller amount for a longer period of time. However, a problem of time inconsistency arises. The monetary authority has an incentive to deviate and not to raise interest rates once fundamentals become weak enough. This emphasizes the importance of commitment devices such as currency boards or a role for international financial institutions such as the IMF.

The model also shows that crises are more likely when interest rates are high, even conditioning on the level of fundamentals. This has important implications for empirical studies on the effectiveness of interest rate defenses against BOP crises (Kraay (1999)). For example, an episode in which interest rates are raised but the monetary authority is nonetheless forced to abandon the peg could be taken as evidence that raising interest rates is not very useful

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in defending a currency under attack. However, in the model presented in this paper pegs are more likely to survive if interest rates are expected to be sharply raised in the future (i.e. strong defense) even though crises are more likely while interest rates are high.[45]

Finally, when there is a high probability that the peg is "viable" if the crisis can be postponed, these results are *stronger*. Such episodes can be associated with liquidity crises.

If the dimension along which investors have private information reflects some "intrinsic" characteristic, the model can be easily extended to account for the phenomenon of contagion. In such an extension, crises would only be transmitted to countries whose fundamentals are sufficiently weak.[46] In addition, an externality would exist between the setting of monetary policy in different countries since, by delaying the crisis in one country, monetary authorities delay the learning process in all other countries as well. This externality implies an important role for the IMF.

In future research, I intend to apply the framework presented in this paper to the areas of asset-markets bubbles and banking crises. The relationship between the rate of return on domestic assets and the speed of information revelation in the model presented in this paper is analogous to the relationship between the high returns to holding an asset with a bubble and the probability that the bubble "bursts." Under the assumption that investors have private information regarding the fundamental value of the asset (i.e. its price when the bubble bursts), bubbles could probably exist even if investors know that the bubble cannot last forever (for example if the price is limited by the size of the economy). In episodes of banking crises, the relationship between the interest rate on deposits and the probability that some investors assign to the bank failing due to other investors' withdrawals poses a similar trade off on investors' actions.

[45] In the aftermath of the Brazilian devaluation in January 1999 the Argentine peso suffered very little pressure. This is likely because investors knew that interest rates would be sharply raised in case of a speculative attack due to the strong commitment to the currency board. After the Mexican devaluation in December 1994, when this commitment had not been previously tested, the pressure on the Argentine peso was much greater.

[46] See Tornell (1998) for evidence that crises are more likely to be transmitted to countries with weak fundamentals.

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A Analysis of Model with Constant r

Here I present a more formal analysis of the model. Since investors are atomistic and there are no transaction costs, investors' actions are taken so as to maximize expected returns pointwise. Thus, the model can be divided in different "stages," depending on the information investors have about each other. At the beginning of the first stage, types are private information. As the stage progresses, information is slowly revealed until a first "crisis" occurs, in which a group of investors pulls out, revealing their type. If the attack is not large enough to force the abandonment of the peg, a second stage begins, which is similar to the first, except that now the type of one of the groups is common knowledge. The "interaction" between the different stages is limited, in the sense that the different stages can be analyzed almost independently.

The first stage of the game lasts until some investors "move" for the first time. To be more precise, let

$$A_i(t) = \int_j (1 - x(i, j, t))\, dj$$

be the proportion of their capital taken out by investors in group i. The first stage of the game ends at time

$$t_1 = \sup \{\tau : \forall \tau' \in (\underline{t}, \tau]\ A_1(\tau') = 0 \text{ and } A_2(\tau') = 0\}$$

since that is the first time at which some investors observe a "movement" by investors in the other group.

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First Stage:

Let

$$\begin{aligned} a_i(t, \nu) &= \{a_i : \sup \{\tau : \forall \tau' \in (\underline{t}, \tau]\ A_i(\tau') = 0 \\ &\qquad \text{if } \forall \tau' \in (\underline{t}, \tau]\ A_{-i}(\tau') = 0\} \in (t - \nu, t + \nu)\} \end{aligned}$$

be the set of types a_i such that, conditional on not having observed any movement by investors in the other group, the earliest time a positive amount of capital from investors in group i leaves the country falls in the interval $(t-\nu, t+\nu)$.[47] Let

$$a_i(t) = \bigcap_{\nu>0} a_i(t,\nu)$$

be the set of types a_i that would start pulling out exactly at t, conditional on the other group not having pulled out before. Let

$$T = \{t : a_i(t) \neq \emptyset \text{ or } a_{-i}(t) \neq \emptyset\}$$

Proposition 7 *In equilibrium, $\exists t_0 \in \left(-\frac{2(a_M-a_m)}{\mu}, 0\right)$ such that, for $i \in \{1,2\}$, $a_i(t)$ is a continuous strictly-decreasing function of t for $t \in [t_0, 0]$, $a_i(0) = a_m$, $a_i(t_0) = a_M$, and $A_i(t) = 0$ for $t < t_0$.*

Proof: The proof contains several intermediate steps:

(i) *For all t and ν, $a_1(t,\nu) = \emptyset$ iff $a_2(t,\nu) = \emptyset$. In addition, $\forall a_i \in a_i(t,\nu)$ $\exists a_{-i} \in a_{-i}(t,\nu)$ such that $a_i + a_{-i} - f(t+\nu) - e_0 > 0$.* This follows from the fact that investors can only condition their actions at t on flows up to $t-\epsilon$ (TA1). As a result, an investor in group i would not pull out at time $t' \in (t-\nu, t+\nu)$ if the probability of the other group pulling out is zero or if, even if the probability is positive, the crisis cannot bring about a positive devaluation.

(ii) *For all t, $a_i(t) = \emptyset$ iff $a_{-i}(t) = \emptyset$. In addition, $\forall a_i \in a_i(t)$ $\exists a_{-i} \in a_{-i}(t)$ such that $a_i + a_{-i} - f(t) - e_0 \geq 0$.* This follows from (i) and the definition of $a_i(t)$.

(iii) *If $a' \in a_i(t)$, $a'' \in a_i(t)$, $a'' > a'$, then $[a', a''] \subseteq a_i(t)$.* This depends on the form of the equilibrium in the second stage of the game, which will be analyzed below. For now, I just need that if a group starts pulling out and $a_i + a_{-i} - f(t) - e_0 > 0$ a crisis with positive devaluation takes place immediately with probability 1. As a result, if an investor of type a' finds

[47] It is not possible for investors to play mixed-like strategies in the first stage of the game in equilibrium. The reason is that if investors are indifferent between moving at two different times when they are of type a_i, they will strictly prefer to move at the earlier (later) time when their type is higher (lower) than a_i. As a result, since the probability of investors being of a certain type is zero (i.e. $g(a)$ has no atoms), the mass of investors who can play mixed-like strategies is zero. The situation is different in the following stages, since the type of one group of investors is common knowledge; hence, that group can play mixed-like strategies.

it optimal to leave (or is indifferent between leaving and staying) then an investor with more liquid investments will strictly prefer to leave. In addition, if an investor of type a'' has not left before time t, then an investor with less liquid assets would have strictly preferred to stay until t.

(iv) *For $i = 1,2$ and for all t $a_i(t)$ is either empty or a single point.* First, $a_i(t)$ cannot have positive measure. If it did, there would be a positive probability of group i reaching its "threshold value" at t. As a result, for all $a_i \in a_i(t)$ and $a_{-i} \in a_{-i}(t)$, $a_i + a_{-i} - f(t) - e_0 \leq 0$ since otherwise there would be a positive probability of crisis with positive devaluation at t which cannot occur in equilibrium. But then there would be $a'_i \in a_i(t)$ such that $a'_i + a_{-i} - f(t) - e_0 < 0$ for all $a_{-i} \in a_{-i}(t)$ which contradicts (ii). Since $a_{-i}(t)$ cannot be empty either due to (ii), I conclude that $a_i(t)$ cannot have positive measure. This, together with (iii) implies (iv).

(v) *T is dense in $T' \equiv [\inf\{T\}, \sup\{T\}]$.* That T is dense at $\inf\{T\}$ and $\sup\{T\}$ is obvious. Now assume $\exists\ \tau_1, \tau_2 \in T'$ such that $[\tau_1, \tau_2] \bigcap T = \emptyset$. Let $\tau'_1 = \sup\{t \in T : t < \tau_1\}$. Now I use the assumption that takes the model to be the limit of a model with transaction costs as these costs tend to zero (TA3). For any positive transaction cost, and regardless of how short $[\tau_1, \tau_2]$ is, $\exists \tau''_1 \in T$ that is so close to τ'_1 that the probability of having a crisis before τ_1 is low enough so that $a_i(\tau''_1)$ has to be empty.[48] This contradicts $\tau''_1 \in T$.

(vi) $T = T'$. Since $a_i(t)$ is either empty or a single point, and for every $a \in [a_m, a_M]$ $\exists t$ such that $a \in a_i(t)$, I can define a function $t_i(a) : [a_m, a_M] \rightarrow T'$ as the inverse of $a_i(t)$. $t_i(a)$ is decreasing and, from (v), its image is dense in T'. This implies $T = T'$.[49]

(vii) *For $i = 1,2$, $a_i(t) : T \rightarrow [a_m, a_M]$ are continuous strictly decreasing functions.* This follows from the fact that $a_i(t)$ is 1-to-1 and decreasing.

(viii) *$T = [t_0, 0]$, where $t_0 \in \left(-\frac{2(a_M - a_m)}{\mu}, 0\right)$.* If $t_i(a_m) < 0$, as $t \rightarrow t_i(a_m) < 0$ the expected devaluation losses would tend to infinite since the

[48] Investors would compare an arbitrarily small probability of crisis with the transaction costs associated with leaving at τ''_1 and coming back right after τ'_1, or the losses associated with being out of the country at a time when a crisis cannot take place (which is at least $r(\tau_2 - \tau_1)$).

[49] This must be a known result but I will present a proof. Assume $\tau \in T'$ but $\tau \notin T$. Since T is dense, $\exists\{\tau_1, \ldots, \tau_n, \ldots\}$ such that $\tau_n \rightarrow \tau$, $\tau_1 < \cdots < \tau_n < \tau_{n+1} \cdots < \tau$, and $\forall n$ $\tau_n \in T$. (For $\tau = \inf\{T\}$ a symmetric argument applies.) Let, for all n, $\alpha_n \equiv a_i(\tau_n)$. Then $\{\alpha_1, \ldots, \alpha_n, \ldots\}$ is a bounded decreasing sequence. Let α be its limit. If $t_i(\alpha) < \tau$, then τ_n is bounded away from τ and $\tau_n \not\rightarrow \tau$. If $t_i(\alpha) > \tau$, then it is impossible that $\tau_n < \tau\ \forall n$.

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hazard rate of crisis tends to infinite while the size of devaluation does not tend to zero. In addition, in equilibrium an investor of type a_m would not leave at $t_i(a_m) > 0$ because the devaluation would be negative with probability 1, and he would prefer to stay longer. Finally, $t_0 < -\frac{a_M - a_m}{\mu}$ is impossible because investors would not leave if, even in the case where $a_1 = a_2 = a_M$, liquid investment are not large enough to exhaust all reserves. □

Proposition 8 *The first stage of the game has a unique and symmetric equilibrium. The equilibrium is characterized by a function $a(t)$, which denotes the type of investors that would take their capital out at time t. $a(t)$ is continuous, strictly-decreasing, differentiable, and is the unique solution to differential equation*

$$r = \frac{g(a(t))}{G(a(t))}(-\dot{a}(t))\left(\frac{2a(t) - f(t)}{a(t)}\right)(2a(t) - f(t) - e_0) \tag{9}$$

that satisfies the boundary condition

$$a(0) = a_m. \tag{10}$$

Note: The function $a(t)$ is only defined for $t \in [t_0, 0]$, where t_0 satisfies $a(t_0) = a_M$.

Proof: The proof contains several intermediate steps:

(i) *For $i = 1,2$, $a_i(t) : T \to [a_m, a_M]$ is differentiable.* Since $a_i(t)$ is monotone it must be differentiable almost everywhere. (See Kolmogorov and Fomin (1970) page 321.) Thus, if $a_i(t)$ is not differentiable at τ, $\exists \nu > 0$ such that $a_i(t)$ is differentiable at all points in $[\tau - \nu, \tau + \nu]$ except at τ. I want to show that $a_i(t)$ must be differentiable from the left and from the right at τ. Assume it is not differentiable from the left. This means that $\exists \epsilon_l$ such that $\forall \nu' > 0$, $[\max\{\dot{a}_i(t) : t \in (\tau - \nu', \tau)\} - \min\{\dot{a}_i(t) : t \in (\tau - \nu', \tau)\}] > \epsilon_l$. But this is not possible because, since $a_{-i}(t)$ is monotone, the hazard rate of group 1's reaching its threshold value (which equals $\mu(-\dot{a}_i(t))$) cannot decrease arbitrarily fast. As a result, the derivatives from the left and from the right must exist at τ. But they cannot be different because $a_{-i}(t)$ is continuous at τ, which implies $a_i(t)$ is differentiable at τ.

(ii) *For $i = 1,2$ all investors in group i move simultaneously and take all their capital out at $t_i(a_i)$.* This follows from the fact that, for all $t > t_i(a_i)$, $\exists a_i' < a_i$ such that investors would leave at $t = t_i(a_i')$ if their type were a_i'.

But that means that for all $t > t_i(a_i)$ investors of type a_i are strictly worse off staying in the country than outside and, as a result, they would leave at $t_i(a_i)$.

(iii) *$a_1(t)$ and $a_2(t)$ are solutions to the system of differential equations*

$$\begin{aligned} r &= \frac{g(a_1(t))}{G(a_1(t))}(-\dot{a}_1(t))\left(\frac{a_1(t)+a_2(t)-f(t)}{a_2(t)}\right)(a_1(t)+a_2(t)-f(t)-e_0) \\ r &= \frac{g(a_2(t))}{G(a_2(t))}(-\dot{a}_2(t))\left(\frac{a_2(t)+a_1(t)-f(t)}{a_1(t)}\right)(a_2(t)+a_1(t)-f(t)-e_0) \end{aligned}$$

and satisfy

$$a_1(0) = a_2(0) = a_m$$

$$a_1(t_0) = a_2(t_0) = a_M$$

for some $t_0 \in \left(-\frac{a_M - a_m}{\mu}, 0\right)$. The boundary conditions were obtained in proposition 7. The form of the differential equations is derived in the main text.

(iv) $\forall t\ a_1(t) = a_2(t)$. The system of differential equations that determine $a_1(t)$ and $a_2(t)$ is symmetric and, in addition, $a_1(t)$ and $a_2(t)$ must satisfy the same initial condition. As a result, $a_1(\cdot) = a_2(\cdot)$. □

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Later Stages:

Without loss of generality, I assume that group 2 is the one that started pulling out at the end of the first stage and, as a result, a_2 is common knowledge. In addition, agents in the second group know that $a_1 \in [a_m, a_2]$. I start the analysis of the second stage assuming that $f(t)$ is high enough when the first stage ends, so that a crisis cannot occur immediately, i.e.

$$f(t_1) > 2a_2 - e_0.$$

This is impossible in equilibrium, but it is easier to start with this case. Let

$$\begin{aligned} a^1(t,\nu) &= \{a_1 : \sup\{\tau : \forall \tau' \in (t_1,\tau]\ A_1(\tau') = 0 \\ &\quad \text{if } \forall \tau' \in (t_1,\tau]\ A_2(\tau') = 0\} \in (t-\nu, t+\nu)\} \end{aligned}$$

be the set of types a_1 such that, conditional on not having observed any movement by investors in group 2, the earliest time a positive amount of capital from investors in group 1 leaves the country falls in the interval $(t - \nu, t + \nu)$.[50] Let

$$a^1(t) = \bigcap_{\nu>0} a^1(t, \nu)$$

be the set of types a_1 that would start pulling out exactly at t, conditional on group 2 not having pulled out before.

The characterization of group 2's play is different from that of group 1, because a_2 is known and, as a result, investors in group 2 can play mixed-like strategies. Let

$$\begin{aligned} d(t,\nu) \quad = \quad & \Pr\left[\sup\left\{\tau : \forall \tau' \in (t_1, \tau]\ A_2(\tau') = 0 \right.\right. \\ & \left.\left. \text{if } \forall \tau' \in (t_1, \tau]\ A_1(\tau') = 0\right\} \in (t - \nu, t + \nu)\right] \end{aligned}$$

be the probability that, conditional on not having observed any movement by investors in group 1, the earliest time a positive amount of capital from investors in group 2 leaves the country falls in the interval $(t - \nu, t + \nu)$. Let

$$d(t) = \lim_{\nu \to 0} \frac{d(t, \nu)}{2\nu}$$

be the probability density of investors in group 2's starting to pull out exactly at t, conditional on group 1 not having pulled out before.[51] Let

$$T = \left\{t : a^1(t) \neq \emptyset \text{ or } d(t) > 0\right\}$$

and

$$t_m = -\frac{a_2 - a_m}{\mu}.$$

[50] As in the first stage, group 1 cannot play a mixed-like strategy in equilibrium because the are no points with positive mass in the distribution of types a_1.

[51] If $\lim_{\nu \to 0} d(t, \nu) > 0$, I can define $d(t)$ like a distribution with positive mass at t, i.e. a "delta function." But it is not necessary to worry much about this because in equilibrium, as will be shown below, the limit always exists.

Proposition 9 *In equilibrium,* $\exists t_0^1 \in \left(-\frac{2(a_2-a_m)}{\mu}, t_m\right)$ *such that* $a^1(t)$ *is a continuous strictly-decreasing function of* t *and* $d(t) > 0$ *for* $t \in (t_0^1, t_m)$, $a^1(t_m) = a_m$, $a^1(t_0^1) = a_2$, $\int_{t_0^1}^{t_m} d(t)\,dt = 1$, *and* $A_i(t) = 0$ *for* $t < t_0^1$.

Proof: It is not necessary to present it because it is very similar to the proof of proposition 7.

Proposition 10 *When* a_2 *is common knowledge,* $a_1 \in [a_m, a_2]$, *and time starts at* $t < -\frac{2(a_2-a_m)}{\mu}$, *the game has a unique equilibrium in which all investors in each group share the same strategies. The equilibrium is characterized by a function* $a^1(t)$, *which denotes the type of investors in group 1 that would take their capital out at time* t, *and* $d(t)$, *which denotes the probability density of investors in group 2's pulling out. The function* $a^1(t)$ *is continuous, strictly-decreasing, differentiable, and is the unique solution to differential equation*

$$r = \frac{g(a^1(t))}{G(a^1(t))}(-\dot{a}^1(t))\left(\frac{a^1(t) + a_2 - f(t)}{a_2}\right)\left(a^1(t) + a_2 - f(t) - e_0\right) \quad (11)$$

that satisfies the boundary condition

$$a^1(t_m) = a_m. \quad (12)$$

The function $d(t)$ *is given by*

$$d(t) = \frac{d}{dt}\left(\frac{e^{\int_{t_1}^{t} h(\tau)d\tau}}{e^{\int_{t_1}^{t_m} h(\tau)d\tau}}\right)$$

where $h(t)$ *is the hazard rate of investor 2's pulling out, which satisfies*

$$r = h(t)\left(\frac{a^1(t) + a_2 - f(t)}{a^1(t)}\right)\left(a^1(t) + a_2 - f(t) - e_0\right). \quad (13)$$

Note: The function $a^1(t)$ *is only defined for* $t \in [t_1, t_m]$, *where* t_1 *satisfies* $a^1(t_1) = a_M$.

Proof: It is not necessary to present it because it is very similar to the proof of proposition 8. The only difference is in step (ii) of the proof. In this case,

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it is not necessary that investors in group 2 move simultaneously if there is a positive response time. However, only the equilibrium in which they move simultaneously survives in the limit as the response time goes to zero (TA1). Also, note that although in the proofs I used $d(t)$, $h(t)$ is a more useful characterization of the equilibrium. □

Now consider a case when

$$f(t_1) < 2a_2 - e_0.$$

In this case there are multiple equilibria; however, one of them strongly *dominates* the others. Let

$$\underline{a}(t; a_2) \equiv \max\{f(t) - a_2 + e_0, a_m\} \tag{14}$$

be the lowest a_1 such that $E_s(t_1) \leq 1$ (or a_m if no such a_1 exists). Also let

$$\bar{a}(t; a_2) \equiv \min\{a_2, a^1(t; a_2)\}.$$

Proposition 11 *When a_2 is common knowledge, $a_1 \in [a_m, a_2]$, and time starts at $t_1 > -\frac{2(a_2-a_m)}{\mu}$, the game has multiple equilibria. The equilibria are characterized by $a^* \in [\underline{a}(t_1; a_2), \bar{a}(t_1; a_2)]$. Investors in group 2 try to take their capital out immediately. Investors in group 1 do the same if $a_1 \in (a^*, a_2]$. If $a_1 \in (a^*, a_2]$, reserves are exhausted and the peg is abandoned immediately. Otherwise, investors in group 2 learn that $a_1 \leq a^*$ and bring their capital back. After that point the game follows the unique equilibrium described in proposition (10). Namely, there is a period in which no attack can occur, which lasts until time $t_0^1(a^*)$ such that $a_1(t_0^1(a^*); a_2) = a^*$. After $t_0^1(a^*)$ the learning process starts, following $a^1(t; a_2)$ and $h(t, a_2)$.*

Proof: In the proposed equilibria no investor has an incentive to deviate. First, at the beginning of the game investors in group 2 have an incentive to leave because there is a positive probability that group 1's type is such that a crisis immediately follows. In addition, an investor in group 1 also has an incentive to leave immediately if $a_1 > a^*$ because he knows that all other investors will leave and, as a result, he would suffer devaluation losses if he stayed. After that point, the proposed strategies constitute a unique equilibrium as proved in proposition (10). That no other equilibria exist follows from the fact that, if $a_1 < f(t_1) - a_2 + e_0$, there would be a *revaluation* if the peg were abandoned immediately and, as a result, investors in group 1 would not leave. □

Out of the continuum of possible equilibria, the one that corresponds to $a^* = \underline{a}(t_1; a_2)$ *dominates* the others. For example consider an equilibrium corresponding to $a^* > \underline{a}(t_1; a_2)$. Imagine, though, that an investor in group 1 thinks there is an arbitrarily small probability $\epsilon > 0$ that the equilibrium is actually the one corresponding to $a^{*\prime} \in [\underline{a}(t_1; a_2), a^*)$. Then if that investor had a type $a_1 \in [a^{*\prime}, a^*)$, he would think there is a *positive* probability ϵ that the devaluation takes place immediately. As a result, he would have an incentive to deviate and leave for a brief moment, to return only after observing that, in fact, the other investors in group 1 did not leave. The equilibrium corresponding to a^* should then not be expected to be played. Note that the equilibrium corresponding to a^* is dominated not only by the one corresponding to $\underline{a}(t_1; a_2)$, but also by all intermediate equilibria. In addition, investors would deviate even if they assign an arbitrarily small probability of deviation by other investors. The equilibrium corresponding to $a^* = \underline{a}(t_1; a_2)$ then strongly dominates all others.[52]

I can now give a full description of the equilibrium of the game when both types are private information.

Proposition 12 *When both a_1 and a_2 are private information, $a_1, a_2 \in [a_m, a_M]$, and time starts at $\underline{t} < -2\frac{(a_M - a_m}{\mu}$, the game has a unique and symmetric equilibrium, with a multi-stage structure. In the first stage, investors stay in the country until time t_1 such that* $\max\{a_1, a_2\} = a(t_1)$, *where $a(t)$ is determined by equations (9) and (10). Without loss of generality, I assume $a_2 > a_1$. At time t_1, investors in group 2 leave. If $a_1 > \underline{a}(t_1; a_2)$, where $\underline{a}(t_1; a_2)$ is defined in equation (14), investors in group 1 also leave, reserves are exhausted and the peg is abandoned. Otherwise, investors in group 2 return and the second stage begins. Investors stay in the country until investors in group 1 pull out when $a_1 = a^1(t)$, where $a^1(t)$ is determined by equations (11) and (12), or until investors in group 2 pull out, which occurs with hazard rate $h(t)$, where $h(t)$ is determined by equation (13). If the "attack" is initiated by investors in group 1, investors in group 2 follow, reserves are exhausted, and the peg is abandoned. If it is initiated by investors in group 2 and $a_1 > \underline{a}(t; a_2)$ investors in group 1 follow, reserves*

[52] This is an extreme form of *risk dominance*. See Harsanyi and Selten (1988) for an introduction to the concept of risk dominance.

are exhausted, and the peg is abandoned. Otherwise, investors in group 2 return and stage 3 begins. The game then continues with all other stages being identical to stage 2.

B Government's Problem with Commitment

In order to solve the problem, I find the first order condition with respect to changes in r at a particular point in time, taking into account the fact that t_0 needs to change accordingly in order for the boundary conditions to be satisfied. For any functional F, let

$$\frac{\partial F[\{r(s)\}]}{\partial r(v)} = \lim_{\epsilon \to 0} \frac{1}{\epsilon} \left. \frac{dF[\{\bar{r}(s; v, r, \epsilon)\}]}{dr} \right|_{r=r(v)}$$

where

$$\bar{r}(s; v, r, \epsilon) = \begin{cases} 0 & \text{for } s < t0(v, r, \epsilon) \\ r(t_0) & \text{for } s \in [t0(v, r, \epsilon), t_0) \\ r(s) & \text{for } s \in [t_0, v - \frac{\epsilon}{2}) \\ r & \text{for } s \in [v - \frac{\epsilon}{2}, v + \frac{\epsilon}{2}] \\ r(s) & \text{for } s \in (v + \frac{\epsilon}{2}, 0] \end{cases}$$

and $t0(v, r, \epsilon)$ is such that the boundary conditions are satisfied. This definition is just a way of formalizing the simple intuition that I look at the effect of changes in the interest rate at times close to v, tracking their effect on t_0.

I start by determining how changes in $r(v)$ affect $a(\tau)$. It can be shown that

$$\frac{\partial a(\tau)}{\partial r(v)} = -\frac{\partial \dot{a}(v)}{\partial r(v)} \phi(\tau, v)$$

where

$$\phi(\tau, v) = \begin{cases} e^{-\int_\tau^v \frac{\partial \dot{a}(l)}{\partial a(l)} dl} & \text{for } \tau < v \\ \frac{1}{2} & \text{for } \tau = v \\ 0 & \text{for } \tau > v \end{cases}$$

Then, I can obtain the effect of changes in $r(v)$ on t_0:[53,54]

[53] Note that $\frac{\partial \dot{a}(v)}{\partial r(v)} = -k(v, a(v))$.

[54] This expression is well defined only if $r(t_0) > 0$. This problem can be solved by

$$\frac{\partial t_0}{\partial r(v)} = \frac{k(v,a(v))}{k(t_0,a(t_0))}\frac{\phi(t_0,v)}{r(t_0)}$$

The effect of $r(v)$ on $L[t,\{r(s)\}]$ is calculated by taking into account the direct effects on $c(r(v))$ and the indirect effects through the path $\{a(\tau)\}_{\tau=t}^{\tau=v}$. It is given by

$$\begin{aligned}
&e^{-\rho t}\frac{\partial L[t,\{r(s)\}]}{\partial r(v)} = \\
&-\frac{k(v,a(v))}{k(t_0,a(t_0))}\frac{\phi(t_0,v)}{r(t_0)}e^{-\rho t_0}\left[c(r(t_0)) + 2g(a(t_0))D(t_0,a(t_0))r(t_0)k(t_0,a(t_0))\right] + \\
&e^{-\rho v}\left[G(a(v))^2 c'(r(v)) + 2g(a(v))G(a(v))D(v,a(v))k(v,a(v))\right] + \qquad (15)\\
&\int_{t_0}^{v}\left[2g(a(\tau))G(a(\tau))c(r(\tau)) + 2g(a(\tau))^2 D(\tau,a(\tau))r(\tau)k(\tau,a(\tau)) + \right.\\
&\quad 2g'(a(\tau))G(a(\tau))D(\tau,a(\tau))r(\tau)k(\tau,a(\tau)) + \\
&\quad 2g(a(\tau))G(a(\tau))D(\tau,a(\tau))r(\tau)k_a(\tau,a(\tau)) + \\
&\quad \left. 2g(a(\tau))G(a(\tau))D_a(\tau,a(\tau))r(\tau)k(\tau,a(\tau))\right]k(v,a(v))\phi(\tau,v)e^{-\rho\tau}d\tau
\end{aligned}$$

where the subscript "a" means partial derivative with respect to a.

I then set $\frac{\partial L[t,\{r(s)\}]}{\partial r(v)} = 0$, use $\phi(\tau,v) = \frac{\phi(t_0,v)}{\phi(t_0,\tau)}$ and $\frac{d}{dv}[k(v,a(v))\phi(t_0,v)] = k_t(v,a(v))\phi(t_0,v)$, and take the derivative with respect to v. After a few cancellations and rearrangements, I obtain

$$\begin{aligned}
\dot{r}(s) \;=\; & \frac{1}{c''(r(s))}\left\{\left[\rho D(s,a(s)) - D_t(s,a(s))\right]2\frac{g(a(s))}{G(a(s))}k(s,a(s)) + \right.\\
& c'(r(s))\left[\rho + r(s)2\frac{g(a(s))}{G(a(s))}k(s,a(s))\right] - \\
& \left. c(r(s))2\frac{g(a(s))}{G(a(s))}k(s,a(s)) + c'(r(s))\frac{k_t(s,a(s))}{k(s,a(s))}\right\}.
\end{aligned}$$

Finally, setting $v = t_0$ in equation (15) and equating to zero, I obtain[55]

$$c(r(t_0)) = r(t_0)c'(r(t_0)).$$

assuming $r(t_0)$ or $r(0)$ rather than t_0 are adjusted to satisfy the boundary conditions. The solution presented here is right nonetheless.

[55] Since $c(0) = 0$, this condition is equivalent to $r(t_0) = 0$.