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"Investment Incentives in Procurement Auctions"

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Abstract

In this paper, we investigate firms' incentives for cost reduction and entry in the first price sealed bid auction, a format largely used for procurement. We find that firms will tend to underinvest in cost reduction, because they anticipate fiercer head-on competition. Moreover, this effect depends on the initial competitive position of the investor. In the first price auction, "laggards" have less incentives to invest and catch up than "leaders." Therefore, our results suggest that the first price auction could reinforce asymmetries between market participants. Finally, our research is related to the strategic investment literature in industrial organization. Though private value first price auctions are not games with increasing best responses, we find that, for comparative statics purposes, they behave like these games. Our results then bear an analogy with the market dominance outcome common in that literature.

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1. Introduction

Consider the following procurement situation. Several firms are competing for a government contract through a sealed bid procedure: the firm with the lowest quote gets the market at the price offered. These firms are not necessarily equally competitive a priori, and, indeed, in many procurement situations, some firms do have a clear cost advantage and are aware of these intrinsic differences among them. (for systematic evidence see, for instance, Carnaghan and Bracewell-Milnes, 1993, Bajari, 1998 and 1999, Porter and Zona, 1999).

In this paper, we want to investigate the incentives for cost reduction and entry in procurement markets.[1] That is, we step back and, rather than focus on the existence of asymmetries in procurement auctions, we ask what the incentives for firms are to improve their competitive situations relative to their rivals. A standard framework to study this question is to start with a two-stage game, where firms invest in cost reduction in the first stage and compete through a procurement auction in the second stage. As Fudenberg and Tirole (1984) and Bulow, Geanakoplos and Klemperer (1985) have shown, it then turns out that the nature of strategic interactions in the second stage game (i.e. whether best responses are increasing or decreasing) is a key element of the analysis. Unfortunately, though the complete information analog of the procurement auction (a Bertrand game) has increasing best response schedules, best responses in the space of strategies for the first price auction are non monotonic. Therefore, the approach based on the nature of strategic interactions in the second stage game does not apply, and we need to compare equilibria directly.

Our approach is based on the characterization of equilibrium behavior in first price auctions as the solution to a system of differential equations (Maskin and Riley, 1996, Lebrun, 1999). Our thought experiment is the following: Consider an initial configuration of bidders for a procurement contract. Suppose that one of them has the opportunity to upgrade his technology in the sense of generating a "lower" ex-ante distribution of costs. What are his incentives to do so if this investment is observed by his competitors? In section 3, we show that the investor's opponents will collectively bid more aggressively after the upgrade than before (propositions 1 and 2). Therefore, any given bid by the investor has a lower chance to win the market after the upgrade. In terms of investment incentives,

[1] In his study of defense procurements, Lichtenberg (1988) finds evidence that competitive procurement stimulates considerable private R&D investment.

this means that bidders will tend to underinvest prior to the procurement stage according to simple static efficiency arguments (holding competitors' strategies fixed). Put differently, firms will invest less in case of observable (overt) investments than if investments were covert.

Another important question for procurement authorities is whether the first price auction format tends to reinforce existing asymmetries between markets participants. In section 4, we provide numerical results that suggest that laggards have lower incentives than market leaders to invest in further cost reduction.[2] As a consequence, we expect asymmetries to grow under the first price auction. Again, the nature of strategic interactions at the procurement stage is key for these results. Indeed, compared to the second price auction, the first price auction provides more incentives for the leader to invest further and less incentives for the laggard to catch up. This means that, in markets where investment prior to the auction is deemed important, the second price auction is likely to be better at fostering competition.

Our research is related to the literature on first price auctions and to the literature on strategic investments in industrial organization. Existence and uniqueness of equilibrium in the independent private value first price auction have been proved under increasingly general assumptions by Lebrun (1996) and Maskin and Riley (1996 and 1999a). Maskin and Riley (1999b) and Li and Riley (1999) provide more precise characterizations of the equilibrium when a stochastic dominance relationship exists among bidders. Lebrun (1998) is closest to our analysis. Our two first propositions extend his comparative statics result in several directions. First, we do not restrict bidders to have a common support for their distribution of costs. Second, we allow for risk aversion and endogenous quantity. Finally, and most importantly, our results apply to $N > 2$ bidders.

This paper also relates to the literature on strategic investments and the evolution of market structure in industrial organization. Our results suggest that, for comparative statics purposes, the first price auction behaves very much like a game with strategic complementarities. Therefore, we expect much of the insight and intuition gained in pricing games under complete information to transpose to the first price auction.

[2] Our numerical simulations are based on Li and Riley's Bidcomp2 program extended to compute bidders' ex-ante expected payoffs. We refer to their paper for technical details about their program.

2. The model

In this section, we present the model and characterize its equilibrium. There is a single buyer (e.g. a government agency) in charge of procuring a given good or service. As in Hansen (1988), we allow quantities to be endogenous. Let $D(b)$ be the buyer's demand at price b. We make the following standard assumptions on demand:[3]

Assumption 1: $D(b) \geq 0, D'(b) \leq 0$ and increasing price eleasticity $\frac{d}{db}[\frac{D'(b)b}{D(b)}] \leq 0$.

$N \geq 2$ firms take part in a first-price, sealed-bid auction for the procurement contract. That is, the contract is awarded to the firm offering to provide the good or service at the lowest price, and the winner is paid the per unit price she bid. Ties are resolved by a random draw among the lowest bidders.

Firms' constant marginal costs have support on $[\underline{c}_i, \overline{c}_i]$, where $0 \leq \underline{c}_i < \overline{c}_i$, and they are independently distributed according to the twice continuously differentiable cumulative distribution function $F_i(.)$, with a density bounded away from zero on its support. These distributions are assumed to be common knowledge. they can be interpreted as representing the technology available to firms. Notice that we do not restrict bidders to have cost levels distributed on a common support. Firm i's profit when its cost is c_i and she makes a bid b is given by:

$$\pi_i(b, c_i) = \begin{cases} V_i((b - c_i)D(b)) & \text{if she wins} \\ 0 & \text{otherwise} \end{cases} \tag{2.1}$$

Assumption 2: For all i, $V_i(0) = 0$, $V_i' > 0$ and $V_i'' \leq 0$.

Lemma 1: *Under assumptions 1 and 2, $\pi_i(b, c)$ is strictly log-supermodular in (b, c), i.e. $\frac{\partial}{\partial c}[\frac{\frac{\partial \pi_i}{\partial p}}{\pi_i}] > 0$ over the domain where $\pi_i > 0$.*

Proof. We first claim that, at any equilibrium, $D(b) + (b - c)D'(b) > 0$ for all b such that b is bid by some firm i. $D(b) + (b - c)D'(b) = 0$ corresponds to the first order condition of the monopolist facing demand $D(b)$. It trades off the marginal benefit of increasing prices with the marginal cost of lost

[3] These guarantee that the complete information monopolist problem is quasiconcave (see, e.g. Caplin and Nalebuff, 1991, proposition 11).

trade. In the procurement setting, increasing prices has an additional cost: the potential loss of the whole market. Therefore, $D(b) + (b-c)D'(b)$ must be strictly positive at any bid b placed in equilibrium by some firm i.

Together with assumptions 1 and 2, this implies that:

$$\begin{aligned}
\frac{\partial}{\partial c}\left[\frac{\frac{\partial \pi_i}{\partial b}}{\pi_i}\right] &= \frac{1}{\pi_i}\frac{\partial^2 \pi_i}{\partial b \partial c} - \frac{1}{\pi_i^2}\frac{\partial \pi_i}{\partial p}\frac{\partial \pi_i}{\partial c} \\
&= \frac{1}{\pi_i}\underbrace{\{-V_i''[D(b)+(b-c)D'(b)]D(b) - V_i'D'(b)\}}_{\text{positive}} \\
&\quad + \frac{1}{\pi_i^2}\underbrace{(V_i')^2 D(b)[D(b)+(b-c)D'(b)]}_{\text{strictly positive}} \\
&> 0 \blacksquare
\end{aligned}$$

The recent results about existence and uniqueness of equilibrium in the first price auction form the basis for our analysis (see, for instance, Maskin and Riley, 1996 and 1999a). An equilibrium in this auction is described by an $N-$tuple of bidding functions $b_i : [\underline{c}_i, \overline{c}_i] \to \mathbb{R}_+$, $i = 1, ..., N$. For our purposes, it will be convenient to look at the inverse bidding functions. We denote them by $\phi_i : \mathbb{R}_+ \to [\underline{c}_i, \overline{c}_i]$, $i = 1, ...N$.

Maskin and Riley (1996 and 1999a) have shown that there exists a unique equilibrium in this environment.[4] The corresponding equilibrium inverse bidding functions $\phi_i(.)$ have support on $[l_i, u]$, $i = 1, ..., N$, and solve the system of differential equations

$$\sum_{j \neq i} \frac{F_j'(\phi_j(b))\phi_j'(b)}{1 - F_j(\phi_j(b))} = \frac{\frac{\partial}{\partial b}\pi_i(b, \phi_i(b))}{\pi_i(b, \phi_i(b))} \qquad i = 1, ...N \qquad (2.2)$$

with boundary conditions $F_i(\phi_i(l_i)) = 0$, and with u determined uniquely by the following lemma:

[4] If one bidder's support is very far to the left of all the other bidders' supports, then the equilibrium is degenerate. We will ignore this case.

For the $N > 2$ case, Maskin and Riley (1996) require an additional condition on the payoff functions to ensure uniqueness. It is satisfied if all bidders are risk neutral or if they have the same CARA or CRRA utility function.

Lemma 2 : Upper bound of the support of the equilibrium distribution of winning bids (adapted from Maskin and Riley, 1996): *Suppose that the distributions* $(F_1, ..., F_N)$ *are ordered so that* $\overline{c}_1 \leq \overline{c}_2 \leq ... \leq \overline{c}_{N-1} \leq \overline{c}_N$. *Then, if* $\overline{c}_1 = \overline{c}_2 = \overline{c}$, *then* $u = \overline{c}$. *Otherwise,* u *solves*

$$\min\{\arg\max_b \pi_1(b, \overline{c}_1) \prod_{i \neq 1} (1 - F_i(b))\} \in (\overline{c}_1, \overline{c}_2) \tag{2.3}$$

If $u < \overline{c}_i$ for some i, we can consider that, for any realization of cost $c_i > u$, firm i bids his own cost (and never wins) or stays out of the auction.

Notice that the lower bounds of the supports of equilibrium bids are endogenously determined by the boundary condition of (2.2). In general, lower bounds to the equilibrium bids need not be common to all firms but they must be common to at least two of them. They depend on the lower bounds of the support of costs, $\underline{c}_i$, and it can be shown that $l_i \leq l_j$ iff $\underline{c}_i \leq \underline{c}_j$. Finally, it can also be shown that the equilibrium inverse bidding functions are strictly increasing and twice differentiable on their support. For further details on the structure of the equilibrium, we refer the interested reader to Maskin and Riley (1996).

To see why $\phi_i(.)$, $i = 1, ..., N$ are indeed equilibrium inverse bidding functions, it suffices to realize that equations (2.2) are the first-order conditions of the firms' pseudo-concave maximization problem. That is, firm i with cost level c_i will choose its bid by solving the problem

$$\max_b \pi_i(b, c_i) \underbrace{\prod_{j \neq i} (1 - F_j(\phi_j(b))}_{\text{probability of winning}}$$

Noting that, at the optimal value of b, we have $c_i = \phi_i(b)$ for all i, equations (2.2) follow.

We want to understand how the equilibrium in the procurement auction is affected by changes in the distribution of cost levels for one firm. For that purpose, we define a proper notion of "better" distribution of costs. The following definition provides such a partial ordering:

Definition 1: Consider two cumulative distribution functions F and $\widetilde{F}$ with bounded support. We shall say that $\widetilde{F} \succ F$ if, for all c, c' such that $c' > c$,

$$\frac{1 - \widetilde{F}(c')}{1 - \widetilde{F}(c)} < \frac{1 - F(c')}{1 - F(c)} \tag{2.4}$$

whenever these expressions are well defined.

The requirement in (2.4) is one of conditional stochastic dominance. It means that, conditioning on any minimum level of costs, it is always more likely for F to yield a higher cost level than it is for $\widetilde{F}$. It can be shown that this condition implies that there is a relation of first-order stochastic dominance between the distributions: $F(c) < \widetilde{F}(c)$ for all c on the interior of their common support. Note that, given our differentiability assumption, (2.4) can be rewritten as

$$\frac{d}{dc}\left(\frac{1-\widetilde{F}(c)}{1-F(c)}\right) < 0$$

or, in terms of hazard rates,

$$\frac{\widetilde{F}'(c)}{1-\widetilde{F}(c)} > \frac{F'(c)}{1-F(c)} \tag{2.5}$$

for all c on their common support.

Definition 1 (or its variant for the standard auction) has become quite common in the asymmetric first price auction literature (see Lebrun, 1998, Maskin and Riley, 1999b, or Li and Riley, 1999, for instance). In practice, it is a little bit stronger than needed and a weak inequality in (2.5) would do for our purposes. However, it would also lengthen the proofs without adding any new insight, hence our decision to stick to the stronger version. Comparing (2.5) with (2.2), it should also be clear that this is the only natural way to order distributions for the first price auction.

In what follows, whenever there is a shift in firm i's distribution of cost levels from F_i to $\widetilde{F}_i \succ F_i$, we will refer to such a shift as an *upgrade*, and to firm i as the *upgrader*. Examples of distributional upgrades that satisfy definition 1 include: additional random draws from the same distribution ($\widetilde{F}(c) = 1-(1-F(v))^x$ for $x > 1$), shifts of distributions to the left (i.e. $\widetilde{F}(c) = F(c+a)$ for $a > 0$) for distributions with a monotone hazard rate $\frac{d}{dc}\left(\frac{F'(c)}{1-F(c)}\right) > 0$,[5] and distributional contractions with a fixed lower end of the support ($\widetilde{F}(c) = \theta F(c)$ for $\theta > 1$ and

[5] Distributions that satisfy this condition include the uniform, the normal, the logistic, the χ^2 distributions, as well as the Weibull, γ and β distributions for some parameter values (see Bagnoli and Bergstrom, 1989).

$c \in (\underline{c}, F^{-1}(1/\theta))$. Distributional stretches with a fixed upper end of the support $(1 - \widetilde{F}(c) = \theta[1 - F(c)]$ for $\theta > 1$ and c in the support of F) satisfy the weaker requirement of first order stochastic dominance and weakly higher hazard rate.

3. Comparing equilibria

We want to understand how a distributional upgrade by one firm affects the resulting equilibrium in the procurement auction. In other words, starting from an initial configuration of firms $(F_1, ..., F_N)$, suppose that firm j has the opportunity to upgrade its distribution of costs to $\widetilde{F}_j \succ F_j$. How does the equilibrium in this new auction $(\widetilde{F}_j, F_{-j})$ compare with that of the initial one, (F_j, F_{-j})?

Referring back to (2.2), it is easy to see that such an investment by firm j shifts its opponents' best response schedules upwards (remember, by lemma 1, the right hand side of (2.2) is increasing in ϕ_i), i.e. they now react more aggressively to firm j's bidding behavior. If auctions were games with increasing best responses, this would be the end of the story. Indeed, j's opponents "commitment" to bidding more aggressively together with increasing best responses would result in more aggressive bidding behavior by all participants in the "post-upgrade" equilibrium (see, e.g., Milgrom and Roberts, 1994 who generalize the earlier analyses by Fudenberg and Tirole, 1984 and Bulow et al., 1985). Unfortunately, first price auctions are not games with monotonic best responses as the following example illustrates. Therefore, we shall attack our question by comparing equilibria directly.

Example 1: Consider the following auction environment. Two risk neutral firms bid for a single object. Firms' costs are distributed uniformly over the interval $[0, 1]$.This is a symmetric first price auction and it is easy to check that the equilibrium bidding functions are $b_i(c_i) = \frac{1+c_i}{2}$ for $i = 1, 2$ (this means that $\phi(b) = 2b - 1$). Now suppose that firm 1 suddenly bids more aggressively: $\widehat{b}_1(c_1) = \sqrt{c_1} < \frac{1+c_i}{2}$ (this corresponds to an inverse bidding function of $\widehat{\phi}_1(b) = b^2$). Firm 2's best response solves $\max_b (b - c_2)(1 - b^2)$. Let $\widehat{\phi}_2(b)$ be the inverse bid function that corresponds to this optimization problem. $\widehat{\phi}_2(b) = \frac{3b^2-1}{2b}$ and has support on $[1/\sqrt{3}, 1]$. The interesting element here is that though firm 1 has become *more aggressive*, $\widehat{\phi}_1(b) > \phi(b)$, firm 2's best response to $\widehat{\phi}_1, \widehat{\phi}_2$, is *less aggressive* than his best response to ϕ.[6] Examples where firm 2 would respond to a more aggressive behavior of firm 1 by being more aggressive can similarly be generated.

Consider the two configurations (F_j, F_{-j}) and $(\widetilde{F}_j, F_{-j})$ with $\widetilde{F}_j \succ F_j$. Denote their respective equilibria by (ϕ_j, ϕ_{-j}) and $(\widetilde{\phi}_j, \widetilde{\phi}_{-j})$. Let l and u (respectively $\widetilde{l}$ and $\widetilde{u}$) be the lower and upper bounds of the equilibrium bids under (F_j, F_{-j}) (respectively $(\widetilde{F}_j, F_{-j})$). For later use, we also define $p_i(b) = F_i(\phi_i(b))$ i.e. the probability that firm i bids under b in configuration (F_j, F_{-j}). $\widetilde{p}_i(b)$ is similarly defined.

In asymmetric first price auctions, we cannot solve for the equilibrium explicitly. Therefore, we need to resort to firms' FOCs to compare equilibria. Though the actual proofs tend to be lengthy, the gist of the argument is actually quite simple. With $N = 2$ and a slight abuse of notation, (2.2) becomes:

$$\frac{p_i'(b)}{1 - p_i(b)} = \frac{\frac{\partial}{\partial b}\pi_j(b, \phi_j)}{\pi_j(b, \phi_j)} \qquad i \neq j \tag{3.1}$$

where the term on the right-hand side is increasing in ϕ_j (from lemma 1). Now suppose that at some point $p_2(\widehat{b}) = \widetilde{p}_2(\widehat{b})$ and $p_2'(\widehat{b}) > \widetilde{p}_2'(\widehat{b})$, that is, p_2 is crossing $\widetilde{p}_2$ from below at $\widehat{b}$. Then, firm 1's FOC (3.1) implies that $\widetilde{\phi}_1(\widehat{b}) > \phi_1(\widehat{b})$. In other words, using firms' FOCs, we are able to deduct from what is happening to one given firm's behavior across configurations, what is happening to the other firms' behavior, too. The rest of the argument usually makes use of the relationship between F_j and $\widetilde{F}_j$.

For $N > 2$ firms, the equations in (2.2) can be rewritten as:

$$\sum_{j \neq i} \frac{p_j'(b)}{1 - p_j(b)} = \frac{\frac{\partial}{\partial b}\pi_i(b, \phi_i)}{\pi_i(b, \phi_i)}$$

Solving for $\frac{p_j'(b)}{1-p_j(b)}$ yields:

$$(n-1)\frac{p_j'(b)}{1 - p_j(b)} = \sum_{i \neq j} \frac{\frac{\partial}{\partial b}\pi_i(b, \phi_i)}{\pi_i(b, \phi_i)} - (n-2)\frac{\frac{\partial}{\partial b}\pi_j(b, \phi_j)}{\pi_j(b, \phi_j)} \tag{3.2}$$

[6] Indeed, $\widehat{\phi}_2(b) = \frac{3b^2-1}{2b}$ is less than $\phi(b) = 2b - 1$ iff $3b^2 - 1 < 4b^2 - 2b$ or $b^2 - 2b + 1 > 0$.

where n is the number of firms who bid down to b. Since the only property of $\frac{\frac{\partial}{\partial b}\pi_i}{\pi_i}$ that is used in the proofs is the fact that it is increasing in ϕ_i, for simplicity we will often write the equivalent of (3.2) for the single object risk neutral case:

$$(n-1)\frac{p_j'(b)}{1-p_j(b)} = \sum_{i\neq j} \frac{1}{b-\phi_i(b)} - (n-2)\frac{1}{b-\phi_j(b)} \tag{3.3}$$

It should be clear that any result proved for this case also holds for the more general case (allowing for risk aversion and endogenous demand).

Our argument proceeds in 3 steps. First, we show that the upper bound to the equilibrium bids must be non increasing i.e. $\widetilde{u} \leq u$ (lemma 3). Second, we show that the lower bound to equilibrium bids is strictly decreasing, $\widetilde{l} < l$ (lemma 5). Finally, we show that, for 2 firms, bidding in the new configuration is more aggressive (in the sense of first order stochastic dominance), $\widetilde{p}_j(b) > p_j(b)$ (proposition 1). For more than two firms and with some additional conditions on the technologies available (the F functions), we prove that, for any b, the probability that the upgrader wins the market with b is lower after the upgrade than in the original configuration (proposition 2).

Lemma 3: *Let $u(F_1, ..., F_N)$ be the upper bound of the equilibrium bids in configuration $(F_1, ..., F_N)$. $u(F_1, ..., F_N)$ is weakly decreasing in its arguments. That is, if $\widetilde{F}_j \succ F_j$, then $u(\widetilde{F}_j, F_{-j}) \leq u(F_j, F_j)$.*

Proof. Let $u = u(F_i, F_{-i})$ and $\widetilde{u} = u(\widetilde{F}_i, F_{-i})$, and assume without loss of generality that $\overline{c}_1 \leq \overline{c}_2 \leq ... \leq \overline{c}_N$. Let $\widetilde{\overline{c}}_i$ be the maximum cost under $\widetilde{F}_i$ ($\widetilde{\overline{c}}_i \leq \overline{c}_i$).

If $\overline{c}_1 = \overline{c}_2$, then $\widetilde{u} \leq u$ follows trivially from Lemma 2. Moreover, the inequality is strict if $\widetilde{\overline{c}}_i < u(F_i, F_{-i})$.

If $\overline{c}_1 < \overline{c}_2$, Lemma 2 implies that u solves:

$$\min\{\arg\max_b \pi_1(b, \overline{c}_1) \prod_{i\neq 1} (1 - F_i(b))\}$$

In particular, u statifies the FOC:

$$\frac{\frac{\partial}{\partial b}\pi_i(b, \overline{c}_1)}{\pi_i(b, \overline{c}_1)} = \sum_{i\neq 1} \frac{F_i'(b)}{1-F_i(b)} \quad \text{for } b \in (\overline{c}_1, \overline{c}_2) \tag{3.4}$$

When $b = \overline{c}_1$, the expression in the left-hand side (LHS) goes to infinity while the expression in the right-hand side (RHS) is equal to zero. When $b = \overline{c}_2$,we have the reverse situation with the term in RHS going to infinity. This means that, at the solution (remember, u is the smallest value that solves the FOC), the RHS crosses the LHS from below.

If $\widetilde{F}_i \succ F_i$ for $i \neq 1$, the RHS of (3.4) increases for all $b \in (\overline{c}_1, \overline{c}_2)$ when firm i upgrades its distribution, and the lowest solution to (3.4) falls: $\widetilde{u} < u$ follows.

If firm 1 is the upgrader, there are two possibilities: Either $\widetilde{\overline{c}}_1 = \overline{c}_1$, in which case $\widetilde{u} = u$, or $\widetilde{\overline{c}}_1 < \overline{c}_1$. In that case, the LHS of (3.4) decreases (using lemma 1) and $\widetilde{u} < u$ follows. Figure 1 illustrates the logic of the proof for the risk neutral single object case where (3.4) can be rewritten as $\overline{c}_1 = b + [\sum\limits_{i \neq 1} \frac{F_i'}{1-F_i}]^{-1}$. ■

[insert figure 1 here]

Lemma 4: *It cannot be that, at any point $\widehat{b} \in [\max\{l, \widetilde{l}\}, \widetilde{u})$, $\widetilde{\phi}_i(\widehat{b}) \leq \phi_i(\widehat{b})$ for all i for whom both functions are defined, including a non-upgrading firm.*

Proof. The proof proceeds in two steps. First, we show that $\widetilde{\phi}_i(\widehat{b}) \leq \phi_i(\widehat{b})$ for all i for whom both functions are defined, implies that $\widetilde{\phi}_i(b) \leq \phi_i(b)$ for all i and for all $b > \widehat{b}$ (strictly close to $\widetilde{u}$). Second, we show that this leads to a contradiction with the fact that $\widetilde{u} \leq u$.

Step 1: Towards a contradiction, suppose that, for some firm j that satisfies $\widetilde{\phi}_j(\widehat{b}) \leq \phi_j(\widehat{b})$ and for some $b^* \geq \widehat{b}$, we have $\widetilde{\phi}_j(b^*) = \phi_j(b^*)$ and $\widetilde{\phi}_i(b) \leq \phi_i(b)$ for all $i \neq j$ and $b \leq b^*$. At b^*, there are potentially three groups of firms bidding:

1. The firms that bid down to $\widehat{b}$ under both configurations, (F_j, F_{-j}) and $(\widetilde{F}_j, F_{-j})$. There are still bidding at b^* and we index them by i.
2. Firms that bid down to b^* only under (F_j, F_{-j}). We index them by q.
3. Firms that bid down to b^* only under $(\widetilde{F}_j, F_{-j})$. We index them by r.

Using firm j's FOC, we have that, at b^*,

$$\frac{1}{b^* - \phi_j(b^*)} = \sum \frac{\widetilde{p}'_r}{1-\widetilde{p}_r} + \sum_{i \neq j} \frac{\widetilde{p}'_i}{1-\widetilde{p}_i} = \sum \frac{p'_q}{1-p_q} + \sum_{i \neq j} \frac{p'_i}{1-p_i} \tag{3.5}$$

(notice that the firms in the second group only appear on the right-hand side since they are active opponents only in the configuration (F_j, F_{-j}) and similarly for the firms in the third group).

Because for all the other firms in group 1, $\widetilde{\phi}_k \leq \phi_k$ at b^*, we also have:

$$\sum \frac{\widetilde{p}'_r}{1-\widetilde{p}_r} + \sum_{i \neq k} \frac{\widetilde{p}'_i}{1-\widetilde{p}_i} \leq \sum \frac{p'_q}{1-p_q} + \sum_{i \neq k} \frac{p'_i}{1-p_i} \tag{3.6}$$

Now, if $\widetilde{\phi}'_j(b^*) > \phi'_j(b^*)$, then $\frac{\widetilde{p}'_j}{1-\widetilde{p}_j} > \frac{p'_j}{1-p_j}$. Moreover, comparing (3.5) with (3.6), we find that, for all the firms in group 1:

$$0 < \frac{\widetilde{p}'_j}{1-\widetilde{p}_j} - \frac{p'_j}{1-p_j} \leq \frac{\widetilde{p}'_i}{1-\widetilde{p}_i} - \frac{p'_i}{1-p_i} \tag{3.7}$$

Therefore, going back to (3.5) we conclude that there must be some firms in group 2. Let p be one of them. We have,

$$\sum \frac{\widetilde{p}'_r}{1-\widetilde{p}_r} + \sum_{i \neq j} \frac{\widetilde{p}'_i}{1-\widetilde{p}_i} > \sum_{q \neq p} \frac{p'_q}{1-p_q} + \sum_{i \neq j} \frac{p'_i}{1-p_i}$$

So, adding $\frac{\widetilde{p}'_j}{1-\widetilde{p}_j} \geq \frac{p'_j}{1-p_j}$

$$\frac{1}{b^* - \widetilde{\underline{c}}_p} > \sum \frac{\widetilde{p}'_r}{1-\widetilde{p}_r} + \sum \frac{\widetilde{p}'_i}{1-\widetilde{p}_i} > \sum_{q \neq p} \frac{p'_q}{1-p_q} + \sum \frac{p'_i}{1-p_i} = \frac{1}{b^* - \phi_p(b^*)} > \frac{1}{b^* - \underline{c}_r} \tag{3.8}$$

where the first inequality follows from the assumption that firm p does not bid down to b^* under $(\widetilde{F}_j, F_{-j})$, the equality corresponds to firm p's FOC and the last inequality comes from the fact that bidding functions are increasing. Expression (3.8) implies that $\widetilde{\underline{c}}_p > \underline{c}_p$, a contradiction. We conclude that $\widetilde{\phi}_i(b) \leq \phi_i(b)$ for all i and for all $b > \widehat{b}$.

To prove the stronger claim that $\widetilde{\phi}_i(b) < \phi_i(b)$ for all i and for all $b \in (\widetilde{u} - \delta, \widetilde{u}]$, we need a few more steps since we also have to rule out $\widetilde{\phi}'_j(b^*) = \phi'_j(b^*)$. In that case, the first inequality in (3.7) is weak. To get a strong second inequality for at least one firm, we need that, for some $i, \widetilde{\phi}_i < \phi_i$ at b^* or that the upgrader belongs to the first group (in which case, even if $\widetilde{\phi}_i = \phi_i$, $\frac{\widetilde{p}'_i}{1-\widetilde{p}_i} > \frac{p'_i}{1-p_i}$ by definition 1). If none of these conditions are satisfied, $\widetilde{\phi}_i = \phi_i$ for all i and for all $b > b^*$ until we meet a firm in category 2 or 3 (and we know that this must happen at some point before $\widetilde{u}$ since the upgrader does not belong to group 1). At that point, we can argue as before (there must be a firm in group 2) and get a contradiction.

Step 2: Consider one of the non-upgrading firm in group 1, say k. Step 1 implies that $\widetilde{\phi}_k(\widetilde{u}) < \phi_k(\widetilde{u})$. However, $\widetilde{\phi}_k(\widetilde{u}) = \min\{\overline{c}_k, \widetilde{u}\}$ and $\phi_k(\widetilde{u}) \leq \phi_k(u) = \min\{\overline{c}_k, u\}$. If $\widetilde{u} = u$, we directly get a contradiction. If $\widetilde{u} < u$, we get the contradiction by using the fact that bidding functions are strictly increasing. ■

Lemma 5: *$l(F_1, ..., F_N)$ is strictly decreasing in its arguments. That is, if $\widetilde{F}_j \succ F_j$, then $l(\widetilde{F}_j, F_{-j}) < l(F_j, F_{-j})$.*

Proof. Towards a contradiction, suppose that $\widetilde{l} \geq l$. Then, for all i where both functions are defined, $\phi_i(\widetilde{l}) \geq \widetilde{\phi}_i(\widetilde{l})$ ($= \underline{c}_i$ or $\widetilde{\underline{c}}_i$ for the upgrader).[7] We first claim that there is at least a non-upgrading firm bidding down to $\widetilde{l}$ under both configurations. Indeed, we know that there are at least two firms bidding to the lower bound l and $\widetilde{l}$. Moreover, we claimed in section 2 that, in any configuration, $l_i \leq l_j$ iff $\underline{c}_i \leq \underline{c}_j$. There is a non-upgrading firm that has the first or second lowest minimum cost under both configurations. We can then apply lemma 4 to get a contradiction. ■

In the procurement first price auction, the upper bound to the equilibrium winning bids, u, is a singularity point for at least one of the differential equations that characterize the equilibrium (since for at least one firm, $p_j(u) = 1$ and so the right-hand side of (3.4) is undetermined). It turns out that we shall need to pin down the behavior of these p functions around the upper bound when $u = \widetilde{u}$.

[7] And, remember that, for the upgrader, $\widetilde{\underline{c}}_i \leq \underline{c}_i$.

From lemma 3, we conclude that $u = \widetilde{u}$ only in two cases: (i) if there are at least two bidders j such that $u = \overline{c}_j$ and $\overline{c}_i, \widetilde{\overline{c}}_i \geq u$; (ii) if $\widetilde{\overline{c}}_i = \overline{c}_i < \min\limits_{j \neq i} \{\overline{c}_j\}$.

The configuration where all firms have the same maximum cost is included in the first case. Bajari (1998) and, for the more general form of the profit function, Maskin and Riley (1996) have found expressions for the first derivative of inverse bidding functions at u. They satisfy $\phi_j'(u) = -\frac{N}{N-1} \frac{\frac{\partial}{\partial b}\pi_i(u,u)}{\frac{\partial}{\partial c}\pi_i(u,u)} < \infty$. We refer to these papers for a proof of this result. Case (i) also includes the possibility that two firms share the same maximum cost equal to u but some other firms have a strictly higher maximum cost. We shall argue that this possibility is a knife-edge case (we can rule it out by imposing the condition that, when some maximum costs differ, then they should all differ) and ignore it.[8]

Lemma 6; *Let i be the upgrader and suppose that $\widetilde{u} = u$. Then*
(a) if $\sum\limits_{j \neq i} \frac{p_j'(b)}{1-p_j(b)} < \sum\limits_{j \neq i} \frac{\widetilde{p}_j'(b)}{1-\widetilde{p}_j(b)}$ for all $b \in (u-\delta, u)$ for some δ positive, then $\prod\limits_{j \neq i} (1-\widetilde{p}_j(b)) > \prod\limits_{j \neq i} (1-p_j(b))$ in $(u-\delta, u)$,
(b) if $\prod\limits_{j \neq i} (1-\widetilde{p}_j(b)) > \prod\limits_{j \neq i} (1-p_j(b))$ in some neighborhood of u, then $\exists\ \delta > 0$ such that $\sum\limits_{j \neq i} \frac{p_j'(b)}{1-p_j(b)} < \sum\limits_{j \neq i} \frac{\widetilde{p}_j'(b)}{1-\widetilde{p}_j(b)}$ for all $b \in (u-\delta, u)$,
and the same claims hold by inverting the roles of p_j and $\widetilde{p}_j$.

Proof. See appendix.

We are now able to prove the main result of this section. We start with the simplest case: two firms. Then, we know from our discussion in section 2 that, at equilibrium, both firms bid on a common support (l, u) (and $(\widetilde{l}, \widetilde{u})$ after the investment). Moreover, lemma 5 implies that $\widetilde{p}_j(b) > p_j(b)$, $j = 1, 2$, for b close to l.

Proposition 1: Let $N = 2$. Then $\widetilde{p}_j(b) > p_j(b)$ *for all i and for all b in the interior of their common support.*

Proof. Let 1 be the upgrader. From lemma 5, $\widetilde{p}_j > p_j$ close to l. In addition, as long as $\widetilde{p}_2(b) > p_2(b)$, $\widetilde{\phi}_2(b) > \phi_2(b)$ and so (using firm 2's FOC) $\frac{\widetilde{p}_1'}{1-\widetilde{p}_1} > \frac{p_1'}{1-p_1}$. Therefore, starting from the left, $\widetilde{p}_1 > p_1$ as long as $\widetilde{p}_2 > p_2$.

[8] A continuity argument would suffice as well..

Now, towards a contradiction, suppose that $\widetilde{p}_2$ and p_2 intersect first at $b_1 < \widetilde{u}$. We have $\frac{p_2'(b_1)}{1-p_2(b_1)} > \frac{\widetilde{p}_2'(b_1)}{1-\widetilde{p}_2(b_1)}$ and (using firm 1's FOC)

$$\phi_1(b_1) > \widetilde{\phi}_1(b_1). \tag{3.9}$$

[insert figure 2 here]

Because $\widetilde{u} \leq u$, $\widetilde{p}_2$ and p_2 must cross at least once more in $(b_1, \widetilde{u}]$. Let b_3 be the next time they cross. At b_3, we must have $\frac{p_2'(b_3)}{1-p_2(b_3)} < \frac{\widetilde{p}_2'(b_3)}{1-\widetilde{p}_2(b_3)}$(this is immediate if $b_3 < \widetilde{u}$; if $b_3 = \widetilde{u}$ then the claim follows from lemma 6 and the fact that $\widetilde{p}_2 < p_2$ close to $b_3 = \widetilde{u} = u$). Using firm 1's FOC, we have that

$$\phi_1(b_3) < \widetilde{\phi}_1(b_3). \tag{3.10}$$

Comparing (3.9) and (3.10), we conclude that there exists $b_2 \in (b_1, b_3)$ such that ϕ_1 and $\widetilde{\phi}_1$ cross with $\widetilde{\phi}_1'(b_2) > \phi_1'(b_2)$. Because $\widetilde{F}_1 \succ F_1$, this means that $\frac{\widetilde{F}_1'(\widetilde{\phi}_1'(b_2))}{1-\widetilde{F}_1(\widetilde{\phi}_1'(b_2))} > \frac{F_1'(\phi_1'(b_2))}{1-F_1(\phi_1'(b_2))}$ and therefore

$$\frac{\widetilde{p}_1'(b_2)}{1-\widetilde{p}_1(b_2)} = \frac{\widetilde{F}_1'(\widetilde{\phi}_1'(b_2))\widetilde{\phi}_1'(b_2)}{1-\widetilde{F}_1(\widetilde{\phi}_1'(b_2))} > \frac{F_1'(\phi_1'(b_2))\phi_1'(b_2)}{1-F_1(\phi_1'(b_2))} = \frac{p_1'(b_2)}{1-p_1(b_2)}$$

Using firm 2's FOCs under both configurations, it follows that $\widetilde{\phi}_2(b_2) > \phi_2(b_2)$, which contradicts the fact that $\widetilde{\phi}_2(b) < \phi_2(b)$ for all $b \in (b_1, b_3)$. ■

When we move to $N > 2$ firms, the system of differential equations that describes the equilibrium puts much less structure on the solution. To get analytical results, we need to impose further conditions. First, we assume that firms have the same utility functions, V_i for all i. Second, we impose that firms' distributions of costs are ordered according to definition 1. This is useful because it can be shown that if $F_i \succ F_j$ then $\phi_i < \phi_j$ and $p_i > p_j$ at equilibrium (see, e.g., Maskin and Riley, 1999b and for a generalization to $N > 2$ bidders, Li and Riley, 1999). Intuitively, firm i, which has a more efficient technology, can afford to take a higher profit margin $b - \phi_i(b)$ at equilibrium. This is because, when trading off between a lower probability of winning and a higher price-cost margin, it takes into account the fact that its opponent is unlikely to have low costs. How strong is this condition? Probably stronger that needed for the claim to hold. On the other hand, it seems that firms participating in an auction have a good idea of their relative cost advantage and, in that case, definition 1 seems appropriate. Finally, the third simplification we make is to restrict the set of possible technological choices for firms (the F functions). This, together with our assumption of common payoff functions implies that, at equilibrium, firms that have the same technology will bid the same way.

Proposition 2: *Suppose there are 3 firms and 2 technologies, $F_H \succ F_L$. Let firm 1 be the upgrader. Define $W_1(b) = \prod_{j \neq 1} (1 - p_j(b))$ i.e. $W_1(b)$ is the probability that firm 1 wins with a bid of b. Define $\widetilde{W}_1(b)$ similarly. Then $\widetilde{W}_1(b) < W_1(b)$ on the interior of their common support.*

Proof. Proposition 2 covers 3 upgrade scenarios:

Before:	L	L	L	L	H	L	L	H	H
After:	H	L	L	H	H	L	H	H	H

where L indicates that the firm has the low efficiency technology, F_L, and H indicates that it has the high efficiency technology, F_H.

We first assume that the supports of winning bids are common to all firms in the post-upgrade configuration. Then, by lemma 5 ($\widetilde{l} < l$), $\widetilde{p}_j > p_j$ for all j, $\widetilde{W}_1 < W_1$, and $\widetilde{\phi}_j > \phi_j$ for all $j \neq i$ close to l.

Claim 1: Starting from the left (i.e. from l onwards), the first $\widetilde{p}$ and p to cross cannot be for the upgrader. Moreover, at that first crossing, it must be that $\widetilde{\phi}_1 < \phi_1$.

Proof: Since $\widetilde{p}_1$ would cross p_1 from above, we must have

$$\frac{\widetilde{p}_1'}{1 - \widetilde{p}_1} < \frac{p_1'}{1 - p_1} \tag{3.11}$$

At the same time, because $\widetilde{\phi}_j \geq \phi_j$ for all $j \neq i$ and $\widetilde{\phi}_1 < \phi_1$ (since $\widetilde{F}_1 > F_1$), we have

$$2\frac{\widetilde{p}_1'}{1 - \widetilde{p}_1} = \sum_{j \neq 1} \frac{1}{b - \widetilde{\phi}_j} - \frac{1}{b - \widetilde{\phi}_1} > \sum_{j \neq 1} \frac{1}{b - \phi_j} - \frac{1}{b - \phi_1} = 2\frac{p_1'}{1 - p_1}$$

a contradiction with (3.11). $\widetilde{\phi}_1 < \phi_1$ follows straightforwardly.

Claim 2: It cannot be that at any single point $b < \widetilde{u}$, $\widetilde{\phi}_j \leq \phi_j$ for all j.

Proof: Proceeds as in lemma 4. A contradiction with $\widetilde{u} \leq u$ obtains.

Claim 3: $\widetilde{W_1}(b) < W_1(b)$ close to u.

Proof: When $\widetilde{u} < u$, this is straightforward. If $\widetilde{u} = u$, suppose towards a contradiction that $\widetilde{W}_1 \geq W_1$ close to $\widetilde{u} = u$. By lemma 5, this implies that $\frac{\widetilde{W}_1'}{\widetilde{W}_1} \leq \frac{W_1'}{W_1}$ close to u, and using firm 1's FOC, $\widetilde{\phi}_1 \geq \phi_1$ close to u. Now, in all three upgrade scenarios, we have $\widetilde{\phi}_1 \leq \widetilde{\phi}_2, \widetilde{\phi}_3$ and $\phi_1 \geq \phi_2, \phi_3$ (some with strict inequalities). Therefore, we must also have $\widetilde{\phi}_2 \geq \phi_2$ and $\widetilde{\phi}_3 \geq \phi_3$ (one of them strict) and $\widetilde{W_1} < W_1$ close to u. A contradiction.

Suppose that p and $\widetilde{p}$ first cross at b_1(starting from the left) for firm 2. Since $\widetilde{u} \leq u$, we know that they must cross again in $(b_1, \widetilde{u}]$. Given lemmas 4 and 5, and claims 1 and 2, this leaves us with 3 generic crossing patterns for the $\widetilde{\phi}$'s and ϕ's.

[insert figure 3 here]

Claim 4: Given our technology assumptions, $\widetilde{\phi}_1 = \phi_1$ implies that $\widetilde{\phi}_2 \geq \phi_2$ and $\widetilde{\phi}_3 \geq \phi_3$ (at least one of them is strict). Therefore, patterns (b) and (c) are impossible.

Claim 5: $\widetilde{W_1} < W_1$ for all b.

Proof: Consider b_2 in pattern (a). Using claim 3 in case $b_2 = u$, we know that $\widetilde{W_1} < W_1$ at b_2 or close to the left of b_2. Now, for all b in (b_1, b_2), $\widetilde{\phi}_1 < \phi_1$, therefore

$$-\frac{\widetilde{W_1'}}{\widetilde{W_1}} = \frac{1}{b - \widetilde{\phi}_1} < \frac{1}{b - \phi_1} = -\frac{W_1'}{W_1}$$

and this rules out any crossing of $\widetilde{W_1}$ and W_1 in (b_1, b_2).

In the appendix, we extend the proof for the case where the upper bound $\widetilde{l}$ is not common to all firms. ■

Comparative statics results on firms' aggregate bidding behavior are all we need to answer questions about investment incentives in the procurement auction.

It is nevertheless useful to remark that proposition 1 implies that, for the non upgrading firms, bidding is also more aggressive for *every cost realization*. Indeed, for them, $\widetilde{p}_j(b) = F_j(\widetilde{\phi}_j(b)) > p_j(b) = F_j(\phi_j(b))$, hence $\widetilde{\phi}_j(b) > \phi_j(b)$. This does not necessarily hold for the upgrader.

Propositions 1 and 2 allow us to answer our initial question concerning the incentives of firms to upgrade their distributions. When firm i upgrades its distribution, it needs to take two effects on its ex-ante expected payoff into account: First, a *direct* effect through an improvement in the ex-ante distribution of its costs (holding its opponents' strategies fixed) and, second, an *indirect* or *strategic* effect through its opponents' adjustments to the new configuration. Propositions 1 and 2 tell us that, under the new configuration $(\widetilde{F}_i, F_{-i})$, firm i's opponents will bid, collectively, more aggressively. This means that the strategic effect is *negative* for distributional upgrades in the first price procurement auction.

At this point, it might be useful to remember that the investments we are considering shift the best response schedule of the investor's opponents upwards. In other words, holding the bidding strategy of the investor fixed, his opponents prefer to bid more agressively after the investment than before (refer to (2.2) if needed). We can then interpret our results as indicating that, for comparative statics purposes, the first price auction behaves as standard games with increasing best response schedules. Firms will tend to invest less in case of overt investments than in case of covert investments.

How strong is the strategic effect? Quite strong as example 2 ilustrates. There, an inefficient firm would be better off avoiding a cost reducing investment, *even if it came at no cost*!

Example 2: Consider the following initial configuration for firms 1 and 2: F_1 is uniform over [0,10], whereas F_2 is uniform over [0,5]. Suppose that firm 1 has a chance to upgrade its distribution to $\widetilde{F}_1 = F_2$. Denote by $U_i(F, \widehat{F})$ firm i's ex-ante payoff when firm 1's distribution is F and firm 2's distribution is $\widehat{F}$. A numerical solution to the first-price auction yields: $U_1(F_1, F_2) = 0.90445$, $U_2(F_1, F_2) = 1.93245$, $U_1(F_2, F_2) = U_2(F_2, F_2) = 0.83333$. The change in its distribution leaves firm 1 worse off.

4. Towards a more symmetric or a less symmetric market

structure?

An important question in industrial organization is whether asymmetries between firms tend to increase or decrease over time. Maintaining a healthy degree of competition is also a concern for procurement authorities. Example 2 illustrated how drastic the negative strategic effect of investment could be for the first price auction. There, the inefficient firm (with cost uniformly distributed over [0,10]) is worse off investing to become as efficient as its opponent than it is remaining inefficient. A natural next question is whether the fact that we were looking at a potential investment for the "laggard" was important in this example. More generally, how do incentives for investment depend on the investor's initial competitive position?

In this section, we return to the standard single object risk neutral procurement auction and focus on two firms. Let $F_2 \succ F_1$ (firm 2 is more efficient). This means that, at equilibrium, firm 2 will be less aggressive than firm 1 and take a larger profit margin over costs —i.e. $\phi_2(b) < \phi_1(b)$. Firm 2 is somewhat insulated from competitive pressures. If such a competitive advantage does indeed protect the leader from head-on competition and allows him to adjust to the weaker bidder, then we can conjecture that incentives for investment upgrades are higher the stronger the initial competitive position of the investor. Below, we provide numerical results that confirm this conjecture.

The distribution of costs for firm 2 is held fixed, and we order the potential distribution of costs for firm 1 by a parameter a with $F_1^a \succ F_1^{a+1}$ (a lower value for a means a more efficient cost distribution). Let $\Pi_1(F, \widehat{F})$ be the ex-ante expected profit for firm 1 when its costs are distributed according to F and firm 2's costs are distributed according to $\widehat{F}$. Define $\Delta = \Pi_1(F_1^a, F_2) - \Pi_1(F_1^{a+1}, F_2)$. Δ is the ex-ante expected increase in firm 1's profit from moving to F_1^{a+1} to F_1^a.

Distributional contraction (fixed lower end): Let F_2 be uniform on $[0, 5]$. F_1^a is uniform on $[0, a]$. Table 1 presents the payoffs that result from the numerical solution to each of the auction for a between 10 and 1. The row in bold type refers to the symmetric configuration.

a	$\Pi_1(F_1^a, F_2)$	Δ
10	0.90445	
9	0.87073	-0.03372
8	0.84941	-0.02132
7	0.83392	-0.01549
6	0.82701	-0.00691
5	**0.83333**	**0.00632**
4	0.85391	0.02058
3	0.91935	0.06544
2	1.00454	0.08519
1	1.12233	0.11779

Table 1

As reflected in the last column, the strategic effect outweighs the direct effect for high levels of a. Catching up leaves firm 1 worse off. However, as soon as firm 1 becomes the most efficient firm, further distributional upgrades always result in an increase in its payoffs (gross of investment costs).

Distributional stretch (fixed upper end): F_2 is uniform on $[5, 10]$ and F_1^a is uniform on $[a, 10]$. The corresponding values are presented in Table 2.

a	$U_1(F_1^a, F_2)$	Δ
9	0.04191	
8	0.15958	0.11767
7	0.33874	0.17916
6	0.56666	0.22792
5	**0.83333**	**0.26667**
4	1.13113	0.29780
3	1.45416	0.32303
2	1.79797	0.34381
1	2.15986	0.36189
0	2.53459	0.37473

Table 2

In this case, the strategic effect is not significant enough to outweigh the direct effect on firm 1's payoffs from any starting distribution. However, the last column exhibits once more that the gains from a one-unit stretch in the distribution are larger the more effcient the original distribution is.

Distributional shift: F_1^a is uniform on $[a-4, a+4]$, while F_2 is uniform on $[12, 20]$. The corresponding values are presented in Table 3.

a	$\Pi_1(F_1^a, F_2)$	Δ
24	0.10255	
23	0.14931	0.04676
22	0.21325	0.06392
21	0.29937	0.08614
20	0.41182	0.11245
19	0.56437	0.15255
18	0.75644	0.19207
17	0.96974	0.21330
16	**1.24499**	**0.27516**
15	1.57223	0.32724
14	1.97229	0.40006
13	2.39907	0.42678
12	2.87819	0.47912
10	3.42465	0.54646
9	4.02154	0.59689
8	4.66895	0.64741
7	5.36238	0.69343

Table 3

Finally, table 4 presents the numerical results of a shift to the left of a truncated normal distribution. Specifically, F_1^a has mean a, whereas F_2 has mean 10. The standard deviation for both distributions is 1 and they are truncated three standard deviations away from the mean.

a	$\Pi_1(F_1^a, F_2)$	Δ
17	0.01813	
16	0.02587	0.00774
15	0.03705	0.01118
14	0.05701	0.01996
13	0.09742	0.04041
12	0.17411	0.07669
11	0.31627	0.14216
10	**0.55835**	**0.24208**
9	0.92902	0.37067
8	1.43440	0.50538
7	2.05971	0.62531
6	2.77621	0.71650
5	3.57305	0.79684
4	4.41764	0.84459
3	5.28299	0.86535

Table 4

All these numerical results support the conjecture that leaders (i.e. the more efficient firms) have greater incentives to upgrade than laggards do. Of course, these results are largely indicative. The numbers in the tables refer to *gross* benefits from the investment. We have left the issue of the origin of the upgrade open on purpose. A possibility is that firms do indeed invest in more efficient technologies. Then, the natural next step would be to model explicitly the investment costs.[9] In particular, marginal investment costs could be increasing. Then, what the results above suggest is that they have to be increasing *fast enough* if they are to offset the growing marginal gains in gross expected profits to the investing firm. Otherwise, once there is some asymmetry between market participants, the first price auction format is likely to reinforce these differences.

Another possibility is that firms' cost advantages result from learning-by-doing. An example of what we have in mind is the repeated procurement auctions for public service provision in the UK. There, it could be argued that the incumbent has an advantage over potential entrants at the end of the contract. Our

[9] For an example of such a model in a symmetric setting see Tan (1992).

results suggest that if firms do take these future advantages of winning a contract today into account, potential entrants have less incentives to bid aggressively because they anticipate the stronger head-on competition in future auctions.

Before concluding this section, we want to emphasize that the *nature of strategic interactions* in the first price auction is key in all these results. The second price auction provides a good benchmark. Because bidding one's own cost is a dominant strategy, the second price auction is void of strategic considerations. Therefore, the negative strategic effect identified in propositions 1 and 2 for the first price auction is trivially absent. In proposition 3, we show that, for two firms, the incentives for upgrading are stronger in the second price auction than in the first price auction for the laggard and they are weaker in the SPA than the FPA for the leader.

Proposition 3: *Let $F_2 \succ F_1$. (1) Suppose firm 1's cost are distributed according to F_1 and firm 2's costs are distributed according to F_2 (i.e. firm 1 is less efficient than firm 2). Then, firm 1 incentives to upgrade its distribution to F_2 (i.e. to catch up on the leader) are greater under the SPA than under the FPA. (2) Suppose that, originally, both firms have their costs distributed according to F_1. Then, the FPA gives more incentives for one of them to upgrade to F_2 than the SPA does.*

Proof. Proposition 3 is an almost direct consequence of Maskin and Riley (1999b)'s proposition 2.6. There, they show that the inefficient firm prefers the FPA format to the SPA auction format. Let $\Pi_1^{FPA}(F, \widehat{F})$ be the ex-ante expected profit of firm 1 when its cost distribution is F and firm 2's cost distribution is $\widehat{F}$. $\Pi_1^{SPA}(F, \widehat{F})$ is the equivalent for the SPA auction. Proposition 2.6 of Maskin and Riley implies that $\Pi_1^{FPA}(F_1, F_2) > \Pi_1^{SPA}(F_1, F_2)$ and $\Pi_2^{FPA}(F_1, F_2) < \Pi_2^{SPA}(F_1, F_2)$. Therefore:

$$\Pi_1^{FPA}(F_2, F_2) - \Pi_1^{FPA}(F_1, F_2) < \Pi_1^{FPA}(F_2, F_2) - \Pi_1^{SPA}(F_1, F_2) = \\ \Pi_1^{SPA}(F_2, F_2) - \Pi_1^{SPA}(F_1, F_2)$$

where the equality of the second and third terms follows from the revenue equivalence theorem. To prove (2), we proceed similarly:

$$\Pi_1^{FPA}(F_2, F_1) - \Pi_1^{FPA}(F_1, F_1) < \Pi_1^{SPA}(F_2, F_1) - \Pi_1^{FPA}(F_1, F_1) = \\ \Pi_1^{SPA}(F_2, F_1) - \Pi_1^{SPA}(F_1, F_1) \blacksquare$$

An implication of proposition 3 is that, in markets where investment prior to the auction is an important feature, the second price auction format is likely to be better at maintaining and enhancing competition among firms.

5. Concluding remarks

Asymmetries between bidders are widespread in procurement situations. They are also often a source of concern for procurement authorities. However, our understanding of these market situations has been largely limited to date by the lack of explicit solutions for the equilibrium in the asymmetric first price auction.

In this paper, we have provided comparative statics results for a class of investments in cost reduction in the first price auction. In section 3, we showed that, after the investment, the investor's opponents will bid collectively more aggressively. In the terminology of industrial organization, this means that investments in the first price auction have a negative strategic effect. In section 4, we found that incentives for investment depend on the initial competitive position of the investor and that laggards have lower incentives to invest than leaders. It is tempting to interpret the low level of competition and of turnover in many procurement markets in light of our results.

More generally, it is interesting to draw an analogy between our results and similar results in the industrial organization literature under complete information. Though first price auctions are not games with increasing best response schedules, we found that, for comparative statics purposes, they behave like such games. In particular, our conclusions are consistent with the insights developed in the context of dynamic games and patent races where a common outcome is that of increasing dominance.[10]

At a purely theoretical level, our results contribute to the current efforts by various researchers to characterize and describe the equilibrium in the asymetric first price auction. Our analysis deals with more than two bidders and provides a systematic treatment of potentially different bidding supports.

In any case, further research is needed. An important open question for the first price auction is the source of bidders' profits. In the meantime, economists will need to rely on numerical methods for gaining understanding of the basic

[10]Notice that the kind of cost reduction considered in this paper corresponds to the "non drastic" innovation in that literature.

forces at play in asymmetric first price auctions (and they have already successfully done so: see Marschall et al., 1994, Athey, 1997, and Li and Riley, 1999).

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6. Appendix

Lemma 6: *Let i be the upgrader and suppose that $\widetilde{u} = u$. Then*
(a) if $\sum\limits_{j \neq i} \frac{p_j'(b)}{1-p_j(b)} < \sum\limits_{j \neq i} \frac{\widetilde{p}_j'(b)}{1-\widetilde{p}_j(b)}$ *for all* $b \in (u-\delta, u)$ *for some* δ *positive, then* $\prod\limits_{j \neq i} (1-\widetilde{p}_j(b)) > \prod\limits_{j \neq i} (1-p_j(b))$ *in* $(u-\delta, u)$,
(b) if $\prod\limits_{j \neq i} (1-\widetilde{p}_j(b)) > \prod\limits_{j \neq i} (1-p_j(b))$ *in some neighborhood of* u, *then* $\exists\ \delta > 0$ *such that* $\sum\limits_{j \neq i} \frac{p_j'(b)}{1-p_j(b)} < \sum\limits_{j \neq i} \frac{\widetilde{p}_j'(b)}{1-\widetilde{p}_j(b)}$ *for all* $b \in (u-\delta, u)$,
and the same claims hold by inverting the roles of p_j *and* $\widetilde{p}_j$.

Proof of lemma 6: (a) To simplify notations, let $W_i(b) = \prod\limits_{j \neq i} (1-\widetilde{p}_j(b))$, i.e. $W_i(b)$ is the probability that all the opponents of bidder i bid above b. Define $\widetilde{W}_i(b)$ similarly. With these notations, $\sum\limits_{j \neq i} \frac{p_j'(b)}{1-p_j(b)} < \sum\limits_{j \neq i} \frac{\widetilde{p}_j'(b)}{1-\widetilde{p}_j(b)}$ becomes $\frac{\widetilde{W}_i'(b)}{\widetilde{W}_i(b)} < \frac{W_i'(b)}{W_i(b)}$. Towards a contradiction, suppose that there exists $\widehat{b} \in (u-\delta, u)$ such that $\widetilde{W}_i(\widehat{b}) < W_i(\widehat{b})$ (this is without loss of generality since if $\widetilde{W}_i(\widehat{b}) = W_i(\widehat{b})$, then $\widetilde{W}_i(\widehat{b}+\varepsilon) < W_i(\widehat{b}+\varepsilon)$ for ε small enough) . $\frac{\widetilde{W}_i'(b)}{\widetilde{W}_i(b)} < \frac{W_i'(b)}{W_i(b)}$ for all $b \in (u-\delta, u)$ implies that $\frac{d}{db}\left(\frac{\widetilde{W}_i(b)}{W_i(b)}\right) < 0$ on the same interval. Then,

$$\int_{\widehat{b}}^{u} \frac{d}{db}\left(\frac{\widetilde{W}_i(b)}{W_i(b)}\right) db = \lim_{b \to u} \frac{\widetilde{W}_i(b)}{W_i(b)} - \frac{\widetilde{W}_i(\widehat{b})}{W_i(\widehat{b})} < 0.$$

That is,

$$\lim_{b \to u} \frac{\widetilde{W}_i(b)}{W_i(b)} < \frac{\widetilde{W}_i(\widehat{b})}{W_i(\widehat{b})} < 1. \tag{6.1}$$

From the discussion in the main text, there are two possibilities to analyze.
(i) All firms have the same maximum cost, u. Using L'Hôpital's rule and the fact that $\phi_j'(u) = \widetilde{\phi}_j'(u) < \infty$, we get that:

$$\lim_{b \to u} \frac{\widetilde{W}_i(b)}{W_i(b)} = 1,$$

and this contradicts (6.1).

(ii) $\overline{c}_i = \widetilde{\overline{c}}_i < \overline{c}_j$ for all $j \neq i$. In such a case,

$$\lim_{b \to u} \frac{\widetilde{W}_i(b)}{W_i(b)} = \frac{\prod\limits_{j \neq i} [1 - F_j(u)]}{\prod\limits_{j \neq i} [1 - F_j(u)]} = 1,$$

since $F_j(u) < 1$ for all $j \neq i$, and we get a contradiction again with (6.1).

The same proof can be used reversing the roles of $\widetilde{W}_i(b)$, $\widetilde{p}_j(b)$ and $W_i(b)$, $p_j(b)$.
(b) Towards a contradiction, suppose that for all b in a neighborhood of u, $\sum\limits_{j \neq i} \frac{p_j'(b)}{1-p_j(b)} \geq \sum\limits_{j \neq i} \frac{\widetilde{p}_j'(b)}{1-\widetilde{p}_j(b)}$. Then, applying the argument of part (a), we get that $\widetilde{W}_i(b) \leq W_i(b)$ in that neighborhood. This contradicts the hypothesis. ■

Proof of proposition 2 when the support of bids need not be common: First, notice that in the first upgrade scenario, lower bounds of equilibrium bids, lower bounds of equilibrium bids must be common to all firms. In the third scenario, the lower bound is common to all firms after the investment but not necessarily before. In particular, it is possible to have $\widetilde{l} < l < l_1$.However, we still have $\widetilde{\phi}_3 > \phi_3$ and $\widetilde{\phi}_2 > \phi_2$ close to l and therefore the crossing patterns identified in main body of the proof can be used to investigate crossing further on.
It remains to examine the second crossing scenario. There, we know that l is common to all firms but it could be that, after the investment, $\widetilde{l} < \widetilde{l}_3$. If $\widetilde{l}_3 < l$, then $\widetilde{p}_j > p_j$ for all j close to l and we can proceed as in themain body of the proof. It remains to rule out $l \leq \widetilde{l}_3$. (intuitively, if firm 3 was happy to bid down to l before the investment, it should be happy to do so afterwards).

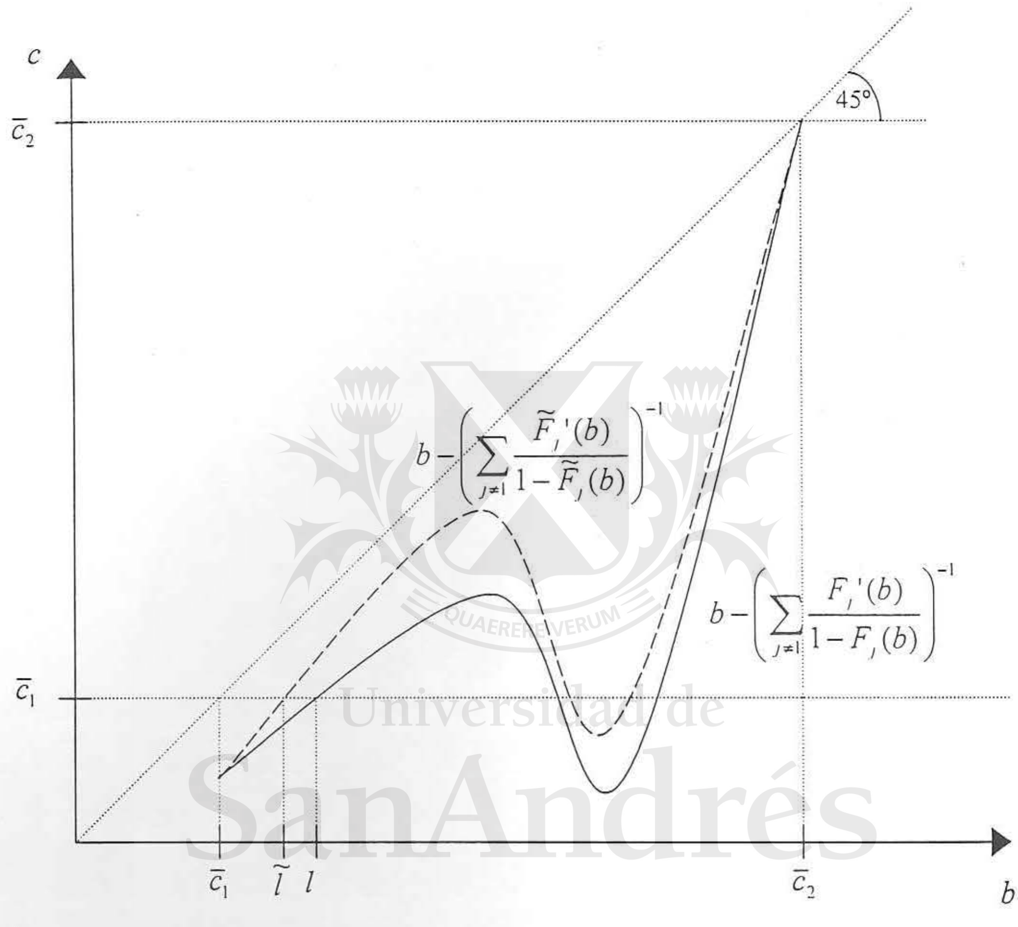
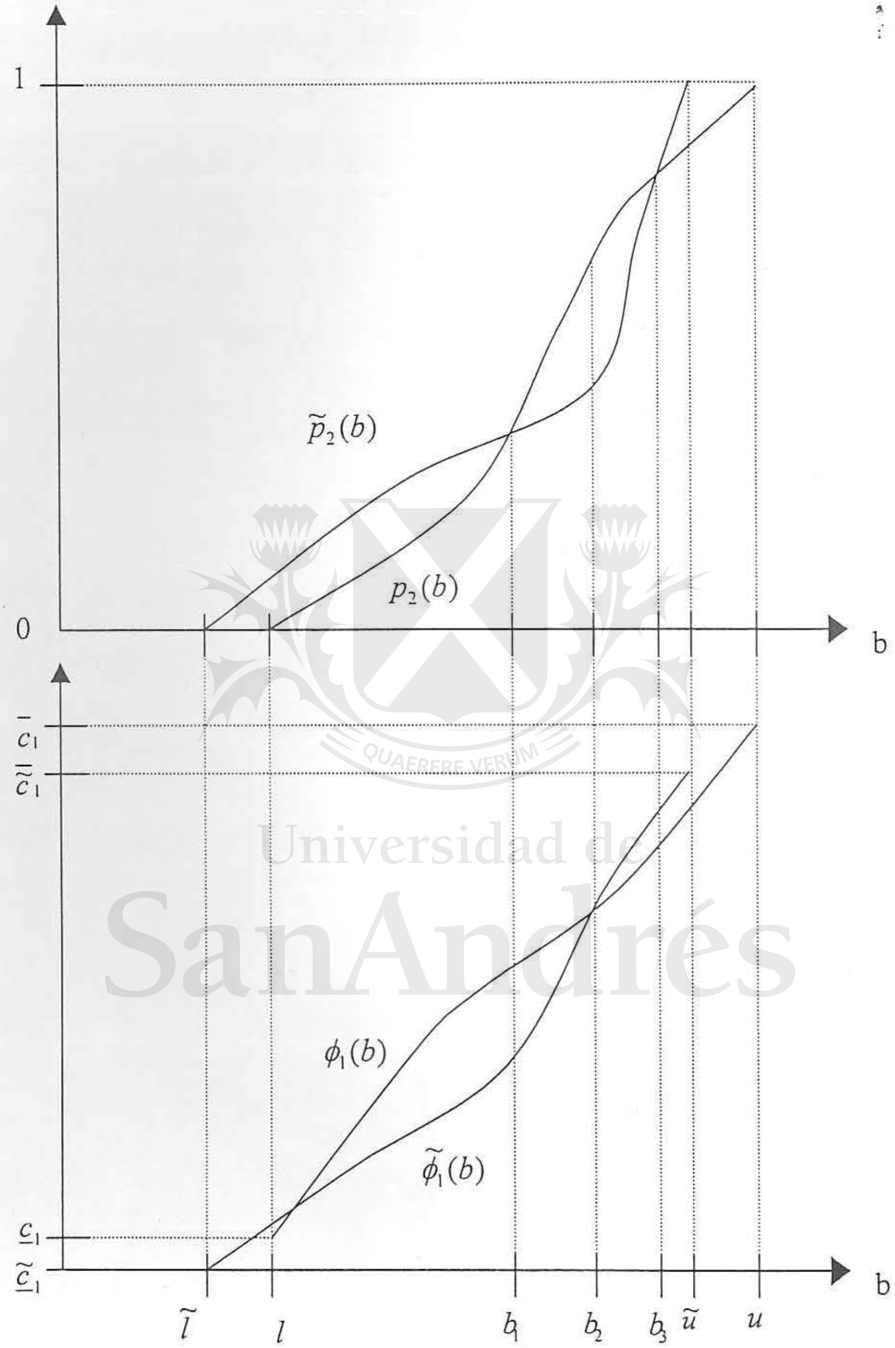


Figure 1

In this example the upper bounds of the distributions of costs for bidders 1 and 2 remain unaltered after the upgrading.



1
0
$\widetilde{p}_2(b)$
$p_2(b)$
b
$\bar{c}_1$
$\widetilde{\bar{c}}_1$
$\phi_1(b)$
$\widetilde{\phi}_1(b)$
$\underline{c}_1$
$\widetilde{\underline{c}}_1$
b
$\widetilde{l}$
l
b_1
b_2
b_3
$\widetilde{u}$
u

Figure 2