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# "Banking Crises, Implicit Government Guarantees, and Optimal Insurance Scheme."

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# Banking Crises, Implicit Government Guarantees, and Optimal Insurance Scheme[1]

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Abstract

After major banking crisis, investors and academics alike are left wondering how it could have been avoided. Crises can take an enormous toll on society. Mexico's 1994 crisis cost almost 10% of GDP. Chile's 1983 crisis was even worse, with the final cost amounting to a stunning 30% of GDP. Moreover, the economy can experience a traumatic recovery process that in some cases lasts several years. The most common explanation of banking crises focuses on the anticipation of government bail out. This mechanism takes place when investors expect that the government will help them cover their losses in case they face a generalized adverse shock.

The paper shows how an insurance scheme eliminates the externality generated by the above government bail out policy. As an example, the paper analyzes the case of liquidity risk, defined as an unexpected cash withdrawal, and it presents a scheme to deal with this risk. This scheme works as an insurance where each bank pays a premium depending on the bank's risk. The scheme used in Argentina where the Central Bank charges to each bank a premium that depends on the bank's liquidity position, for an insurance which the Central Bank acquires in the international markets is an empirical example.

Besides the insurance scheme, a competitive tool to deal with liquidity risk is the existence of a lender of last resort. This paper introduces a lender of last resort and it shows what the optimal punitive interest rate is, which is derived from the model developed in the paper.

In addition, a new procedure is developed to estimate the social cost of a bank crisis which is different from the net transfer from the government to the banking sector and independent of the existence of the crisis.

# 1 Introduction

After a banking crisis, a question that arises among investors and academics is, could it have been avoided? First, right after a banking crisis, the government needs an enormous amount of resources to avoid a generalized bankruptcy. As an example, the Mexican crisis in 1994 cost almost 10% of GDP, while Chile's government during the 1983 banking crisis needed around 30% of GDP.

Note that the costs mentioned above are the net transfers from the government to the financial sector at the time of the crisis. In addition to this government transfer, the economy is typically involved in a traumatic recovery process, that could even last several years. The painfulness of this recovery process is due not only to the government policies to raise resources in order to overcome the crisis (i.e. tax increase, inflation or some other creative policies), but also to the economy's restructuring (i.e. financial system reorganization and other resource allocation). Therefore, the estimation of the total cost should include more than the net transfer from the government to the financial system.

Besides the cost generated by a financial crisis, the financial system has particular characteristics. As shown by Edwards and Vegh (1996), it can transmit a shock generated in one economic sector to the whole economy in a short period.

To formalize the bank's problem, the paper borrows from Freixas and Rochet (1997) a risk classification for financial institutions, where the bank's uncertainty can divided in three types: a) Default risk, when borrowers are not able to repay the debt; b) Liquidity risk, banks must make unexpected cash payment such as deposit withdrawal or pay back loans; and, c) Market risk, when some external shock affects the portfolio of marketable assets of the bank. Taking this classification into account, the paper analyzes only the second case, when banks have to honor their liabilities in a short period. The reason to model only liquidity risk is that introducing other types of risk will not make a significant contribution to the analysis.

During a banking crisis, though an economy faces countless problems, one of the most remarkable is a liquidity constraint. Moreover, from time to time this liquidity constraint can drive the country into a generalized crisis. For example, after the Mexican Crisis in December of 1994, the financial system in Argentina faced a strong liquidity constraint that almost brought down the stabilization plan implemented in March of 1991. If the Argentinean Central Bank had not reduced banks' reserve requirements to inject liquidity into the system, plus the support program by the International Monetary Fund, the resolution of the crisis could have been totally different. This liquidity risk might not be the cause of a banking crisis itself, but without doubt it plays a significant role during the crisis. The paper will use this risk to introduce the government bail out policy.

Among the generating mechanisms of financial crises, the anticipated ex-post government bail out policy emerges as a competitive explanation. A bail out is when investors know that in case of a generalized negative shock, the government will help them pay for the losses. This paper will show how an insurance scheme can eliminate bank crises generated by a government bail out.

Concerning this government intervention policy in recent episodes, Krugman (1998) pointed out that the implicit government guarantee to financial intermediaries was responsible for the South Korean crisis in 1997. According to Krugman, financial intermediaries knew that the government would help them in case of a crisis. Hence, they did not consider, in their investment decision, a negative shock as a possible outcome. In addition, he mentioned that this implicit subsidy could generate a social cost. With respect to the social cost, this paper develops an alternative methodology to measure it.

The first part of the paper analyzes the effect of the asset liquidity degree on the bank's profit maximization problem. In other words, how the liquidity degree of assets affects the bank's investment decision. The liquidity problem is formulated as follows. After a bank receives deposits, it has to decide whether to invest in loans or government bonds. At the time of the investment decision, the bank knows that with certain probability depositors will withdraw their deposits at the end of the period. In case of a bank run, the bank can sell its bond holdings in the market at face value to honor depositors. If the bank does not have enough bond holdings to honor depositors, it will need to sell loans in the market at a discount, generating a loss.

The natural question at this stage should be, why would a bank be willing to provide loans if it could generate a loss in case of a bank run? The reason is that the loan interest rate is higher than the return on government bonds.[1] Driven by this trade-off between risk and return, banks should find an optimum relation between loans and government bonds. So far, there is nothing new because a bank, as any other firm in the economy, has revenues and costs from its transactions. So, these financial institutions can determine the optimum holding of assets from this trade-off between costs

[1]This assumption will be shown explicitly later.

and revenues. The problem arises when there is an expected government guarantee on bank operations. In this case, the bank would not consider the assets discounted price as a loss in the maximization process, because the bank knows that the government will pay for it.

With respect to the alternative ways liquidity was modeled in the literature, the following methodologies come to mind: A) Diamond and Dybvig (1983) motivated one direction of research, where the basic model assumes two periods and two assets. In addition, the authors assume that the long run asset has a higher return than the short run asset and banks do not know if they will face a run at the end of the first period. Now, suppose banks face a run at the end of the first period. If banks do not have enough liquid assets to honor depositors, they will need to sell long run assets at a discounted price generating a capital loss.

In the Diamond and Dybvig (D&D) model, any mechanism that can stop the run in the first period increases the wealth of the economy. The increase in wealth is explained by holding higher return (long run) assets. A difference between D&D and the model presented in this paper is that D&D did not introduce a probability distribution for the bank run; they assume that the run is generated by a change in investor's expectations. On the other hand, the model presented in this paper introduces a probability distribution for the bank run. The justification for introducing a probability distribution for the bank run is grounded in the following example. Suppose that an investor has a CD in an emerging market. When US interest rates rise, the investor would like to withdraw the money from the emerging market to invest it in US financial market. Thus, the probability distribution of the bank run would come from the probability distribution of the US interest rate.

B) Ad-Hoc liquidity cost. This methodology introduces an ad-hoc cost for being illiquid in the bank profit function. In this case, the illiquidity cost is determined by the amount of the liquidity shortage times a predetermined punitive interest rate. The optimal bank liquidity position is found by maximizing the profit function. In the same fashion, this paper presents the lender of last resort, but this time the punitive interest rate is determined by the model. It means that the interest rate is determined by the liquidity risk position taken by the bank.

C) Banks as Market makers. Following the idea of a broker role in the exchange market, where the bid-ask prices are derived from the broker's profit maximization problem, this methodology derives the interest rate spread from the bank's profit maximization problem.[2]

Summarizing, the model developed in this paper is a combination of both, the Diamond and Dybvig structure and of banks as market makers. It is a one period, two assets model. Banks do not know if they will face a run at the end of the period, but they know the probability distribution of a bank run. In the case that banks face a run, and they do not have enough liquid assets, they will need to sell illiquid assets at a discount to honor depositors. Next, the paper introduces a lender of a resort as a competitive tool to deal with liquidity risk, and it shows what is the optimal punitive pre-determined interest rate.

The model presents several attractive characteristics. Despite its simplicity, it is flexible enough to simulate the effects on the economy of a wide variety of shocks.

Second, it models government bail out. The paper studies a regime where banks know that in case of a generalized crisis, the government will help them to avoid a generalized bankruptcy in the banking system. In addition, the behavior of the loan and deposit interest rates and of the spread between them is analyzed. On the other hand, the paper develops an insurance scheme to eliminate the externality generated by the government bail out policy.

Third, the paper derives a new procedure to estimate the social cost of the government intervention.

In section II, the model is derived. Section III analyzes the comparative statics. This section shows how the equilibrium reacts to: a capital inflow, an increase in the probability of bank run, an increase of the asset liquidity degree, and an increase in the power of the run. Section IV examines the welfare cost of a government intervention and shows how an insurance scheme could eliminate the government externality, in addition, this section presents the optimal intervention of a lender of last resort. Section V concludes.

# 2 Section II

## 2.1 Agents in the economy

The model includes the following participants: the Central Bank, a private bank, and a collection agency. For the sake of clarity, this paper abstracts

[2] In the model presented in the paper, spread should be interpreted as the difference between loan interest rate and the deposit interest rate. And the asset characteristic considered would be the asset liquidity degree.

from the problem of production and consumption by assuming an ad-hoc demand for loans and an ad-hoc supply of deposits to complete the deposit and loan market.

In this economy, the Central Bank imposes a reserve requirement on private banks, sets the exchange rate devaluation and acts as an insurance company. The private bank receives deposits, and uses its net worth plus the difference between deposits and reserve requirements to invest in government bonds, and loans to firms.

The collection agency buys financial assets sold by banks and recovers the face value of these assets after paying a cost associated to the liquidity level of the asset.

The model assumes a small open economy integrated with the rest of the world. This, together with a predetermined exchange rate, implies that the nominal interest rate is the real interest rate plus the devaluation rate $(i = r + \epsilon)$.

## 2.2 Bank Profit Maximization Problem

The banking literature typically refers to the mismatching problem as the difference in timing between assets and liabilities. Instead, this paper will focus on the mismatch of the liquidity levels of assets and liabilities. The reason for this is the fact that the disparity in liquidity levels and not the time structure determines the capital loss when an asset is sold immediately. For example, there should be no problem in investing in a 30 year US T-bill if banks, at the time they need cash, could go to the market, and sell it without a capital loss.

This liquidity mismatch is crucial in the case of bank run, because the liquidity shortage can generate a huge loss. This possible cost raises the following question: Why would banks prefer to invest in illiquid assets if this can create a loss in the case of bank run?

As a possible answer, Mendelson and Amihud(1986) showed that asset return is an increasing and concave function of the asset illiquidity degree. In other words, when the asset liquidty degree is decreasing, the asset return is increasing but at a lower rate. Therefore, banks can increase their profit by holding more illiquid assets[3].

What happens if depositors want their money back? If banks do not have enough liquid assets to honor depositors, they need to sell other assets

[3]Here, the model assumes that loans are the illiquid assets and government bonds and deposits are the liquid assets and liquid liabilities respectively.

to obtain resources. Selling illiquid assets will generate a huge cost. And sometimes, this loss can produce the bankruptcy of the institution.

Note that given the trade-off between the gain from a higher interest rate and the cost of liquidation, banks are able to find an optimum liquidity ratio. Using this trade-off, banks should determine the optimal composition of the bank's portfolio in government bonds, loans, and reserve requirements.

Next equation presents the bank's balance sheet.

$$a = h + b + z - d \tag{1}$$

Here, $a$ is net assets, $b$ government bonds, $h$ reserve requirements, $z$ loans, and $d$ deposits. The paper assumes that the Central Bank imposes a reserve requirement $h = \delta d$.

The investment opportunities faced by the bank are loans ($z$) and government bonds ($b$). The asset characteristics are summarized in its rate of return, and the only difference between the investment opportunities (loan and bonds) are their liquidity degree. The interest rate for z (less liquid asset) is higher than the interest rate of government bond (liquid asset), according to stylized facts in Mendelson and Amihud(1986).

The sequence of events in the model will help to clarify the problem faced by banks.

At the beginning of period, banks receive deposits, invest in loans and government bonds, and hold reserve requirements. At the end of the period, banks discover whether or not they face a bank run.

In case of a bank run, if banks have enough liquid assets (government bonds plus reserve requirements[4]) to honor depositors, they will not need to sell assets and no loss occurs. However, if they do not have enough liquid resources, they have to sell their assets to honor depositors, which generates a loss.

Banks's profits are given by

$$\pi = [(1+i)a + (i^L - i)z + (i - i^d)d - ih] - \sum_{k=1}^{2} q_k \max(0, (\frac{\alpha_k d - \delta d - b}{P(w)}))[1 - P(w)] \tag{2}$$

[5]

Where

[4] The paper assumes that in case of a bank run the Central Bank allows banks to use the reserve requirement to honor depositors.

[5] The derivation of this equation is in Appendix A.

$i^d$ Deposit interest rate.

$i^L$ Loan interest rate.

$i$ Bond interest rate.

$k = 1$ indicates a bank run and $k = 2$ indicates no bank run.

$P(w)$ Fire price of assets.

$\alpha_k$ the percentage of deposits that banks have to honor in state of nature k. Where $\alpha_2 = 0$, and $\alpha_1 = \alpha, 0 \preceq \alpha \preceq 1$.

$\delta$ is the percentage of reserve requirements.

$q_1$ Probability of a bank run.

The first terms in brackets of equation (2) represents the bank operational revenue, which consists of: a) nominal interest rate earning on bank net worth, b) The loan return over the opportunity cost $(i^L - i)$ times the amount of loan, c) The nominal interest rate minus the deposit rate $(i - i^d)$ times the amount of deposit, and d) The cost of the reserve requirement.

The second part of equation (2) analyzes the bank's uncertainty. This uncertainty is a cost that will depend on the revealed state of nature, where the state is determined by whether or not a bank run occurs.

In the case of a bank run, the bank should honor a percentage $\alpha$ of the deposit. If $\alpha d > \delta d + b$, then, the bank would need to sell assets to honor depositors. The amount of assets sold, $(\frac{\alpha_k d - \delta d - b}{P(w)})$, times the loss per asset sold, $[1 - P(w)]$, is the total capital loss[6]. In the case of no bank run, or $\alpha d < \delta d + b$, this cost is zero because banks do not need to sell assets.

At period zero banks do not know for sure the outcome at the end of the period, but they know the expected value of the loss, which is the loss in each state times the probability of that state of the nature.[7]

Now, it is clear how the trade off between assets works. From the first part of the equation (2), an increase in loans, given a constant $d$, will increase the bank's profit, because the loan interest rate $(i^L)$ is higher than the bond interest rate $(i)$. Nevertheless, this reallocation can generate a capital loss in case of bank run, because the difference $\alpha d - \delta d - b$, will be higher. In other words, banks will need to sell more assets to honor depositors.

Using the basic insurance literature and assuming that the bank is risk neutral, the second part of the profit function can be rewritten as the cost of an insurance that covers possible capital losses. From the microeconomic

[6] $P(w)$ is price of the asset when banks need to sell it immediately. $w$ is the amount of resources, above the available liquid assets held by banks, needed to honor depositors. The derivation of $P(w)$ and $w$ will be shown later.

[7] It is important to emphasize that, although a constant $q$ is assumed, I do not believe that this probability is invariable. Moreover, I think that understanding $q$ further will help us to design better policies to manage this type of shocks.

literature, it is a well-known result that a risk neutral individual is indifferent between playing a deterministic game or buying an insurance for the fair price. Both problems give the same expected result.

The possible outcomes of the game are: no loss, if there is no bank run; and a capital loss if there is a bank run. The equivalent insurance scheme means that banks should pay a premium, and the insurance company will pay the loss in case of a bank run. Introducing the insurance scheme in the bank maximization problem will not change the result, but this procedure will facilitate the mathematical exposition and the interpretation of the results.

Plugging the insurance scheme in the bank problem, its profit function becomes

$$\pi = (1+i)a + (i^L - i)z + (i - i^d)d - h - J(w), \tag{3}$$

where $J(w)$ is the insurance premium paid by banks and $w$ is the bank's risk type. A lower $w$ means a lower risk.

### 2.3 What is $J(w)$?

The model assumes that the Central Bank not only imposes a reserve requirement ($\delta$), but also plays the role as an insurance company. As such, the Central Bank will charge a premium to banks depending on their liquidity position. In the event of a bank run, the Central Bank should pay for the bank's capital loss. If the bank run does not occur, the Central Bank's payment is zero because the loss does not occur.[8]

The risk level associated to a bank is determined in the following way.

Assuming the bank has zero net worth, its balance sheet is:

| Assets | Liabilities |
|---|---|
| $z$ | $d$ |
| $\delta d$ | |
| $b$ | |

In case of a bank run, the bank has to honor the amount $\alpha d$ of deposits. Then, if $\alpha d < \delta d + b$, the bank will have enough liquid resources to honor depositors. However, if $\alpha d > \delta d + b$, the bank should sell assets because its liquid resources are not enough to honor depositors.

[8] This insurance scheme works exactly as any typical insurance. Using as an example the case of the car market insurance, the event of a car accident is equivalent, in this paper, to a bank run. The insurance payment instead of being for fixing the car is for the bank capital loss.

The risk level associated to a bank is the amount of resources over the liquid asset holdings that it needs to honor depositors in case of a bank run.

Define $w$ as the amount of resources, above the available bank's liquid assets, needed to honor deposits:

$$w \equiv \alpha d - \delta d - b \tag{4}$$

Adding and subtracting $z$ in the first part of the equation, and plugging the bank's balance sheet identity $d = \delta d + b + z$, $w$ can be rewritten as,

$$w = (\alpha - 1)d + z \tag{5}$$

Using the above equation, the following expression is obtained:

$$m = \frac{w}{z} = \frac{d(\alpha - 1)}{z} + 1 \tag{6}$$

$m$ measures the amount of resources needed per unit of loan to honor depositors in the event of a bank run. Note that this variable depends on the ratio of deposits and loans. Intuitively, when the ratio $\frac{d}{z}$decreases, the bank needs more resources per loan to avoid bankruptcy. This relationship can be observed in graph 1.

The y-axis is a measure of the Central Bank's cost due to the run. As we move closer to 0 in the x-axis, the cost is higher. If $\frac{d(\alpha-1)}{z} \leq -1$ the cost would be 0 because the bank has enough liquid resources to cover the run (it is the case when $\alpha d < \delta d + b$). The higher $m$, the higher the Central Bank cost, which should indicate a higher premium for the bank.

Formally, the insurance premium is obtained as a fair insurance. This fair insurance premium is the value of $J(w)$ that makes the following equation hold:

$$\{J(w) - [\frac{1 - P(w)}{P(w)}]w\}q + J(w)(1 - q) = 0 \tag{7}$$

where $\frac{[1-P(w)]}{P(w)}w$ is the amount of the capital loss in a bank run.

Hence, the premium is given by:

$$J(w) = [\frac{1 - P(w)}{P(w)}]wq \tag{8}$$

Taking price P(w) as given, but replacing w by (5), the premium can then be written as[9]

[9] Banks chose $w$, but take $P(w)$ as given. Next section shows how this price is determined in the market.

$$J(w) = \{(\alpha - 1)d[\frac{1-P(w)}{P(w)}] + z[\frac{1-P(w)}{P(w)}]\}q \tag{9}$$

The insurance premium depends on $z, d, \alpha, P(w)$ in the following ways:

$$\frac{\partial J(w)}{\partial z} = [\frac{1-P(w)}{P(w)}]q > 0 \tag{10}$$

$$\frac{\partial J(w)}{\partial d} = (\alpha - 1)[\frac{1-P(w)}{P(w)}]q < 0 \tag{11}$$

$$\frac{\partial J(w)}{\partial \alpha} = d[\frac{1-P(w)}{P(w)}]q > 0 \tag{12}$$

$$\frac{\partial J(w)}{\partial P(w)} = -\frac{[(\alpha-1)d + z]q}{P(w)^2} < 0 \tag{13}$$

According to equation (10), the premium increases when $z$ increases. The intuition is that an increase of bank loans increases the possible loss in case of a bank run, therefore, the bank that takes a higher risk pays a higher premium. From equation (11), the premium is lower when the amount of deposit increases. The increase in deposits, given a determined amount of loans, means that the bank's investment in liquid assets increases, and this allows the bank to sell a smaller amount of assets in case of a bank run. Equation (12) establishes that the premium is higher if the bank run is stronger. From Equation (13), the premium is higher when the liquidation price that the bank can get for the assets sold is lower.

Having defined the term $J(w)$ , the next step is to derive the first order condition of the bank's profit maximization problem. Assuming a binding reserve requirement, the first order conditions are[10]:

$$i^L - i = [\frac{1-P(w)}{P(w)}]q \tag{14}$$

$$i - i^d - \delta i = (\alpha - 1)[\frac{1-P(w)}{P(w)}]q \tag{15}$$

Note that the terms on the right hand side of (14) and (15) represent the liquidity risk component of the maximization process.

[10] Remember that bank decides over $z$ and $d$.

## 2.4 How $P(w)$ is determined

The liquidation price of the asset, as mentioned above, is determined endogenously in the model. The equilibrium level of this price is one that yields equal levels of supply and demand of $w$. The supply of $w$ comes from the banking system when it sells assets to raise cash, and the demand is determined by the collection agency.

The collection agencies buy assets sold by banks and, after paying a certain cost, these agencies can recover the full amount of the loan. Note that the paper models only liquidity risk, and assumes no default risk.

The profit function of these collection agencies would be,

$$\Pi = [\frac{1 - P(w)}{P(w)}]w - C(w) \tag{16}$$

Revenue is given by $[\frac{1-P(w)}{P(w)}]w$. It means that these agencies recover the face value of the loan, and pay $P(w)$ for each asset bought. In addition, they have to pay a cost of $C(w)$ to recover the full amount of the loan.

So, the FOC of the collection agencies is,

$$P(w) = \frac{1}{1 + C'(w)} \tag{17}$$

>From this FOC, the demand of assets sold by banks is obtained.

The following two factors can be expected to affect the liquidation price $P(w)$: First, liquidity level: The lower the asset liquidity degree, the lower the liquidation price. For example, the loss generated when a house is sold immediately should be higher than the loss generated when government bonds are sold at the same times. Second, the amount of assets to be sold: the larger the amount of assets to be sold in the market, the lower the liquidation price.

In particular, the cost function assumed is: $C(w) = e^{\tau w} - 1$ for all $w \geqslant 0$ and $\tau \geqslant 0$, where $w$ represents the amount the assets traded in the market and $\tau$ the liquidity parameter. In this setting a higher $\tau$ means a lower liquidity degree. For example, if the government bonds are totally liquid, then $\tau = 0$, and its price would always be 1. The paper uses this cost function because it contains both characteristics specified above, and facilitates the interpretation of the results.

Then, considering the above cost function, the liquidation price would be:

$$P(w) = \frac{1}{1 + \tau e^{\tau w}}$$

### 2.5 Equilibrium Condition

The exogenous variables in the model are $z, d, \delta, \alpha, i, q, \tau, \varepsilon$. Even though $z$ and $d$ are assumed exogenous in this section, these variables will be endogenized in the welfare section where ad-hoc deposit supply and loan demand are incorporated. This exogeneity assumption yields simpler expressions without changing the results in qualitative terms.

The endogenous variables so far are $P(w), i^L, i^d, J(w), w$. The equilibrium can be found using the following five equations.

i) $(\alpha - 1)d + z = w$

$w$ is derived for a given $z, d$, and $\alpha$.

ii) $P(w) = \frac{1}{1+C'(w)}$

Plugging $w$ and $\tau$ in the $C\prime$, the liquidity price is obtained

iii) $J(w) = \{[(\alpha-1)d[\frac{1-P(w)}{P(w)}] + z[\frac{1-P(w)}{P(w)}]\}q$

Then, for a given $q$ and $\alpha$ the value of the premium is determined

iv) $i^L - i = \frac{\partial J(w)}{\partial z}$

v) $i - i^d = \delta i + \frac{\partial J(w)}{\partial d}$

With $i$ and the bank's first order conditions, the interest rates are derived.

# 3 Comparative Statics

This section analyzes how the equilibrium changes when one of the parameters of the economy varies, specifically for the cases of $d, q, \tau$, and $\alpha$. Throughout this section the exogeneity of $d = d_0$ and $z = z_0$ is maintained.

The following table summarizes the results of this section.[11]

Summary of the Comparative Static

Table 1

| | $P(w)$ | $w$ | $J(w)$ | $i^L$ | $i^d$ | $i^L - i^d$ |
|---|---|---|---|---|---|---|
| Increase $d$ | $\Uparrow$ | $\Downarrow$ | $\Downarrow$ | $\Downarrow$ | $\Downarrow$ | $\Downarrow$ |
| Increase $q$ | = | = | $\Uparrow$ | $\Uparrow$ | $\Uparrow$ | $\Uparrow$ |
| Increase $\tau$ | $\Downarrow$ | = | $\Uparrow$ | $\Uparrow$ | $\Uparrow$ | $\Uparrow$ |
| Increase $\alpha$ | $\Uparrow$ | $\Uparrow$ | $\Uparrow$ | $\Uparrow$ | ? | $\Uparrow$ |

### 3.1 Case 1- Increase in $d$

Suppose deposits increase from $d_0$ to $d_1$. From (5), $(\alpha-1)d + z = w$, and $(\alpha - 1)$ is less than zero, so this increase will reduce $w$.

[11] All the results are derived in the Appendix.

Next, the liquidation price is obtained plugging the new value of $w$ in equation (17), $P(w) = \frac{1}{1+C'(w)}$. This increment in the price comes from the reduction in the marginal cost.

Equation (14) and (15) are used to find the effect on the interest rates.

$$\frac{\partial i^d}{\partial d} = -\frac{\partial^2 J}{\partial d^2} < 0$$

$$\frac{\partial i^L}{\partial d} = \frac{\partial^2 J}{\partial d \partial z} < 0$$

In addition, the change in spread is

$$\frac{\partial (i^L - i^d)}{\partial d} = \frac{\partial^2 J}{\partial d^2} + \frac{\partial^2 J}{\partial d \partial z} < 0$$

The same result would be obtained if instead of an increase in deposits, the banking system faced a reduction in loans. A graphical intuition of this event was shown in graph 1, where the bank's risk type is given by a ratio between deposits and loans. In this case, the increase in deposits improves the liquidity position of the banking system.

Edwards and Vegh (1996) using a general equilibrium model, derived this result, explaining the mechanism that generates the change in the deposit-loan ratio. Here, the change is the ratio is exogenously given. However, a contribution of this model with respect to Edwards and Vegh is that this model gives a strong intuition of what could generate the cost function assumed in their model, namely liquidity risk.



## 3.2 Case 2- Increase in $q$

Suppose that the probability of a bank run ($q$) increases. From (5), if $\alpha$, $d$ and $z$ are constant, $w$ should not change either. In other words, $w$ would be constant, which according to equation (17), $P(w) = \frac{1}{1+C'(w)}$, would imply a constant liquidity price $P(w)$. It is clear from (8), $J(w) = [\frac{1-P(w)}{P(w)}]wq$, that the insurance premium will increase.

The effect on the interest rate can be studied though the partial derivative of the different interest rates with respect to $q$. Using (14) and (15),

$$\frac{\partial i^d}{\partial q} = -\frac{\partial^2 J}{\partial d \partial q} > 0$$

$$\frac{\partial i^L}{\partial q} = \frac{\partial^2 J}{\partial z \partial q} > 0$$

And the change in spread is

$$\frac{\partial (i^L - i^d)}{\partial q} = \frac{\partial^2 J}{\partial d \partial q} + \frac{\partial^2 J}{\partial z \partial q} > 0$$

Thus, the increase in the probability of a bank run will increase not only the loan and deposit interest rate but also the spread between them.

An example of this event is the so-called "tequila effects ". After the Mexican crisis in (1994), the probability of a bank run in Argentina could have increased because investors believed that the Argentina economy could face some problem too. As the model predicts, and the data showed, Argentina faced an increase in both the loan and deposit interest rates, and the spread between them.

### 3.3 Case 3. Decrease in $\tau$

The liquidity degree of the asset is defined by $\tau$,where a higher $\tau$ means a lower liquidity degree.. For example, if a government bond is 100 % liquid, $\tau = 0$, an investor gets its face value when he sells the government bond in the market. On the other hand, if a house has $\tau$ bigger than zero, then, the price of the house will suffer a discount. The above price refers to the liquidation price, in other words, when the asset is sold immediately.

The paper analyzes two different issues concerning the parameter $\tau$:

a) **Secondary market.** $\tau$ can be interpreted as an indication of the secondary market trading cost. For example, if banks can create a new security with its assets, where this new security has a higher liquidity degree than the aggregate liquidity when assets are sold separately[12], then, the liquidation price of this new security will be higher than the price received when each asset is sold separately. Formally, this procedure of security design could be introduced as an asset with a lower $\tau$. Then, a reduction in $\tau$ could be interpreted as a more efficient secondary market.

b) **Implicit subsidy.** Let the liquidity parameter faced by the banking system be $\eta$, where $\eta = \tau - s$, where $s$ is an indication of the government subsidy. In this setting, the extreme values of $s$ are given by: $s = 0$ when the banking system does not have any subsidy, and $s = \tau$ when the banking system has the highest subsidy.

[12] An empirical example of this situation is to transform a real estate loans in a mortgage.

Banks will consider $\eta$ as the private asset liquidity degree instead of its social liquidity degree $\tau$. In the special case that $s = \tau$, bank would consider all its assets as being totally liquid. Let me postpone the analysis of this government policy to the next section.

Continuing with the derivation of the comparative statics, a change in $\tau$ does not affect $\alpha$, $d$, and $z$. Using the relation $(\alpha - 1)d + z = w$, $w$ will be constant.

Although $w$ does not change, equation (17), $P(w) = \frac{1}{1+\tau e^{\tau w}}$, shows that the liquidation price decreases due to the increment in $\tau$.

The change in $P(w)$ affects the premium paid by banks, which according to equation (8), $J(w) = [\frac{1-P(w)}{P(w)}]wq$, increases.

The effect of the change in the asset liquidity degree on the interest rate can be calculated from the derivatives of (14) and (15) with respect to $\tau$. As shown in the appendix, not only the loan and the deposit interest rate increase, so does the spread.

Returning to case (a) "secondary markets ", as mentioned above, a smaller $\tau$ (asset is more liquid) implies a higher asset liquidation price. Then, any mechanism that reduces $\tau$ could be interpreted as a reduction of trading costs. As a consequençe, there would be a reduction in the deposit and loan interest rates, and in the spread between them. There are several ways to improve the secondary market efficiency. An example could be the creation of a mortgages market in emerging markets.

Table 2 shows, for the United States, the capital loss suffered by banks when their assets were sold immediately. The difference between the liquidation cost for real estate and mortgages can be interpreted as an improvement of the liquidation price generated by the formalization of the real estate secondary market.

Latin-American countries do not have this type of market as advanced as developed countries. According to the model, if the mortgage market is created not only the loan and the deposit interest rate, but also the spread between them, should decrease.

### 3.4 Case 4- Increase in $\alpha$

$\alpha$ is defined as the percentage of deposits that depositors withdraw in the event of a bank run. Thus, an increase in $\alpha$ indicates a stronger bank run. Given $(\alpha - 1)d + z = w$, an increase in $\alpha$ means an increase of $w$, because $d$ and $z$ are constant.

Then, the liquidation price should decrease because the amount of assets that banks sell increases. This result can be seen deriving the liquidation price function with respect to $\alpha$.

$$\frac{\partial P(w)}{\partial \alpha} = \frac{-\tau^2 d e^{\tau w}}{(1+\tau e^{\tau w})^2} < 0$$

In addition, the premium, $J(w) = \frac{1-P(w)}{P(w)} wq$, increases because the liquidation price decreases and $w$ increases. This result can be seen formally by taking the partial derivative of the premium with respect to $\alpha$.

$$\frac{\partial J(w)}{\partial \alpha} > 0$$

The effect $\alpha$ on the interest rate can be determined by taking the derivative of equations (14) and (15) with respect to $\alpha$.

$$\frac{\partial i^d}{\partial \alpha} = -\frac{\partial^2 J}{\partial d \partial \alpha}$$

$$\frac{\partial i^L}{\partial \alpha} = \frac{\partial^2 J}{\partial z \partial \alpha} > 0$$

The effect on the spread is

$$\frac{\partial (i^L - i^d)}{\partial \alpha} = \frac{\partial^2 J}{\partial d \partial \alpha} + \frac{\partial^2 J}{\partial z \partial \alpha} > 0$$

Therefore, according to the model, if the economy faces the possibility of a stronger bank run, the loan interest rate and the spread will go up. On the other hand, the model is not conclusive in regard to the deposit interest rate.

# 4 Government Bail Out

This section analyzes three aspects of an anticipated government bail out: i) the welfare effect, ii) the optimal bank's asset mismatch, iii) the lender of last resort, and iv) the effect of an imposed reserve requirement.

## 4.1 Welfare analysis of an implicit government subsidy

### 4.1.1 Setting up the problem

To study the welfare effect, the paper endogenizes the determination of loans and deposits. Formally, the paper introduces ad-hoc deposit supply and loan demand. The introduction of ad-hoc functions is for the sake of simplicity, otherwise, the model should formalize the production and consumption problem. The bank faces an upward sloping supply for deposit and a downward sloping demand for loans.

To determine the equilibrium in the loan market, let me assume the following ad-hoc loan demand

$$i^L = c - vz \tag{18}$$

where $c > 0$ and $v > 0$. The loan supply comes from the bank's FOC. Loan supply is,

$$i^L = i + \tau q e^{\tau w} \tag{19}$$

Note that $w = (\alpha - 1)d + z$. Then, equation (19) is not a supply function, because that should include only $z$ and prices, and this equation depends also on $d$, which is endogenous. Therefore, it is not possible to find the equilibrium of $z$ and $i^L$ using solely equations (18) and (19). The equilibrium determination in the deposit market would have the same inconvenience.

One way to find the equilibrium in both markets is solving a system that includes these two markets together. This new system has two equations and two unknowns.

To complete the equations of the system, it is necessary to include the ad-hoc supply function for deposits

$$i^d = k + hd \tag{20}$$

where $k > 0$ and $h > 0$

Remember that the other bank's FOC is

$$i^d = (1 - \delta)i - \tau q(\alpha - 1)e^{\tau w} \tag{21}$$

The first equation of the system comes from the deposit market equilibrium condition, when equation (20) is equal to equation (21),

$$(1 - \delta)i - \tau q(\alpha - 1)e^{\tau w} = k + hd \tag{22}$$

The other equation of the system is derived from the loan market equilibrium condition, when equation (18) is equal to equation (19)

$$\tau q e^{\tau w} = c - vz - i \tag{23}$$

Using equations (22) and (23\), the equilibrium relation between $z$ and $d$ can be found

$$d = \phi + \lambda z \tag{24}$$

Where $\phi = [(\alpha - \delta)i - c(\alpha - 1) - k]/h$

$\lambda = [v(\alpha - 1)]/h$

Now, plugging (24) in (19), the optimal amount of $z$ and $i^L$ can be determined

$$i + \tau q e^{\{\tau[(\alpha-1)\phi + [(\alpha-1)\lambda+1]z]\}} = c - vz \tag{25}$$

Note that equation (25) has a unique equilibrium. The reason of this uniqueness is that the first term of equation (25) is increasing with respect to $z$ and the second term is decreasing with respect $z$.

In the same way, the equilibrium in the deposit market is derived. Defining $\varphi = -\phi/\lambda$ and $\beta = 1/\lambda$, the relation (24) between $d$ and $z$ can be rewrite as

$$z = \varphi + \beta d \tag{26}$$

Plugging (26) in (22), the optimal amount of deposits and $i^d$ can be found using the following equation,

$$(1 - \delta)i - \tau q(\alpha - 1)exp\{\tau[[(\alpha - 1) + \beta]d + \varphi]\} = k + hd \tag{27}$$

Before introducing the government bail out policy, it would be convenient to show the comparative statics of the liquidity parameter $\tau$. In the loan market, this effect can be shown using equation (25). The partial derivative of the first part of the equation (25) with respect to $\tau$ shows a reduction of the loan supply. Then, the equilibrium loan interest rate increases when $\tau$ increases.

$$\frac{\partial i^L}{\partial \tau} = q e^{\tau w}(1 + \Psi\tau) > 0$$

Where $\Psi = (\alpha - 1)\phi + [(\alpha - 1)\lambda + 1]z$

This effect can be seen in graph (2), where a decrease in $\tau$, from $\tau_1$ to $\tau_0$, (so the asset is more liquid) would increase the optimal amount of $z$ and decrease $i^L$.

On the other hand, concerning the deposit market, the effect can be analyzed deriving the first part of equation (27) with respect to $\tau$.

$$\frac{\partial i^d}{\partial \tau} = -q(\alpha-1)e^{\tau w}(1+\mu\tau) > 0 \tag{28}$$

Where $\mu = [(\alpha - 1) + \beta d] + \varphi$

Graph 3 presents the static comparative of a change in $\tau$. According to this graph, a decrease of $\tau$, from $\tau_1$ to $\tau_0$, reduces the amount of deposits. Concluding from graphs (2) and (3), a reduction of $\tau$ (an increase in the asset's liquidity degree) would result in an increase in the amount of loans and a reduction in the amount of deposit.

Now, the next step is to show the formalization of a government bail out policy and how the social cost can be measured. This bail out policy is when the government pays for part of the asset liquidity degree. In other words, the asset price received by banks is higher than its social price. In the context of the model, this policy is represented through the liquidity parameter. Defining the private asset liquidity degree faced by banks as $\eta = \tau - s$, where $s$ indicates the government subsidy. In this setting, if $s$ is bigger than zero, banks will consider in its maximization problem the asset liquidity degree as more liquid than its true or "social " liquidity degree. As a result, banks would consider in its demand for assets a liquidation price determined by the formula, $P(w,\eta) = \frac{1}{1+\eta e^{\eta w}}$.

### 4.1.2 How can the social cost of the mismatch be measured?

The social cost is measured using supply and demand functions. The inconvenience to measure the social cost, so far, is that the deposit and loan market do not present a supply and demand equation as required to calculate a social cost. A solution could be measuring the social cost using the created variable $w$. The procedure to estimate this cost would consist in, first, estimating the market equilibrium of $w$ without distortion in the economy. And second, taking into account this equilibrium as a benchmark, a government bail out policy is introduced by defining $P(w,\eta) = \frac{1}{1+\eta e^{\eta w}}$ with $s > 0$.

The supply function for $w$ is deriving using the system of equations presented before. But this time, the liquidation price would not be replaced in its formula. Thus, the system of equation is,

$$\text{Loan market} \qquad i + q[\frac{1-p(w)}{p(w)}] = c - vz \tag{29}$$

$$\text{Deposit market} \qquad (1-\delta)i - q(\alpha-1)[\frac{1-p(w)}{p(w)}] = k + hd \tag{30}$$

>From this equation system, the supply function for $w$ can be found by solving (29) for $z$, and (30) for $d$. So, using the equation $w = (\alpha - 1)d + z$, the supply equation is,

$$w = \frac{-i - q[\frac{1-p(w)}{p(w)}] + c}{v} + (\alpha - 1)\frac{(1-\delta)i - q(\alpha - 1)[\frac{1-p(w)}{p(w)}] - k}{h} \tag{31}$$

The next equation shows that equation (31) behaves as a supply function (upward sloping).

$$\frac{\partial w}{\partial P(w)} = \frac{q}{vp(w)^2} + \frac{(\alpha - 1)^2}{hp(w)^2} > 0 \tag{32}$$

As mentioned above, the demand for $w$ comes from the collection agencies, which is equation (17). The optimal $w$ and $P(w)$ are determined where supply is equal to demand. This equilibrium is represented by $b$ in graph 4.

Next, the paper introduces the government bail out policy. This government policy, as any other subsidy, generates a gap between the social price and private price for asset liquidation. This policy means that the liquidity price faces by banks, $P(w, \eta)$, is higher than the social liquidity price, $P(w, \tau)$, in other words, $P(w, \eta) = \frac{1}{1+\eta e^{\eta w}} > P(w, \tau) = \frac{1}{1+\tau e^{\tau w}}$ when $\eta < \tau$.

The new equilibrium with the implicit subsidy would be at $w_1$ and the liquidation price received by banks would be $P(w_1)$. Given that the social price is $P(w_2)$, the social cost is the area $abc$. Note that this cost is independent to the existence of a bank run. This cost is generated by a misallocation of resources every period.

## 4.2 How does the implicit subsidy affect the asset mismatch?

Let me define a liquidity ratio as the ratio between liquid assets ($b + \delta d$) and liquid liability ($d$).

$$\kappa = \frac{b + \delta d}{d} \tag{33}$$

Thus, a lower $\kappa$ is interpreted as a bigger mismatch because the gap between liquid assets and liquid liabilities increases. Using the bank's balance sheet, the above equation can be written as

$$\kappa = 1 - \frac{z}{d} \tag{34}$$

Now, before introducing the government bail out policy, equation (34) determines the **optimal mismatch** once the equilibrium values of $z$ and $d$

are plugged. On the other hands, a government bail out policy, as was shown above, increases $z$ and decreases $d$, these new values of $z$ and $d$ generate a reduction of ratio, which means a higher mismatch between liquid assets and liabilities.

This result illustrates a difference between the model presented in this paper and the model proposed by Diamond and Dybvig. According to Diamond and Dybvig, any policy that can stop the bank run would increase the economy's wealth. This result is explained by the fact that banks could invest their resources in less liquid assets whose return is higher.

The model in this paper shows that an optimal mismatch exists. To reach the asset diversification proposed by Diamond and Dybvig (all money invested in the higher return less-liquid asset) the government would have to pay for the liquidity risk. In other words, the Central Bank should subsidy the liquidity risk taken by banks. The source of the difference of these two models is how uncertainty is generated. While the uncertainty in Diamond and Dybvig is represented by a change in expectation, in this paper the uncertainty has a well-defined distribution.

In addition, this section makes a contribution to the endless discussion between narrow banking and free banking. The result of this model shows an optimal mismatch between assets and liabilities. This result implies that narrow banking is too restricted, because this market structure does not allow the banking system to get as much risk as it wants. On the other hand, if there exist an explicit or an implicit government bail out policy, a free banking system implies an over-expansion of the banking system.

### 4.3 Lender of Last Resort

This sub-section discuses, on the one hand, the optimal intervention of a lender of last resort, and on the other, how introducing the assumption made by D&D (net worth equal to zero and probability of bank run equal to zero) increase the vulnerability of the bank system. Note that the last conclusion, as was pointed out before, does not depend on the existence of moral hazard, this result is based on how the uncertainty is introduced.

In contrast to the D&D model, where the uncertainty is due to a change in the depositor's expectation on the bank's health, this paper supposes a pre-determined distribution for the uncertainty. The following example helps to clarify this uncertainty. Suppose a bank in a small open economy receives a deposit of one dollar. At the time the bank invests the money received, it knows that there is a probability $q$ the US Federal Reserve will raise the interest rate. This increase in the US interest rate would generate a bank run in the small open economy, because depositors would want to invest their portfolio in the US financial market.

To introduce the lender of last resort, the model assumes that in case of bank run a government agency will provide as much liquidity as banks need, but charging a punitive interest rate. Recalling that the bank is risk neutral, the profit function is

$$\pi = (i^l - i)z + (i - i^d)d - ih + (i - i^p)q(\alpha d - \delta d - b) \tag{35}$$

Where $i^p$ is the interest rate charged by the liquidity provider.[13]

Using the bank's balance sheet equivalence, the profit function can be written as,

$$\pi = (i^l - i)z + (i - i^d)d - ih + (i - i^p)q[(\alpha - 1)d + z - NW] \tag{36}$$

The first order condition are

$$\frac{\partial \pi}{\partial z} = (i^l - i) + q(i - i^p) \tag{37}$$

$$\frac{\partial \pi}{\partial d} = (i - i^d) - i\delta + (i - i^p)q(\alpha - 1) \tag{38}$$

And the spread is

$$i^l - i^d = i\delta + (i^p - i)\alpha q \tag{39}$$

If the liquidity provider charges the interest rate $i^p = i + \frac{1-p}{p}$, the FOCs of the benchmark case, equations (14) and (15) are recovered.

On the other hand, if $i^p < i + \frac{1-p}{p}$, the loan supply would be higher than the social optimal loan supply, and the deposit demand would be lower than the social optimal deposit demand. The liquidity position of the banking system would decrease. In other words, the vulnerability of the banking system increases.

As an empirical example, the Central Bank of Argentina implemented a combination between the lender of last resort and the liquidity insurance scheme presented in this paper. The Central Bank of Argentina charges a monthly premium to each bank for a liquidity insurance that the Central Bank has acquired in the international market. In case of a bank run, the

[13] Having more liquidity than needed to finance the bank run is pareto dominated, so, $\alpha d - \delta d - b \succeq 0$.

international banks will make a repo transaction at the interest rate of $i$. Therefore, the Argentina case can be interpreted as follows. During tranquil periods, the Argentina Central Bank pays something that represents $\frac{1-p}{p}$ and during the crisis it pays the difference $i$. Then, in expected value, the interest rate scheme paid by the Argentina banking system looks like $i^p = i + \frac{1-p}{p}$, which is the benchmark case.

A fact to emphasize from the Argentina experience is that the private sector provides the contingent credit line, where sometimes it is believed that only international organization can offer this type of contract.

As mentioned above if $i^p < i + \frac{1-p}{p}$, the banking system would be more vulnerable, because its liquidity position decreases. Therefore, if any institution offers a contingent credit line at the interest rate lower than $i^p = i + \frac{1-p}{p}$, according to this result, this institution would increase instead of decreasing the financial system fragility.

The next step is to study the effect of introducing the D&D's assumptions. The incorporation of these assumptions, zero net worth ($NW = 0$) and zero probability of bank run ($q = 0$), would transform the first order conditions to:

$$i^l = i \tag{40}$$

$$i^d = i(1 - \delta) \tag{41}$$

The loan supply would be even higher and the deposit demand even lower, so the liquidity position of the financial institutions will be worse. Remember that the uncertainty assumed is that the US Federal Reserve could increase the interest rate with probability $q$. When the Federal Reserve actually increases the interest rate, this banking system will suffer a bank run and given that banks do not have net worth, they will go bankruptcy. Note that this result does not depend on the Moral Hazard problem. Financial system instability comes from the way uncertainty is modeled.[14]

[14]""Calvo", Reinhart, etc mentioned that the fluctuation of the U.S. interest rate is a main factor for capital flows in emerging markets""

## 5 Conclusion

The paper shows an alternative solution to the moral hazard problem generated by an expected government bail out the financial system. This solution is an insurance scheme that depends on the risk faced by banks. As an example, the model presents the case where the financial system faces only liquidity risk. In this case, the policy recommendation is that the Central Bank should charge a premium to each bank depending on its liquidity position. An example of this type of scheme is found in Argentina, where the Central Bank charges a premium to each bank for liquidity insurance that it acquires in the international market.

Before introducing the government bail out policy, the paper analyzes the effect on the economy, that has liquidity risk, of shocks such as: i) capital inflow, ii) development of secondary markets for bank's assets, iii) increases in the probability of a bank run, and iv) increases in the power of the run.

Once the government bail out policy is introduced, the model shows how this policy can i) reduce the lending interest rate, ii) reduce deposit interest rates, and iii) reduce the spread between them.

Further, the paper derives a new procedure to estimate the social cost of the externality generated by the government bail out policy. This social cost is different from what is considered so far as cost, which is the transfer from the government to the financial sector, and is independent of the occurrence of a crisis.

Next, the model predicts an increase in the fragility of the financial system due to the government bail out policy. This is measured by the ratio between liquid assets and liquid liabilities, where the lower the ratio, the higher the fragility. An interesting extension of the fragility variable is its link with the discussion between narrow and free banking. The model shows that before introducing the government bail out policy, there is an optimum mismatch. In other words, in equilibrium, there is an optimum amount of assets that, in case of an unexpected withdrawal, banks should sell at a discount. Once the government bail out policy is introduced, the model predicts that the mismatch, as well as the social cost, increase.

Finally, the paper introduces a lender of last resort, which is a competitive devise to deal with liquidity risk. Using the the model developed early, it is shown the optimal punitive interest rate. Whithin this idea, if a government has access to a contingent credit line at the interest rate lower than the optimal interest rate, this credit line would increase instead of decrease the financial system fragility.

# 6 Appendix A

The profit function used in the paper is the following,

$$\pi = (\frac{1+i^L}{1+\varepsilon})z-(\frac{1+i^d}{1+\varepsilon})d+(\frac{1+i}{1+\varepsilon})b+\frac{h}{1+\varepsilon}-\frac{1}{1+\varepsilon}\sum_{k=1}^{2}q_k \max(0,(\frac{\alpha_k d-\delta d-b}{P(w)}))[1-P(w)] \tag{42}$$

Where[15]

$i^d$ Deposit interest rate.

$i^L$ Loan interest rate.

$i$ Bond interest rate.

$k=1$ indicates a bank run and $k=2$ indicates no bank run.

$P(w)$ Fire price of assets.

$\alpha_k$ the percentage of deposits that banks have to honor in state of nature k. Where $\alpha_2 = 0$.

$\delta$ is the percentage of reserve requirements.

$q_1$ Probability of a bank run.

The first terms in brackets of the above equation represents the bank operational revenue, which consists of: a) nominal interest rate earning on bank net worth, b) The loan return over the opportunity cost $(i^L - i)$ times the amount of loan, c) The nominal interest rate minus the deposit rate $(i - i^d)$ times the amount of deposit, and d) The cost of the reserve requirement.

The second part of the same equation analyzes the bank's uncertainty. This uncertainty is a cost that will depend on the revealed state of nature, where the state is determined by whether or not a bank run occurs.

In the case of a bank run, the bank should honor a percentage $\alpha$ of the deposit. If $\alpha d > \delta d + b$, then, the bank would need to sell assets to honor depositors. The amount of assets sold, $(\frac{\alpha_k d-\delta d-b}{P(w)})$, times the loss per asset sold, $[1-P(w)]$, is the total capital loss[16]. In the case of no bank run, or $\alpha d < \delta d + b$, this cost is zero because banks do not need to sell assets. At period zero banks do not know for sure the outcome at the end of the period,

[15]This profit function is expressed in nominal term to facilitate the derivation of the solution. Given that $\varepsilon$ is constant, the solution of both, the nominal and the real profit function are the same.

[16]$P(w)$ is price of the asset when banks need to sell it immediately. $w$ is the amount of resources, above the available liquid assets held by banks, needed to honor depositors. The derivation of $P(w)$ and $w$ will be shown later.

but they know the expected value of the loss, which is the loss in each state times the probability of that state of the nature.

Using the the bank's balance sheet, b=a+d-z-h, the profit function could be written as

$$\pi = (\frac{1+i^L}{1+\varepsilon})z - (\frac{1+i^d}{1+\varepsilon})d + (\frac{1+i}{1+\varepsilon})(a+d-z-h) + \frac{h}{1+\varepsilon} - \frac{1}{1+\varepsilon}\sum_{k=1}^{2} q_k \max(0,(\frac{\alpha_k d - \delta d - b}{P(w)}))[1-P( \tag{43}$$

Rearrangement tems,

$$\pi = (\frac{1+i}{1+\varepsilon})a + (\frac{i^L - i}{1+\varepsilon})z + (\frac{i-i^d}{1+\varepsilon})d - \frac{ih}{1+\varepsilon} - \frac{1}{1+\varepsilon}\sum_{k=1}^{2} q_k \max(0,(\frac{\alpha_k d - \delta d - b}{P(w)}))[1-P(w)] \tag{44}$$

Multiplying both term by $(1+\varepsilon)$

$$\pi(1+\varepsilon) = (1+i)a + (i^L - i)z + (i-i^d)d - ih - \sum_{k=1}^{2} q_k \max(0,(\frac{\alpha_k d - \delta d - b}{P(w)}))[1-P(w)] \tag{45}$$

And defining $\Pi = \pi(1+\varepsilon)$, the profit function is found

$$\Pi = (1+i)a + (i^L - i)z + (i-i^d)d - ih - \sum_{k=1}^{2} q_k \max(0,(\frac{\alpha_k d - \delta d - b}{P(w)}))[1-P(w)] \tag{46}$$

# 7 Appendix B

In this section the equations used in the static comparative section are developed.

The cost function assumed was $C(w) = e^{\tau w} - 1$, for all $w>0$ and $\tau>0$.

Remember from (10) that $w = (\alpha - 1)d + z$

>From the collection firm's profit maximization problem we get

$P(w) = \frac{1}{1+C'(w)}$ (19)

Plug in the cost function in equation (19), we get

$$\frac{1-P(w)}{P(w)} = te^{\tau w} \tag{47}$$

Moreover, plugging equation (43) in the premium function we have,

$$J = q\tau exp(\tau w)[(\alpha-1)d+z] \tag{48}$$

To facilitate the reading, we order the partial derivatives in the following order: First, we will show the partial derivatives that solve the first order condition of the bank. Second, we will derive the needed partial derivative for each endogenous variable.

1- Partial derivatives for the bank first order condition.

$$\frac{\partial J(w)}{\partial d} = \tau e^{\tau w} q(\alpha-1) < 0 \tag{49}$$

$$\frac{\partial J(w)}{\partial z} = \tau e^{\tau w} q > 0 \tag{50}$$

2- Groups of derivatives for each endogenous variable.

a) Derivatives for the insurance premium

$$\frac{\partial J^2}{\partial d^2} = \tau^2 e^{\tau w} q(\alpha-1)^2 > 0$$

$$\frac{\partial J^2}{\partial d \partial z} = \tau^2 e^{\tau w} q(\alpha-1) < 0$$



$$\frac{\partial J^2}{\partial z^2} = \tau^2 e^{\tau w} q > 0$$

With respect to $\tau$

$$\frac{\partial J^2}{\partial d \partial \tau} = e^{\tau w} q(\alpha-1)[1+\tau w] < 0$$

$$\frac{\partial J^2}{\partial z \partial \tau} = e^{\tau w} q[1+\tau w] > 0$$

With respect to $\alpha$

$$\frac{\partial J^2}{\partial d \partial \alpha} = \tau q e^{\tau w}[1+\tau d(\alpha-1)]$$

$$\frac{\partial J^2}{\partial z \partial \alpha} = \tau^2 q e^{\tau w} d > 0$$

With respect to q

$$\frac{\partial J^2}{\partial d \partial q} = \tau e^{\tau w}(\alpha - 1) < 0$$

$$\frac{\partial J^2}{\partial z \partial q} = \tau e^{\tau w} > 0$$

b) Spread

The spread is defined as $i^L - i^d$,

$$i^L - i^d = \delta i + \frac{\partial J}{\partial d} + \frac{\partial J}{\partial z}$$

Replacing for (45) and (46) we should get

Now, we develop how the interest rate spread change when some of the parameter changes.

$$\frac{\partial(i^L - i^d)}{\partial d} = \tau^2 q e^{\tau w}(\alpha - 1)\alpha < 0$$

$$\frac{\partial(i^L - i^d)}{\partial z} = \tau^2 \alpha q e^{\tau w} > 0$$

$$\frac{\partial(i^L - i^d)}{\partial \tau} = \alpha q e^{\tau w}[1 + \tau w] > 0$$

$$\frac{\partial(i^L - i^d)}{\partial \alpha} = \tau \alpha q e^{\tau w}[1 + \tau \alpha d] > 0$$

$$\frac{\partial(i^L - i^d)}{\partial q} = \tau \alpha e^{\tau w} > 0$$

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