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"Inflación y Crecimiento: el rol de la incertidumbre y la información asimétrica"

Marcelo Sánchez
(University of California at Berkeley)

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Inflation and Growth: The Role of Uncertainty and Asymmetric Information

Marcelo Sánchez*
Department of Economics
University of California at Berkeley

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Abstract

This paper introduces inflation uncertainty and an informational asymmetry into an otherwise standard growth model and examines its implications for the relationship between the rate of inflation and steady-state output. Equilibria in this model fall into either a full information or a private information regime. Under either regime, higher inflation brings about financial disintermediation and a fall in investment. Moreover, the paper shows how the economy can transit between the full information regime and the private information regime. In particular, it examines how inflation contributes to both financial and real fluctuations by eventually making regime transitions more likely and more frequent.

Keywords: inflation, inflation uncertainty, growth, private information, credit rationing, regime switching

JEL classification: O16, O41, E32, E44

Correspondence to: Department of Economics, University of California at Berkeley, 549 Evans 3880, Berkeley CA 94720-3880. Phone: (510) 664-2756, Fax: (510) 642-6615, e-mail: msanchez@econ.berkeley.edu.

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1 Introduction

In this paper I introduce inflation uncertainty and an informational asymmetry into an otherwise standard growth model and examine its implications for the relationship between inflation and steady-state output. Typically, this relationship is analyzed in the context of monetary growth models. In many contributions the steady-state levels of per capita output and of the capital-labor ratio are either positively related to the steady-state rate of inflation, or else money is "superneutral" and the steady-state level of real activity is unaffected by changes in the rate of inflation.[1] Thus monetary growth models are unable to account for the well-documented negative relationship between inflation and growth.

The other stylized fact standard growth models cannot explain is that the relationship between inflation and growth seems to depend strongly on the initial level of the inflation rate. High inflation, say, above 40 percent annually, is widely believed to be bad for growth, but there is much less agreement on the effects of less severe inflation. For example, Barro (1995) finds evidence of a negative impact of inflation on growth. Statistically significant results emerge only when high-inflation experiences are included in the sample. Bruno and Easterly (1998) demonstrate that there is no evidence of any consistent relationship between growth and inflation for countries with low to moderate inflation rates. However, they find that there is a negative correlation between inflation and growth for high-inflation countries (countries where inflation is above 40 percent annually). Bullard and Keating (1995), using structural vector autoregressions, find that a permanent increase in the rate of inflation raises the long-run level of output in countries where inflation is initially low. For economies experiencing moderate initial rates of inflation, the same kind of change in inflation seems to have no significant effect on long-run real activity. In countries whose initial inflation rates are fairly high, further increases in inflation significantly reduce the long-run level of output. Along similar lines, Ghosh and Phillips' (1998) results suggest that at very low inflation rates (less than 2-3 percent) inflation and growth are positively correlated. Otherwise, they are negatively related in a nonlinear fashion. Judson and Orphanides (1996), using cross-country panel data for

[1] Examples of the former kind of models include Mundell (1965), Tobin (1965), Shell *et al.* (1969), and Drazen (1981). Monetary growth models like that of Sidrauski (1967) differ in that the steady-state per capita output level and capital-labor ratio are unaffected by the inflation rate.

the past 30 years, find a strong negative correlation between inflation and growth for all but low-inflation countries. Furthermore, they report that inflation volatility is also robustly negatively correlated with growth even after the effect of the level of inflation is controlled for. Finally, Sarel (1996) shows that when inflation is low (below 8 percent), it has no effect on economic growth. But when inflation is high, it has a negative effect on growth.[2]

Thus there is a consensus in the empirical literature about the existence of a negative relationship between inflation and economic performance in high-inflation experiences. For low to moderate inflation countries, there is no agreement about the nature of that relationship. Any successful theory of how inflation affects real activity must account for the nonmonotonicity in the relation between inflation and long-run output. A similar threshold pattern seems to take place in the relationship between inflation and financial deepening (see Boyd *et al.*, 1996). This motivated the monetary growth models of Azariadis and Smith (1996) and Choi *et al.* (1996), which are based on the Diamond (1965) overlapping generations model. Their models differ from the latter in that they assume there is private information on the ability of the private sector to repay the debts with financial intermediaries. Due to this modification, their models can address the issue of threshold effects of inflation on long-run output and financial activity.[3] They derive a threshold level above which inflation has a negative effect on both financial activity and long-run output. Below the threshold, inflation is good for growth.

A crucial feature of these models is that inflation is negatively associated with the real rate of return to savings. However, there does not seem to be an agreement about whether this association takes place in practice. Choi *et al.* (1996) mention casual evidence in favor of a threshold effect (inflation has to be high enough to adversely affect the real rates of return to equity).[4]

[2]Fischer (1993) also reports evidence of a negative association between inflation and growth. Unlike the other authors, he detected exceptions given by countries where high growth took place despite high inflation.

[3]Before them, development economists such as McKinnon (1973) and Shaw (1973) already believed that high inflation rates interfere with the efficient operation of capital markets and thus with capital formation. The main channel was thought to be the strain placed by high inflation on the operation of financial markets extending medium- to long-term loans. More recently, Stockman (1981) acounted for the negative relationship between inflation and real activity in his model with cash-in-advance constraints applied to capital investments. His approach fails to explain the threshold effect of inflation on growth and do not explicitly deal with financial contracts.

[4]Their preferred examples are Korea and Taiwan, which —according to the authors—

However, their regressions show that such inverse relationship holds for all countries (not only for those experiencing high inflation). The dependent variable is the *ex post* real rate of return to stocks, while the main explanatory variable is the *ex post* rate of inflation. Instead, Geweke (1986) used low-frequency movements in inflation as a proxy for anticipated inflation. He found no evidence that anticipated inflation had any effect on output or *ex post* real interest rates. Finally, Chari *et al.* (1996) argue that changes in inflation have trivial effects on real rates of return.

On the other hand, a substantial body of evidence suggests that higher rates of inflation are typically accompanied by larger inflation variability, as shown in Friedman (1992) and Levine and Renelt (1992).[5] The empirical evidence also indicates that the relationship between the level and the variability of inflation is subject to a nonlinear pattern.[6]

This paper extends the research done by Azariadis and Smith (1996), and Choi *et al.* (1996), taking into account the positive association between the level and the variability of inflation. I keep the assumption of asymmetric information on the ability of the private sector to repay its debts, but I drop the negative effect of inflation on the real rate of return to savings. In its place I introduce the effect of inflation uncertainty on the private sector's saving decisions.

A second difference is that I suppress the perfect substitutability between money and bank deposits. In order to focus on the effect of inflation on the demand for deposits, I abstract from the presence of money in the model.[7] I derive an arbitrage condition relating the real rates of return to deposits and the alternative asset ("storage"), which includes the inflation risk premium as an additional component. Moreover, I analyze the conditions under which

experienced fairly pronounced jumps in their rates of inflation in the late eighties.

[5]Since there is a negative relationship between inflation and growth, and a positive relationship between the level and the variability of inflation, it is not surprising that some studies have found a negative relationship between inflation volatility and growth (theoretically, as in Gomme, 1991; or empirically, as in Kormendi and Meguire, 1985; Fischer, 1993; and Easterly *et al.*, 1994), or between inflation volatility and investment (see, for example, the empirical study by Huizinga, 1993).

[6]Arnold and den Hertog (1995) provide an empirical study based on Ball's (1992) theoretical model. In Ball's model the nonlinear relationship takes the form of a threshold effect (which in principle is country specific).

[7]Incorporating money would complicate the analysis without producing additional insights. As a result of the absence of money in my model, inflation is not the result of money creation, but it is assumed to be exogenous.

a higher inflation rate will make depositors reduce their holdings of deposits due to the fact that the real return to the latter becomes more uncertain.

As a consequence of these differences in the basic assumptions between the models, while Azariadis and Smith, and Choi *et al.* conclude that, below the threshold level, inflation promotes growth, I find that the two variables are unrelated for low enough inflation rates.

This paper examines an overlapping generations framework. I neglect technical progress so that steady state per capita income is constant. Households are assumed to have access to two classes of assets: bank deposits and autarkic unintermediated assets ("storage"). The real rate of return to deposits is subject to uncertainty about inflation. Storage bears a lower rate of return than physical capital but has the advantage of privacy since it involves no transactions. Instead, holdings of physical capital, which require production and market activity, are assumed to be publicly observable. Neither storage nor capital is subject to the inflation tax.

In addition, there are two categories of borrowers. Legitimate or high-quality borrowers use credit to produce capital. Illegitimate or low-quality borrowers are not capable of converting current resources into future capital and hence will ultimately be detected. To escape detection, dissembling agents simply abscond with the money. Therefore, loans to these borrowers are never repaid.[8]

It turns out that the solution to the implied adverse selection problem is that intermediaries offer loan contracts such that no potential depositors will increase their utility by becoming counterfeit entrepreneurs. A higher rate of inflation raises inflation uncertainty, and thus turns the real rate of return to deposits more uncertain. This makes more people want to be borrowers and fewer people want to be savers. However, people who were not initially getting credit represent "lower quality borrowers." To deter false claims, markets may ration credit, in which case the economy is said to be operating in the private information regime. Otherwise, a Walrasian regime prevails.

The analysis demonstrates that, no matter what regime the economy is

[8] The essential feature of any model of credit rationing based on moral hazard or adverse selection is that different agents have different probabilities of loan repayment and hence have different attitudes toward the magnitude of the interest rate charged on a loan. Mine is the simplest version of such a scenario: type 1 agents repay loans with probability zero, while type 2 agents repay with probability one. It is straightforward to modify the analysis to allow each type to repay with a probability strictly between zero and one, but this adds complication without introducing any substantive issues.

in, inflation will most likely generate a process of financial disintermediation and a weaker investment activity. In the long run, higher inflation is to blame for lower levels of both financial and real activity. This adverse effect of inflation on financial and capital deepening results from an exacerbation of the informational friction in capital markets.

Since both the Walrasian and the private information regime are consistent with equilibrium, equilibrium is indeterminate: the economy can follow either the full information or the private information law of motion. Moreover, there exist equilibria in which the economy can switch from one law of motion to the other in either a deterministic or a stochastic manner.[9]

The remainder of the paper proceeds as follows. Sections 2 and 3 lay out a theoretical model that illustrates the argument just given. Section 4 studies Walrasian equilibria with slack incentive constraints and no credit rationing. Section 5 introduces private information and examines how inflation affects the level of real activity when the financial market friction is operative and the rationing of credit that the binding incentive constraints require. Section 6 shows how the economy can transit between Walrasian regimes and regimes of credit rationing. The final section offers conclusions.

2 The Model

2.1 Environment

I consider a discrete-time economy populated by an infinite sequence of two-period-lived overlapping generations. Each generation is identical in size and composition, consisting of a continuum of agents with unit mass. Time is indexed by $t = 0, 1, ...$

Each period a single "final commodity" is produced using a constant returns to scale technology with capital and labor as inputs. A producer using K units of capital and N units of labor produces $F(K, N)$ units of the good. For purposes of exposition, I assume that F has the constant elasticity

[9] The analysis of regime transitions which is closest to mine is Azariadis and Smith (1998). See also Schreft and Smith (1998). The main contribution here is to endogenize the switching thresholds, which in general depend on inflation.

of substitution (CES) form[10]

$$F(K,N) = [aK^\gamma + bN^\gamma]^{\frac{1}{\gamma}} \tag{1}$$

where γ can in principle take any value below one. I will concentrate on the case when $\gamma \geq 0$. It can be shown that when $\gamma \geq 0$, the elasticity of substitution between capital and labor, $\sigma \equiv 1/(1-\gamma)$, is larger than or equal to one. [11] Defining $k \equiv K/N$ to be the capital-labor ratio, it will be convenient to work with the intensive production function $f(k) = F(k,1)$. Clearly, here

$$f(k) = [ak^\gamma + b]^{\frac{1}{\gamma}} \tag{2}$$

I assume that $f(k) = 0$. Finally, I assume that capital depreciates completely in the process of production.

Within each generation agents are divided into two types. Type 1 agents, who comprise a fraction $\lambda \in (1/2, 1)$ of the population, are endowed with one unit of labor when young, and no labor when old. Young labor generates no disutility. In addition, type 1 agents are endowed with a constant returns to scale technology for storing goods between periods. One unit of the good stored at t returns $1-r \in (0,1)$ units of consumption at $t+1$. Let ξ_t indicate the fraction of young type 1 agents who mimic type 2 agents.

Type 2 agents, who are as fraction $1-\lambda$ of the population, are endowed with one unit of labor (for which they have no alternative use) when old and no labor when young. They are not able to use the storage technology just

[10] A form of the CES function usually given is

$$F(K,N) = \phi\,[\delta K^\gamma + (1-\delta)N^\gamma]^{\frac{\nu}{\gamma}}$$

where ϕ is the parameter of efficiency (or scaling) and ν is the parameter indicating the degree of returns to scale. Equation (1) is equivalent to the expression in this footnote for $a \equiv \phi^\gamma \delta$, $b \equiv \phi^\gamma(1-\delta)$, and constant returns to scale ($\nu = 1$).

[11] When $\gamma < 0$, σ is less than one. Azariadis and Smith (1996) and (1998) consider a general production function and assume that this function exhibits some properties which, in the CES case, correspond to $\gamma \geq 0$. Instead, Choi *et al.* (1996) analyze the case when $\gamma < 0$ in a CES framework. The empirical evidence indicates that the parameters of the CES production function are highly sensitive to changes in the data, measurement of variables, and methods of estimation. The only tentative conclusion is that most of the time-series estimates of σ are below unity, while the cross-section estimates are close to unity. Econometric studies based on the CES function are surveyed in Griliches (1967), Jorgenson (1974), Kennedy and Thirlwall (1972), Nadiri (1970), and Nerlove (1967).

described, but they do have access to a technology that converts one unit of the final good at t into one unit of capital at $t+1$. Type 1 agents cannot use this technology. Any agent who owns or rents capital at t can operate the final goods production process at $t+1$. Thus type 2 agents are "producers" in old age. I assume (with no real loss of generality) that type 2 agents must work for themselves; their labor is not traded.

An agent's type is assumed to be known by the agent but to be private information. However, all market transactions are assumed to be observable. The information structure is quite simple: household type and input into storage are private information; age and market transactions like working and borrowing are observable. Thus, young type 2 agents who have no labor endowment cannot claim to be type 1 agents and work when young.[12] On the other hand, young type 1 agents can credibly claim to be of type 2. If they do so, they will borrow when young as type 2 agents do, and they must supply no labor. However, type 1 agents have no ability to create physical capital and therefore no ability to operate the production process when old. They would then be discovered as having misrepresented their type, and we assume that they can be punished prohibitely. Therefore, type 1 agents who borrow in youth will avoid detection only if they "abscond" with their loan. An agent who absconds never repays the bank and becomes autarkic (that is, he goes underground). An absconding agent's old-age consumption must be financed strictly by using his own storage technology. Since type 2 agents have no access to the storage technology, they choose never to abscond.

Agents' preferences are also quite simple: everyone cares only about old-age consumption. Thus all youthful income is saved in some form by investing in bank deposits or in storage.[13] In addition, I assume that type 1 agents are risk-averse,[14] while type 2 are risk-neutral. Let $C^i_{2,t+1}$ denote agent i's second-

[12]This is the purpose of assuming that type 2 agents have no labor endowment when young. In particular, this assumption implies that there can never be a binding incentive constraint requiring type 2 agents not to want to mimic type 1 agents. In addition, the assumption that type 2 agents have no young period labor endowment frees me from having to worry about issues related to the internal financing of capital investments.

[13]The assumption that young agents' saving rate is one is easily relaxed in the case of type 2 agents. That is not the case regarding type 1 agents due to the presence of risk-aversion. The model would lose its analytical tractability if it were to allow type 1 agents' period 1 consumption to be endogenously determined and positive (along the lines of Sandmo, 1968, 1969, 1970; Levhari and Srinivasan, 1970; Hansen and Menezes, 1975; Sproule, 1985; and Monticelli, 1991).

[14]In allowing type 1 agents to be risk-averse I deviate from Azariadis and Smith's (1996)

period consumption. Thus type 2's utility is just $E[C^2_{2,t+1}]$. Type 1's utility is $E[U(C^1_{2,t+1})]$, where U is assumed to be of the constant relative risk aversion (CRRA) type. This means that $E[U(C^1_{2,t+1})] = E\left[(C^1_{2,t+1})^{1-\rho}/(1-\rho)\right]$, where $\rho > 0$ is the relative risk aversion coefficient.

The underlying uncertainty is given by the randomness in the inflation rate. Uncertainty in the inflation rate causes both the real rate of interest on bank deposits and that on loans to have a stochastic distribution.

In addition to young agents, there is an initial old generation at $t = 0$. These agents are each endowed with one unit of labor and a capital stock of $K_0 > 0$.

2.2 Trading

There are three types of trades that can take place in this economy. First, old producers hire the labor of young type 1 agents. Let w_t be the real wage rate in period t. Second, young type 1 agents save their entire labor income, some of which is lent to young type 2 agents and possibly to dissembling type 1 agents. It will make sense to think of this lending as being intermediated. There is free entry into the activity of intermediation, and I let $1 + x_{t+1}$ be the gross real rate of interest offered on savings by intermediaries between t and $t + 1$. Similarly, $1 + X_{t+1}$ is the gross real rate of interest charged by intermediaries on loans made at t and maturing at $t + 1$. I assume that contemporaneous inflation is observed by all agents, so that X_{t+1} and x_{t+1} are known as of time $t+1$. However, inflation at time $t+1$ is not known as of time t. Thus, agents form expectations at date t on the value of X_{t+1} and x_{t+1}. They are assumed to know the probability distribution of these variables, which have means $\mu_{X,t+1}$ and $\mu_{x,t+1}$ and variances σ^2_X and σ^2_x, respectively.[15] Simple relationships between $\mu_{X,t+1}$ and $\mu_{x,t+1}$, or between σ^2_X and σ^2_x, can be found by means of the expression

$$(1 + X_{t+1}) = \left[1 + \frac{\lambda}{1-\lambda}\xi_t\right](1 + x_{t+1})$$

The variability of inflation is responsible for the behavior of σ^2_x. Indeed,

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and Choi *et al.*'s (1996) models, which assume all agents to be risk-neutral.

[15] I suppress the time subscript from the variances because they are exogenous in my model. The only comparative statics exercise I do consist of once-and-for-all changes in σ^2_x.

the empirical evidence shows that inflation variability is positively correlated with the rate of inflation. Therefore, in what follows, a higher σ_x^2 is interpreted to be the result of higher inflation.[16]

I denote by b_t the real value of borrowing by (purported) young type 2 agents at t. Clearly, all type 2 agents will invest in capital the resources they obtain in youth because they cannot store goods and they are not interested in consuming when young. Hence each old producer at t+1 will have a capital stock

$$K_{t+1} = b_t \tag{3}$$

At time t each producer has an inherited capital stock of K_t, which he combines with L_t units of young type 1 labor and with his own single unit of labor. Thus his total labor input is $N_t = L_t + 1$. The producer's total real income and old-age consumption is then

$$C_{2,t}^2 = F(K_t, L_t + 1) - w_t L_t - (1 + X_t) b_{t-1}$$

since the agent incurred an interest obligation of $(1 + X_t)b_{t-1}$ when young. Hired labor services L_t are chosen to maximize the value of this expression, which means that

$$\begin{aligned} w_t &= F_2(K_t, L_t + 1) = f(k_t) - k_t f'(k_t) \\ &= b\left[ak_t^\gamma + b\right]^{\frac{1-\gamma}{\gamma}} \\ &\equiv w(k_t) \end{aligned} \tag{4}$$

must hold at any interior maximum. It is straightforward to see that, when $\gamma \geq 0$, $w(.)$ is an increasing, strictly concave function of k_t. Then the producer's consumption is

$$C_{2,t}^2 = F_1(.)K_t + [F_2(.) - w_t]L_t + F_2(.) - (1 + X_t)b_{t-1}$$

[16] One key result of the model is that there is a threshold effect of inflation on growth via inflation variability. The presence of a threshold in the effect of inflation on inflation variability, and thus on σ_x^2, might constitute an additional source of nonmonotonicity in the relationship between inflation and growth. Though relevant, this idea is not pursued further here.

or[17]

$$C^2_{2,t} = [F_1(.) - (1 + X_t)]b_{t-1} + w_t \tag{5}$$

by Euler's law and equations (3) and (4).

At date t the supply of young labor is $\lambda(1 - \xi_t)$ while the measure of producers is $1 - \lambda$. Since all producers are identical, labor market clearing requires that

$$L_t = \frac{(1 - \xi_t)\lambda}{1 - \lambda} \tag{6}$$

Therefore, the capital-labor ratio is

$$k_t \equiv \frac{K_t}{L_t + 1} = \frac{1 - \lambda}{1 - \lambda\xi_t}K_t \tag{7}$$

Finally, I analyze the behavior of intermediaries in the economy. They are assumed to be risk-neutral, and to consist of a fraction of type 2 agents. Free entry into intermediation means that intermediaries earn zero profits in equilibrium. Intermediaries make loans of measure $1 - \lambda$ to type 2 agents at t —who repay their loans— and of measure $\lambda\xi_t$ to type 1 agents —who do not. Since young type 1 agents who mimic type 2 agents also borrow b_t at t, the zero profit condition for intermediaries requires that

$$(1 + \mu_{X,t+1}) = \left[1 + \frac{\lambda}{1 - \lambda}\xi_t\right](1 + \mu_{x,t+1}) \tag{8}$$

This equation says that honest agents compensate lenders for dishonest agents who borrow and default. Moreover, it determines the relationships between $\mu_{X,t+1}$ and $\mu_{x,t+1}$.

Of course, if $\xi_t = 0$, (8) reduces to

$$(1 + \mu_{X,t+1}) = (1 + \mu_{x,t+1}) \tag{9}$$

[17] If inflation at t is much lower than expected the real burden of the interest rate on the loans taken by type 2 agents in period $t - 1$ may make them default. I assume this possibility does not actually occur and that type 2 agents repay the entire amount of their loans. However, the introduction of the possibility of default on the part of legitimate entrepreneurs is a very interesting extension of the analysis here.

I maintain the typical assumption of economies with adverse selection: on the loan side intermediaries are Nash competitors who announce loan contracts consisting of pairs $(\mu_{X,t+1}, b_t)$ at t. These announcements are made by each active intermediary taking the announcements of other active intermediaries as given. On the deposit side intermediaries are assumed to be competitive, taking the cost of deposits x_{t+1} as given at t. There are no other costs of converting deposits into loans.

Before I discuss an equilibrium, I need to describe the portfolio decision of young type 1 workers. These agents earn w_t at t, all of which they save. Savings can either be deposited with an intermediary or stored.

Let s_t denote storage per capita at t. Then, total savings at t is $(1-\xi_t)\lambda w(k_t)$, total borrowing is $(1-\lambda)K_{t+1} + s_t$, and

$$(1-\xi_t)\lambda w(k_t) = (1-\lambda)K_{t+1} + s_t \tag{10}$$

must hold. This equation simply asserts that total savings equals capital formation plus storage. In principle, storage can come from both dissembling and non-dissembling type 1 agents. However, it turns out that, in equilibrium, only honest type 1 agents may decide to store.

And finally, type 1 agents are willing to supply funds to intermediaries if and only if the return they receive is at least as large as the return available on the alternative savings instrument (storage) plus an inflation risk premium term. This requires that[18]

$$1 + \mu_{x,t+1} \geq 1 - r + \alpha_t \rho \sigma_x^2 \tag{11}$$

where α_t is the fraction of type 1 agents' wages put in bank deposits. If (11) holds as an inequality then $\alpha_t = 1$, while if (11) holds at equality then $\alpha \in (0,1]$.

2.3 Loan contracts

Suppose that not all young type 1 agents misrepresent their type at t and hence that $\xi_t < 1$ holds.[19] It follows that type 1 agents must do at least as well

[18] Equation (11) results from the second-order Taylor approximation of the first-order condition for $\alpha_t \in (0,1]$ around $x_{t+1} = r = 0$, and letting cross-products and squares of $\mu_{x,t+1}$, r and σ_x^2 drop if applicable. That first-order condition is $E[U'(.)(1+x_{t+1})] \geq (1-r)E[U'(.)]$, where the argument of U' is $w_t[\alpha_t(1+x_{t+1}) + (1-\alpha_t)(1-r)]$.

[19] If $\xi_t = 1$, no young agents work, and there is no saving supplied in the marketplace at t. Hence $k_{t+1} = 0$, and it follows that $k_{t+1} = 0, \forall t \geq 0$. Thus, if $\xi_t = 1$ at any date, the

by revealing his type (working) as by mimicking type 2 agents. A young type 1 agent who works earns w_t when young, all of which is saved. Therefore, $C^1_{2,t+1} = w_t\left[\alpha_t(1+x_{t+1}) + (1-\alpha_t)(1-r)\right]$. This agent's lifetime expected utility is $E\left[(C^1_{2,t+1})^{1-\rho}/(1-\rho)\right] = (w_t)^{1-\rho}E\left\{\left[\alpha_t(1+x_{t+1}) + (1-\alpha_t)(1-r)\right]^{1-\rho}\right\}$ $/(1-\rho)$. A second-order approximation of this expression around $x_{t+1} = r = 0$ yields:[20]

$$E\left[\frac{1}{1-\rho}(C^1_{2,t+1})^{1-\rho}\right]$$

$$= \frac{1}{1-\rho}(w_t)^{1-\rho}\left[1-(1-\alpha_t)(1-\rho)r + \alpha_t(1-\rho)\mu_{x,t+1} + \frac{1}{2}(\alpha_t)^2\rho(\rho-1)\sigma_x^2\right]$$

where the expression in square brackets on the RHS must be nonnegative.

On the other hand, a young type 1 agent who misrepresents his type borrows b_t. All of it is stored, giving a lifetime expected utility of $(b_t)^{1-\rho}(1-r)^{1-\rho}/(1-\rho)$. Thus $\xi_t < 1$ requires that

$$\frac{1}{1-\rho}(w_t)^{1-\rho}\left[1-(1-\alpha_t)(1-\rho)r + \alpha_t(1-\rho)\mu_{x,t+1} + \frac{1}{2}(\alpha_t)^2\rho(\rho-1)\sigma_x^2\right]$$

$$\geq \frac{1}{1-\rho}(b_t)^{1-\rho}(1-r)^{1-\rho} \qquad (12)$$

Competition among intermediaries implies that, in any Nash equilibrium, b_t must be chosen to maximize the lifetime expected utility of young type 2 agents, subject to the self-selection constraint (12). From (5), the lifetime expected utility of a young type 2 agent is given by

$$E\left[C^2_{2,t+1}\right] = \left[F_1(k_{t+1},1) - (1+\mu_{X,t+1})\right]b_t + w_{t+1}$$

or, using (8),

$$E\left[C^2_{2,t+1}\right] = \left\{F_1(k_{t+1},1) - (1+\mu_{x,t+1})\left[1+\frac{\lambda}{1-\lambda}\xi_t\right]\right\}b_t + w_{t+1}$$

economy jumps to the trivial steady-state equilibrium with no capital stock.

[20] Again, I let cross-products and squares of $\mu_{x,t+1}$, r and σ_x^2 drop if applicable.

Moreover, by inverting (4) I obtain $k_{t+1} = \phi(w_{t+1})$, so that

$$F_1(k_{t+1}, 1) = F_1\left[\phi(w_{t+1}), 1\right] \equiv \psi(w_{t+1})$$

Then the expected utility of a young type 2 agent at t is given by

$$E\left[C^2_{2,t+1}\right] = \left\{\psi(w_{t+1}) - (1+\mu_{x,t+1})\left[1 + \frac{\lambda}{1-\lambda}\xi_t\right]\right\} b_t + w_{t+1} \qquad (13)$$

Intermediaries must choose b_t to maximize the expression in (13) subject to (12), taking w_{t+1}, $\mu_{x,t+1}$ and ξ_t as given.

In any nontrivial equilibrium, the maximizing choice of b_t must be positive and finite. Therefore, both (12) and

$$f'(k_{t+1}) = \psi(w_{t+1}) \geq (1+\mu_{x,t+1})\left[1 + \frac{\lambda}{1-\lambda}\xi_t\right] \qquad (14)$$

must hold, and at least one with equality. Supposing that (14) holds with equality, and that $\xi_t = 0$, then $f'(k_{t+1}) = (1+\mu_{x,t+1})$, which coincides with the outcome that would obtain if agents' types were fully observable. On the other hand, if $f'(k_{t+1}) > (1+\mu_{x,t+1})\left[1 + \lambda\xi_t/(1-\lambda)\right]$, then young type 2 would like to borrow more than banks allow and are rationed in equilibrium. In this case, (12) is an equality that determines b_t.

To summarize succinctly the properties of a Nash equilibrium in the loan market, I substitute equations (3), (4), and (7) into (12) to obtain an alternative expression of the incentive constraint:

$$\frac{1}{1-\rho}(w(k_t))^{1-\rho}\left[1 - (1-\alpha_t)(1-\rho)r + \alpha_t(1-\rho)\mu_{x,t+1} + \frac{1}{2}(\alpha_t)^2\rho(\rho-1)\sigma_x^2\right]$$

$$\geq \frac{1}{1-\rho}\left.\frac{1-\lambda\xi_{t+1}}{1-\lambda}\right)^{1-\rho}(k_{t+1})^{1-\rho}(1-r)^{1-\rho} \qquad (15)$$

Expressions (14) and (15), at least one with strict equality, summarize the restrictions on the sequence $\{k_{t+1}, \alpha_t, \mu_{x,t+1}, \xi_t\}$ imposed by a Nash equilibrium in the loan market.

3 Markets

Nontrivial equilibria will satisfy three sorts of requirements at each date:

- Self-selection is observed in the credit market, and the incentive constraint (15) is satisfied.
- The arbitrage condition (11) holds for individuals and the arbitrage condition (14) holds for producers and financial intermediaries.
- Markets for loans, labor, and capital clear. In particular, the amount of a loan to a type 2 agent equals the amount invested in physical capital, as in equation (3); labor supply equals labor demand, as in equation (6); and aggregate household wealth equals the total value of asset portfolios, as in equation (10).

To express dynamical equilibrium in a compact manner, I combine the market clearing conditions (6) and (10), as well as (7) and the definition $s_t \equiv (1-\alpha_t)\lambda w_t$, into

$$(\alpha_t - \xi_t)\lambda w(k_t) = (1 - \lambda\xi_{t+1})k_{t+1} \tag{16}$$

Then the relevant equilibrium conditions are (14), (15) and (16). Except for the borderline case, only one of the two relations (14) and (15) will hold as an equality. When the incentive constraint (15) is binding, producers are credit rationed; I call this situation a private information equilibrium. When the arbitrage condition (14) is tight, the provision of credit is competitive in the usual sense; I call this state of affairs a Walrasian equilibrium. I examine each case in turn.

4 Walrasian Equilibria

In this section I analyze dynamical equilibria free from credit rationing, which are sequences $\{k_{t+1}, \alpha_t, \mu_{x,t+1}, \xi_t\}$ that satisfy equation (16), equation (14) as an equality, and equations (11) and (15) for each $t = 0, 1, ...$, given the initial condition $k_0 \geq 0$. As it turns out, nontrivial pooling equilibria do not exist in this case (see Appendix A). Thus, in this section I focus on separating equilibria in which $\xi_t = 0$, and hence $\mu_{X,t+1} = \mu_{x,t+1}$ for all t. This means that equations (14) and (16) can be rewritten as

$$f'(k_{t+1}) = 1 + \mu_{x,t+1} \tag{17}$$

$$\alpha_t \lambda w(k_t) = k_{t+1} \tag{18}$$

Using (18), the incentive constraint (15) becomes

$$\frac{1}{1-\rho}\left[1-(1-\alpha_t)(1-\rho)r+\alpha_t(1-\rho)\mu_{x,t+1}+\frac{1}{2}\,(\alpha_t)^2\rho(\rho-1)\sigma_x^2\right]$$
$$\geq \frac{1}{1-\rho}(\alpha_t)^{1-\rho}\left(\frac{\lambda}{1-\lambda}\right)^{1-\rho}(1-r)^{1-\rho} \tag{19}$$

Thus, the system that characterizes Walrasian equilibria is given by (17), (18), (19) and (11). There are two possible cases, depending on whether the portfoiio arbitrage condition (11) holds at equality or not. It is useful to calculate the threshold value of k_t, $\overline{k}_c^{*P}$ (henceforth "Walrasian portfolio threshold"), at which the economy switches from a Walrasian equilibrium in which (11) is slack to a Walrasian equilibrium in which (11) is binding.

The Walrasian portfolio threshold of k_t is determined by three conditions: (i) equation (11) holds at equality with $\alpha_t = 1$; (ii) equation (17) holds; (iii) equation (18) holds with $\alpha_t = 1$. Thus

$$f'(\overline{k}^{*P}) = 1 - r + \rho\sigma_x^2 \tag{20}$$
$$\lambda w(\overline{k}_c^{*P}) = \overline{k}^{*P} \tag{21}$$

where, in order to get (20), I have plugged the value of $\mu_{x,t+1}$ from (17) into (11). I denote by $\overline{k}^{*P}$ the lowest "Walrasian capital stock" consistent with a binding condition (11), while $\overline{k}_c^{*P}$ is the value of the capital stock that maps into $\overline{k}^{*P}$ under (18) with $\alpha_t = 1$.

For a given inflation rate, the Walrasian equilibrium is consistent with a slack portfolio arbitrage condition only when k_t is below $\overline{k}_c^{*P}$. Values of k_t at or above $\overline{k}_c^{*P}$ imply that equation (11) binds in Walrasian equilibrium.

Finally, from equations (20) and (21) $d\overline{k}_c^{*P}/d\sigma_x^2 < 0$ since $w'(.)$ is positive for all γ.

In the remainder of this section I will solve for Walrasian equilibria. To do so I need to find the relationship between k_t and k_{t+1} both when the portfolio arbitrage condition (11) is slack and when it is binding. I label that locus K^*K^*, which is composed of two parts: K^*K^{*S} when (11) is slack and K^*K^{*B} when (11) is binding.

a) Case when the portfolio arbitrage condition is slack

When $k_t < \overline{k}_c^{*P}$, equation (11) is not binding, and thus $\alpha_t = 1$ for all t. Equation (18) becomes dynamic in terms of k_{t+1} only. Given k_{t+1}, equation (17) determines $\mu_{x,t+1}$. For values of k_t to the left of $\overline{k}_c^{*P}$, Figure 1a represents that part of the locus K^*K^* which I label

$$K^*K^{*S} = \left\{(k_t, k_{t+1}) : k_{t+1} = \lambda w(k_t) \text{ and } k_t < \overline{k}_c^{*P}\right\} \tag{22}$$

where superscript S is a reminder that (11) is "slack."[21] It is easy to show that, in this case, the slope of K^*K^{*S} is positive and decreasing in k_t.

A steady state equilibrium is determined where the 45^0 degree line in Figure 1a intersects the locus K^*K^*. This intersection can occur either in the K^*K^{*S} part or in the K^*K^{*B} part. In the former case, which is the one represented in Figure 1a, the steady-state level of the capital-labor ratio, k^*, satisfies the equation

$$\lambda w(k^*) = k^* \tag{23}$$

If the steady state equilibrium lies on the K^*K^{*S} part of locus K^*K^*, it will be stable since $w'(.)$ is an increasing, strictly concave function of k_t. Given the value of k^* from (23), equation (17) determines the steady-state value of μ_x, μ_x^*. This, together with the steady-state value of b_t,, b_t^*, which results from using (3) and (7) and plugging k^*, characterizes the Nash equilibrium loan contracts of the economy.

b) Case when the portfolio arbitrage condition is binding

The remaining part of the locus K^*K^* corresponds to the situation where the portfolio arbitrage condition (11) is binding. I label this part K^*K^{*B}, where superscript B is a reminder that (11) is "binding."

In this case the equilibrium behavior of the economy is described by (17), (18), (19) and the portfolio arbitrage condition (11) at equality. Except for the borderline situation where k_t equals its Walrasian portfolio threshold value, $\overline{k}_c^{*P}$ —in which case $\alpha_t = 1$—, the value of α_t will be strictly below 1. Because of (18), when $\alpha_t < 1$ K^*K^{*B} will lie strictly below the continuation

[21] Strictly speaking, Figure 1a represents the case when $\gamma = 0$ only. When $\gamma > 0$, the origin is still part of the locus because of the assumption that $f(k) = 0$. However, k_{t+1} does not approach the origin as k_t goes to 0, but a positive value equal to $\lambda b^{1/\gamma}$.

of K^*K^{*S} (see the dashed line in Figure 1a) derived before for $\alpha_t = 1$. Moreover, at $k_t = \overline{k}_c^{*P}$ an intersection between K^*K^{*S} and K^*K^{*B} occurs.

Using (11), (17) and (18), K^*K^{*B} turns out to be

$$K^*K^{*B} = \left\{ (k_t, k_{t+1}) : f'(k_{t+1}) = 1 - r + \frac{\rho}{\lambda}\sigma_x^2 \frac{k_{t+1}}{w(k_t)} \text{ and } k_t \geq \overline{k}_c^{*P} \right\} \quad (24)$$

Inspection of equations (11), (17) and (18) permits to assess the shape of K^*K^{*B}. From (17) and a binding (11), k_{t+1} and α_t are negatively related. This, together with (18) and the fact that $w'(.) > 0$ for all k_t, establishes that a higher k_t will be associated with a higher k_{t+1} and a lower α_t. This means that K^*K^{*B} is an increasing, strictly concave function of k_t.

When the 45^0 degree line intersects the locus K^*K^* in the K^*K^{*B} part, the steady-state level of the capital-labor ratio is given by

$$f'(k^{*'}) = 1 - r + \frac{\rho}{\lambda}\sigma_x^2 \frac{k^{*'}}{w(k^{*'})} \quad (25)$$

For all values of γ, the LHS of (25) is a decreasing, strictly convex function of $k^{*'}$. Moreover, $f'(k)$ tends to $+\infty$ as k goes to 0, and it tends to 0 as k goes to $+\infty$.

On the other hand, the behavior of the RHS of (25) depends on the properties of the function $k/w(k) = (1/b)k\left[ak^\gamma + b\right]^{-(1-\gamma)/\gamma}$. The latter can be characterized by its first and second derivatives with respect to k :

$$\frac{d}{dk}\left[\frac{k}{w(k)}\right] = \frac{1}{b}\left[ak^\gamma + b\right]^{-\frac{1}{\gamma}}\left[\gamma a k^\gamma + b\right] \quad (26)$$

$$\frac{d^2}{dk^2}\left[\frac{k}{w(k)}\right] = -\frac{a(1-\gamma)}{b}k^{-(1-\gamma)}\left[ak^\gamma + b\right]^{-\frac{(1+\gamma)}{\gamma}}\left[\gamma a k^\gamma + b(1+\gamma)\right] \quad (27)$$

When $\gamma \geq 0$, it is easy to see that

$$\frac{d}{dk}\left[\frac{k}{w(k)}\right] > 0 \text{ and } \frac{d^2}{dk^2}\left[\frac{k}{w(k)}\right] < 0$$

That is, $k/w(k)$ is an increasing, strictly concave function of k. Moreover, $k/w(k)$ tends to 0 as k goes to 0, and it tends to $+\infty$ as k goes to $+\infty$.

These properties, together with those of $f'(k)$ described above, imply that the steady state equilibrium defined in equation (25) exists and is unique.

Given the value of $k^{*'}$ in equation (25), one can determine unique steady-state values for α_t and $\mu_{x,t+1}$ by the following equations

$$\alpha^{*'}\lambda w(k^{*'}) = k^{*'} \tag{28}$$
$$f'(k^{*'}) = 1 + \mu_x^{*'} \tag{29}$$

The dynamical properties of this regime can be analyzed in terms of the shape of K^*K^{*B}, which is the part of the phase diagram that corresponds to a binding portfolio arbitrage condition (11). I have showed that K^*K^{*B} is an increasing, strictly concave function of k_t. Moreover, for a sufficiently low value of $\overline{k}_c^{*P}$ the 45^0 degree line will intersect the locus K^*K^* in the K^*K^{*B} part at a unique steady-state level of the capital-labor ratio, $k^{*'}$. This establishes the stability of $k^{*'}$.

It is interesting to analyze the effect of inflation on $k^{*'}$ and $\alpha^{*'}$. The former effect can be analyzed by totally differentiating (25):

$$\left\{ f''(k^{*'}) - \frac{\rho}{\lambda}\sigma_x^2 \frac{d}{dk^{*'}}\left[\frac{k^{*'}}{w(k^{*'})}\right]\right\}\frac{dk^{*'}}{d\sigma_x^2} = \rho\alpha^{*'} \tag{30}$$

where I have used (28).

Given that $d[k/w(k)]/dk > 0$ when $\gamma \geq 0$, equation (30) implies that $dk^{*'}/d\sigma_x^2 < 0$. This, together with use of (28), has the implication that $d\alpha^{*'}/d\sigma_x^2 < 0$.[22] Figure 1b illustrates the effect of inflation on the economy. An increase in σ_x^2 amounts to a downward shift in the K^*K^{*B} part of the locus K^*K^*. As a result of this, the steady-state level of the capital-labor ratio goes down (from k^* to $k^{*'}$), and so does the steady-state level of the deposit to savings ratio (from 1 to $\alpha^{*'}$).[23] Regarding the transitional dynamics of the economy, inflation generates a reduction of per capita investment as well as a process of financial disintermediation.

[22] From (26) and (28), using the second line in (7), it is possible to see that the percentage response of $\alpha^{*'}$ to a change in σ_x^2 is smaller than that of $k^{*'}$ since $\gamma < 1$.

[23] From (29), the effect of σ_x^2 on $\mu_x^{*'}$ is positive.

c) Summary

I now put together the results that have been obtained both when the portfolio arbitrage condition (11) is slack and when it is binding. A Walrasian steady state equilibrium can take place either in the K^*K^{*S} part or in the K^*K^{*B} part of locus K^*K^*. My analysis has showed that, no matter on what part of the locus K^*K^* the Walrasian steady state equilibrium lies, it will be unique and globally stable. Inflation has no effect on the economy when the current level of the capital-labor ratio is below its Walrasian portfolio threshold value, $\overline{k}_c^{*P}$. In this case, $\alpha_t = 1$.

However, inflation reduces the value of the threshold value $\overline{k}_c^{*P}$, thereby eventually making the economy switch to a situation where equation (11) binds. In this case, inflation has a negative impact on the steady-state levels of both the capital-labor ratio and the deposit to savings ratio.

5 Private Information Equilibria

Producers will be rationed in the credit market whenever the incentive constraint (15) binds and the producers' arbitrage condition (14) holds as a strict inequality. Furthermore, the market clearing condition (16) and the portfolio arbitrage condition (11) must hold. As with the Walrasian regime, it turns out that the steady-state fraction of dissembling type 1 agents, $\hat{\xi}$, equals 0 (see Appendix A). In summary, the sequence $\{k_{t+1}, \alpha_t, \mu_{x,t+1}\}$ is a private information equilibrium if, for each t :

$$\alpha_t \lambda w(k_t) = k_{t+1} \tag{31}$$

$$(w(k_t))^{1-\rho}\left[1-(1-\alpha_t)(1-\rho)r+\alpha_t(1-\rho)\mu_{x,t+1}+\frac{1}{2}(\alpha_t)^2\rho(\rho-1)\sigma_x^2\right] = \left(\frac{1-r}{1-\lambda}\right)^{1-\rho}(k_{t+1})^{1-\rho} \tag{32}$$

$$f'(k_{t+1}) > 1+\mu_{x,t+1} \tag{33}$$

Substituting $w(k_t)/k_{t+1}$ from (31) into (32) yields

$$1-(1-\alpha_t)(1-\rho)r+\alpha_t(1-\rho)\mu_{x,t+1}+\frac{1}{2}(\alpha_t)^2\rho(\rho-1)\sigma_x^2$$

$$= \quad \frac{\lambda}{1-\lambda}\Big)^{1-\rho}(1-r)^{1-\rho}(\alpha_t)^{1-\rho} \tag{34}$$

Thus, the system that characterizes private information equilibria is given by (31), (33), (34) and (11). Unlike Walrasian case b), the private information equilibrium value of α_t, $\hat{\alpha}'$, is determined independently from k_{t+1}. Indeed, the equilibrium values $\hat{\alpha}'$ and $\hat{\mu}_x'$ are given by the system of equations

$$1-(1-\hat{\alpha}'^L)(1-\rho)r+\hat{\alpha}'^L(1-\rho)\hat{\mu}_x'+\frac{1}{2}(\hat{\alpha}'^L)^2\rho(\rho-1)\sigma_x^2$$

$$=(\hat{\alpha}'^L)^{1-\rho}\quad\frac{\lambda}{1-\lambda}\Big)^{1-\rho}(1-r)^{1-\rho} \tag{35}$$

$$1+\hat{\mu}_x'=1-r+\hat{\alpha}'^L\rho\sigma_x^2 \tag{36}$$

where superscript L stands for "lower bound" and is a reminder that the actual equilibrium value of $\hat{\alpha}'$ will be the minimum of $\hat{\alpha}'^L$ and 1.

Plugging the value of $\hat{\mu}_x'$ from (36) into (35) yields

$$\frac{1}{1-\rho}\left[1-(1-\rho)r++\frac{1}{2}(\hat{\alpha}'^L)^2\rho(1-\rho)\sigma_x^2\right]$$

$$=\frac{1}{1-\rho}(\hat{\alpha}'^L)^{1-\rho}\quad\frac{\lambda}{1-\lambda}\Big)^{1-\rho}(1-r)^{1-\rho} \tag{37}$$

The RHS of (37) is an increasing, strictly concave function of $\hat{\alpha}'^L$. When $\hat{\alpha}'^L=0$, the RHS of (37) is at the origin. When $\rho>1$, the LHS of (37) is an increasing, strictly convex function of $\hat{\alpha}'^L$. When $\hat{\alpha}'^L=0$, the LHS of (37) is below the origin. Thefore, for $\rho>1$ there will be a unique value of $\hat{\alpha}'^L$ that satisfies (37).

I will maintain the assumption that $\rho>1$ as a reasonable one. Mehra and Prescott 's (1985) seminal paper on the "equity premium puzzle" reported estimates of ρ from many studies, only one of which was consistent with a value of ρ below 1. Since then the debate has been focused not on whether the magnitude of the equity premium requires a value of ρ which is above or below 1, but on whether the high values of ρ that could justify the level of the equity premium are consistent with plausible behavior on the part of investors. In a survey of the literature, Kocherlakota (1996) argues that a vast majority of economists believe that values of ρ above 5 (or even somewhat lower) imply highly implausible behavior on the part of the individuals. However, this author also mentions that, because of the arguments offered by Kandel and Stambaugh (1991) and Kocherlakota (1990), some economists believe that individuals are more risk averse than is commonly thought.

Given the unique equilibrium values of $\hat{\alpha}'$ and $\hat{\mu}_x'$, I need to analyze two possible cases, depending on whether the portfolio arbitrage condition (11) holds at equality or not. It is useful to calculate the threshold value of k_t, $\bar{\hat{k}}_c^P$ (henceforth "private information portfolio threshold"), at which the economy switches from a private information equilibrium in which (11) is slack to a private information equilibrium in which (11) is binding.

In the remainder of this section I will solve for private information equilibria. To do so I need to find the relationship between k_t and k_{t+1} both when the portfolio arbitrage condition (11) is slack and when it is binding. I label that locus $\hat{K}\hat{K}^S$ when (11) is slack and $\hat{K}\hat{K}^B$ when (11) is binding.[24]

a) Case when the portfolio arbitrage condition is slack

When equation (11) is not binding the value of $\hat{\alpha}'$ equals 1. This requires that $\hat{\mu}_x'$ be sufficiently high. In this case, the economy behaves in much the same way as in case *a*) of the Walrasian regime. The part of the locus $\hat{K}\hat{K}$ which I label

$$\hat{K}\hat{K}^S = \{(k_t, k_{t+1}) : k_{t+1} = \lambda w(k_t)\} \tag{38}$$

is basically the same as $K^{*S}K^{*S}$ in Figure 1a. In particular, as in Walrasian K^*K^{*S} the slope of $\hat{K}\hat{K}^S$ is positive and decreasing in k_t. An important difference between $K^{*S}K^{*S}$ and K^*K^{*S}, however, is that the maximum private information level of k_t must be lower than the Walrasian threshold, $\bar{k}_c^{*P}$. Comparison of (17) and (36), using (31) —which holds in either regime—

[24] Note that, unlike the case of the Walrasian regime, $\hat{K}\hat{K}^S$ and $\hat{K}\hat{K}^B$ are two mutually exclusive loci, and not parts of a common locus.

with $\alpha_t = 1$, establishes this result.[25]

The only other difference with respect to Walrasian case a) is that the steady-state level of $\mu_{x,t+1}$, $\hat{\mu}_x$, is no longer determined by (17), but by (34) with $\alpha_t = 1$, that is:

$$1 + (1-\rho)\mu_{x,t+1} + \frac{1}{2}\rho(\rho-1)\sigma_x^2 = \quad \frac{\lambda}{1-\lambda}\Big)^{1-\rho} (1-r)^{1-\rho} \qquad (39)$$

As results from (39), inflation has a positive effect on the level of $\mu_{x,t+1}$. Moreover, higher inflation will reduce the value of the Walrasian threshold, $\overline{k}_c^{*P}$, and thus may have a negative impact on maximum private information level of k_t too.

A steady state equilibrium is determined where the 45^0 degree line intersects the locus $\hat{K}\hat{K}$. This intersection can occur either in the $\hat{K}\hat{K}^S$ part or in the $\hat{K}\hat{K}^B$ part. In the former case, which is the one analogous to Figure 1a, the steady-state level of the capital-labor ratio, $\hat{k}$, satisfies the equation

$$\lambda w(\hat{k}) = \hat{k} \qquad (40)$$

If the steady state equilibrium lies on the locus $\hat{K}\hat{K}^S$, it will be globally stable since $w'(.)$ is an increasing, strictly concave function of k_t.

b) Case when the portfolio arbitrage condition binds

The locus $\hat{K}\hat{K}^B$ corresponds to the situation where the portfolio arbitrage condition (11) is binding. In this case the equilibrium behavior of the economy is described by (31), (33), (34) and the portfolio arbitrage condition (11) at equality. Clearly, except for a borderline situation where $\alpha_t = 1$, α_t will lie below 1. All in all, $\hat{K}\hat{K}^B$ is given by

$$\hat{K}\hat{K}^B = \Big\{(k_t, k_{t+1}) : k_{t+1} = \hat{\alpha}'\lambda w(k_t)\Big\} \qquad (41)$$

and, clearly, lies strictly below the locus $\hat{K}\hat{K}^S$ derived for $\alpha_t = 1$.

Given the unique value of $\hat{\alpha}'$, it is easy to analyze the dynamical properties of this regime captured in (41). If there is an intersection of the 45^0 degree line

[25] This idea is developed further in subsection 6.1. Furthermore, in this subsection I will analyze the existence of a private information lower bound for k_t.

with the locus $\hat{K}\hat{K}^B$, then the steady-state level of k, $\hat{k}'$, will be determined by

$$\hat{k}' = \hat{\alpha}'\lambda w(\hat{k}') \tag{42}$$

Moreover, the system will be stable, again, since $w'(.)$ is an increasing, strictly concave function of k_t.

So far I have focused on the behavior of the economy when σ_x^2 is constant. In order to analyze the effect of inflation on $\hat{k}'$ and $\hat{\alpha}'$, I totally differentiate (37), which gives

$$\left\{\rho\hat{\alpha}'\sigma_x^2 - \left(\hat{\alpha}'\right)^{-\rho}\left(\frac{\lambda}{1-\lambda}\right)^{1-\rho}(1-r)^{1-\rho}\right\}\left[\frac{d\hat{\alpha}'}{d\sigma_x^2}\right]^L = \frac{1}{2}\rho\hat{\alpha}' \tag{43}$$

where L stands for "lower bound." The reason is that the actual value of $d\hat{\alpha}'/d\sigma_x^2$ is given by

$$\frac{d\hat{\alpha}'}{d\sigma_x^2} = \min\left\{\left[\frac{d\hat{\alpha}'}{d\sigma_x^2}\right]^L, 0\right\} \tag{44}$$

because the maximum possible value of $\hat{\alpha}'$ is 1. If the value of $\left[d\hat{\alpha}'/d\sigma_x^2\right]^L$ resulting from (43) is nonnegative at $\hat{\alpha}' = 1$, then the interpretation is that $\hat{\alpha}'$ stays constant at 1. In this case, inflation affects neither $\hat{k}'$ nor $\hat{\alpha}'$, but it has a positive effect on $\hat{\mu}_x'$.[26]

On the other hand, if $d\hat{\alpha}'/d\sigma_x^2 < 0$ at any initial $\hat{\alpha}'$, this means that inflation generates financial disintermediation. The sign of the RHS of (43) is positive. This has two important implications. First, that the sign of $d\hat{\alpha}'/d\sigma_x^2$ will be negative if and only if the expression in curly brackets on the LHS of (43) is too. Second, that $\left[d\hat{\alpha}'/d\sigma_x^2\right]^L$ is increasing in $\hat{\alpha}'$ since the expression in curly brackets on the LHS of (43) is too. Thus, if $d\hat{\alpha}'/d\sigma_x^2$ is negative for some initial $\hat{\alpha}'$ it will also be so for a lower initial value.

[26] At some point $1 + \hat{\mu}_x'$ will equal $f'(\hat{k}')$, violating (37). A switch to the Walrasian regime eventually occurs. See the analysis in the following section.

A relatively high value of ρ[27] is consistent with a positive value for the expression in curly brackets on the LHS of (43) under a favorable configuration of the other parameters and variables (high enough λ and σ, low enough r, and an initial value of $\hat{\alpha}'$ close to one). However, for more plausible values of ρ and/or a different configuration of the other parameters and variables, the expression in curly brackets on the LHS of (43) will be negative. In what follows, I assume the latter alternative prevails.

When $d\hat{\alpha}'/d\sigma_x^2 < 0$, then, from (??) and the fact that (26) is positive, I conclude that $d\hat{k}'/d\sigma_x^2 < 0$. In this case, above the portfolio threshold level, inflation has a negative impact on the steady-steady state levels of both the capital-labor ratio and the ratio of deposits to saving.[28] With regard to the transitional dynamics of the economy, inflation generates a reduction of per capita investment as well as a process of financial disintermediation.

c) Summary

The analysis has showed that, no matter on whether the portfolio arbitrage condition (11) is slack or binding, the private information steady state equilibrium is unique and globally stable. Inflation has no effect on the economy when (11) is slack. On the other hand, inflation has a negative impact on the steady-state levels of both the capital-labor ratio and the deposit to savings ratio when (11) is binding.

6 Endogenous Regime-Switching Mechanisms

Sections 4 and 5 considered equilibria corresponding to the Walrasian and the private information regimes, respectively. In either of the two regimes, there are two possible cases: *a*) the portfolio arbitrage condition (11) is slack; and *b*) equation (11) is binding. Thus, there are in total four states in which the economy can be in equilibrium, depending on what regime prevails and on whether the portfolio arbitrage condition is slack or binding. Sections 4 and 5 considered both the situation when the economy stayed in any of the four possible equilibrium states permanently, and that when the economy switches between cases *a*) and *b*) *within* a given regime due to a change in the rate of inflation. However, it is also possible —or even necessary— that

[27] See the discussion above for a more precise meaning of "high" ρ.

[28] For the same reasons as in the previous section, the effect of σ_x^2 on $\hat{\mu}_x'$ is indeterminate.

there exist equilibria in which the economy transits *between* the Walrasian and the private information regime at a given inflation rate. These regime transitions can occur in either a deterministic or stochastic manner.

In subsection 6.1 I calculate the boundaries between Walrasian and private information regimes for the four possible states of the economy mentioned above.[29] Section 6.2 analyzes regime transitions. These can occur only between the Walrasian regime (both when (11) is slack and binding) and the private information regime when (11) is binding. No matter which of the two different cases of Walrasian regime I take, the analysis is esentially the same. Thus, for concreteness, I focus on the case when the economy transits between a Walrasian regime of slack porfolio constraint and one of credit rationing and tight portfolio constraint.[30]

6.1 Computation of the switching thresholds

i) Walrasian regime when the portfolio arbitrage condition is slack

The portfolio arbitrage condition (11) will be slack if the value of k_t is below the Walrasian portfolio threshold, $\overline{k}_c^{*P}$. I define critical values for the capital stock (at t and at $t+1$) and for σ_x^2. Let $\overline{k}^{*I}$ and $\overline{k}_c^{*I}$ satisfy

$$\left\{1+(1-\rho)\left[f'(\overline{k}^{*I})-1\right]+\frac{1}{2}\rho(\rho-1)\sigma_x^2\right\} = \left(\frac{\lambda}{1-\lambda}\right)^{1-\rho}(1-r)^{1-\rho} \quad (45)$$

$$\overline{k}^{*I} \equiv \lambda w(\overline{k}_c^{*I}) \quad (46)$$

where superscript I is a reminder that the incentive constraint is binding.

Then $\overline{k}^{*I}$ is the largest "full information capital stock" that can satisfy the incentive constraint, and $\overline{k}_c^{*I}$ is the capital stock that maps into $\overline{k}^{*I}$ under (23). Equation (45) implicitly establishes a negative relationship between $\overline{k}^{*I}$ and σ_x^2. Moreover, this equation indicates that if k_t is as high as $\overline{k}_c^{*I}$ the incentive constraint will be binding. Values of the capital stock larger

[29]In general, the switching thresholds for the capital stock depend on inflation. In this sense, the model here endogenizes thresholds similar to the ones studied by Azariadis and Smith (1998).

[30]The analysis here is somewhat informal. Azariadis and Smith (1998) study an analogous regime transition in a more technical way.

than that are inconsistent with a Walrasian allocation because they imply excessively low interest rates. At these interest rates it is not possible to deter type 1 agents from misrepresenting their type without the existence of some credit rationing. Thus k_t cannot exceed $\overline{k}_c^{*I}$ without inducing some rationing of credit at time t.

In sum, it is feasible to be in a Walrasian regime while the portfolio arbitrage condition is slack if two conditions are met: (a) the level of economic activity is not too high in the following sense:

$$k_t < \overline{k}_c^{*I} \tag{47}$$

and (b) the investment projects available to low-quality borrowers (type 1 agents) are sufficiently less productive than those open to high-quality borrowers (type 2 agents).

ii) Walrasian regime when the portfolio arbitrage condition is binding

When k_t is above or at the Walrasian portfolio threshold, $\overline{k}_c^{*P}$, the portfolio arbitrage condition is binding. Let $\overline{k}^{*I'}$ and $k_c^{*I'}$ satisfy

$$\alpha^{*'}\lambda w(\overline{k}_c^{*I'}) = \overline{k}^{*I'} \tag{48}$$

$$\left[1-(1-\alpha^{*'})(1-\rho)r+\alpha^{*'}(1-\rho)f'(\overline{k}^{*I'})+\frac{1}{2}(\alpha^{*'})^2\rho(\rho-1)\sigma_x^2\right]$$
$$=(\alpha^{*'})^{1-\rho}\left(\frac{\lambda}{1-\lambda}\right)^{1-\rho}(1-r)^{1-\rho} \tag{49}$$

The presence of α_t as a new variable in the present case makes it very hard to analyze the capital-stock switching thresholds for all possible equilibria. Therefore, my analysis focuses on the feasibility of the steady state $\left\{k^{*'},\alpha^{*'}\right\}$.

The threshold $\overline{k}^{*I'}$ is the largest "full information steady-state capital stock" that can satisfy the incentive constraint,[31] and $\overline{k}_c^{*I'}$ is the capital stock that maps into $\overline{k}^{*I'}$ under (27). Equation (49) implicitly establishes a relationship between $\overline{k}^{*I'}$ and σ_x^2, and thus, from (46), between $\overline{k}_c^{*I'}$ and σ_x^2,

[31] That a steady-state level $k^{*'}$ above $\overline{k}^{*I'}$ is not compatible with the incentive constraint follows from a comparison between (19) and (49), taking into account (28) and (29).

given the relation between $\alpha^{*'}$ and σ_x^2 obtained in (46). From (49), this effect can be expressed as

$$\alpha^{*'} f''(\overline{k}^{*'}) \left[\frac{d\overline{k}^{*'}}{d\sigma_x^2}\right] = \frac{1}{2}\rho(\alpha^{*'})^2 - \frac{1}{\alpha^{*'}}\left[1 - \frac{1}{2}(\alpha^{*'})^2\rho\sigma_x^2\right]\left[\frac{d\alpha^{*'}}{d\sigma_x^2}\right] \tag{50}$$

The sign of $d\overline{k}^{*I'}/d\sigma_x^2$ is negative if $d\alpha^{*'}/d\sigma^2 = 0$. In this case, inflation reduces the level of both the steady-state level $k^{*'}$ and the switching threshold $\overline{k}^{*I'}$. Therefore, $k^{*'}$ will not be feasible if higher inflation brings about reductions in $\overline{k}^{*I'}$ that are sufficiently larger than those in $k^{*'}$. In that case, inflation will bring about some credit rationing, which amounts to a switch from the Walrasian regime to the private information regime.

In case $d\alpha^{*'}/d\sigma_x^2$ is negative,[32] $d\overline{k}^{*I'}/d\sigma_x^2$ will be negative as long as $(\alpha^{*'})^2\rho\sigma_x^2 < 2$, which holds for reasonable parameter values. With $d\overline{k}^{*'}/d\sigma_x^2 < 0$, the condition under which the steady-state value of k_{t+1} hits the switching threshold is qualitatively the same as when $d\alpha^{*'}/d\sigma_x^2 = 0$. On the other hand, if $d\overline{k}^{*'}/d\sigma_x^2 \geq 0$, then $k^{*'}$ never hits the switching threshold. Higher inflation makes it even more likely that the Walrasian steady state be feasible.

In short, it is feasible to be in a Walrasian steady state $\left\{k^{*'}, \alpha^{*'}\right\}$ while the portfolio arbitrage condition is binding if two conditions are met: (a) the steady-state level of economic activity is not too high in the following sense:

$$k_t < \overline{k}_c^{*I'} \tag{51}$$

and (b) the investment projects available to low-quality borrowers (type 1 agents) are not much less productive than those open to high-quality borrowers (type 2 agents). I argued that item (a) requires that —in case it is negative— the effect of higher inflation rates on the threshold $\overline{k}^{*I'}$, and thus on $\overline{k}_c^{*I'}$, be not too strong compared to that on $k^{*'}$.

iii) Private information when the portfolio arbitrage condition is slack

I now define critical values for the capital stock (at t and at $t+1$) when the portfolio arbitrage condition (11) will be slack under private information.

[32]This was found to be the most likely situation, according to the discussion of case $b.1$) in section 5.

Let $\bar{\hat{k}}$ and $\hat{\underline{k}}$ satisfy

$$f'(\bar{\hat{k}}) = 1 + \hat{\mu}_x \tag{52}$$

$$f'(\hat{\underline{k}}) = \frac{1 + \hat{\mu}_x}{1 - \lambda} \tag{53}$$

while $\bar{\hat{k}}_c$ and $\hat{\underline{k}}_c$ are defined by

$$\bar{\hat{k}} \equiv \lambda w(\bar{\hat{k}}_c) \tag{54}$$

$$\hat{\underline{k}} \equiv \lambda w(\hat{\underline{k}}_c) \tag{55}$$

Then $\bar{\hat{k}}$ and $\hat{\underline{k}}$ are, respectively, the largest and the smallest "private information capital stocks" consistent with expression (A.1) in Appendix A. In turn, $\bar{\hat{k}}_c$ and $\hat{\underline{k}}_c$ are the values of the capital stock that map into $\bar{\hat{k}}$ and $\hat{\underline{k}}$, respectively, under (34). Since there is no relationship between $\hat{\mu}_x$ and σ_x^2 when (11) is slack, equations (52) and (53) imply that inflation impacts neither $\bar{\hat{k}}$ nor $\hat{\underline{k}}$.

Equation (52) indicates that if k_{t+1} is as high as $\bar{\hat{k}}$ the producers' arbitrage condition (37) will be binding. That high level of k_{t+1} corresponds to relatively large loans b_t, which no longer are constrained by (12). In other words, when $k_{t+1}(k_t)$ attains $\bar{\hat{k}}$ ($\bar{\hat{k}}_c$) the economy no longer exhibits rationing of credit at time t.

On the other hand, when k_t is as small as $\hat{\underline{k}}_c$ condition (53) will hold. In this case, the marginal product of capital is so high relative to the rate of interest that it is not possible to deter lenders from pooling type 1 and type 2 agents. Financial disintermediation is complete, and the economy collapses. Thus, both too large and too small values of the capital stock are inconsistent with a private information equilibrium allocation.

In sum, it is feasible to be in a nontrivial private information ; equilibrium while the portfolio arbitrage condition is slack if two ; conditions are met: (a) the level of economic activity is neither too high nor too low in the following sense:

$$\hat{\underline{k}}_c < k_t < \bar{\hat{k}}_c \tag{56}$$

and (b) the investment projects available to low-quality borrowers (type 1 agents) are sufficiently less productive than those open to high-quality borrowers (type 2 agents).

iv) Private information regime when the portfolio arbitrage condition is binding

I now calculate the switching thresholds for the case when the portfolio arbitrage condition is binding. Let $\bar{\hat{k}}'$ and $\underline{\hat{k}}'$ satisfy

$$f'(\bar{\hat{k}}') = 1 - r + \hat{\alpha}'\rho\sigma_x^2 \tag{57}$$

$$f'(\underline{\hat{k}}') = \frac{1 - r + \hat{\alpha}'\rho\sigma_x^2}{1-\lambda} \tag{58}$$

while $\bar{\hat{k}}'_c$ and $\underline{\hat{k}}'_c$ are defined by

$$\bar{\hat{k}}' \equiv \hat{\alpha}'\lambda w(\bar{\hat{k}}_c) \tag{59}$$

$$\underline{\hat{k}}' \equiv \hat{\alpha}'\lambda w(\underline{\hat{k}}_c) \tag{60}$$

The thresholds $\bar{\hat{k}}'$ and $\underline{\hat{k}}'$ are, respectively, the largest and the smallest "private information steady-state capital stocks" consistent with expression (A.1) in Appendix A.[33] In turn, $\bar{\hat{k}}_c$ and $\underline{\hat{k}}'_c$ are the values of the capital stock that map into $\bar{\hat{k}}'$ and $\underline{\hat{k}}'$, respectively, under (35).

Equation (57) implicitly establishes a relationship between $\bar{\hat{k}}'$ and σ_x^2, while (58) similarly determines a relationship between $\underline{\hat{k}}'$ and σ_x^2. Given that an increase in inflation will most likely induce a reduction in $\hat{\alpha}'$, use of equations (57) and (58) indicates that there is no way to know what is the sign of the effect of σ_x^2 on the switching thresholds $\bar{\hat{k}}'$ and $\underline{\hat{k}}'$, respectively.

Since the response of the switching thresholds to inflation is indeterminate, I assume that it is the behavior of the steady-state level of the capital-labor ratio, $\hat{k}'$, that determines whether the latter hits a given threshold or not. The analysis of case *b*.1) in section 5 shows that higher inflation rates

[33] This follows from a comparison between (A.1) on the one hand, and (57) and (58) on the other, taking (11) at equality into consideration.

bring about a fall in $\hat{k}'$. Thus, for a sufficiently high inflation rate $\hat{k}'$ hits the lower threshold, $\underline{\hat{k}}'$. The analysis implies that $\hat{k}'$ will never hit the upper threshold, $\overline{\hat{k}}'$.

In short, it is feasible to be in a private information steady state $\left\{\hat{k}', \hat{\alpha}'\right\}$ while the portfolio arbitrage condition is binding if two conditions are met: (a) the steady-state level of economic activity is neither too high nor too low in the following sense:

$$\underline{\hat{k}}_c' < k_t < \overline{\hat{k}}_c' \tag{61}$$

and (b) the investment projects available to low-quality borrowers (type 1 agents) are not much less productive than those open to high-quality borrowers (type 2 agents).

6.2 Regime transitions

This subsection focuses on the case when the economy transits between a Walrasian regime of slack porfolio constraint —case i) in the previous subsection— and one of credit rationing and tight portfolio constraint —case iv) in the previous subsection.

I start with the following result:

PROPOSITION 1. When the Walrasian steady state with slack porfolio constraint, k^*, and the private information steady state with tight portfolio constraint, $\left\{\hat{k}', \hat{\alpha}'\right\}$, are simultaneously feasible, the Walrasian switching threshold $\overline{k}^*$ is larger than the lower private information switching threshold $\underline{\hat{k}}'$. That is,

$$\underline{\hat{k}}' < \overline{k}^* \tag{62}$$

Proposition 1 is proved in Appendix B.

In the analysis of the switches between regimes I will refer to Figures 3.a and 3.b.[34] Figure 3.a is drawn under the assumption that

[34] I assume throughout that $\overline{\hat{k}}_c' > \overline{k}_c^{*I}$, so that transitions from the Walrasian regime to that of credit rationing are always feasible.

$$\hat{\underline{k}}_c' < \hat{k}' < k^* < \overline{k}_c^{*I} \tag{63}$$

Figure 3.b, on the other hand, excludes the steady-states from its domain by assuming that

$$\hat{k}' < \hat{\underline{k}}_c' < \overline{k}_c^{*I} < k^* \tag{64}$$

Intuitively, inequality (63) holds when storage technology is much less productive than the neoclassical technology at k^*, while at $\hat{k}'$ banks are able to offer contracts separating high-quality borrowers from low-quality ones. Inequality (64) on the other hand, means that the storage technology is only a little less productive than the neoclassical technology at k^*, and banks are completely unable to offer contracts permitting them to distinguish low- from high-quality borrowers at $\hat{k}'$.[35]

Dynamic equilibria are solutions to the discontinuous, set-valued difference equation represented by the solid lines in Figures 3.a and 3.b. Formally, that equation is given by

$$k_{t+1} = \lambda w(k_t); \qquad k_t < \hat{\underline{k}}_c' \tag{65}$$

$$k_{t+1} \in \left\{\lambda w(k_t), \hat{\alpha}' \lambda w(k_t)\right\}; \qquad \hat{\underline{k}}_c' \leq k_t \leq \overline{k}_c^* \tag{66}$$

$$k_{t+1} = \hat{\alpha}' \lambda w(k_t); \qquad \overline{k}_c^* < k_t < \overline{\hat{k}}_c' \tag{67}$$

where $\hat{\alpha}_t'$ denotes the value of α_t under the private information regime when (11) is binding.

To ensure that equilibria exist I require that the critical values $(\overline{k}_c^{*I}, \hat{\underline{k}}_c')$ satisfy

$$\hat{\underline{k}}_c' < \overline{k}_c^{*I} \tag{68}$$

Otherwise the solid graph in Figures 4.a and 4.b will contain a hole, and a deterministic k_{t+1} will be undefined[36] if k_t were to lie in the interval $\left[\hat{\underline{k}}_c', \overline{k}_c^{*I}\right]$. A necessary and sufficient condition for (68) is

[35] Azariadis and Smith (1998) also consider two other cases in which one of the steady states is within the relevant domain of definition and the other is outside.

[36] As pointed out by Azariadis and Smith (1998), a stochastic equilibrium may exist for k_{t+1} even if the deterministic map is undefined in some region. This solution may require mixed strategies from banks, *e.g.*, stochastic credit rationing.

$$\hat{\underline{k}}' < \hat{\alpha}'\overline{k}^* \tag{69}$$

which means that $\overline{k}^{*I}$ must be sufficiently larger than $\hat{\underline{k}}'$. Since $\hat{\alpha}' \leq 1$, (69) is in general more restrictive than (62).

Given that (69) holds, it is easy to see that the economy in Figures 3.a and 3.b have an *invariant set*, that is, a subset in its state space which traps all sequences that start in it. That set is the interval $\left\{\hat{k}', k^*\right\}$ in Figure 3.a and the interval $\left\{\hat{\underline{k}}', \overline{k}^{*I}\right\}$ in Figure 4.b. The classes of sequences which are trapped in the former invariant set include:

(a) Walrasian equilibria converging monotonely to k^* from below;

(b) private information equilibria converging monotonely to $\hat{k}'$ from above;

(c) a very large set of deterministic cycles, an example of which is given by the periodic two-cycle depicted in Figure 3.a;[37]

(d) a very large set of stochastic shifts between the two regimes in a Markovian manner, with the probability of regime transitions depending potentially on time, history, or the state of the system;

(e) any combination of (c) and (d);

(f) equilibria converging to a steady state (either Walrasian or private information) after experiencing a given number of regime transitions (either deterministic or stochastic, or a combination of the two).

Thus, in economies having the configuration of Figure 3.a, both the indeterminacy of equilibrium and undamped fluctuations are a very real possibility. Regime transitions may occur despite the fact that economic fundamentals are consistent with a continuation of the present regime at each

[37]The periodic two-cycle is a special case of an "(m, n) cycle." Loosely speaking, an (m, n) cycle is defined to be a deterministic cycle of m periods spent in the Walrasian regime, followed by n periods spent in the private information regime (and so on). It is appropriate to mention that (m, n) cycles by no means exhaust the set of possible deterministic periodic equilibria. For example, there may be equilibria with m periods of expansion followed by n periods of contraction, p periods of expansion again, and then q periods of contraction (and so on), with $m \neq p$ and $n \neq q$. See Azariadis and Smith (1998).

date. In addition, the different equilibria can be attained starting from a variety of initial capital stocks. In economies having the configuration of Figure 3.b, there are neither fully monotone equilibrium sequences —such as the varieties (a) and (b) above— nor equilibrium paths equivalent to those described in item (f) above. The reason is that all such sequences eventually violate either the bound in equation (47) or the lower bound in (61). Thus all equilibrium sequences must display transitions between the Walrasian and the private information regimes, and therefore all economies having the configuration in Figure 3.b *necessarily* exhibit fluctuations despite the absence of any variations in economic fundamentals.

The explanation for these regime transitions lies in self-fulfilling beliefs of savers about the behavior of intermediaries and the rate of return on deposits. Suppose, for instance, that at $t-1$ the economy is in the Walrasian regime while the portfolio arbitrage condition (11) is slack. Suppose further that savers at t pessimistically believe that the rate of return to savings will be low, or —in other words— that (11) will be binding. These low interest rate expectations imply that it is not feasible to have a Walrasian allocation at t, so that banks must ration credit. At the same time, the yield on deposits persuades savers to transfer some savings out of the banking system, and into storage. This disintermediation forces banks to ration credit. Moreover, the existence of credit rationing breaks the link between the marginal product of capital and the equilibrium rate of interest. This makes it possible for the low returns that depositors expect to actually be observed in equilibrium. Thus, a transition from the Walrasian to the private information regime is associated with a self-confirming prophecy of falling interest rates, disintermediation, and credit restrictions. Transitions from the private information to the Walrasian regime occur similarly. If depositors expect (11) to be slack, they will have optimistic expectations of high yields and all savings will be channelled through the banking system, ruling out credit rationing. In equilibrium a full-information allocation must now prevail. In sum, switches from the private information to the Walrasian regime are accompanied by a self-fulfilling prophecy of rising interest rates and an availability of funds that allows the credit market to clear.

In the situation represented in Figure 3.b, an economy that remains in the Walrasian regime long enough will eventually have a capital stock that is too high —and a real interest rate that is too low— to be consistent with a full information equilibrium allocation of resources. Thus an economy that starts in the Walrasian regime must ultimately transit into the private information regime. By the same token, an indefinite continuation in the private information regime will eventually lead to a capital stock that is so low —and a marginal product of capital that is so high— that lenders will have an incentive to pool all borrowing agents, so that self-selection can no longer be sustained. At this point the economy cannot continue in the private information regime, and must switch to the Walrasian regime.

In Figure 3.b, these transitions must clearly repeat themselves. Economic fluctuations associated with these regime transitions are not only possible, but actually inevitable. Continuation in either the Walrasian or the private information regime will ultimately violate the ceiling in equation (47) or the floor in (61), respectively. When these thresholds satisfy (64), neither the Walrasian nor the private information steady state constitute a legitimate competitive equilibrium. Therefore, the scope of savers' beliefs is limited by ruling out persistently optimistic or persistently pessimistic rational expectations equilibria. The result is that all competitive equilibria must display endogenous volatility which does not vanish asymptotically. Thus, both financial and real activity will fluctuate even if fundamentals do not vary over time.[38]

Finally, I demonstrate that higher inflation can be responsible for a change in the economic context from the one showed in Figure 3.a to the one presented in Figure 3.b. Indeed, higher inflation brings about a fall in the value of $\overline{k}^{*I}$, while it does not affect k^*. Therefore, for high enough values of σ_x^2 there no longer is a monotone equilibrium sequence leading to the Walrasian steady state. Therefore, at some point k_{t+1} must jump downwards.[39] The magnitude of the jump itself is related to inflation because the higher the latter, the lower $\hat{\alpha}'$ and thus the larger the distance between the loci K^*K^{*S} and $\hat{K}\hat{K}^B$. Thus, the first result is that higher inflation will eventually trigger a switch from the Walrasian to the private information regime.

Suppose that, after this switch occurs, the inflation rate stays constant. In the absence of other deterministic or stochastic oscillations, the economy will start approaching the private information steady-state level $\hat{k}'$. If the current value of σ_x^2 is large enough, $\hat{k}'$ will not be feasible. Thus, it must be the case that at some point the economy switches back to the Walrasian regime.

[38] In addition to the equilibrium sequences described before, the case when (64) holds is consistent with other deterministic cycles and with stochastic transitions as described in items (c) and (d) above, respectively.

[39] I assume throughout that, despite the increase in inflation, inequality (75) —or, for that matter, (76)— still holds.

Otherwise, a monotone equilibrium sequence leading to $\hat{k}'$ exists. However, my discussion of case *iv*) in subsection 6.1 implies that further increases of inflation may turn a transition to the Walrasian regime inevitable (given the assumption in footnote 39). In fact, for high enough values of σ_x^2 the floor given by $\hat{\underline{k}}_c'$ will be larger than $\hat{k}'$. In other words, not only will the last inequality in (71) hold, but also the first. In this case, the economy behaves in exactly the same way as the one represented in Figure 3.b. Finally, if inflation keeps rising the distance between the floor and the ceiling in Figure 4.b will shrink. In sum, sufficiently high inflation make regime transitions more likely and eventually more frequent.[40]

7 Conclusions

This paper has examined how inflation uncertainty and adverse selection in the credit markets affect rational expectations equilibria in an otherwise standard, one-sector overlapping generations model of growth in a closed economy.

Equilibria in this model fall into two possible regimes. When equilibrium loan contracts ration legitimate entrepreneurs, the economy will operate in the private information regime. Otherwise, a Walrasian regime prevails. In both of the cases, incentive constraints induce all of the low-quality borrowers to reveal their true characteristics.

The analysis has demonstrated that, no matter what regime the economy is in, inflation will most likely generate a process of financial disintermediation and a reduction in purchases of capital goods. In the long run, higher inflation is to blame for lower levels of both financial and real activity.

Moreover, the potential for switching between the Walrasian and the private information regimes has been analyzed. For some range of current capital stocks either regime is consistent with the existence of an equilib-

[40] In case the Walrasian regime is of the (ii) type, increases in the inflation rate would make transitions to the private information regime inevitable if the effect of inflation on the switching threshold $\overline{k}_c^{*'}$ is both negative and sufficiently stronger than that on the steady state $k^{*'}$. In case the private information regime is of the (iii) type, increases in the inflation rate would affect neither the switching thresholds $\hat{\underline{k}}_c$ and $\hat{\overline{k}}_c$ nor the steady-state level of the capital stock, $\hat{k}'$. What higher inflation would do is eventually hit the portfolio threshold value $\hat{\sigma}_x^{2P}$, and thus trigger a switch —within the private information regime— to the (iv) variety.

rium. Therefore, it is possible to observe equilibria in which deterministic or stochastic transitions occur between regimes. Moreover, for some configurations of parameters, the *only* equilibria that can be observed display oscillations that do not vanish asymptotically. In such economies regime transitions are not only possible, but indeed are a necessary feature of any equilibrium. In sum, the financial system can be a source not only of indeterminacy, but also of "excessive fluctuations."

In the one case that was studied in some detail, large enough increases in inflation were found to eventually make transitions inevitable and more frequent. It was possible to analyze this effect because the present model endogenizes the capital-stock regime-switching thresholds analyzed by Azariadis and Smith (1998), which in general depend here on inflation. More specifically, I compared the effect of inflation on the steady-state level of the capital-labor ratio with that on the corresponding regime-switching thresholds.

The results of the model have been obtained in the context of a highly stilized and simplified model of the financial system. A natural topic for further investigation is to analyze how general these results are. So far there have been attempts to examine overlapping generations growth models under asymmetric information in the cases where external finance is subject to a costly state verification problem (Boyd and Smith, 1998) and where the informational friction takes the form of limited comunication (Schreft and Smith, 1997 and 1998). It would be interesting to investigate the possibility of extending the present model in similar ways.

Appendix A: Existence of Separating and Pooling Equilibria

The loan contracts derived in the text are generically denoted (X_{t+1}, b_t), or taking the expected value of X_{t+1}, $(\mu_{X,t+1}, b_t)$. Along the lines of Rothschild and Stiglitz (1976), I assume that financial intermediaries are Nash competitors who take the loan contracts announced by other banks as given. In addition, I assume that intermediaries take $\mu_{x,t+1}$ as given. Therefore, competition will make any Nash equilibrium contract earn zero profits, so that (8) will hold.

I wish to state conditions under which such contracts are genuine Nash equilibrium contracts. To do so, it is sufficient to derive conditions implying that, in the presence of the contract $(\mu_{X,t+1}, b_t)$, no intermediary has an incentive to offer an alternative contract $(\tilde{\mu}_{X,t+1}, \tilde{b}_t)$. I show that there cannot be such an incentive both if $f'(k_{t+1}) = 1 + \mu_{x,t+1}$ and if $f'(k_{t+1}) > 1 + \mu_{x,t+1}$ and λ is sufficiently large.

First, suppose that all active intermediaries are announcing the separating equilibrium contracts $(\mu_{x,t+1}, b_t)$ at t. I now ask whether any (potential) intermediary has an incentive to offer an alternative contract $(\tilde{\mu}_{X,t+1}, \tilde{b}_t) \neq (\mu_{x,t+1}, b_t)$.

Clearly, the incentive does not exist if $\tilde{\mu}_{X,t+1} = \mu_{x,t+1}$. Then suppose $\tilde{\mu}_{X,t+1} > \mu_{x,t+1}$. If $f'(k_{t+1}) = 1+\mu_{x,t+1}$, such a contract will not be accepted by any young type 2 agents and hence will not be offered. Therefore, $(\mu_{x,t+1}, b_t)$ is a Nash equilibrium contract if $f'(k_{t+1}) = 1 + \mu_{x,t+1}$. It is a separating equilibrium contract since, from (8), $\mu_{X,t+1}$ equals $\mu_{x,t+1}$ if and only if $\xi_t = 0$. This corresponds to the Walrasian equilibrium regime of section 4.

Suppose now that $f'(k_{t+1}) > 1 + \mu_{x,t+1}$. This means that the economy is operating in the private information regime. Therefore, (12) must bind. If $1 + \tilde{\mu}_{X,t+1} \in (1 + \mu_{x,t+1}, (1 + \mu_{x,t+1})/(1 - \lambda))$, then from (8) $\xi_t = \tilde{\xi} \in (0, 1)$. However, $\tilde{\mu}_{X,t+1}$ cannot be part of an equilibrium contract.[41] If it were the current lending rate, any individual intermediary would have an incentive to lower the interest rate he charges on loans while also reducing b_t by a tiny amount. He would only attract type 2 agents since dissembling type 1 agents only care about the volume aspect of the loan. Provided the interest rate set by the intermediary is above $\mu_{x,t+1}$, he would be strictly better off by deviating. This means that contracts induce self-selection, and the possibility of pooling is ruled out.

If there is any contract that offers $1 + \mu_{X,t+1} \geq (1 + \mu_{x,t+1})/(1 - \lambda)$, then it must be a pooling contract with $\xi_t = 1$. In this case, the choice of b_t is no longer constrained by (12). This pooling equilibrium would have to occur only in a trivial case, where all type 1 agents borrow and abscond. Hence, no saving is supplied to the market, and $b_t = 0$. It follows that $k_{t+1} = 0, \forall t \geq 0$.

To summarize, a separating Walrasian equilibrium exists if $f'(k_{t+1}) = 1 + \mu_{x,t+1}$ and a nontrivial separating private information equilibrium exists in the credit market if

$$1 + \mu_{x,t+1} < f'(k_{t+1}) \leq \frac{1 + \mu_{x,t+1}}{1 - \lambda} \tag{A.1}$$

[41] The proof follows Azariadis and Smith (1996). It assumes that banks treat the deposit rate $\mu_{x,t+1}$ as given, along the lines of Kreps (1990, ch. 17).

that is, if λ is large enough.

Appendix B: Proofs of Proposition 1

When $\sigma_x^2 \geq \sigma_x^{2*I}$, the discussion for case i) in subsection 6.1 establishes the feasibility of a Walrasian regime with slack portfolio arbitrage condition (11). Thus,

$$k^* \leq \overline{k}^* \qquad (A.2)$$

When $\alpha^* = 1 > \hat{\alpha}'$, a comparison between equations (23) and (42) yields

$$\hat{k}' < k^* \qquad (A.3)$$

Moreover, (44) and the second inequality in (A.1) give

$$\underline{\hat{k}}' \leq \hat{k}' \qquad (A.4)$$

Finally, (A.2) through (A.4) clearly imply

$$\overline{k}^* > \underline{\hat{k}}'$$

which establishes Proposition 6.

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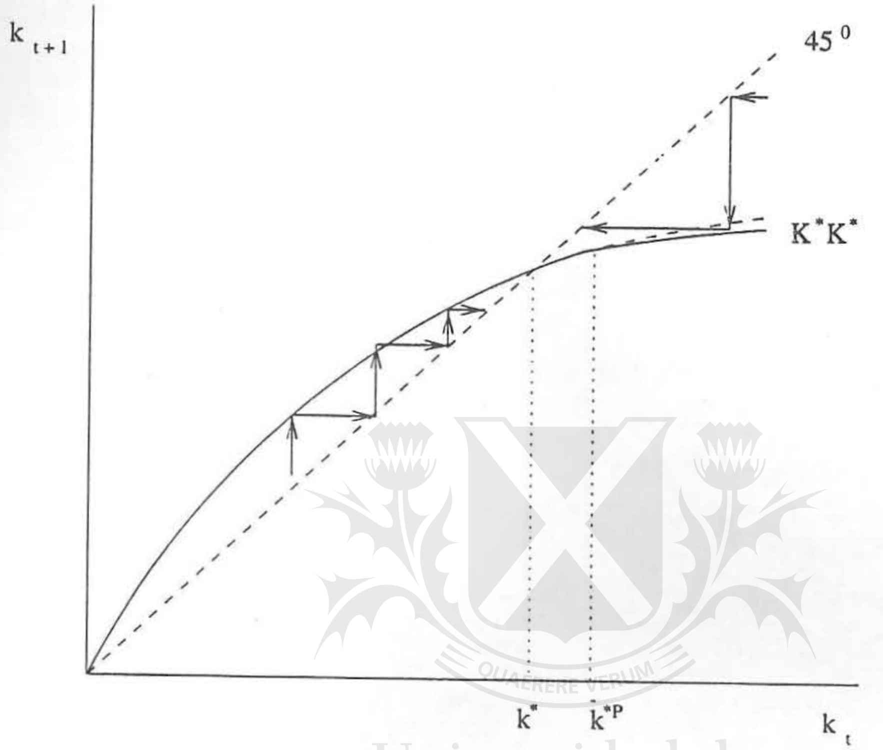

Figure 1.a. The Walrasian regime

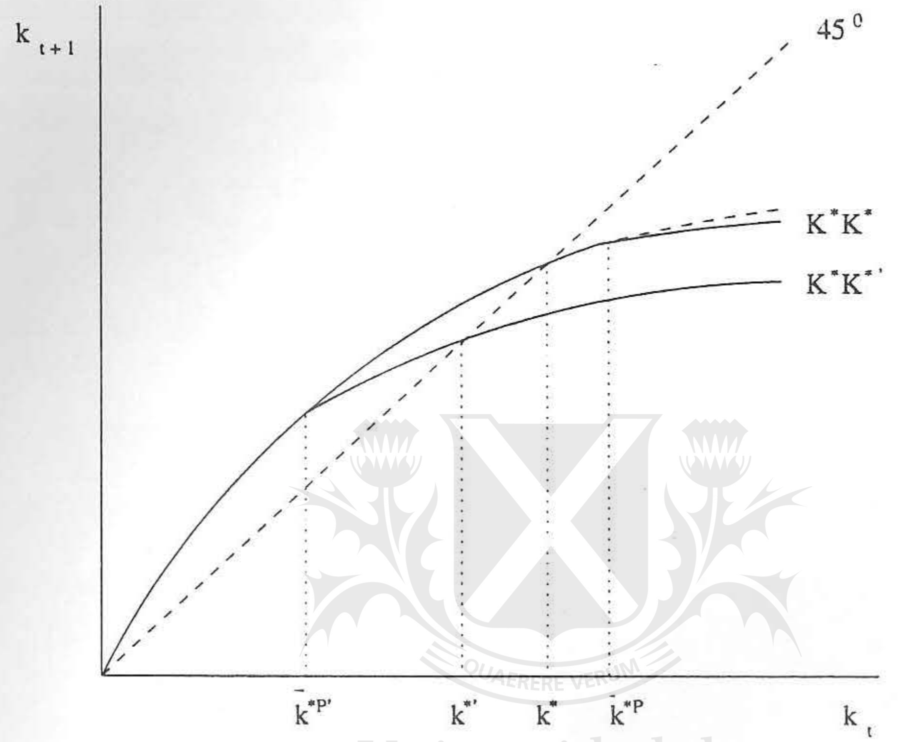

Figure 1.b. The effect of inflation under the Walrasian regime

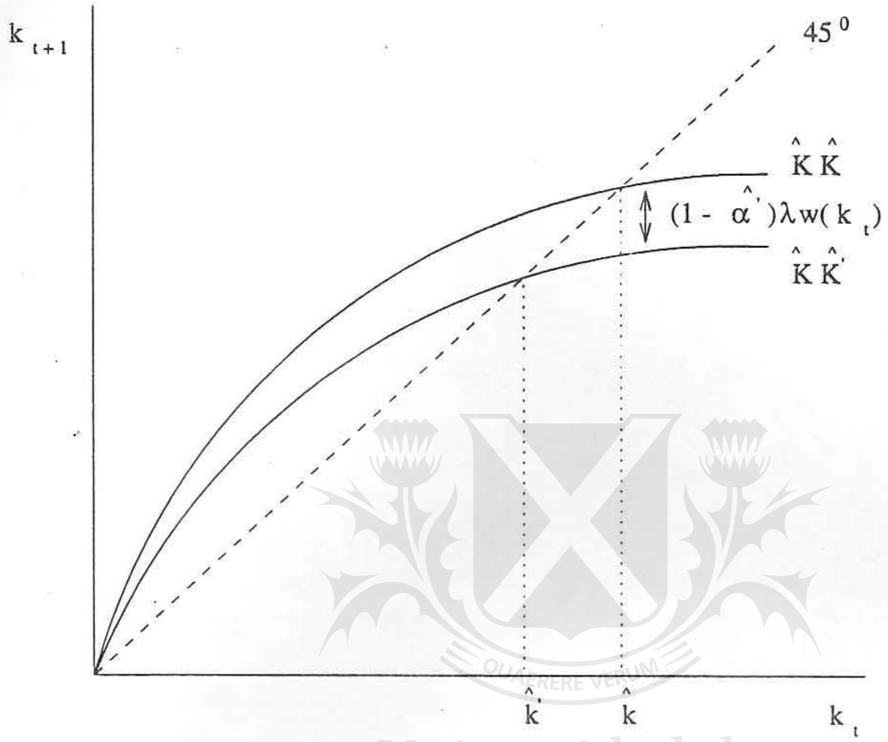

Figure 2. The private information case

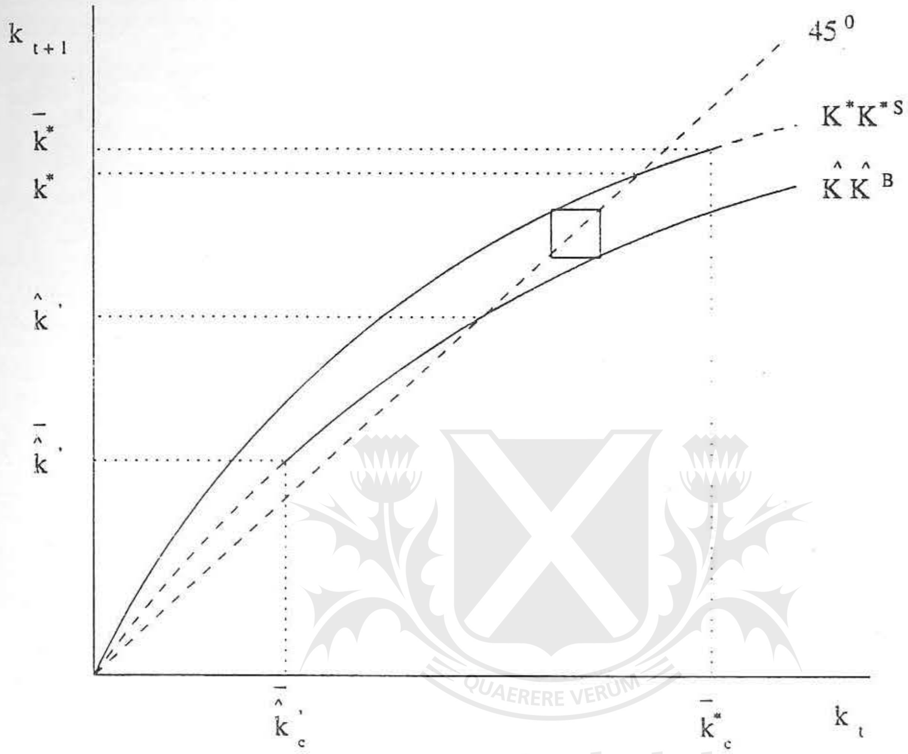

Figure 3.a

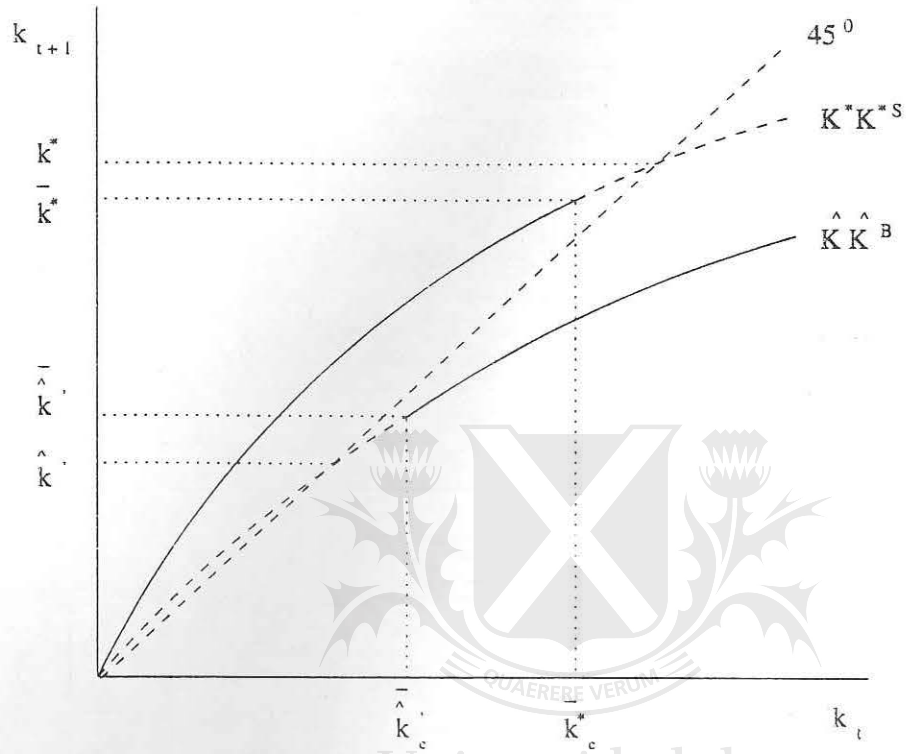

Figure 3.b