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"Bank Regulation and Lender of Last Resort in a small open economy."

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Abstract

In a two - currency version of the Bryant - Diamond and Dybvig model I study three different arrangements that prevent self fulfilling bank runs. The first case is a the international lender of last resort. This has been under discussion in the past year. This last regime might prevent runs implementing the planner's solution although this depends on the consumption allocations. Moral hazard considerations cannot be important as long as perfect debt repayment and common objectives between managers and depositors are assumed. I also analyze a combination of an international lender of last resort and a local lender of last resort. This implements the planner's allocation and prevents runs always, regardless of how that allocation looks like. The third is a regime that imposes narrow banking on foreign currency investments, coupled with a lender of last resort in home currency. I show that this regime prevents runs. However this can do it at the expense of excess liquidity in the short run.

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1. Introduction

Since the Asian Financial Crisis in 1997 [1] many discussions about banking regulation have taken place at different levels. One of the most recent ones has been related to the role of international institutions (such as the IMF) in preventing financial crises. In October 1998 a new credit line was approved by the G7 to *help* "vulnerable but essentially healthy nations" (see New York Times [16]). Indeed, the G7 group declared on October 30, 1998 that

> The statement...reflects the shared determination of the U.K. and G7 to modernize the financial system and to put in place new rules and procedures that will promote stability and growth. It affirms that the G7 commit themselves to: (...)
>
> - develop improved procedures for managing crises and *preventing* them from spreading, including an enhanced IMF financing mechanism supported by private and bilateral finance as appropriate..

The chancellor 's statement following the G7 declaration affirms explicitly the creation of a "supplementary reserve facility which would provide a contingent short term line of credit for countries pursuing strong IMF approved policies. This facility could be drawn upon in times of need and would entail appropriate interest rates along with shorter maturities. "[2]

Although this statement also includes issues such as *global regulatory regimes*, which are not clarified yet, it is explicit in terms of creating a reserve for short term credit in order to prevent / solve financial distress for those economies following an *IMF approved* policy. This implicitly states that the distress usually should be mostly associated with liquidity rather than solvency issues, at least from an official point of view. If the IMF tends to approve policies that do not induce risky investments by the financial sector, this then leads to the conclusion that this line would hardly be applied to countries with financial crises due to solvency problems.

Although the last remark is debatable, it certainly states that if a financial sector has only a liquidity problem, then it is likely to be beneficiary of such a credit line. The objective of the present work is to show that this credit line would be useful in preventing bank runs in small open economies with potential short run liquidity problems but that it will not always succeed. Unless coupled with

[1] For surveys on the Asian Crisis see [6] and [8], among others.
[2] See Chancellor's statement, including in the G7 statement.

a local lender of last resort, the international institution may fail to eliminate a run. This essay also presents alternative ways of preventing such runs, although not all of them lead to the efficient allocation.

This chapter uses the same version of the Bryant - Diamond - Dybvig model ([3] and [11]) in a small open economy[3] used in chapter 3 to analyze the issue considered above. I study three main different regulation regimes, all of them involving a lender of last resort. The first case is a regime with an *international lender of last resort* (which is what the G7 group ultimately had created). This could prevent runs every time there is illiquidity in F money only. If there are patient consumers who behave as impatients in the interim period the international lender takes care of them. This implies that the banking system does not have to make early liquidations of the long asset. If the bank is illiquid in both currencies, then depending upon the differences between the foreign and the local returns on long term investments this institution could also implement the planner's allocation without runs. It can be shown that this is the case when foreign long term returns are greater than local long run returns. If the contrary is true the effectiveness of this institution is more limited. When the funds provided by the international institution are used to solve illiquidity problems in both currencies it can happen that in the last period the revenues from investments are not enough to repay deposits plus all debt. Therefore, only if the lent amount is small enough, can the international lender alone prevent runs.

I demonstrate that within this setup (with perfect enforcement of repayment) there are no moral hazard issues (since there is no separation of objectives between managers and depositors). This entails a lesson in terms of policy. The moral hazard issues currently discussed for the implementation of an international lender of last resort may be reduced as long as the managers and depositors coincide on the objectives.

I next show that an international lender of last resort providing liquidity in foreign currency coupled with a local lender of last resort (providing liquidity in the local currency) *always* prevents runs, so long as the interest rate is not too high, implementing the social planner's solution. This suggests that both lenders can complement each other.

Among other bank-run-preventing regimes, I concentrate on the case in which narrow banking is imposed on the foreign currency coupled with a local lender of last resort. This can be interpreted as an extreme case of consolidated banking systems in which foreign investment is limited (usually the Central Banks could

[3]For a survey on the bank runs literature see [12], chapter 7.

invest their reserves in short run safe assets only) but where the Central Bank acts as a lender of last resort in the local currency. It is shown that this policy eliminates runs. However this regime is not efficient since it involves excess liquidity in the short run, due to narrow banking in the foreign currency.

I then analyze the situation in which the proportion of impatient consumers is not known ex-ante (the aggregate uncertainty case). Wallace [17] has already shown that several regulatory policies cannot be implemented in this context. He has also shown [18] under specific assumptions that partial suspension of convertibility is optimal. Under these circumstances the properties of the three main policies analyzed without aggregate uncertainty do not change even though aggregate uncertainty is assumed, demonstrating that those results do not depend on the ex-ante knowledge of short term withdrawals.

In all these results the international institution only had a run-preventing role. However it can also be important in terms of the planner's solution itself, particularly when aggregate uncertainty is relevant. I show that if the planner has available a credit line from lenders of last resort (both international and local) then partial suspension does not hold anymore in the implementable solution (although some extra assumptions should be made to insure incentive compatibility constraints). The result also states that when the liquidity shock is large (meaning that the amount of withdrawals in the short run is larger than the mean) the planner actively uses the credit line from both lenders. This is consistent with casual observations of credits extended by international institutions after a financial distress event.

Section 2 discusses the role of lenders of last resort in preventing runs within the chapter 2 framework. It studies two alternative systems. The first only has an international institution serving as a lender of last resort. The second regime includes both an international and lender of last resort. Section 3 discusses other alternative regulations, including narrow banking. These are more extensively discussed in Kawamura [14]. Section 4 introduces the assumption of ex-ante unknown patient customers. Section 5 adds lenders of last resort to the planner's problem of the economy in section 4. Section 6 discusses some policy implications. Finally section 7 presents concluding remarks and points of future research.

2. Related Literature

As already mentioned, the framework used here is essentially identical to the ones in Kawamura [14] and [15]. These two papers follow the Bryant - Diamond and

Dybvig traditional framework ([3], [11]). The main feature is the presence of two currencies instead of the typical unique type of money in the literature. However the two papers share with the standard literature the potential existence of two equilibria, one involving runs.

The two main antecessors are the papers by Chang and Velasco (see [4] and [5]). They construct a Bryant - Diamond - Dybvig model in a small open economy. In [5] the long term investment is financed partially by international borrowing (to be paid in the last period). The consumption for impatients is financed by short term international funds to be paid at the end of the economy. In my model there is only international borrowing at the beginning of the economy, to be paid in the last period. The only *short term borrowing* is the case when the lenders of last resort funds the system to prevent runs.

In the second half of this paper I present a device to treat the aggregate uncertainty case taken from Wallace ([17] and [18]). That is to say, the amount of short run withdrawals is itself stochastic. In the first paper [17] he shows that the two run-preventing regulatory regimes studied by Diamond and Dybvig cannot be implemented, due to the non-observability of proportion of impatient consumers. In the second paper [18] Wallace presents a special case in which the banking system 's manager can learn the proportion through the order in which consumers withdraw in the interim period. He shows that declaration of partial suspension of convertibility of deposit contracts is part of the planner's allocation. I use this special case to redo the exercise with two currencies.

3. The Economy without aggregate uncertainty.

The economy lasts for three periods: $t = 0, 1, 2$. There are two currencies, called the *home* (non-tradable) and the *foreign* currency(tradable). I also use the term money interchangeably with the term currency. There is a storage technology for each type of money. This technology returns one unit in period $t + 1$ for each unit of the currency invested in period t, where $t = 0, 1$. On the other hand there are two long-term technologies, one for the foreign currency and the other for the home currency. For each unit of currency h invested in the long term technology written in h at period 0 it returns $R^h > 1$ units of the same type of money in period 2, but only $r^h \in (0, 1]$ in period 1. A spot market for the foreign currency is open. The economy takes the price of the foreign currency as given since it is small. I assume that this price to be one.

There is a large number N of ex-ante identical consumers. At the beginning of period 1 each person receives an idiosyncratic preference shock. This determines whether the consumer survives until period 2 or dies at period 1. The ex-ante probability of dying in period 1 is π. This probability is known ex-ante. The assumption will be partially relaxed later in the paper. Since N is large, by the law of large numbers π is also the proportion of consumers who die at $t = 1$.

If the consumer turns out to die at period 1 then she is *impatient*. Otherwise the consumer is *patient*. I assume that, if she is impatient, her utility function is $u\left(c_1^F\right) + v\left(c_1^H\right)$. If she is patient her utility function is $u\left(c_2^F\right) + v\left(c_2^H\right)$. Hence the ex-ante expected utility for the representative consumer in period 0 is just $\pi\left[u\left(c_1^F\right) + v\left(c_1^H\right)\right] + (1-\pi)\left[u\left(c_2^F\right) + v\left(c_2^H\right)\right]$[4]. I assume the usual properties for the utility functions: u and v are C^2, strictly increasing, strictly concave and satisfy standard Inada conditions.

3.1. A characterization of the planner's allocation.

This subsection characterizes the social planner's optimal allocation. I assume a *technology* that converts one unit of F into one unit H at any period. This captures the fact that most banking systems are governed by a Central Bank that creates domestic currency from foreign currency at some fixed exchange rate. I set this to unity.

The planner borrows from abroad a certain amount d of the foreign currency in period 0. Next she uses part of borrowed funds to get a certain amount θ^H of the home currency, using the remaining part as foreign currency. The planner invests θ^h, $h \in \{F, H\}$ in both the storage technology and the long term technology to finance consumption of both currencies. The storage technologies are used to finance period 1 consumption of both currencies for the impatient consumer and the long term technology is used to finance period 2 consumption of both types of money for those who turn out to be patient. This is true as long as the foreign currency deposits are repaid using entirely foreign assets. In the case of the long term asset written in foreign types of money this is also used to return the amount borrowed in period 0 at the international interest rate ρ.

The social planner's problem can be formalized as follows.

$$\max \quad \pi\left[u\left(c_1^F\right) + v\left(c_1^H\right)\right] + (1-\pi)\left[u\left(c_2^F\right) + v\left(c_2^H\right)\right]$$

subject to

[4] Although this seems a strong assumption, a larger family of utility functions can be generated through transformations of $u\left(c_t^F\right) + v\left(c_t^F\right)$. I thank Guido Cozzi for making this remark.

$$\theta^H + \theta^F \le d \tag{3.1}$$

$$\begin{aligned} b^F + X^F &\le \theta^F \\ b^H + X^H &\le \theta^H \end{aligned} \tag{3.2}$$

$$\pi\left(c_1^F + c_1^H\right) \le b^F + b^H \tag{3.3}$$

$$\pi c_1^F \le b^F \tag{3.4}$$

$$(1-\pi)\left(c_2^F + c_2^H\right) + \rho d \le X^F R^F + X^H R^H \tag{3.5}$$

$$(1-\pi)\, c_2^F + \rho d \le X^F R^F \tag{3.6}$$

$$\begin{aligned} d &\ge 0, \quad \theta^h \ge 0 \\ c_t^h &\ge 0, \quad t = 1,2 \quad h = H,F \\ b^h &\ge 0, \quad h = H,F \\ X^h &\ge 0, \quad h = H,F \end{aligned} \tag{3.7}$$

This is just the standard planner's problem in the bank runs literature. The only difference is the presence of two budget constraints in each period $t = 1,2$, the second of which captures the fact that H assets cannot finance F money consumption. This is because of the technological assumptions.

It can be shown that the solution to this problem is well defined as long as $\rho = R^F$. In this case d is indefinite. If $\rho > R^F$ the optimal d is 0, while if $\rho < R^F$ then the optimal $d = \infty$ and then there is no solution. If $R^F > \rho$ then I assume that $d \le \bar{d}$, where $\bar{d} > 0$. In this way we assure that there is a solution to the planner. For the rest of the paper we will assume that $\rho < R^F$ so I impose the constraint $d \le \bar{d}$[5].

Hence, the solution to the planner's problem can be characterized as usual. First, from the Kuhn-Tucker conditions we have the following proposition.

[5]This has been used by Chang and Velasco[5]. It can be easily shown that this is equivalent to a world rate of interest ρ which is a non linear function of d. For example, an interest rate rule might be $\rho = \underline{\rho}$ where $\underline{\rho} < R^F$, as long as $d \le \bar{d}$ and $\rho = \bar{\rho}$, where $\bar{\rho} > R^F$ when $d > \bar{d}$.

Proposition 3.1. *The planner's allocation contract is characterized by* $c_1^h < c_2^h$, *for* $h = F, H$. *Moreover, the optimal* $X^H > 0$ *if* $R^H \geq R^F$. *Otherwise* $X^H = 0$.

Proof. See Kawamura [14] and [15].

This result states that the standard incentive compatible constraint must hold under the planner's allocation. This allows for implementation of the social planner's solution as an equilibrium. This also assures under which condition the banking system will not invest in the long term H money asset.

3.2. Equilibrium behavior in a decentralized consolidated banking system.

This subsection presents a decentralized consolidated banking system which behaves competitively. Consumers are called *depositors*. These individuals run institutions called *commercial banks*. These banks allow the depositors to withdraw certain amount of currencies H and F at period 1 if they turn out to be impatient and the amount of consumption of both currencies if they are patient. They are essentially depositor-managed mutual funds. These banks will offer a consumption allocation $\left(\left(c_1^H, c_1^F\right), \left(c_2^H, c_2^F\right)\right)$ to the consumers-customers. This allocation maximizes the ex-ante utility of the customer (assuming perfect competition for the banks that induce them to offer the best possible contract to the consumer) subject to the constraints stated above and the incentive compatibility constraint:

$$u\left(c_2^F\right) + v\left(c_2^H\right) \geq u\left(c_1^F\right) + v\left(c_1^H\right) \tag{3.8}$$

Separability of preferences allows to get the first obvious result.

Proposition 3.2. *The planner's allocation can be implemented as an equilibrium contract.*

Proof. It follows from proposition 3.1. ■

However, the contract that implements the planner's solution can be subject to runs, as a second equilibrium. This is not surprising since the basic structure is totally borrowed from the original DD model. However the (sufficient) condition to have such run equilibrium is that the banking system is illiquid in both currencies.

Proposition 3.3. *Suppose that* $c_1^F > b^F + r^F\left(X^F - \left(\rho/R^F\right)d\right)$ *and* $c_1^H > b^H + r^H X^H + b^F - \pi c_1^F$ *Suppose that* N *is large enough. Then there is an equilibrium where all consumers run against the bank system and this closes in period* 1.

Proof. See Kawamura [14] and [15]. ■

The intuition is as follows. Liquidity is now taken on a currency by currency basis. This means that the last condition can be read as a bank having illiquidity problems in both money types. This seems to be a strong condition. The following proposition shows that if the bank is only illiquid in one currency, then there could be no run equilibrium.

Proposition 3.4. *Suppose that the optimal contract satisfies* $c_1^F > b^F + r^F\left(X^F - \left(\rho/R^F\right)d\right)$, $c_1^H \leq b^H + r^H X^H + b^F - \pi c_1^F$ *and* $v\left(c_2^H\right) - v\left(c_1^H\right) \geq u\left(c_1^F\right) - u\left(0\right)$. *If* $R^H \geq R^F$ *then the contract is not subject to runs.*

Proof. See Kawamura [14] and [15]. ■

This result states that the planner's solution might not be subject to runs even if there is an illiquidity problem in only one currency. The result is interpreted as follows. If the bank is illiquid in one money type but liquid in the other, and if the gain of waiting until period 2 in terms of the liquid money for the patient more than compensates the 0 consumption of the illiquid currency in period 2 then it is not optimal for the patient to lie, and then there cannot be a run equilibrium with this contract.

Notice that if $R^H < R^F$ then $X^H = 0$. In this case, if $c_1^F > b^F + r^F\left(X^F - \left(\rho/R^F\right)d\right)$ then the banking system has a run equilibrium. Therefore the rate of return differential has some role in financial fragility, through the portfolio that induces. If $c_1^F > b^F + r^F\left(X^F - \left(\rho/R^F\right)d\right)$, $c_1^H \leq b^H + r^H X^H + b^F - \pi c_1^F$ and $R^H \geq R^F$ but $v\left(c_2^H\right) - v\left(c_1^H\right) < u\left(c_1^F\right) - u\left(0\right)$ then runs still occur. This shows that the way the planner's solution looks like is very important for financial fragility.

4. Lenders of Last Resort

Since the last section emphasizes the possibility of runs through self-fulfilling prophecies, concerns of how to eliminate the coordination failure naturally arise. Given the motivation of this chapter, I introduce in this section different types of institutions tending to eliminate the possibility of such runs. It is important to remark on the nature of bank runs here. I focus on runs originated in coordination failure problems, so I ignore any other information - based (solvency) type of runs.

4.1. International Lender of Last Resort and the prevention of bank runs.

An institution called *international lender of last resort* is introduced. In my model it is just an entity outside the economy (not explicitly modelled) that lends any amount of currency F that the financial system needs to borrow at $t = 1$. I am assuming here a perfectly elastic supply for this credit line, emphasizing the small open economy description.

The main result of this section is that, if the net interest rate of the loan is zero, this institution may not always be able to prevent self - fulfilling liquidity-based runs. The following proposition provides sufficient conditions for this to be true [6].

Proposition 4.1. *If the banking system experience illiquidity problems at the interim period, the existence of an international lender of last resort that lends enough resources to eliminate illiquidity is sufficient to implement the planner's solution without runs whenever $R^F > R^H$ or otherwise when $\left(c_1^H + c_1^F\right) \leq c_2^F$ (if there is illiquidity in both), or $c_1^H \leq c_2^F$ (if there is illiquidity in H but not in F) or when there is only illiquidity in F but not in H.*

The intuition is as follows. If $R^F > R^H$ then banks only possess long term assets written in the foreign currency to pay both the domestic and the foreign period 2 consumption allocations stated in the deposit contracts. This implies that banks have enough foreign assets at the last period to repay the debt from the lender of last resort. If the domestic long term return is greater than the foreign return, banks will back part of the domestic money deposits with the domestic currency long term investment. Hence foreign long term investment is not as large as in the first case. This means that only when the lent amount in period 1 (in the event of panics) is small enough then the banking system is able to repay to the international institution. This is true when period 1 per-person specified withdrawals are not too big. Otherwise banks might not be able to return the short term credit line.

The main lesson is that this institution may prevent illiquidity problems, but the extent to which this is true depends on the deposit contracts design and the rates of returns of long term investment. In a sense, this institution is not very useful when the banking system has a high level of short run deposits and a big

[6] Proofs of propositions of subsequent sections are in the appendix, unless specified.

portion of its long term securities invested in local currency assets. In this case the amount to be lent is too high so that long run deposits cannot be repaid, and so banks still fail.

The assumption of zero net interest rate is not necessarily a limitation. As long as the interest rate to be paid for this credit line is not too large, then the same properties hold. The following proposition shows this: (proof is again in the appendix).

Proposition 4.2. *When $R^F > R^H$ then an international lender of last resort can prevent runs if the net interest rate to be paid is less than or equal to $\left(c_2^H + c_2^F\right) / \left(c_1^H + c_1^F\right)$. If $R^H \geq R^F$, the international lender of last resort prevents runs when illiquidity on both currencies if the interest rate is less than or equal to $\frac{c_2^F}{c_1^H + c_1^F}$. If the illiquidity is on the F money only the interest rate must be less than or equal to $\frac{c_2^F}{c_1^F}$, and if the illiquidity is on the H money, then the interest rate must be less than or equal to $\frac{c_2^F}{c_{1H}}$.*

This constitutes a generalization of the former proposition. The result sets upper bounds on interest rates for those contingent credit lines. This is important in terms of policy applications. Although this should not be taken literally, it gives some guideline to see what is a reasonable interest rate for these credit lines. It basically states that the actual amount of deposits to be withdrawn in different periods matter to set such interest rates. As a general interpretation one can say that, the higher is the amount of long run deposits, the higher could be the interest rate charged to the borrowers.

4.2. International Lender of Last Resort and Moral Hazard.

The result stated above ignores any moral hazard issue related to the presence of such a mechanism. With perfect commitment and perfect observability of actions, then the bankers *must* use the funds coming from the international lender to finance any difference between the withdrawals at period 1 and the revenue from the liquid investment. If actions were not observable to the lender of last resort, and if the managers of the banking system commit not to let the banks to fall into bankruptcy then it is possible to show that there is still no incentive to deviate from the strategy of paying . The following proposition shows this.

Proposition 4.3. *Even if there is no explicit commitment to sustain the financial system, as long as repayment to the international lender of last resort is perfectly enforced, then there is no incentive for banks to deviate funds at $t = 1$.*

The last paragraph implies that, if the banking system commits to repay all debt perfectly and also commits perfectly to use at least partially the funds from the international lender, then there is no moral hazard problem under the rest of the standard assumptions of the Diamond and Dybvig model.

The intuition is that when the objectives of managers and depositors are the same, there are no incentives to divert funds from the main purpose. Since the debt contract with the international lender is perfectly enforceable there is no point in not sustaining the financial system when threatened by a run. Given that the banking system must repay the debt, by saving the lent amount and not paying to the customers running at $t = 1$ the managers do not gain anything relative to the full payment.

If there is no perfect commitment of debt repayment the story is different. Without commitment of repayment, there might not be any credit line from the lender, unless there is some punishment mechanism if the banking system defaults on its debt. This is discussed in the concluding remarks section [7]. Another key assumption is the absence of any investment technology between $t = 1$ and $t = 2$ except for the storage technology. A relaxation of this (including for example a short term technology between $t = 1$ and $t = 2$ with gross return greater than one) could affect the result, although it is not clear in what circumstances.

Notice that this proposition may also break down if there is a separation between managers and depositors. When the objective of the managers is not to maximize the ex-ante welfare of customers, then the possibility of borrowing from abroad and not using this to satisfy liquidity needs is much more reasonable. In such a case incentive compatible contracts should be offered in order to induce the managers not to misbehave with those funds. [8]

[7] Suppose that there is no debt repayment commitment. If the creditors were able to seize all of revenues coming from foreign long term investment in period 2, then in some circumnstances it may be incentive compatible to repay. In particular, if the actual proportion of the population withdrawing at date 1 is less than half of the theoretical proportion of patient customers (that is, if $\hat{\pi} < (1 - \pi) \ / \ 2$) then it is incentive compatible to repay given the threat. However when $\hat{\pi}$ is larger (in fact, when this is almost 1) then the incentive compatibility argument breaks down. Hence in this case moral hazard problem becomes an issue.

[8] In the context of solvency regulations, Dewatripont and Tirole ([9] and [10]) put the control rights allocation at the center of the analysis. Although in my paper I do not consider solvency

4.3. International and Local Lenders of last resort.

Consider a variation of the case above. In addition to the international institution the Central Bank acts as a local lender of last resort, providing local currency if there are illiquidity problems. In order for this to work I assume that, only at period 1, an institution called *local lender* of last resort could create extra units of H up to some upper bound $\bar{c}^H$. This institution may produce this extra amount by lending them to the banking system at some interest rate. The version of the model considered here can be viewed as a reduced form of a banking system that commits to create local currencies out of foreign currencies to build reputation, although with a local lender of last resort clause that might enhance liquidity in local currency for the banks in the event of a financial distress.

I show that coupling these two institutions prevent runs at period 1 in any case (for not very high interest rates).

Proposition 4.4. *If there is perfect commitment in repaying debts, then the international lender of last resort plus a local lender of last resort prevents runs as long as the interest rate on the foreign loan is less than or equal to* $\min\left\{\frac{c_2^F}{c_1^F}, \frac{c_2^F+c_2^H}{c_1^F+c_1^H}\right\}$ -1 *and the interest rate on the local loan is at most* $\frac{c_2^H}{c_1^H}-1$.

This result suggests that complementing the international line short run credit with a local lender is useful to prevent runs in any situation, as long as interest rates on those credit lines are not large. However this proposition also states that in order to have such complementarity between the two lenders of last resort, the local institution should lend local currency only when this will be consumed and not exchanged for foreign currency. In section 7, I discuss in more detail the difficulties of implementing this regime.

5. Other regimes: narrow banking and local lender of last resort.

This section is devoted to an analysis of extra menu of regimes, concentrating on one including narrow banking in the foreign currency plus a local lender of last resort in the local currency. This may be interpreted as a special case of several actual policies currently applied by Central Banks.

problems, this literature might be useful to build models in which liquidity problems and moral hazard considerations could be studied.

5.1. Narrow Banking on the foreign currency and local lender of last resort.

In the bank runs literature narrow banking proposals received considerable attention. In my model narrow banking could be applied to both currencies or to any of them individually. I consider the case in which narrow banking is imposed only on the F currency. Even if the bank is illiquid in both currencies, narrow banking in the F currency only coupled with a local lender of last resort might prevent self fulfilling runs. The next lines explore this issue.

On the other hand, even if the H liquidity condition is not satisfied, coupling this regime with a local lender of last resort that lends in H money only also prevents runs.

Proposition 5.1. *Suppose that the F narrow banking regime is complemented by a local lender of last resort, who lends H currency to the banking system to meet the liquidity demands in period* 1. *Then runs are also prevented.*

In a sense, the F narrow banking per se ensures liquidity in the F currency, while the local lender ensures liquidity in H money. Still this is not an efficient way of eliminating runs. This regime causes excess liquidity of foreign money in the interim period, which does not exist under the planner's solution.

This analysis suggests that imposing very tight regulations on the portfolio in foreign currencies together with a local lender of last resort eliminates the risk of self fulfilling liquidity based runs. However, this also tells us that the cost of implementing this regulatory policy is the excess of liquidity in foreign currencies in the short run, which could have been invested in more profitable longer term assets. This has some similarity with the current situation in countries such as Argentina, in which the banking system has faced a growing amount of foreign currency deposits during the period 1997 - 1999. Although part of the excess liquidity in dollars present in this case could also be explained by risk project issues, the current Central Bank investment policy may tend to help in generating this excess liquidity.

5.2. Other Alternative Regulatory Regimes.

The regimes considered above are only three out of other many alternatives to prevent runs. I consider other possible arrangements, most of whom are just re-adaptations of standard regulatory regimes studied in the literature.

5.2.1. Narrow Banking on both currencies.

I have already discussed a regime with narrow banking on the foreign currency coupled with a local lender of last resort. A more extreme policy is to impose narrow banking on both currencies. In this case the regulatory regime forces to finance all period 1 consumption with the liquid asset. The banking system faces the following constraints:

$$c_1^H + c_1^F \leq b^H + b^F \tag{5.1}$$

$$c_1^F \leq b^F \tag{5.2}$$

$$(1-\pi)\left(c_2^F + c_2^H\right) + \rho d \leq X^F R^F + X^H R^H + b^H + b^F - \pi\left(c_1^H + c_1^F\right) \tag{5.3}$$

$$(1-\pi)\, c_2^F + \rho d \leq X^F R^F + b^F - \pi c_1^F \tag{5.4}$$

The problem for the banking system is to maximize the ex-ante utility subject to the constraints 3.1, 3.2, 3.7 , $d \leq \bar{d}$, the incentive compatibility constraint and 5.1, 5.2, 5.3, 5.4. These inequalities imply the absence of runs, so the amount of withdrawals in period 1 is strictly less than the total amount of liquid assets, the remaining of which is used to finance period 2 consumption.

Clearly this policy impedes any type of run by construction.. It is obviously less efficient than the planner's solution since the bank experiences excess liquidity (relative to the planner's allocation) at period 1.

5.2.2. Minimum Liquidity Requirements (War Chests)

An alternative to complete narrow banking is a war chest policy. Suppose for this case that $r^h = 1$. The banking system is forced to reserve a sum T^h of the date 0 supply of currency h received in period 0, where $T^h \leq \theta^h$. This should be used to prevent a run so that $T^H + b^H + T^F + b^F = c_1^H + c_1^F$ and $c_1^F \leq b^F + T^F$. If $R^H \geq R^F$, T^h ($h \in \{H, F\}$) are invested in the long term investment, so that, if there are no runs, this is used to finance consumption of currency h at period 2 through the return R^h. If $R^F > R^H$ then only T^F units are invested so that $b^H + T^F + b^F = c_1^H + c_1^F$. In a sense, the regulation assures that the amount of long term investment is large enough to prevent runs.

The problem for the bank is thus to maximize the ex-ante utility subject to $d \leq \bar{d}$, 3.1, 3.3, 3.4, the incentive compatibility constraint and

$$b^h + X^h \leq \omega^h - T^h, \quad h \in \{H, F\} \tag{5.5}$$

$$(1-\pi)\left(c_2^F + c_2^H\right) + \rho d \leq R^F\left(X^F + T^F\right) + R^H\left(X^H + T^H\right) \tag{5.6}$$

$$(1-\pi)\, c_2^F + \rho d \leq R^F\left(X^F + T^F\right) \tag{5.7}$$

Also T^h must be such that $T^H + b^H + T^F + b^F = c_1^H + c_1^F$ and $c_1^F \leq b^F + T^F$. Note that the incentive compatibility constraint can again be ignored since the first order conditions of the problem without the IC constraint implies that $c_1^h < c_2^h$ for both h and then IC is satisfied automatically.

By the condition state above there is no need of liquidating the long term asset in period 1. Hence there is no incentive to lie for a patient consumer and no runs occur. However we will show that the welfare properties of this regime are different from the implications of the narrow banking regime.

5.2.3. Comparison between Narrow Banking and War Chests.

It is possible to compare the welfare properties of the last policy with respect to the complete narrow banking proposal. Under both proposals all constraints can be reduced to the following expression:

$$(1-\pi)\, c_2^h = R^h \theta^h - \left(R^h - (1-\pi)\right) c_1^h$$

This means that the *marginal rate of transformation* [9] between c_1^h and c_2^h (for $h = H$ or F) is just $\frac{\left(R^h - (1-\pi)\right)}{(1-\pi)}$. On the other hand, from the first order conditions it can be shown that the marginal rate of substitution between c_1^h and c_2^h is equal to the marginal rate of transformation under both proposals. However war chests policy in this model does not imply excess liquidity, as opposed to complete narrow banking.

This constitutes a major difference with respect to Chang and Velasco 's analysis [4]. In fact this proposition shows that their result stating that narrow banking

[9] I define the marginal rate of transformation as the derivative $\partial c_2^h / \partial c_1^h$ when the other two consumption allocations are kept fixed.

is preferable to large international reserves depends upon the assumption of liquidation costs for the long term asset if liquidated in period 1. This is what impedes to invest T^h in the long term asset. In the case of this subsection I assume no liquidation costs, so T^h is invested in the long term asset. What the monetary authority is doing is just to accumulate enough long term assets to avoid panics. The securities that the banking system has (even if liquidating part of the long asset) is large enough to prevent banking failures. In a sense, it is equivalent to "reduce" the early payments for the impatient consumers by reducing b^h (which is implied by the amount T^h taken away from θ^h). Therefore this reduces fragility.

If $r^h < 1$ (which is the more common assumption in the literature) still the latter policy could be applied as long as r^h is sufficiently close to 1. In this case the conditions are

$$\begin{aligned} r^H T^H + b^H + r^F T^F + b^F &= c_1^H + c_1^F \\ c_1^F &\leq b^F + r^F T^F \end{aligned}$$

which for r^h close enough to 1 still this has similar properties as in the paragraphs above. However when r^h is not close to 1, then a war chest policy of the Chang - Velasco type may be preferable. This emphasizes the role of r^h in determining war chests.

5.2.4. Total Suspension on the Foreign currency deposit and a local lender of last resort.

The Diamond and Dybvig paper claimed the usefulness of total suspension clauses in order to prevent runs. It can be easily seen that imposing such a clause together with a local lender of last resort is equivalent alternative to the regime in which there are both an international and a local lender of last resort. As long as the regulator commits perfectly to the total suspension clause, the bank never runs out of foreign money type at period 1. On the other hand, any attempt of run against the home currency is prevented by the local lender of last resort. As a result patient customers never run since bank failures at $t = 1$ cannot occur.

Although this states that the mentioned regime prevents runs efficiently, it is not implementable whenever the amount of early withdrawals $N\pi$ is random and not observable ex-ante. The reason is that this policy needs to specify when the banking system stops paying at $t = 1$ in terms of currency F. To do this the regulator must know π, which is feasible under absence of aggregate uncertainty only. Another problem with this type of regulation is the possible effects on expectations.. Although this has not been studied yet, some believe that a total suspension clause could exacerbate any pessimistic expectations that customers may have. The clause might work as a *bad signal* that might not be necessarily accurate.

6. The economy with aggregate uncertainty.

Some of the regulatory regimes depend on the knowledge of the proportion of impatients in period 1. It is not only that π is known but also that the actual proportion of people behaving as impatients is known. The following question is how these results change when π is not known ex-ante. This section explores the planner's allocation, financial fragility and regulation in an economy in which π is stochastic.

Suppose that in period 0 the proportion of impatients is a random variable. For simplicity I adopt the device presented by Wallace ([17] and specially [18]). Assume that π can be either $p\alpha + (1-p)$, with probability q_1 and $p\alpha$, with probability q_2. The banking system knows at least $N\alpha p$ people are impatient (and $N(1-\alpha)p$ are patient consumers). There could be still other $N(1-p)$ impatient consumers (with probability q_2). Otherwise they are all impatient (with probability q_1). Any person is within the first group with probability p and within the second group with probability $(1-p)$. Hence the ex-ante utility is now:

$$p\left[\alpha\left[u\left(c_1^{F1}\right)+v\left(c_1^{H1}\right)\right]+(1-\alpha)\left(\sum_{s=1}^{2} q_s\left(u\left(c_2^{F1}(s)\right)+v\left(c_2^{H1}(s)\right)\right)\right)\right] \quad (6.1)$$
$$+(1-p)\left(q_1\left(u\left(c_1^{F2}(1)\right)+v\left(c_1^{H2}(1)\right)\right)+q_2\left(u\left(c_2^{F2}(2)\right)+v\left(c_2^{H2}(2)\right)\right)\right)$$

where $c_t^{hj}(s)$ denotes consumption of currency h in period t, state s and position-in-line j. Here $s=1$ denotes the state in which all the people in the second group are impatient, and $s=2$ corresponds to the state in which all of them are patient. Similarly, $j=1$ denotes the (individual) state in which the consumer is in the first group, while $j=2$ denotes the state in which the consumer is in the second group. Note that consumption of impatient consumers within the first group does not depend on s, i.e., c_1^{h1} is independent of s. This is because I assume that the banking system does *not* observe the aggregate state s. They must infer it from the number of impatients showing up at date 1. If only $N\alpha p$ impatient consumers show up, the bankers infer (correctly) that the proportion of impatient people is αp and then the state is $s = 1$. If the total number exceeds $N\alpha p$, it must be clearly $N(\alpha p + (1-p))$ (provided that nobody lies). Hence whenever the banker observes that there are more impatient consumers than $N\alpha p$, then she infers that the state is $s = 2$. However, in any case, people who were lucky showing up first (among the first $N\alpha p$) should receive the same consumption in period 1 since the bankers cannot know the state at that stage[10].

6.1. The planner's problem when π is stochastic.

The problem for the planner is to maximize 6.1 subject to the constraints 3.1, 3.2, $d \leq \bar{d}$, and

$$p\alpha\left(c_1^{F1} + c_1^{H1}\right) + (1-p)\left(c_1^{F2}(1) + c_1^{H2}(1)\right) \leq b^H + b^F \tag{6.2}$$

$$p\alpha c_1^{F1} + (1-p)\, c_1^{F2}(1) \leq b^F \tag{6.3}$$

$$p(1-\alpha)\left(c_2^{F1}(1) + c_2^{H1}(1)\right) + \rho d \leq R^F X^F + R^H X^H \tag{6.4}$$

$$p(1-\alpha)\, c_2^{F1}(1) + \rho d \leq R^F X^F \tag{6.5}$$

$$\begin{aligned} & p(1-\alpha)\left(c_2^{F1}(2) + c_2^{H1}(2)\right) + (1-p)\left(c_2^{F2}(2) + c_2^{H1}(2)\right) + \rho d \\ \leq\ & R^F X^F + R^H X^H + b^F + b^H - p\alpha\left(c_1^{F1} + c_1^{H1}\right) \end{aligned} \tag{6.6}$$

$$\begin{aligned} & p(1-\alpha)\, c_2^{F1}(2) + (1-p)\, c_2^{F2}(2) + \rho d \\ \leq\ & R^F X^F + b^F - p\alpha c_1^{F1} \end{aligned} \tag{6.7}$$

Notice that if $s = 2$, the period 1 constraints would be $p\alpha\left(c_1^{F1} + c_1^{H1}\right) \leq b^H + b^F$ and $p\alpha c_1^{F1} \leq b^F$. In equilibrium, $c_1^{h2}(1) > 0$, which implies that the two constraints mentioned above are non-binding and hence ignored.

The first order conditions give the following result.

[10] This is a consequence of the sequential service assumption.

Proposition 6.1. *The planner's solution implies that for both $h \in \{H, F\}$:*

$$c_1^{h2}(1) < c_1^{h1}$$

Proof. See Kawamura [14] and [15]. ■

This proposition is a version of the optimality-of-partial-suspension result by Wallace [18]. In his article he showed that partial suspension is optimal in a one currency economy. In a two-currency world partial suspension is still optimal but now imposed on both currencies. That is, in an economy with two types of money it is optimal to pay less of each commodity at period 1 after $Np\alpha$ consumers show up claiming impatience.

6.2. Equilibrium

It is not difficult to show the conditions under which the planner's allocation presented above can be implemented. The following proposition proven in [14] and [15] shows this.

Proposition 6.2. *Under the stated assumptions the planner's solution (with partial suspension of convertibility) can be offered in a decentralized banking system equilibrium if $R^F \geq R^H$. Otherwise it is implementable if either p is sufficiently close to 1 or if $b^F = \alpha p c_1^{1F} + (1-p)\, c_1^{2F}(1)$.*

Notice that this implementation requires the declaration of total suspension of convertibility of certificates in the interim period after $N(p\alpha + 1 - p)$ people show up. This is the key element that allows to implement the planner's allocation *without* runs. However, if the partial suspension is sustained for *all* people who show up after the first $N\alpha p$ show up, then runs are still possible. The following proposition shows this.

Proposition 6.3. *If $p\alpha\left(c_1^{F1}\right) + (1-\alpha p)\, c_1^{F2}(1) > b^F + r^F\left(X^F - \left(R^F/\rho\right) d\right)$, and $p\alpha\left(c_1^{FH}\right) + (1-\alpha p)\, c_1^{FH}(1) > b^H + r^H X^H + b^F - p\alpha\left(c_1^{F1}\right) - (1-p)\, c_1^{F2}(1)$ then there is an equilibrium with runs.*

Proof. See Kawamura[14] ■.

Having stated these standard results, I show next that the main result on the effectiveness of lenders of last resort to prevent bank runs in this set-up does not depend on the ex-ante knowledge of π.

6.3. Lenders of Last Resort with aggregate uncertainty.

An alternative preventive way to avoid runs is still the presence of an international lender of last resort. The arguments are very similar to the ones given in section 5. Suppose the following arrangement. The banking system pays to the first $N\alpha p$ consumers who claim impatience a total of c_1^{h1} of currency h. To the rest of consumers the banking system pays $c_1^{h2}(1)$ units of h. Notice that the short term investment in currency h is sufficient to pay to the first $N(\alpha p + 1 - p)$ impatient consumers. If even more consumers show up in the interim period the banks pay them by getting funds from the international lender at a net interest rate of zero.

The next proposition affirms that this arrangement prevents runs essentially under the same assumptions as in proposition (4.1).

Proposition 6.4. *If $R^F > R^H$ then an international lender of last resort charging zero net interest rate allows for implementation of the planner's allocation without runs. If $R^H \geq R^F$ the international institution can avoid runs if there is only illiquidity in F, or if $\left(c_1^{2F}(1) + c_1^{2H}(1)\right) \leq c_2^{1F}(1)$ if there is illiquidity in both, or $c_1^{2H}(1) \leq c_2^{1F}(1)$ if there is illiquidity only in terms of H.*

This result demonstrates that not knowing the proportion of impatient consumers ex-ante is not an impediment to implement an international lender of last resort as a mechanism to eliminate the run equilibrium. In other words, the coordinating property of the international lender (if it is effective) does *not* depend on the knowledge of impatient consumers.

On the other hand it is perfectly possible to get similar results on international and local lenders to the ones obtained in section 5 when aggregate uncertainty is introduced.

Proposition 6.5. *If $R^F > R^H$ then an international lender of last resort allows for implementation of the planner's allocation without runs if the net interest rate is less than or equal to $\frac{c_2^{1F}(1)+c_2^{1H}(1)}{c_1^{2F}(1)+c_1^{2H}(1)} - 1$. If $R^H \geq R^F$ the international institution can avoid runs when there is only illiquidity in F if the interest rate is at most $\frac{c_2^{1F}(1)}{c_1^{2F}(1)} - 1$. When there is illiquidity in both currencies, then interest rate must be less than or equal to $\frac{c_2^{1F}(1)}{\left(c_1^{2F}(1)+c_1^{2H}(1)\right)} - 1$, and when there is illiquidity only in terms of H the interest rate must be at most $\frac{c_2^{1F}(1)}{c_1^{2H}(1)}$.*

Proposition 6.6. *If both an international lender of last resort and a local lender of last resort exist, then the planner's allocation can be implemented without runs even when there is aggregate uncertainty whenever the gross interest rate charged by the international lender of last resort is less than or equal to min* $\left\{\frac{c_2^{1F}(1)+c_2^{1H}(1)}{c_1^{2F}(1)+c_1^{2H}(1)}; \frac{c_2^{1F}(1)}{c_1^{2F}(1)}\right\}$, *and the one charged by the local lender is at most* $\frac{c_2^{1H}(1)}{c_1^{2H}(1)}$.

This series of propositions imply that the effectiveness of such lenders of last resort to prevent runs does not depend on the ex-ante observability of period 1 withdrawals. Therefore the implementation of these regimes is independent of such considerations. In the next section I informally discuss which factors are really important to apply the results in policy making.

7. The planner's problem with lenders of last resort

In the last case of uncertain number of impatient customers there is room for adding lenders of last resort in the planner's problem itself. The main reason is that in small open economies the effective use of such lenders (specially the international institutions) is often observed. On the other hand the version of the model in the last section does not imply effective use of these lenders in equilibrium. In this section I show that there are sufficient conditions under which adding these lenders of last resort avoid partial suspension of convertibility, replaced by the use of funds from these lenders of last resort. I assume that the net interest rate for these funds is zero.

In this case the planner faces the problem of maximizing 6.1 subject to the constraints 3.1, 3.2, $d \leq \bar{d}$, and

$$\begin{aligned}
[\alpha p + 1 - p]\left(c_1^{1F} + c_1^{1H}\right) &\leq b^H + b^F + \lambda_1^F(1) + \lambda_1^H(1) \\
[\alpha p + 1 - p]\, c_1^{1F} &\leq b^F + \lambda_1^F(1)
\end{aligned}$$

$$\begin{aligned}
\alpha p\left(c_1^{1F} + c_1^{1H}\right) &\leq b^H + b^F \\
\alpha p c_1^{1F} &\leq b^F
\end{aligned}$$

$$\begin{aligned}
p(1-\alpha)\left[c_2^F(1) + c_2^H(1)\right] &\leq R^F X^F - \rho d + R^H X^H - \left(\lambda_1^F(1) + \lambda_1^H(1)\right) \\
p(1-\alpha)\, c_2^F(1) &\leq R^F X^F - \rho d - \lambda_1^F(1)
\end{aligned}$$

$$
\begin{aligned}
(p(1-\alpha)+1-p)\left[c_2^F(2)+c_2^H(2)\right] &\leq R^F X^F-\rho d+R^H X^H \\
&\quad +b^H+b^F-\alpha p\left(c_1^{1F}+c_1^{1H}\right) \\
(p(1-\alpha)+1-p)\, c_2^F(2) &\leq R^F X^F-\rho d+b^F-\alpha p c_1^{1F}
\end{aligned}
$$

and if $R^H > R^F$ I also impose $\lambda_1^H(1) \leq (1-p)\, c_1^{1H}$.

This is the same problem as above (section 6) except for the presence of the variables $\lambda_1^h(1)$, which represents funds borrowed from the corresponding lender of last resort delivering units of currency h (when $h = F$ it is the international lender of last resort, when $h = H$ the institution is the local lender of last resort). The last constraint is added since in the case of $R^H > R^F$ and $\lambda_1^F(1) > 0$ the amount of $\lambda_1^H(1)$ can grow unboundedly without that condition.

The following result shows the sufficient conditions under which the solution to the problem can be decentralized as an equilibrium.

Proposition 7.1. *If $R^F \geq R^H$ the solution to the planner's problem with lenders of last resort can be sustained as an equilibrium in a banking system of the type of section 6. If $R^F < R^H$ the same is true given that $(p(1-\alpha)+1-p)\, c_2^F(2) = R^F X^F - \rho d + b^F - \alpha p c_1^{1F}$ and $b^F = \alpha p c_1^{1F}$, or else if p is sufficiently close to 1. If the lenders of last resort are available for the banking system, and if the local lender could lend more than $(1-p)\, c_1^{1H}$ only to cover liquidity needs, then there is no run equilibrium.*

The proof is in the appendix. Note the similarity of this with proposition 6.2, except in the condition concerning the short run currency F asset. The main conclusion is that the extra short run liquidity needs at state 1 in period 1 can be solved by the lenders of last resort, avoiding any kind of partial suspension.

This result formalizes the intuition that the lenders of last resort can cover transitory illiquidity situations instead of reducing the real amount of withdrawals by the second group of impatient customers. This is consistent with several situations in which international institutions actually must lend funds to countries whose financial institutions are under liquidity distress.

8. Policy Implications.

The results in this chapter allow for a discussion about how to implement contingent credit lines such as the one discussed in the introduction. Since the decision made by the G7 countries has been attacked from several points of view, it is useful to see what such propositions teach us about their effectiveness.

First, as long as the funds from this credit line are used to help *transitorily illiquid* financial systems, then propositions 4.1 and 6.4 specify under which conditions an international lender of last resort charging zero net interest rate could work. Basically those conditions demand to check the returns differential in different currencies. Those propositions also point out the importance of calculating ratios of long versus short run deposits in different currencies.

Second, propositions 4.2 and 6.5 suggests upper bounds for the interest rate that the international lender of last resort must charge in order to make repayment feasible. Once more the main problem here is to measure deposits in different currencies and horizons so that interest rates on these credit lines are not too high. I do not suggest to take these ratios literally, but they constitute a major guide for interest rate negotiations.

Third, proposition 4.3 states the necessity of working on managerial incentives in the banking sector, so that bank managers do not have any willingness to let the system fall. As long as the debt contract with the lender of last resort is perfectly enforceable, common objectives between managers and depositors assure absence of any moral hazard consideration. In order to induce such objectives coordination, explicit or implicit incentives could be made to managers and stockholders so that it is on their interest to use those funds on behalf of depositors. Alternatively regulation could be applied in order to ensure this objective.

Fourth, propositions 4.4 and 4.6 state that an international lender could be coupled with a suitable local lender of last resort. In a sense these results suggest that both institutions tend to complement, not to substitute, each other. However such a local lender cannot have a loose behavior. Its main purpose is to lend local currency to the financial institutions in the short run whenever it is proved that the banking system can be threatened by a *liquidity* crisis. However these results also state that the local lender is useful as long as there is illiquidity in local currency. That is, it cannot help in situations where the illiquidity is essentially in foreign currencies. The main danger of having a local lender in this situation is to worsen the foreign reserves situation having more customers with local currency running against the Central Bank. Thus the purpose of the local institution must limited only to cover local currency liquidity needs.

A special remark about the local lender is the fact that it is only used when the illiquidity in local currency arises. This assumes a large degree of commitment by the institution acting as a local lender (usually the Central Bank). This also has policy implications. Implementing such mechanism in this way implies the creation of institutions or legal systems that prevents *irresponsible* behavior by the acting local lender (*creating* liquidity when there is no need of it). Hence the setting up of a local lender of last resort demands the creation of very solid laws and institutions to avoid local lender misbehavior. Another possible way is to have international institutions such as the IMF monitoring the functioning of such local lenders.

From the paragraphs above it is clear that implementing such institutions is not easy. Monitoring costs (in the sense of keeping track of deposits) and the problem of measuring the liquidity needs in each currency are difficult. This does not mean that they are infeasible in practice, but it gives a warning in terms of how to implement them. The present essay still has a last message. Even if the conditions for an international lender are not met, as a kind of *second best* the Central Bank could declare narrow banking on deposits written in foreign currency, while working as a local lender of last resort in local currency in the way described before. This is the main conclusion of proposition 5.1.

From proposition 7.1 we have learned that partial suspension of convertibility need not hold in the aggregate uncertainty case. Instead the lenders of last resort take care of the liquidity of the banking system when extra impatient customers withdraw in period 1. This means that the availability of those institutions not only prevent runs, by threatening the patients, but also provides funds when there are extra withdrawals in the short run, so that withdrawals do not have to be suspended. In terms of evidence, some facts from the banking distress situation in the case of Argentina, in 1995, suggest that the funds coming from the Inter American Development Bank and the World Bank had as one of the main purposes to enhance liquidity for the healthy banks of this country.

In any case all these regulatory regimes implied by the results deal with *liquidity* problems. It does not say anything in terms of solvency issues. The main challenge in practice is to discover whether certain financial distress phenomena were caused by liquidity or solvency problems. This still remains an open question for the policy makers.

9. Concluding remarks and possible extensions

This paper has presented an extension of the Diamond - Dyvbig framework to a banking system with two currencies. I have done that with the two alternative assumptions of the proportion of impatient consumers known and unknown (ex-ante).

The main message is that an international lender of last resort per se might not be able to prevent runs even when runs are mostly liquidity-based and when perfect commitment of repayment is possible. Complementing this institution with a local lender might be more effective. Another message is that interest rates charged by these institutions cannot be too high. The results above indicates that in order to set these rates it is important to know the ratio of long term over short term deposits. This indicator can give an important guideline for policy makers to set suitable interest rates for these credit lines. Another remark is that these institutions do not have to know precisely the amount of withdrawals in the short run. Knowledge of the distribution is enough. However other elements such as perfect commitment of repayment and common objectives between bank managers and depositors are key features for these institutions to work.

The introduction of liquidation costs as the more recent bank run literature includes (see [4], [7] and [12], among others) does not affect essentially the nature of the problem. The main results could be easily gotten with a long term asset h that gives $r^h < 1$ per unit invested in the long asset if early liquidation occurs [11]. However this is not true in terms of the minimum reserve requirements policy, since in this case, for sufficiently low values of r^h, the amount set apart at the beginning of the economy will not be invested in the long term asset[12].

Another possible direction for future work is the construction of a version of this model in a world integrated economy with two tradable currencies, following also similar ideas as in Allen and Gale [2]. There are several issues that can be addressed with this framework. Perhaps one of the most discussed issues is the incentive to constitute the reserves for the international lender of last resort. In the paper I have presented such problem could not be studied since the economy was of the small open type. A world integrated economy with well-defined participants could help to see when each country is willing to deposit funds in an international institution.

Fundamental shocks can be introduced, making either the short term rate (as in Chang and Velasco [5]) or the long investment return (as in Allen and Gale [1]) stochastic. This would allow to study solvency- based runs and the role of the lenders of last resort to prevent such runs, if these are not optimal.

[11] The condition under which runs occur is basically the same as in proposition 3.2 except that in this case the total value of asset income at period 1 is equal to $b^F + b^H + r^F X^F + r^H X^H$.

[12] Obviously if $r^h < 1$ but sufficiently close to 1 most of the arguments in subsection 5.2.2. still hold.

Nevertheless, problems of asymmetric information could worsen here. The reason is that when returns are risky, adverse selection may not allow for availability of an international lender of last resort. This issue should be studied in combination with a world-integrated environment.

A related topic to the solvency problem is the explicit separation between managers and depositors. By study a version of this banking model in which managers do not have the same objective as the depositors the moral hazard considerations mentioned above could be seriously addressed. That is, moral hazard considerations are to be studied in settings where those objectives are discordant, since it is obvious that when they are the same hidden action problems cannot arise. There are several alternatives for modelling this. There is a vast literature on incomplete contracts in banking (see [9] and [10]). Also Chang and Velasco [5] present a model in which the banking sector is monopolistic. Any of these frameworks could be helpful to study moral hazard and lenders of last resort.

A. Proofs

A.1. Proof of Proposition (4.1)

Suppose that the planner's allocation implies an illiquidity problem in the banking system. Assume moreover that in period 1 there is an international entity that lends to the financial system any amount needed to satisfy the withdrawals in period 1 for those contracts written in foreign currency without having to liquidate the long term asset and to return the borrowed amount $\left(\rho/R^F\right)d$. Hence if a proportion $\hat{\pi} > \pi$ withdraws $\left(c_1^F + c_1^H\right)$ the amount lent in period 1 is $\hat{\lambda} = (\hat{\pi} - \pi)\left(c_1^F + c_1^H\right)$.

Suppose first that $R^F > R^H$. Period 2 income is $R^F X^F - \hat{\lambda} - \rho\bar{d}$. I claim that this is at least $(1-\hat{\pi})\left(c_2^F + c_2^H\right)$. Suppose not. Then: $R^F X^F - \hat{\lambda} - \rho\bar{d} < (1-\hat{\pi})\left(c_2^F + c_2^H\right)$. However $R^F X^F - \rho\bar{d} = (1-\pi)\left(c_2^H + c_2^F\right)$. Using this and the definition of $\hat{\lambda}$ we obtain

$$(1-\pi)\left(c_2^H + c_2^F\right) - (\hat{\pi} - \pi)\left(c_1^F + c_1^H\right) < (1-\hat{\pi})\left(c_2^F + c_2^H\right)$$

or

$$(\hat{\pi} - \pi)\left(c_2^F + c_2^H\right) < (\hat{\pi} - \pi)\left(c_1^F + c_1^H\right)$$

which implies either $c_2^F < c_1^F$ or $c_2^H < c_1^H$. However this contradicts proposition 4.2. Therefore it must be true that $R^F X^F - \hat{\theta}^F - \rho \bar{d} \geq (1-\hat{\pi})\left(c_2^F + c_2^H\right)$. Thus no patient consumer has incentive to lie, which means that in the equilibrium of this economy $\hat{\pi} = \pi$. Hence the banking system in equilibrium does not need to use the credit line. This implies that the *availability* of an international lender of last resort is sufficient to prevent runs by implementing the planner's solution.

When $R^H > R^F$ results are not identical. Suppose that the banking system is illiquid in both currencies. Then the amount lent is still $(\hat{\pi} - \pi)\left(c_1^F + c_1^H\right)$. Notice that this amount is in the F currency, so in period 2, the H currency income is just $R^H X^H$, which is more than enough to pay $(1-\hat{\pi})\, c_2^H$. On the other hand, in terms of F money, net income in period 2 is $R^F X^F - \rho \bar{d} - \hat{\lambda} = (1-\pi)$ $c_2^F - (\hat{\pi} - \pi)\left(c_1^F + c_1^H\right)$. If $\left(c_1^F + c_1^H\right) \leq c_2^F$ then the expression $(1-\pi)\ c_2^F - (\hat{\pi} - \pi)\left(c_1^F + c_1^H\right) \geq (1-\pi)\, c_2^F - (\hat{\pi} - \pi)\, c_2^F = (1-\hat{\pi})\, c_2^F$. This gives a sufficient condition for the international institution to prevent runs.

Suppose that there is only illiquidity in the F currency but not in the H currency. I claim that if the international lender lends $\hat{\lambda} = (\hat{\pi} - \pi)\, c_1^F$ then there are no runs. In this case, the banking system liquidates $\hat{l}^H = (\hat{\pi} - \pi)\ c_1^H$. On the other hand, period 2 net income in the F currency is $R^F X^F - \rho \bar{d} - \hat{\lambda} = (1-\pi)\, c_2^F - (\hat{\pi} - \pi)\, c_1^F \geq (1-\pi)\, c_2^F - (\hat{\pi} - \pi)\, c_2^F = (1-\hat{\pi})\, c_2^F$. Hence they also get c_2^F. No incentives to run are in this case.

If the contract implementing the planner's solution has illiquidity in the H currency only (but it is liquid in the foreign currency), then the banking system might lend $\hat{\lambda} = (\hat{\pi} - \pi)\, c_1^H$. Clearly the long asset in the H currency must not be liquidated, and so the amount c_2^H can repaid to the remaining patient customers in period 2. On the other hand, if $c_1^H \leq c_2^F$ then it can be shown following similar arguments as in previous paragraphs that the remaining patients also get c_2^F. Therefore this also constitutes a sufficient condition under which an international lender of last resort prevents panics in the presence of illiquidity in currency H and $R^H \geq R^F$. ■

A.2. Proof of Proposition (4.2)

Consider first the case in which $R^F > R^H$. Let $\hat{\lambda}$ be defined as above. Next, suppose that the gross interest rate on this loan is $\zeta^F \leq \frac{c_2^F + c_2^H}{c_1^H + c_1^F}$. The period 2 net revenue of F money is $R^F X^F - \zeta^F \hat{\lambda} - \rho \bar{d} = (1-\pi)\left(c_2^F + c_2^H\right) - \zeta^F (\hat{\pi} - \pi)\left(c_1^H + c_1^F\right)$, which is greater than or equal to $(1-\pi)\ \left(c_2^F + c_2^H\right) - (\hat{\pi} - \pi)\ \left(c_2^H + c_2^F\right) =$

$(1-\hat{\pi})\left(c_2^H + c_2^F\right)$. Therefore the banking system is able to pay both c_2^H and c_2^F to the patient customers who did not lie.

Assume now that $R^H \geq R^F$. Suppose that there is an illiquidity problem in both currencies, so that still $\hat{\lambda} = (\hat{\pi}-\pi)\left(c_1^H + c_1^F\right)$. At period 2 the net revenue in terms of foreing currency is $R^F X^F - \rho\bar{d} - \zeta^F(\hat{\pi}-\pi)\left(c_1^H + c_1^F\right)$. Assuming that $c_1^H + c_1^F \leq c_2^F$, and since $\zeta^F \leq \frac{c_2^F}{c_1^H + c_1^F}$, then $R^F X^F - \rho\bar{d} - \zeta^F(\hat{\pi}-\pi)\left(c_1^H + c_1^F\right) = (1-\pi)c_2^F - \zeta^F(\hat{\pi}-\pi)\left(c_1^H + c_1^F\right) \geq (1-\pi)c_2^F - (\hat{\pi}-\pi)c_2^F = (1-\hat{\pi})c_2^F$. If the illiquidity problem is in terms of the F currency only, it is true that $\hat{\lambda} = (\hat{\pi}-\pi)$ c_1^F and if $\zeta^F \leq \frac{c_2^F}{c_1^F}$ then by identical arguments it is clear that net revenues in terms of currency F are enough to repay c_2^F for the $N(1-\hat{\pi})$ patient customers.

A.3. Proof of Proposition (4.3)

Assume that the banking system is illiquid in both currencies (without loss of generality). The arguments are identical in the case of illiquidity in only one currency. Suppose first that $\hat{\lambda}_1^F = (\hat{\pi}-\pi)\left(c_1^F + c_1^H\right)$ and also that $R^F > R^H$. (Otherwise, if $R^H \geq R^F$ suppose that the condition $\left(c_1^H + c_1^F\right) \leq c_2^F$ is met so that the international institution is able to prevent runs). Assume that, instead of using this entirely to satisfy the period 1 withdrawals the banking system uses $\hat{\lambda}_1^F - l_1^F$ and liquidates l_1^F units of the long term F investment in period 1 (where $0 \leq l_1^F \leq X^F$). Hence the banking system does not close in the interim period. In period 2, total income is equal to $R^F\left(X^F - l_1^F\right) - \rho\bar{d} - \left(\hat{\lambda}_1^F - l_1^F\right) = (1-\pi)\left(c_2^F + c_2^H\right) - (\hat{\pi}-\pi)\left(c_1^F + c_1^H\right) + \left(1 - R^F\right)l_1^F < (1-\pi)\left(c_2^F + c_2^H\right) - (\hat{\pi}-\pi)\left(c_1^F + c_1^H\right)$, which is income available in period 2 if the banking system decides to use $\hat{\theta}_1^F$ and not to liquidate early the long term investment. Hence the patient consumer cannot be better off by doing this. The argument when $R^F \leq R^H$ is similar. In this case it is clear that this does not affect the constraint corresponding to the H currency. For the foreign currency constraint in period 2, notice again that the total income is $R^F\left(X^F - l_1^F\right) - \rho\bar{d} - \left(\hat{\lambda}_1^F - l_1^F\right) = (1-\pi)c_2^F - (\hat{\pi}-\pi)\left(c_1^F + c_1^H\right) + \left(1 - R^F\right)l_1^F$ $< (1-\pi)c_2^F - (\hat{\pi}-\pi)\left(c_1^F + c_1^H\right)$. Again, by misbehaving the banking system gets less income than by using all the lent funds properly.

Suppose that the banking system decides to borrow $\hat{\lambda}_1^F > (\hat{\pi}-\pi)\left(c_1^F + c_1^H\right)$

and still $R^F > R^H$. Here the funds are greater than the needs of liquidity in the banking system. Suppose that it is used $\hat{\lambda}_1^F - l_1^F$, and η_1^F units of the long term investment is liquidated in period 1. Hence $(\hat{\pi} - \pi)\left(c_1^F + c_1^H\right) = \hat{\lambda}_1^F - l_1^F + r^F \eta_1^F$. In period 2 total income is equal to $R^F\left(X^F - \eta_1^F\right) - \left(\hat{\lambda}_1^F - l_1^F\right) = (1-\pi)\left(c_2^F + c_2^H\right) + \rho\bar{d} - \left(R^F \eta_1^F + \hat{\lambda}_1^F - l_1^F\right) \leq (1-\pi)\left(c_2^F + c_2^H\right) + \rho\bar{d} - \left(\eta_1^F + \hat{\lambda}_1^F - l_1^F\right) \leq (1-\pi)\left(c_2^F + c_2^H\right) - (\hat{\pi} - \pi)\left(c_1^F + c_1^H\right)$. Hence total income in period 2 is less than or equal to period 2 income under the original case in which $\hat{\lambda}_1^F = (\hat{\pi} - \pi)\left(c_1^F + c_1^H\right)$ and is totally used to satisfy period 1 liquidity needs. If $\hat{\lambda}_1^F < (\hat{\pi} - \pi)\left(c_1^F + c_1^H\right)$, and $R^F > R^H$ then long term investment must be liquidated. Suppose that the amount to be liquidated to satisfy the banks liquidity needs is strictly less than X^F. Then $(\hat{\pi} - \pi)\left(c_1^F + c_1^H\right) = \hat{\lambda}_1^F + r^F l_1^F$, where $l_1^F < X^F$. In period 2 total income is $R^F\left(X^F - l_1^F\right) - \rho\,\bar{d} - \hat{\lambda}_1^F = (1-\pi)\left(c_2^F + c_2^H\right) - \left(R^F l_1^F + \hat{\theta}_1^F\right) < (1-\pi)\,c_2^F - \left(r^F l_1^F + \hat{\theta}_1^F\right) = (1-\pi)\,c_2^F - (\hat{\pi} - \pi)\left(c_1^F + c_1^H\right)$. The same conclusion as before applies. (The cases in which $R^H \geq R^F$ are similar and left to the reader).

I have used the assumption of perfect commitment in this argument. However the perfect commitment to sustain the financial system and prevent runs is not necessary. Suppose that the managers of banks decide to borrow some amount $\hat{\lambda}_1^F$ in period 1 from abroad, but instead of using this to pay $\left(c_1^F, c_1^H\right)$ to those who claim to be impatient they store it. If the conditions of proposition 3.2 are true then there is a run and everybody gets either at most $\left(c_1^F, c_1^H\right)$, depending upon the order in line. This is true even for those patients who behave as impatients. In period 2 the manager - depositors must return $\hat{\lambda}_1^F$, and also there is no extra consumption to be paid. But this is clearly worse than the situation under which $\hat{\lambda}_1^F$ is used to pay to the depositors who withdraw in period 1, since under this case all patients who behave as such get more than $\left(c_1^F, c_1^H\right)$. Therefore there is no incentive to let the bank close in the interim period. ■

A.4. Proof of Proposition (4.4)

Since when $R^F > R^H$ an international lender alone always prevents runs as long as the interest rate is less than or equal to $\frac{c_2^F + c_2^H}{c_1^H + c_1^F}$ then it suffices to show that this regime prevents runs when $R^H \geq R^F$. Suppose that a proportion $\hat{\pi} > \pi$ decides to withdraw from the banks at $t = 1$. The following happens. The banking system borrows from the international lender of last resort an amount of currency F equal to $(\hat{\pi} - \pi)\, c_1^F$. It also borrows from the local lender an amount of currency H equal to $(\hat{\pi} - \pi)\, c_1^H$. Let the gross interest rate of the first loan be ζ^F and the second ζ^H. Then at period 2 the net amount of revenue in currency F is $R^F X^F - \rho \bar{d} - \zeta^F\ (\hat{\pi} - \pi)\, c_1^F$. Similarly at $t = 2$ net revenue in home currency is $R^H X^H - \zeta^H\ (\hat{\pi} - \pi)\, c_1^H$. The first expression is equal to $(1 - \pi)\ c_2^F - \zeta^F\ (\hat{\pi} - \pi)\, c_1^F$. Since $\zeta^F \leq \min\left\{\frac{c_2^F}{c_1^F}; \frac{c_2^F + c_2^H}{c_1^H + c_1^F}\right\}$ then $(1 - \pi)\ c_2^F - \zeta^F\ (\hat{\pi} - \pi)\, c_1^F \geq (1 - \pi)\ c_2^F - (\hat{\pi} - \pi)\, c_2^F = (1 - \hat{\pi})\, c_2^F$. Similarly since $\zeta^H \leq \frac{c_2^H}{c_1^H}$ then $R^H X^H - \zeta^H\ (\hat{\pi} - \pi)\, c_1^H = (1 - \pi)\ c_2^H - \zeta^H\ (\hat{\pi} - \pi)\, c_1^H \geq (1 - \pi)\ c_2^H - (\hat{\pi} - \pi)\, c_2^H = (1 - \hat{\pi})\, c_2^H$. Hence under the stated conditions both lenders of last resort prevent runs.

A.5. Proof of Proposition (5.1)

In this case we again know that there are no illiquidity problems in F. Suppose that a proportion $\hat{\pi} > \pi$ withdraws in period 1. The local lender of last resort will be lending an amount equal to

$$\begin{aligned}
\tau^H &= \hat{\pi}\left(c_1^H + c_1^F\right) - \left(b^H + b^F\right) - \left(b^F - \hat{\pi} c_1^F\right) \\
&= (\hat{\pi} - \pi)\left(c_1^H + c_1^F\right) - \left(b^F - \hat{\pi} c_1^F\right)
\end{aligned}$$

Suppose $R^H \geq R^F$ first. Then in period 2 the patient consumers who did not run get c_2^F since none of the F – money long asset must be liquidated in $t = 1$. In fact total income is $R^F X^F - \rho \bar{d} = (1 - \pi)\, c_2^F > (1 - \hat{\pi})\ c_2^F$. This means that there is an excess of F currency in period 2 equal to $(\hat{\pi} - \pi)\, c_2^F$. This is converted to H currency. Hence the total net income is $(\hat{\pi} - \pi)\, c_2^F + R^H X^H - \tau^H = (1 - \pi)\, c_2^H - (\hat{\pi} - \pi)\left(c_1^H + c_1^F\right) + \left(b^F - \hat{\pi} c_1^F\right) + (\hat{\pi} - \pi)\, c_2^F \geq (1 - \pi)\, c_2^H - (\hat{\pi} - \pi)\left(c_1^H + c_1^F\right) + \left(c_1^F - \hat{\pi} c_1^F\right) + (\hat{\pi} - \pi)\, c_2^F = (1 - \pi)\, c_2^H + (\hat{\pi} - \pi)\left(c_2^F - c_1^F\right) + c_1^F\,(1 - \hat{\pi}) - (\hat{\pi} - \pi)\, c_1^H > (1 - \pi)\, c_2^H - (\hat{\pi} - \pi)\, c_1^H > (1 - \pi)\, c_2^H - (\hat{\pi} - \pi)\, c_2^H = (1 - \hat{\pi})\, c_2^H$. Therefore the system is able to repay $\left(c_2^H, c_2^F\right)$ when $R^H \geq R^F$.

Suppose now that $R^F > R^H$. Hence $X^H = 0$. The amount lent by the local lender is still $\tau^H = (\hat{\pi} - \pi)\ \left(c_1^H + c_1^F\right) - \left(b^F - \hat{\pi} c_1^F\right)$. In period 2 total net income is $R^F X^F - \rho \bar{d} - \tau^H = (1 - \pi)\left(c_2^H + c_2^F\right) - (\hat{\pi} - \pi)\ \left(c_1^H + c_1^F\right) + \left(b^F - \hat{\pi} c_1^F\right) >$

$(1-\pi)\left(c_2^H+c_2^F\right)-(\hat{\pi}-\pi)\left(c_1^H+c_1^F\right)\geq(1-\pi)\left(c_2^H+c_2^F\right)-(\hat{\pi}-\pi)\left(c_2^H+c_2^F\right)$ $=(1-\hat{\pi})\left(c_2^H+c_2^F\right)$. This means that the bank is also able to repay c_2^H and c_2^F to those patients who did not run. Henceforth this policy prevents runs. ■

A.6. Proof of Proposition (6.4)

Suppose that $R^F > R^H$. Let the total consumers claiming impatience showing up in period 1 be $N(\alpha p+1-p+\hat{\pi})$ where $\hat{\pi}\leq p(1-\alpha)$. In this case the total amount lent is $\hat{\lambda}_1=\hat{\pi}\left(c_1^{2F}(1)+c_1^{2H}(1)\right)$. We show that the corresponding period 2 income is sufficient to pay $\left(c_2^{1H}(1)+c_2^{1F}(1)\right)$ to the remaining patient consumers who did not show up in period 1.

Income in period 2 is $R^FX^F-\rho\bar{d}-\hat{\lambda}_1=p(1-\alpha)\left(c_2^{1F}(1)+c_2^{1H}(1)\right)-\hat{\pi}\left(c_1^{2F}(1)+c_1^{2H}(1)\right)$. Suppose by way of contradiction that this is strictly less than $(p(1-\alpha)-\hat{\pi})\left(c_2^{1F}(1)+c_2^{1H}(1)\right)$. Then either $c_2^{1F}(1)<c_1^{2F}(1)$, or $c_2^{1H}(1)<c_1^{2H}(1)$ which contradicts the incentive compatibility constraint. Hence it must be true that $R^FX^F+R^HX^H-\rho\bar{d}-\hat{\lambda}_1\geq(p(1-\alpha)-\hat{\pi})\left(c_2^{1F}(1)+c_2^{1H}(1)\right)$. Therefore the banking system is able to pay $\left(c_2^{1F}(1)+c_2^{1H}(1)\right)$ to the remaining patient consumers. Hence, under $s=1$ there is no incentive to lie on behalf of the patient consumers.

If $s=2$ the reasoning is similar. The banking system does not close in the interim period due to the lender of last resort. Hence the consumer who claims impatience gets either $\left(c_1^{1F},c_1^{1H}\right)$ or $\left(c_1^{2F}(1),c_1^{2H}(1)\right)$ in period 1. The patient consumer gets either $\left(c_2^F(2),c_2^H(2)\right)$ or $\left(c_2^{1F}(1),c_2^{1H}(1)\right)$. She gets the first amount if only $N\alpha p$ impatient consumers show up in period 1. She gets the second amount if more than $N\alpha p$ impatient consumers withdraw in the interim period. The proof that the banking system is able to pay these amounts follow the same lines than the last two paragraphs. Thus, in any case, the second period consumption is greater than both $\left(c_1^{1F},c_1^{1H}\right)$and $\left(c_1^{2F}(1),c_1^{2H}(1)\right)$. This means that when $s=2$ there is no incentive for the patient consumers to lie about their types.

On the other hand, if $X^H>0$ (which happens when $R^H\geq R^F$) the sufficient conditions are similar to the case in which π was known. If there is illiquidity in both currencies still the amount to be lent is $\hat{\pi}\left(c_1^{2F}(1)+c_1^{2H}(1)\right)$. If this is at most $c_2^{1F}(1)$ then the banking system can satisfy the late withdrawals in period 2 by the remaining patient customers. This is because total net income in terms of currency F is equal to $p(1-\alpha)\ c_2^{1F}(1) - \hat{\pi}\left(c_1^{2F}(1) + c_1^{2H}(1)\right) \geq (p(1-\alpha) - \hat{\pi})$ $c_2^{1F}(1)$. Henceforth in this case the banking system is saved by the international institution. If there is only illiquidity in F the international lender is able to sustain liquidity since $c_1^{2F}(1) < c_2^{1F}(1)$. Finally, if there is illiquidity in H but not in F the sufficient condition is $c_1^{2H}(1) \leq c_2^{1F}(1)$. ■

A.7. Proof of Proposition (6.5)

I follow essentially the same arguments as in the last subsection. Suppose that $R^F > R^H$. Let the total consumers claiming impatience showing up in period 1 be $N(\alpha p + 1 - p + \hat{\pi})$ where $\hat{\pi} \leq p(1-\alpha)$. In this case the total amount lent is $\hat{\lambda}_1 = \hat{\pi}\left(c_1^{2F}(1) + c_1^{2H}(1)\right)$. We show that the corresponding period 2 income is sufficient to pay $\left(c_2^{1H}(1) + c_2^{1F}(1)\right)$ for the given interest rate to the remaining patient consumers who did not show up in period 1.

Income at period 2 is $R^F X^F - \rho\bar{d} - \chi\hat{\lambda}_1 = p(1-\alpha)\left(c_2^{1F}(1) + c_2^{1H}(1)\right) - \chi\hat{\pi}\left(c_1^{2F}(1) + c_1^{2H}(1)\right)$, where χ is the gross interest rate. Since $\chi \leq \frac{\left(c_2^{1F}(1)+c_2^{1H}(1)\right)}{\left(c_1^{2F}(1)+c_1^{2H}(1)\right)}$, then $p(1-\alpha)\left(c_2^{1F}(1) + c_2^{1H}(1)\right) - \chi\hat{\pi}\left(c_1^{2F}(1) + c_1^{2H}(1)\right) \geq p(1-\alpha)\left(c_2^{1F}(1) + c_2^{1H}(1)\right) - \hat{\pi}\left(c_2^{1F}(1) + c_2^{1H}(1)\right) = \left(c_2^{1F}(1) + c_2^{1H}(1)\right)(p(1-\alpha) - \hat{\pi})$. This means that the banking system is able to pay $\left(c_2^{1F}(1) + c_2^{1H}(1)\right)$ to the remaining patient consumers. Hence, under $s = 1$ there is no incentive to lie on behalf of the patient consumers.

If $s = 2$ the reasoning is similar to the proof in the last subsection and thus omitted..

On the other hand, suppose $X^H > 0$ (which happens when $R^H \geq R^F$). Suppose that there is illiquidity in both currencies. The amount to be lent is still $\hat{\pi}$ $\left(c_1^{2F}(1) + c_1^{2H}(1)\right)$. Total net income in terms of currency F is equal to $p(1-\alpha)$ $c_2^{1F}(1) - \chi\hat{\pi}\left(c_1^{2F}(1) + c_1^{2H}(1)\right)$. In this case $\chi \leq \frac{c_2^{1F}(1)}{\left(c_1^{2F}(1)+c_1^{2H}(1)\right)}$. Therefore it must be true that $p(1-\alpha)\ c_2^{1F}(1) - \chi\hat{\pi}\left(c_1^{2F}(1) + c_1^{2H}(1)\right) \geq (p(1-\alpha) - \hat{\pi})\ c_2^{1F}(1)$. Henceforth in this case the banking system is also saved by the international institution.

Following similar arguments it can be shown that when there is only illiquidity in F the international lender could charge a gross interest rate $\chi \leq \frac{c_2^{1F}(1)}{c_1^{2F}(1)}$. Finally, when there is illiquidity in H but not in F the gross interest rate is $\chi \leq \frac{c_2^{1F}(1)}{c_1^{2H}(1)}$. ■

A.8. Proof of Proposition (6.6)

This follows almost literally the same steps as proposition 4.4. Since when $R^F > R^H$ an international lender alone always prevents runs as long as the interest rate is less than or equal to $\frac{c_2^{1F}(1)+c_2^{1H}(1)}{c_1^{2H}(1)+c_1^{2F}(1)}$ then it suffices to show that this policy prevents runs when $R^H \geq R^F$. Suppose that a proportion $(\alpha p + 1 - p + \hat{\pi})$ (where $\hat{\pi} \leq p(1-\alpha)$) decides to withdraw from the banks at $t = 1$. Then the banking system borrows from the international lender of last resort an amount of currency F equal to $\hat{\pi} c_1^{2F}(1)$. It also borrows from the local lender an amount of currency H equal to $\hat{\pi} c_1^{2H}(1)$. Let the gross interest rate of the first loan be ζ^F and the second ζ^H. Then at period 2 the net amount of revenue in currency F is $R^F X^F - \rho \bar{d} - \zeta^F \hat{\pi} c_1^{2F}(1)$. Similarly at $t = 2$ net revenue in home currency is $R^H X^H - \zeta^H \hat{\pi} c_1^{2H}(1)$. The first expression is equal to $p(1-\alpha)\, c_2^{1F}(1) - \zeta^F \hat{\pi}\, c_1^{2F}(1)$. Since $\zeta^F \leq \min\left\{\frac{c_2^{1F}(1)}{c_1^{2F}(1)}; \frac{c_2^{1F}(1)+c_2^{1H}(1)}{c_1^{2H}(1)+c_1^{2F}(1)}\right\}$ then $p(1-\alpha)\, c_2^{1F}(1) - \zeta^F \hat{\pi} c_1^{2F}(1) \geq p(1-\alpha)\, c_2^{1F}(1) - \hat{\pi}\, c_2^{1F}(1) = (p(1-\alpha) - \hat{\pi})\, c_2^F$. Similarly since $\zeta^H \leq \frac{c_2^{1H}(1)}{c_1^{2H}(1)}$ then $R^H X^H - \zeta^H (\hat{\pi} - \pi)\, c_1^H = p(1-\alpha)\, c_2^{1H}(1) - \zeta^H \hat{\pi} c_1^{2H}(1) \geq p(1-\alpha)\, c_2^{1H}(1) - \hat{\pi} c_2^{1H}(1) = (p(1-\alpha) - \hat{\pi})\, c_2^H$. Hence under the stated conditions both lenders of last resort prevent runs. ■

A.9. Proof of Proposition 7.1

The first order conditions of the planner's problem implies the following conditions.

$$
\begin{aligned}
& (\alpha p + (1-p)\, q_1)\, u'\left(c_1^{1F}\right) \\
= \; & R^F \alpha p \left[q_1 u'\left(c_2^F(1)\right) + q_2 u'\left(c_2^F(2)\right)\right] + (1-p)\, q_1 u'\left(c_2^F(1)\right)
\end{aligned}
$$

$$\alpha p \left(c_1^{1F} + c_1^{1H}\right) = b^H + b^F$$

$$
\begin{aligned}
& p(1-\alpha)\left(c_2^F(1) + c_2^H(1)\right) + (1-p)\left(c_1^{1F} + c_1^{1H}\right) \\
= \; & R^F X^F - \rho \bar{d} + R^H X^H
\end{aligned}
$$

$$(p(1-\alpha) + (1-p))\left(c_2^F(2) + c_2^H(2)\right) = R^F X^F - \rho \bar{d} + R^H X^H$$

These last two conditions can be rewritten

$$\begin{aligned} & p(1-\alpha)\left(c_2^F(1)+c_2^H(1)\right)+(1-p)\left(c_1^{1F}+c_1^{1H}\right) \\ = & \ (p(1-\alpha)+(1-p))\left(c_2^F(2)+c_2^H(2)\right) \end{aligned} \tag{A.1}$$

I must consider two subcases.

- **Case 1:** $R^F \geq R^H$.

In this case it is true that

$$\begin{aligned} u'\left(c_1^{1F}\right) &= v'\left(c_1^{1H}\right) \\ u'\left(c_2^F(s)\right) &= v'\left(c_2^H(s)\right) \end{aligned}$$

for $s = 1, 2$. Under this assumption there is no loss of generality in making $\lambda_1^H(1) = 0$ (the local lender of last resort is completely superfluous).

$$\begin{aligned} & (\alpha p+(1-p)q_1)\, v'\left(c_1^{1H}\right) \\ = & \ R^F \alpha p\left[q_1 v'\left(c_2^H(1)\right)+q_2 v'\left(c_2^H(2)\right)\right]+(1-p)\, q_1 v'\left(c_2^H(1)\right) \end{aligned}$$

Suppose that $c_1^{1F} > c_2^F(1)$. Therefore we have

$$u'\left(c_1^{1F}\right) < u'\left(c_2^F(1)\right)$$

and then

$$q_1(1-p)\, u'\left(c_1^{1F}\right) < q_1(1-p)\, u'\left(c_2^F(1)\right)$$

Therefore, given the equality above, it must be true that

$$\alpha p u'\left(c_1^{1F}\right) < R^F \alpha p\left[q_1 u'\left(c_2^F(1)\right)+q_2 u'\left(c_2^F(2)\right)\right]$$

or

$$q_1\left(u'\left(c_1^{1F}\right)-R^F u'\left(c_2^F(1)\right)\right) < q_2\left[R^F u'\left(c_2^F(2)\right)-u'\left(c_1^{1F}\right)\right]$$

Since $u'\left(c_1^{1F}\right) < u'\left(c_2^F(1)\right)$, then $u'\left(c_1^{1F}\right) < u'\left(c_2^F(1)\right) R^F$, which means that

$$\left(u'\left(c_1^{1F}\right)-R^F u'\left(c_2^F(1)\right)\right) < 0$$

and so is $\left[R^F u'\left(c_2^F(2)\right) - u'\left(c_1^{1F}\right)\right]$, which means that $R^F u'\left(c_2^F(2)\right) < u'\left(c_1^{1F}\right)$, which also implies that $u'\left(c_2^F(2)\right) < u'\left(c_1^{1F}\right)$. This implies also that $c_2^F(2) > c_1^{1F}$. Since $u'\left(c_1^{1F}\right) = v'\left(c_1^{1H}\right)$ and $u'\left(c_2^F(s)\right) = v'\left(c_2^H(s)\right)$ then all this also implies that if $c_1^{1F} > c_2^F(1)$ then $c_1^{1H} > c_2^H(1)$ and by the same argument $c_2^H(2) > c_1^{1H}$. But then $c_2^F(2) > c_2^F(1)$ and $c_2^H(2) > c_2^H(1)$. Therefore

$$\begin{aligned}
& p(1-\alpha)\left(c_2^F(1) + c_2^H(1)\right) + (1-p)\left(c_1^{1F} + c_1^{1H}\right) \\
< \ & (p(1-\alpha) + (1-p))\left(c_2^F(2) + c_2^H(2)\right)
\end{aligned}$$

which contradicts the equality A.1. Therefore it must be true that $c_1^{1F} < c_2^F(1)$ (if they are equal a similar type of contradiction is gotten). Then also $c_1^{1H} < c_2^H(1)$. On the other hand, by the equality A.1 we must have that $c_1^{1h} < c_2^h(2) < c_2^h(1)$. This is because otherwise the equality is violated with either strict sign. (If $c_1^{1h} < c_2^h(1) < c_2^h(2)$ then a similar inequality would be true. If $c_2^h(2) < c_1^{1h} < c_2^h(1)$ then the reverse inequality is true). This shows that under this case the solution can be implemented as an equilibrium.

- **Case 2**: $R^H > R^F$

In this case we have

$$\begin{aligned}
& (\alpha p + (1-p) q_1) v'\left(c_1^{1H}\right) \\
= \ & R^H \alpha p \left[q_1 v'\left(c_2^H(1)\right) + q_2 v'\left(c_2^H(2)\right)\right] + (1-p) q_1 v'\left(c_2^H(1)\right)
\end{aligned}$$

but now we have

$$\begin{aligned}
u'\left(c_1^{1F}\right) &> v'\left(c_1^{1H}\right) \\
u'\left(c_2^F(s)\right) &> v'\left(c_2^H(s)\right)
\end{aligned}$$

so the equalities between marginal utilities does not hold anymore. However, since $R^H > R^F$ it must be true that

$$\begin{aligned}
p(1-\alpha) c_2^H(1) &= R^H X^H - \lambda_1^H(1) \\
p(1-\alpha) c_2^F(1) &= R^F X^F - \rho d - \lambda_1^F(1)
\end{aligned}$$

and also that $\lambda_1^H(1) = (1-p)\,c_1^{1H}$ (the constraint is binding), and then $\lambda_1^F(1) = (1-p)\,c_1^{1F}$. On the other hand, if $\alpha p c_1^{1F} = b^F$ and $(p(1-\alpha)+1-p)\,c_2^F(2) = R^F X^F - \rho d + b^F - \alpha p c_1^{1F}$ then it must be true that

$$\begin{aligned}(p(1-\alpha)+1-p)\,c_2^F(2) &= R^F X^F - \rho d\\ (p(1-\alpha)+1-p)\,c_2^H(2) &= R^H X^H\end{aligned}$$

Then we must have

$$\begin{aligned}p(1-\alpha)\,c_2^F(1)+(1-p)\,c_1^{1F} &= (p(1-\alpha)+1-p)\,c_2^F(2)\\ p(1-\alpha)\,c_2^H(1)+(1-p)\,c_1^{1H} &= (p(1-\alpha)+1-p)\,c_2^H(2)\end{aligned}$$

Arguing again by contradiction it gives first that $c_1^{1h} < c_2^h(1)$. The argument is the same as above. Then it must also be true that $c_1^{1h} < c_2^h(2) < c_2^h(1)$, so that each equality holds. This again shows (under the conditions stated in the second part of the proposition) that the solution to this problem can be implemented by a banking system as described in section 6 when lenders of last resort are added to the planner's problem.

If the two conditions do not hold but p is close enough to 1 the solution to the problem is also implementable. The reason is as before. If $p = 1$ we are back in the no aggregate uncertainty case. We know that this is implementable and that the consumption for the impatients is strictly less (for each currency) than the consumption for the patients. Since the solution to the problem is continuous in p (by concavity and convexity assumptions) then when p is sufficiently close to 1 those properties are maintained.

Finally we must prove that if these institutions are available to the banking sector then this equilibrium is implementable without runs. To do this, I follow the same steps as in the other propositions. Suppose that the state is $s = 1$ (arguments are identical if $s = 2$). Suppose that there is a proportion $\alpha p + (1-p) + \hat{\pi}$ of impatients who want to withdraw. Here $\hat{\pi} \le p(1-\alpha)$.

Suppose first that $R^F \ge R^H$. In this there is no need to use any local lender of last resort. Then the total amount lent from abroad is equal to

$$\lambda_1^{*F}(1) = (1-p+\hat{\pi})\left(c_1^{1H}+c_1^{1F}\right)$$

In period 2 the net income is

$$\begin{aligned}
& R^F X^F - \rho d + R^H X^H - \lambda_1^{*F}(1) \\
= \; & R^F X^F - \rho d + R^H X - (1 - p + \hat{\pi}) \left(c_1^{1H} + c_1^{1F} \right) \\
= \; & p(1-\alpha) \left(c_2^F(1) + c_2^H(1) \right) + (1-p) \left(c_1^{1H} + c_1^{1F} \right) - (1 - p + \hat{\pi}) \left(c_1^{1H} + c_1^{1F} \right) \\
= \; & p(1-\alpha) \left(c_2^F(1) + c_2^H(1) \right) - \hat{\pi} \left(c_1^{1H} + c_1^{1F} \right)
\end{aligned}$$

Since $c_1^{1h} < c_2^h(1)$ then

$$\begin{aligned}
& p(1-\alpha) \left(c_2^F(1) + c_2^H(1) \right) - \hat{\pi} \left(c_1^{1H} + c_1^{1F} \right) \\
> \; & p(1-\alpha) \left(c_2^F(1) + c_2^H(1) \right) - \hat{\pi} \left(c_2^F(1) + c_2^H(1) \right) \\
= \; & (p(1-\alpha) - \hat{\pi}) \left(c_2^F(1) + c_2^H(1) \right)
\end{aligned}$$

Therefore the net income is more than sufficient to pay $\left(c_2^F(1) + c_2^H(1) \right)$ to the remaining patients.

If $R^H > R^F$, we have that the amount lent from the international lender of last resort is $\lambda_1^{*F} = (1 - p + \hat{\pi}) c_1^{1F}$, while the amount lent from the local lender of last resort is (if allowed to violate the constraint in the event of a potential run) $\lambda_1^{*H} = (1 - p + \hat{\pi}) c_1^{1H}$. In this case the net income in terms of currency F is

$$\begin{aligned}
& R^F X^F - \rho \bar{d} - \lambda_1^{*F}(1) \\
= \; & R^F X^F - \rho \bar{d} - (1 - p + \hat{\pi}) c_1^{1F} \\
= \; & p(1-\alpha) c_2^F(1) + (1-p) c_1^{1F} - (1 - p + \hat{\pi}) c_1^{1F} \\
= \; & p(1-\alpha) c_2^F(1) - \hat{\pi} c_1^{1F} \\
> \; & (p(1-\alpha) - \hat{\pi}) c_2^F(1)
\end{aligned}$$

and in terms of currency H

$$\begin{aligned}
& R^H X^H - \lambda_1^{*H}(1) \\
= \; & R^H X^H - (1 - p + \hat{\pi}) c_1^{1H} \\
= \; & p(1-\alpha) c_2^H(1) + (1-p) c_1^{1H} - (1 - p + \hat{\pi}) c_1^{1H} \\
= \; & p(1-\alpha) c_2^H(1) - \hat{\pi} c_1^{1H} \\
> \; & (p(1-\alpha) - \hat{\pi}) c_2^H(1)
\end{aligned}$$

Therefore the patients do not have any incentive to run at date 1. (banks are perfectly able to pay to those patients remaining at home until period 2). Hence this mechanism prevents runs and allows for implementation of the planner's allocation. ■

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