Cross Border Nominal Assets and International Monetary Interdependence.

Pablo Andrés Neumeyer*†
Department of Economics
University of Southern California

Makoto Yano
Department of Economics
Keio University

Abstract
This paper studies the equilibrium of a two-country monetary economy in which there are cross-border holdings of nominal assets that create a monetary linkage between the two economies. It is demonstrated that the equilibrium response to an unexpected monetary shock is completely different from the predictions obtained in standard models of international finance. Our results provide new explanations for certain real world phenomena related to changes in the supply of U.S. dollars, the currency in which most of the existing cross-border nominal assets are denominated.
JEL Classification Numbers: F3, F4.
Keywords: International Interdependence, Nominal Assets, Monetary Policy.

*Corresponding author: Pablo A. Neumeyer, Department of Economics, University of Southern California, Los Angeles, CA 90089-0253, Tel. (213) 740-2110, Fax (213) 740-8543, e-mail: pablo@rcf.usc.edu.

†The authors express their appreciation to Roger Farmer, Philip Lane, Maurice Obstfeld, Carlos Vegh, and Michael Woodford, seminar participants at Columbia University, UCLA, University of Southern California and the Latin American Meetings of the Econometric Society (Rio, 1996) for valuable comments. We are grateful to the Department of Economics at UCLA that hosted Neumeyer in Fall 1996 and to the Center for International Business and Research of the School of Business Administration at USC for financial support for Yano's visit to USC.

1 Introduction

Between 1982 and 1995, the US switched from having dollar denominated assets equivalent to 1% of its income to having dollar denominated debts that ascended to 21% of national income. On the other hand, between 1983 and 1994 Japan's net accumulation of long term bonds denominated in foreign currency added up to 26% of its 1994 income. Many developing countries have large external debts denominated in foreign currencies. For example, in the 1980's the debt-income ratio in Latin America was about 60%, while in Sub-Saharan Africa that number climbed to just under 75% in 1995. In such environments, the real value of initial cross-border nominal assets may have a significant effect on the international distribution of real wealth that is akin to the real balance effect (Pigou, 1943, and Patinkin, 1965).

The purpose of the present study is to investigate the nature of this international redistributive effect, in particular to characterize the equilibrium response to an unexpected monetary shock in the standard framework of monetary models with infinitely-lived consumers that use money to economize on transaction costs (Sidrauski, 1967, and Brock, 1974). This framework, in which money is neutral in the absence of cross-border holdings of nominal assets, is suitable for highlighting two important features of the initial cross-border holdings of nominal assets. That is, (i) they provide a channel through which the neutrality of money may break down, and (ii) they bring a country's domestic real wealth and absolute price level under the influence of foreign economic factors, in particular, of foreign monetary policy. These features have not yet been examined in a dynamic general equilibrium model of international finance.[1]

Although our model is a natural extension of the standard model, it is new in that it incorporates international nominal assets positions explicitly. Recently, there has been a renewed interest in the literature on monetary economics to examine the role of nominal bonds in models with dynamically optimizing agents. We follow this new trend by extending the closed-economy model of Woodford (1995) to a two-country framework. Like most studies of monetary economies with infinitely-lived consumers, we adopt a rather simple model, which enables us to explicitly capture the most fundamental equilibrium effect of a monetary policy change.

[1] As a dynamic general equilibrium treatment of international financial issues, our study follows the line of research originating from such studies as Stockman (1980), Obstfeld (1981), Calvo (1982) and Lucas (1982).

Despite the model's simplicity, the overall effect of an unexpected monetary shock is fairly complicated. We explain it by decomposing it into two: (i) the direct effect of an unexpected domestic monetary expansion is to redistribute wealth in favor of the country with liabilities denominated in the domestic currency and (ii) the indirect (feedback) effect that arises because the initial redistribution of wealth induces a change in the foreign price level that will further redistribute wealth if nominal assets denominated in the foreign currency are also held across the border. We show that for reasonable levels of initial cross-border asset holdings the direct effect dominates[2].

The dominant direct effect can be further decomposed into its domestic part and its spill-over effect on the foreign country. While each of these components can be given a simple intuitive explanation, the total effect is again complicated and, in some cases, quite surprising. One such effect appears in the response of the exchange rate to an unexpected monetary shock. In our model, for example, a "rich" nation's unexpected monetary expansion, even if it has a minor domestic effect, can cause a significantly large appreciation of the exchange rate of a poor nation if the poor nation holds initial nominal liabilities denominated in the rich nation's currency.[3] This result may cast new light on the observed instability in the exchange rates of highly indebted developing countries.

Another interesting finding, which has not been captured in the existing literature, is that the depreciation of the domestic currency will permanently shift the current account in the deficit direction for domestic-currency debtors and in the surplus direction for creditors. Given the current US international asset position, this implies that, contrary to the elasticities approach, a depreciation of the US dollar should increase the US current account deficit and reduce that of the rest of the world. Thus, this effect may help to explain why empirical tests of the elasticities approach (Rose and Yellen, 1989) fail to uncover a statistically significant relationship between exchange rates and the current account.[4]

[2]When the feedback effect outweighs the direct effect it is possible for an unexpected domestic monetary expansion to redistribute wealth in favor of the country with assets denominated in the home currency. Such a situation arises if both countries have assets denominated in their own currency that exceed the present value of their consumption.

[3]In the less realistic case of the poor nation's holding initial nominal credits (deonominated in the rich nation's currency), our model demonstrates an even more paradoxical possibility that the rich nation's monetary expansion gives rise to an appreciation of its currency against the poor nation's currency.

[4]Sticky prices may be another factor contributing the elusiveness of the empirical re-

The real effect of a monetary shock that our model captures may be attributed to the debt revaluation effect of an unexpected monetary shock, *i.e.*, to the fact that an expected monetary shock affects the real value of initial debts by changing the absolute price level. The debt revaluation effect has been extensively studied in the Mundell-Fleming model (Boyer, 1977, Rodriguez, 1979, and Frenkel and Razin, 1987).[5] Those studies conclude that the direct debt revaluation effect of a monetary policy change is transitory[6] and that money is neutral in the long run. The sharp contrast between this conclusion and ours is attributable to the fact that consumers are assumed to seek to attain a target level of wealth in the Mundell-Fleming model (as summarized in Frenkel and Razin, 1987), whereas consumers' decisions are derived from dynamic optimization in our model.

Modern general equilibrium treatments of the transfers in international wealth that the revaluation of internationally traded nominal assets generates can be found in Helpman and Razin (1982), Svensson (1989), Neumeyer (1993) and Kim (1996). Those studies focus on the role of nominal securities as insurance devices in economies where asset markets are incomplete.

2 The Model

Take a two-country monetary economy where there is a single perishable commodity. Call the countries H and F. Let p_t and p_t^* be the (nominal) prices of the consumption good in period t in countries H and F, respectively. Denote by e_t the exchange rate in period t. Since the good is freely traded, the purchasing power parity condition holds, *i.e.*, for $t = 0, 1, ...$,

$$p_t = p_t^* e_t \tag{1}$$

lation between exchange rates and the current account. Dynamic two country general equilibrium monetary models with pre-set prices, but no cross-border nominal asset holdings, predict the same statistical relation between exchange rates and the current account as our model (Obstfeld and Rogoff, 1995).

[5] Also see Henderson and Rogoff (1982), who treat a similar issue in a dynamic framework.

[6] In Rodriguez (1979) debt revaluations have a long-run indirect effect due to the impact of transitory current account imbalances on the long-run asset position of a country.

2.1 Consumers

The preferences of both country's consumers are represented by a single utility function. Denote this utility function by $U = \sum_{t=0}^{\infty} \beta^t u(c_t, m_t)$, where c_t is the consumption in period t and where $m_t = M_t/p_t$ denotes the real money balances demanded at the end of period t.[7] (Later, we will adopt a specific form of utility function in order to obtain a closed-form equilibrium in a two-country model. In presenting the basic structure of our model, however, it is more convenient to write preferences in a general form.)

At the end of each period, the representative consumer can hold wealth in three types of nominal assets: H's currency denominated bonds (B_{Ht}), F's currency denominated bonds (B_{Ft}) and H's currency (M_t). Let i_{t-1} and i_{t-1}^* be the nominal interest rates earned on H's currency bonds and F's currency bonds, respectively. The value of assets at the beginning of period t is

$$A_t = M_{t-1} + (1 + i_{t-1})B_{Ht-1} + (1 + i_{t-1}^*)e_t B_{Ft-1}. \tag{2}$$

The flow budget constraint faced by the representative consumer is, for $t = 0, 1, ...,$

$$M_t + B_{Ht} + e_t B_{Ft} = p_t (y_t - c_t - \tau_t) + A_t \tag{3}$$

where τ_t represents the lump-sum taxes levied by the government and y_t is the income received by consumers at each moment of time. Assume that the sequence of outputs y_t is exogenously given and uniformly bounded in t, and that the government is required to set values of its policy variables in such a way that taxes τ_t are uniformly bounded in t.

In the present setting, in which there is perfect foresight and in which bonds are freely traded internationally, H's currency bonds and F's currency bonds are perfect substitutes. Despite this, it is convenient to explicitly distinguish two currency bonds, B_{Ht} and B_{Ft}, so as to capture the effects that may appear from the initial asset holdings, $B_{H,-1}$ and $B_{F,-1}$, that are given. The perfect substitutability of domestic and foreign bonds implies the interest rate parity condition, *i.e.*,

$$1 + i_t = (1 + i_t^*) \frac{e_{t+1}}{e_t}, \tag{4}$$

[7]This utility function is of the type introduced by Sidrauski (1967) and Brock (1974). The inclusion of real money balances in the utility function is meant to capture the value of resources (time) that are economized by the use of money as a medium of exchange.

for $t = 0, 1, \ldots$ The real interest rates r_t and r_t^* can be defined by

$$1 + i_t = (1 + r_t)\frac{p_{t+1}}{p_t} \text{ and } 1 + i_t^* = (1 + r_t^*)\frac{p_{t+1}^*}{p_t^*}. \tag{5}$$

By (1) and (4), it must hold, for any t, that

$$r_t = r_t^*. \tag{6}$$

By using (2), (4) and (5), the flow budget constraint, (3), can be rewritten as

$$\frac{A_{t+1}}{p_{t+1}} = (1 + r_t)\left[\frac{A_t}{p_t} + y_t - c_t - \tau_t - \delta_t \frac{M_t}{p_t}\right] \tag{7}$$

for $t = 0, 1, \ldots$, where $\delta_t \equiv \frac{i_t}{1+i_t} \equiv 1 - \frac{p_t}{p_{t+1}} \frac{1}{1+r_t}$. In addition to the flow budget constraint, the consumer is assumed to face a sequence of borrowing constraints that prevents its debt $(-A_t)$ from exceeding the present value of their future disposable income, i.e.,

$$\frac{A_{t+1}}{p_{t+1}} \geq -\sum_{s=0}^{\infty}\left(\prod_{j=1}^{s}\frac{1}{1 + r_j}\right)(y_{t+j} - \tau_{t+j}) \tag{8}$$

for $t = 0, 1, 2, \ldots$ The infinite sum on the right-hand side of this condition is well defined under the condition $\sum_{t=0}^{\infty}\left(\prod_{s=0}^{t-1}\frac{1}{1+r_s}\right) < \infty$.

Given t and sequence x_{t+s}, $s = 0, 1, 2, \ldots$, let $\tilde{x}_t = \sum_{s=0}^{\infty}\left(\prod_{j=0}^{s-1}\frac{1}{1+r_{t+j}}\right) x_{t+s}$; that is, $\tilde{\cdot}$ is the operator taking the present discounted value. Define the real wealth at date t as

$$w_t = \frac{A_t}{p_t} + \tilde{y}_t - \tilde{\tau}_t. \tag{9}$$

It is assumed that at time $t = -1$ asset holdings are such that $w_0 > 0$. The flow budget constraints (7) can then be transformed into one intertemporal budget constraint, with the aid of the borrowing constraints (8), as follows:

$$\sum_{s=0}^{\infty}\left(\prod_{j=0}^{s-1}\frac{1}{1 + r_{t+j}}\right)(c_{t+s} + \delta_{t+s} m_{t+s}) \leq w_t. \tag{10}$$

As constraint (10) shows, δ_t represents the opportunity cost of holding money. This opportunity cost can be called the *nominal (current) rental price of* *money* at date t.[8] In this sense, $\delta_t M_t$ may be thought of as the (nominal) *expenditure on money*.

In summary, for each t, the consumer's behavior can be expressed as

$$\{c_s, m_s\}_{s=t}^{\infty} = \arg \max_{\{\xi_s, \mu_s\}_{s=t}^{\infty} \geq 0} \sum_{s=t}^{\infty} \beta^s u(\xi_s, \mu_s) \text{ subject to (10).} \tag{11}$$

The consumer's optimization problem over the entire time horizon is described by (11) with $t = 0$.

2.2 Government

Let M_t^g be the monetary liabilities of H's government (money supply) and B_{Ht}^g and B_{Ft}^g be its non-monetary liabilities denominated in the currencies of H and F, respectively.[9] The total debt of H's government is

$$D_t^g = M_{t-1}^g + B_{Ht-1}^g (1 + i_{t-1}) + e_t B_{Ft-1}^g \left(1 + i_{t-1}^*\right). \tag{12}$$

Let g_t be the government's consumption, which is assumed to be uniformly bounded in t. The government's flow budget constraint is

$$M_t^g + B_{Ht}^g + e_t B_{Ft}^g = p_t (g_t - \tau_t) + D_t^g \tag{13}$$

for $t = 0, 1, \ldots$. In the same manner as the consumer's flow budget constraint, this can be transformed into

$$\frac{D_{t+1}^g}{p_{t+1}} = \left(\frac{D_t^g}{p_t} + g_t - \tau_t - \delta_t \frac{M_t^g}{p_t}\right)(1 + r_t) \tag{14}$$

for $t = 0, 1, \ldots$ Observe that the term $\delta_t M_t^g$ represents the government's revenues from supplying money, i.e., its seignorage.

[8]This fact is more clearly shown by transforming (10) in nominal terms into

$$\sum_{s=0}^{\infty} \left(\prod_{j=0}^{s-1} \frac{1}{1 + i_{t+j}}\right) (p_{t+s} c_{t+s} + \delta_{t+s} M_{t+s}) \leq p_t w_t$$

by using (5).

[9]Notice that we allow a government to issue debt denominated in a foreign currency.

We assume that the government is required to satisfy an intertemporal budget constraint similar to that of the consumer, which is[10]

$$\frac{D_t^g}{p_t} = \sum_{s=0}^{\infty}\left(\prod_{j=0}^{s-1}\frac{1}{1+r_{t+j}}\right)\left(\tau_{t+s} - g_{t+s} + \delta_{t+s}\frac{M_{t+s}^g}{p_{t+s}}\right). \tag{15}$$

Following Woodford (1995), moreover, we assume that the government sets its supply of bonds in such a way that their present value falls to zero in the infinite future[11].

$$\lim_{T\to\infty}\left(\prod_{t=0}^{T-1}\frac{1}{1+r_t}\right)\left(\frac{B_{HT}^g}{p_T} + \frac{B_{FT}^g}{p_T^*}\right) = 0. \tag{16}$$

2.3 Market Clearing

The behavior of country F's representative consumer and its government are analogous to that of the domestic ones. We denote F's consumer's decision variables with the superscript * and F's government's policy variables with the superscript $^{g^*}$. There are four markets: the markets for H's money, for F's money, for goods and for bonds. Assume that at date 0 the total assets of the private sector equal the overall liabilities of the domestic and the foreign governments, i.e.,

$$A_0 + e_0 A_0^* = D_0^g + e_0 D_0^{g^*}. \tag{17}$$

[10] An alternative assumption, that in equilibrium implies (15) is to assume that the government is required not to let the real value of the government's lending, $-D_t^g/p_t$, grow to infinity, *i.e.*, that there is an $\alpha > 0$ such that

$$\frac{D_t^g}{p_t} > -\alpha$$

for any $t = 0, 1, ...$ This assumption implies that the present value of total government debt as $t \to \infty$ is non-positive and is a surrogate for an optimizing government. For simplicity we assume (15).

[11] A suuficient condition for (16) is a tax rule of the form

$$T_t = p_t g_t - [M_t - M_{t-1}] + \gamma(1+i_{t-1})(B_{Ht-1} + e_{t-1}B_{Ft-1}),$$

for some $\gamma > 0$, where the government raises enough tax revenue to repay part of its debt every period.

The market clearing conditions of the world economy can be described by the money market clearing conditions in the respective countries and the goods market clearing condition, which can be written, for $t = 0, 1, ...$, as follows.

$$M_t = M_t^g; \tag{18}$$

$$M_t^* = M_t^{g^*}; \tag{19}$$

$$c_t + c_t^* + g_t + g_t^* = y_t + y_t^*. \tag{20}$$

Once these conditions are satisfied, given (17), the markets for bonds are cleared at every t. By (17) together with the consumer's and government's budget constraints (7) and (14), Walras's law means that $A_t + e_t A_t^* = D_t^g + e_t D_t^{g^*}$ for $t = 0, 1, ...$ It follows from (18) and (19) that this condition is equivalent to the bonds market clearing condition

$$B_{Ht} + e_t B_{Ft} + B_{Ht}^* + e_t B_{Ft}^* = B_{Ht}^g + e_t B_{Ft}^g + B_{Ht}^{g^*} + e_t B_{Ft}^{g^*}. \tag{21}$$

3 Equilibrium

In an equilibrium, the representative consumer takes an optimal plan of consumption and money holding, solving the optimization problem (11), and the markets for goods and each country's money are cleared. Each government must set its policy variables in such a way that constraints (14), (15) and (16) are satisfied.

In the present setting, a government can control two types of independent policy instruments. We choose these independent policy instruments to be the streams of levels of government consumption, $\{g_t\}_{t=0}^{\infty}$ and $\{g_t^*\}_{t=0}^{\infty}$, which represent fiscal policies, and the sequences of money supplies, $\{M_t\}_{t=0}^{\infty}$ and $\{M_t^*\}_{t=0}^{\infty}$, which represent monetary policies.

Once values of these policy instruments are set, all the other variables are determined endogenously. It is convenient to think of the stream of real interest rates $\{r_t\}_{t=0}^{\infty}$ to be determined in the market for goods and those of the two countries' price levels $\{p_0\}$ and $\{p_t^*\}_{t=0}^{\infty}$ to be determined in the respective country's money markets. Given the equilibrium values of real interest rates and price levels, nominal interest rates and exchange rates follow from the interest rate parity and purchasing power parity conditions, (4) and (1).

3.1 Consumer Behavior

In order to obtain a closed-form equilibrium, we adopt the specific forms of utility functions: $u(c_t, m_t) = \ln c_t + \gamma \ln m_t$ and $u^*(c_t^*, m_t^*) = \ln c_t^* + \gamma \ln m_t^*$. Given these utility function, it follows from the first order conditions of the consumer's optimization problem (11) that the demands for goods and money can be written as follows:

$$c_t = \frac{1-\beta}{1+\gamma} w_t; \tag{22}$$

$$m_t = \frac{\gamma(1-\beta)}{1+\gamma}\frac{w_t}{\delta_t}. \tag{23}$$

Consumers allocate the proportion $1/1{+}\gamma$ of their permanent income $(1-\beta)\,w_t$ to consumption and the proportion $\gamma/1+\gamma$ to liquidity services.

By using (22), (23) and (9), the flow budget constraint (7) can be rewritten as

$$w_{t+1} = \beta(1+r_t)w_t. \tag{24}$$

Since, by (24), $w_{t+s} = \beta^s \prod_{j=0}^{s-1}(1+r_{t+s})w_t$, by (22) and (23), the sums of present values of consumptions and those of services from money can be expressed as follows.

$$\widetilde{c}_t = \frac{1}{1+\gamma} w_t; \tag{25}$$

$$\widetilde{(\delta_t m_t)} = \frac{\gamma}{1+\gamma} w_t. \tag{26}$$

Since, in equilibrium, the intertemporal budget constraint (10) must hold with equality, the following transversality condition holds.

$$\lim_{T\to\infty}\left(\prod_{t=0}^{T}\frac{1}{1+r_t}\right)\frac{A_{T+1}}{p_{T+1}} = 0. \tag{27}$$

Country F's demands for goods and money, c_t^* and m_t^*, the time path of wealth w_{t+1}^*, and the transversality condition can be expressed in a similar manner.

3.2 Interest Rates and Inflation Rates

Once the streams of government consumptions, $\{g_t\}$ and $\{g_t^*\}$, are set, the goods market clearing conditions determine the stream of equilibrium real interest rates. To demonstrate this, note $(c_{t+1} + c_{t+1}^*)/(c_t + c_t^*) = \beta(1 + r_t)$, which follows from $r_t = r_t^*$, (22) and (24). By this equality, (20) implies

$$1 + r_t = \frac{1}{\beta}\frac{y_{t+1} + y_{t+1}^* - g_{t+1} - g_{t+1}^*}{y_t + y_t^* - g_t - g_t^*} \tag{28}$$

for $t = 0, 1, \ldots$ This shows that, as in the standard model with no initial cross-border nominal asset holdings, real interest rates depend on the fiscal policies of both countries but not on their monetary policies.

The inflation rate of a country depends on the monetary policy of that country but not on that of the foreign country. In order to demonstrate this fact, it is convenient first to determine the opportunity cost of holding money, which is also independent of the monetary policy of the foreign country. For the sake of discussion, take country H. By rewriting (23) as a difference equation in m_t and solving it forwards, the money market clearing conditions, $M_t = M_t^g$ for $t = 0, 1, 2, \ldots$, can be transformed into

$$\frac{M_t^g}{p_t} = \frac{\gamma(1-\beta)}{1+\gamma} w_t \sum_{s=0}^{\infty} \beta^s \left(\prod_{j=0}^{s-1} \frac{1}{1 + \mu_{t+j}^g} \right), \tag{29}$$

where $\mu_t^g = M_{t+1}^g / M_t^g - 1$ (see the Appendix for derivation).[12] It follows from (23) and (29) that, in equilibrium, the opportunity cost of holding money, δ_t, is

$$\delta_t = 1 / \sum_{s=0}^{\infty} \beta^s \left(\prod_{j=0}^{s-1} \frac{1}{1 + \mu_{t+j}^g} \right). \tag{30}$$

This expression demonstrates that the opportunity cost of holding money in a country is determined completely by that country's monetary policy, or more precisely, the choice of a stream of growth rates of money supply, $\{\mu_t^{\infty}\}_{t=0}^{\infty}$.

Once the opportunity cost of holding money, $\{\delta_t\}_{t=0}^{\infty}$, is determined in H's money market, the inflation rate of country H is determined by the

[12]Note that in order for this condition to be well defined, we need to assume that $\prod_{j=0}^{s-1} 1/(1 + \mu_{t+1+j}^g)$ is uniformly bounded in s.

interaction of the monetary and real sides of country H's economy only. That is, it follows from (23), (24) and (28) that

$$\frac{p_{t+1}}{p_t} = (1+\mu_t^g)\frac{\delta_{t+1}}{\delta_t}\frac{y_t + y_t^* - g_t - g_t^*}{y_{t+1} + y_{t+1}^* - g_{t+1} - g_{t+1}^*}. \tag{31}$$

This demonstrates that if the growth rate of money supply is kept constant, *i.e.*, if $\mu_t^g = \mu^g$ for all t, the inflation rate will be equal to the difference between the rate of growth of money supply, μ^g, and the rate of growth of "net output" (*i.e.*, gross output minus government consumption); if μ_t^g is kept constant over time, so is δ_t. Temporary changes in monetary policy that affect δ_t but not δ_{t+1} will also affect the rate of inflation.

Equilibrium nominal interest rates are obtained by substituting r_t and p_{t+1}/p_t for their equilibrium values in the Fisher equations (5), that is,

$$1 + i_t = \beta^{-1}(1+\mu_t)\frac{\delta_{t+1}}{\delta_t} \tag{32}$$

Country F's opportunity cost of holding money, $\{\delta_t^*\}_{t=0}^{\infty}$, and its inflation rate, $\{p_{t+1}^*/p_t^*\}_{t=0}^{\infty}$, are determined analogously. Interest rates and inflation rates are independent of the initial cross border holdings of nominal assets and of foreign monetary policy.

3.3 Simultaneous Determination of an International Equilibrium

In the present model, as shown above, all the real variables can be represented by period-0 real wealth levels w_0 and w_0^* whereas all the nominal variables can be represented by period-0 absolute price levels p_0 and p_0^*. Among them, p_0 and w_0 are the domestic variables whereas p_0^* and w_0^* are the foreign variables.

An important feature of our model is that these variables interact with each other. In other words, the nominal and real variables are not dichotomized, nor are the domestic and foreign variables. In order to characterize the interaction of variables, denote $\mathcal{M} = \delta_0 M_0^g$ and $\mathcal{M}^* = \delta_0^* M_0^{g^*}$. Then, country H's money market clearing condition can be written, by using (29) and (30), as

$$\frac{\mathcal{M}}{p_0} = \frac{\gamma(1-\beta)}{1+\gamma}w_0. \tag{33}$$

For the foreign country, similarly, we have

$$\frac{\mathcal{M}^*}{p_0^*} = \frac{\gamma(1-\beta)}{1+\gamma} w_0^*. \tag{34}$$

By $M_0 = M_0^g$, (15) implies $\widetilde{\tau}_0 = \frac{D_0^g}{p_0} + \widetilde{g}_0 - \widetilde{(\delta_0 m_0)}$. Thus, by (26) and the definitions of A_0 and D_0^g, we may transform (9) into

$$w_0 = (1+\gamma)\left(\frac{H_0}{p_0} + \frac{F_0}{p_0^*} + \widetilde{y}_0 - \widetilde{g}_0\right), \tag{35}$$

where $H_0 = (1+i_{-1})\left(B_{H,-1} - B_{H,-1}^g\right)$ and $F_0 = \left(1+i_{-1}^*\right)\left(B_{F,-1} - B_{F,-1}^g\right)$ are the domestic and foreign currency components of the home country's consumers' total net assets. Similarly, w_0^* can be expressed as

$$w_0^* = (1+\gamma)\left(\frac{H_0^*}{p_0} + \frac{F_0^*}{p_0^*} + \widetilde{y}_0^* - \widetilde{g}_0^*\right), \tag{36}$$

where $H_0^* = (1+i_{-1})\left(B_{H,-1}^* - B_{H,-1}^{g^*}\right)$ and $F_0^* = \left(1+i_{-1}^*\right)\left(B_{F,-1}^* - B_{F,-1}^{g^*}\right)$. Since the governmental supply of each country's currency bonds must be equal to the private demand for them in any period, we may assume

$$H_0 + H_0^* = 0 \text{ and } F_0 + F_0^* = 0. \tag{37}$$

Equations (33), (34), (35) and (36) are an equilibrium system, which determines the price levels and real wealth levels, p_0, p_0^*, w_0 and w_0^*. The other variables can be treated as exogenous variables. By (30), $\mathcal{M} = \delta_0 M_0^g$ and $\mathcal{M}^* = \delta_0^* M_0^{g^*}$ can be expressed as

$$\mathcal{M} = \frac{M_0^g}{\sum_{s=0}^{\infty} \beta^s \prod_{j=0}^{s-1} \frac{1}{1+\mu_j^g}} \text{ and } \mathcal{M}^* = \frac{M_0^{g^*}}{\sum_{s=0}^{\infty} \beta^s \prod_{j=0}^{s-1} \frac{1}{1+\mu_j^{g^*}}}. \tag{38}$$

This demonstrates that, in the above system, $\mathcal{M}$ and $\mathcal{M}^*$ contain all the information (relevant for the determination of period-0 real wealth levels and absolute prices) about a country's monetary policy, which can be captured by the period-0 money supply, M_0^g and $M_0^{g^*}$, and the streams of growth rates of future money supply, $\{\mu_t^g\}$ and $\{\mu_t^{g^*}\}$.

3.4 Real Wealth, Absolute Prices and the Exchange Rate

As the above equilibrium system indicates, the initial cross-border holdings of nominal assets have two important effects. That is, (i) they provide a channel through which the neutrality of money may break down, and (ii) they bring a country's domestic real wealth and absolute price under the influence of foreign economic factors, in particular, of foreign monetary policy.

In order to characterize these effects of nominal assets, it is useful to compare the equilibrium of our model with that which would hold in the autarky case. The autarky equilibrium for, say, country H can be obtained by setting $H_0 = F_0^* = 0$ in (33) and (35). That is, the autarky real wealth and price level of H are, respectively,

$$w_0^A = (1+\gamma)\left(\widetilde{y}_0 - \widetilde{g}_0\right) \text{ and } p_0^A = \frac{\mathcal{M}}{\gamma(1-\beta)(\widetilde{y}_0 - \widetilde{g}_0)}. \tag{39}$$

Similarly, for country F, the autarky real wealth and price levels are, respectively,

$$w_0^{A^*} = (1+\gamma)\left(\widetilde{y}_0^* - \widetilde{g}_0^*\right) \text{ and } p_0^{A^*} = \frac{\mathcal{M}^*}{\gamma(1-\beta)(\widetilde{y}_0^* - \widetilde{g}_0^*)}. \tag{40}$$

These expressions exhibit the standard features of the monetary model with a dynamically optimizing representative agent. As they show, in the closed economy case, the price level is proportionate to the money supply (the neutrality of money), and the real wealth level is determined by the domestic resources for consumption, $\widetilde{y}_0 - \widetilde{g}_0$.

In the free-trade case, by using (37), we solve the above equilibrium system (33)-(36) and obtain

$$w_0 = \varphi \cdot w_0^A \text{ and } p_0 = \frac{1}{\varphi} \cdot p_0^A, \tag{41}$$

where[13]

$$\varphi = \frac{1 - \frac{F_0^*}{p_0^* \widetilde{c}_0^*}\left(1 + \frac{\widetilde{y}_0^* - \widetilde{g}_0^*}{\widetilde{y}_0 - \widetilde{g}_0}\right)}{1 - \frac{H_0}{p_0 \widetilde{c}_0} - \frac{F_0^*}{p_0^* \widetilde{c}_0^*}}. \tag{42}$$

[13]This expression can be obtained by using $\frac{\mathcal{M}}{\gamma(1-\beta)} = p_0 \widetilde{c}_0$ and $\frac{\mathcal{M}^*}{\gamma(1-\beta)} = p_0^* \widetilde{c}_0^*$. The former expression, for example, can be derived by (33) and (25).

Since $H_0 = F_0 = 0$ implies $\varphi = 1$, in the absence of initial cross-border holdings of nominal assets, a country's real wealth and price level, w_0 and p_0, are equal to those in the autarky case. This shows that, in this case, the real wealth is determined only by the domestic factors while the price level p_0 is proportionate to the money supply, $\mathcal{M}$.

In the presence of initial cross-border holdings of nominal assets, in general, $\varphi \neq 1$ and φ depends on $\mathcal{M}^*$ and $\widetilde{y}_0^* - \widetilde{g}_0^*$. Therefore, a country's real wealth $w_0 = \varphi w_0^A$ is dependent on foreign factors and different from the real wealth level in the closed economy. Moreover, the price level $p_0 = (1/\varphi)p_0^A$ is no longer proportional to the money supply, $\mathcal{M}$. The coefficient φ, which may be called the *coefficient of interdependence,* captures the extent to which real wealth and price levels are sensitive to the interplay between the initial cross-border holdings of nominal assets and domestic and foreign monetary policies.

Similar discussions can be made for country F. The counter-part of (41) in country F can be expressed as follows.

$$w_0^* = \varphi^* \cdot \bar{w}^{A^*} \text{ and } p_0^* = \frac{1}{\varphi^*} \cdot \bar{p}_0^{A^*}, \tag{43}$$

where

$$\varphi^* = \frac{1 - \frac{H_0}{p_0 \widetilde{c}_0}\left(1 + \frac{\widetilde{y}_0 - \widetilde{g}_0}{\widetilde{y}_0^* - \widetilde{g}_0^*}\right)}{1 - \frac{H_0}{p_0 \widetilde{c}_0} - \frac{F_0^*}{p_0^* \widetilde{c}_0^*}}. \tag{44}$$

Since $e_0 = p_0/p_0^*$, the exchange rate in the initial period, e_0, can be determined as follows.

$$e_0 = \frac{\varphi^*}{\varphi} \cdot \frac{\widetilde{y}_0^* - \widetilde{g}_0^*}{\widetilde{y}_0 - \widetilde{g}_0} \cdot \frac{\mathcal{M}}{\mathcal{M}^*}, \tag{45}$$

This expression captures the way in which the initial cross-border holdings of nominal assets separate the period-0 exchange rate e_0 from that which would hold in their absence, *i.e.*, $e_0^A = \frac{\widetilde{y}_0^* - \widetilde{g}_0^*}{\widetilde{y}_0 - \widetilde{g}_0} \cdot \frac{\mathcal{M}}{\mathcal{M}^*}$. In particular, $e_0 \lesseqgtr e_0^A$ if and only if $\varphi^*/\varphi \lesseqgtr 1$. The domestic country's currency is stronger than that which would hold in the absence of cross-border holdings if and only if the home country is a net borrower.

Remark: In order to have a well-defined equilibrium, as the above characterization shows, $\varphi > 0$ and $\varphi^* > 0$ must be guaranteed. It may be demonstrated that these conditions are satisfied if and only if

$$\left(\theta - \frac{F_0^*}{p_0^*\widetilde{c}_0^*}\right)\left(\theta^* - \frac{H_0}{p_0\widetilde{c}_0}\right) > 0, \tag{46}$$

where $\theta = \frac{\widetilde{y}_0 - \widetilde{g}_0}{\widetilde{y}_0 - \widetilde{g}_0 + \widetilde{y}_0^* - \widetilde{g}_0^*}$ and $\theta^* = \frac{\widetilde{y}_0^* - \widetilde{g}_0^*}{\widetilde{y}_0 - \widetilde{g}_0 + \widetilde{y}_0^* - \widetilde{g}_0^*}$.[14]

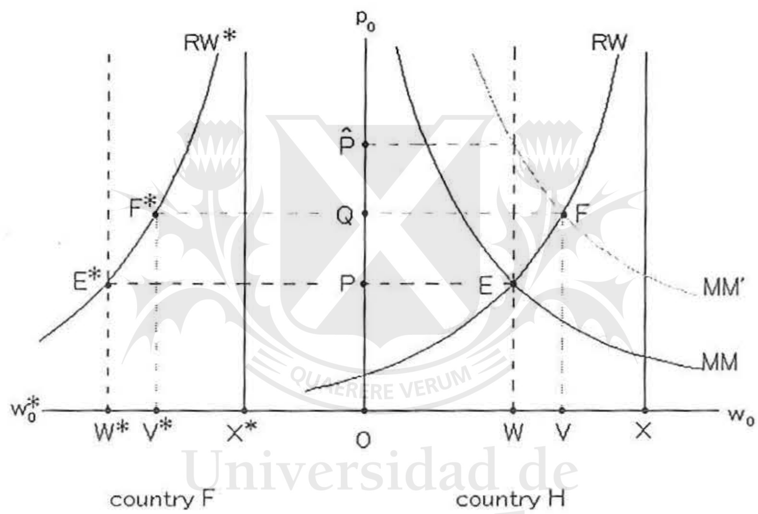

Figure 1 ($H_0 < 0$)

3.5 Diagrammatic Exposition

As demonstrated above, money is no longer neutral in the presence of initial cross-border holdings of nominal assets. This is because a change in a country's price level facilitates a partial default on the nominal debts denominated in that country's currency that were accumulated prior to the initial

[14]In order to obtain this condition, think of the case in which the two terms on the left-hand side of (46) are both positive. In that case, since $\theta + \theta^* = 1$, $1 - \frac{H_0}{p_0\widetilde{c}_0} - \frac{F_0^*}{p_0^*\widetilde{c}_0^*} > 0$, which implies $\varphi > 0$ and $\varphi^* > 0$. If, instead, those terms are both negative, then $1 - \frac{H_0}{p_0\widetilde{c}_0} - \frac{F_0^*}{p_0^*\widetilde{c}_0^*} < 0$, which implies $\varphi > 0$ and $\varphi^* > 0$, again. The converse can also be proved in a similar manner.

period. In this subsection, we explain the working of this partial default effect diagrammatically.

The partial default effect of a price change appears through the intertemporal budget constraint of a country. In order to illustrate this, in Figure 1, curve RW illustrates the relationship between H's price level p_0 and its real wealth w_0 in the expression for H's real wealth, (35), for the case of $H_0 < 0$. That is, by (35); *i.e.*, the RW curve is

$$\text{RW:} \qquad w_0 = (1+\gamma)\left(\frac{H_0}{p_0} + \frac{F_0}{p_0^*} + \widetilde{y}_0 - \widetilde{g}_0\right).$$

Similarly, curve RW* illustrates the relationship between H's price level p_0 and F's real wealth w_0^* in the expression for country F's real wealth, (36); *i.e.*,

$$\text{RW}^*: \qquad w_0^* = (1+\gamma)\left(\frac{H_0^*}{p_0} + \frac{F_0^*}{p_0^*} + \widetilde{y}_0^* - \widetilde{g}_0^*\right).$$

These curves give the real wealth levels of countries for each given price level p_0 and, for this reason, are called real wealth loci.

The fundamental function of initial cross-border holdings of nominal assets is captured by the fact that real wealth loci, RW and RW*, are not vertical. In the case in which no nominal assets denominated in H's currency are held across the border (*i.e.*, if $H_0 = 0$), each country's real wealth level is determined independently of H's price level p_0. In that case, the real wealth loci are vertical lines, as shown in Figure 1.

If H's currency denominated assets are held internationally (*i.e.*, if $H_0 \neq 0$), the real wealth loci are no longer vertical, *i.e.*, each country's real wealth level becomes dependent of H's price level, as curves RW and RW* show. This is because an increase in H's price level facilitates a partial default on the initial debts of the country holding initial liabilities denominated in H's currency. This creates a redistributive effect on real wealth in favor of that country. If, for example, country H holds liabilities denominated in H's currency (*i.e.*, $H_0 < 0$), H's price increase facilitates H's partial default on its debt and, as a result, redistributes real wealth from F into H. That is, as p_0 increases, H's real wealth increases along H's real wealth locus, curve RW, and F's real wealth decreases along that of F, curve RW*.

Notice that the intertemporal budget loci of countries H and F relate their respective wealth levels not to their respective domestic price levels but to the price level of country H. In this way, we can highlight the fact that an exogenous change in the home country's price level has a *spill-over effect* on the real variables of a foreign country, if nominal assets denominated in the home currency are held across the border. In other words, as the left-hand side panel of Figure 1 shows, a change in p_0 affects foreign real wealth w_0^*.

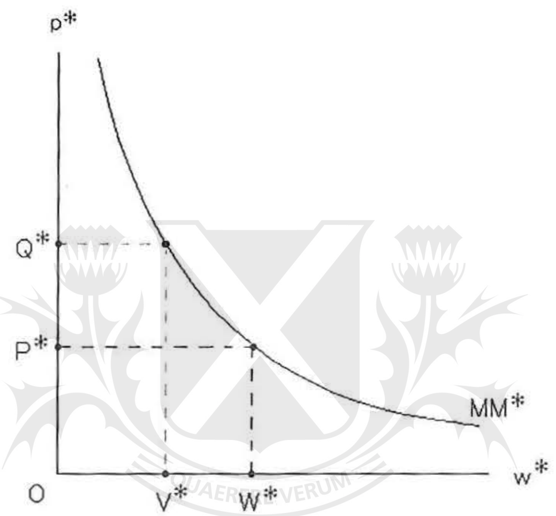

Figure 2

An equilibrium is determined by an interaction between the intertemporal budget constraints, (35) and (36), and the countries' money market clearing conditions, (33) and (34). In order to illustrate an equilibrium, in Figure 1, curve MM illustrates the relationship between p_0 and w_0 that satisfies the money market clearing condition, (33). That is, curve MM, which we call a money market clearing locus, is

$$\text{MM}: \qquad w_0 = \frac{1+\gamma}{\gamma(1-\beta)} \cdot \frac{\mathcal{M}}{p_0}.$$

Then, H's equilibrium price level and real wealth level can be captured at the intersection between H's real wealth locus, RW, and its money market clearing locus, MM. Moreover, F's real wealth level is determined at the intersection between F's real wealth locus, RW*, and the horizontal line through P. Given its real wealth level, country F's price level is determined by its money market clearing condition, which is illustrated by the money market clearing locus,

$$\text{MM}^*: \qquad w_0^* = \frac{1+\gamma}{\gamma(1-\beta)} \cdot \frac{\mathcal{M}^*}{p_0^*},$$

in Figure 2.

It is important to note that Figures 1 and 2 are interrelated since F's price level determined in Figure 2 must be consistent with the positions of the vertical asymptotes of curves RW and RW*, which depend on F's price level. This captures the fact that, if nominal assets denominated in the foreign currency are held across the border as well, the spill-over effect of a change in the home country's price level on the foreign country creates a *feedback* effect on the home country.

4 The Effect of an Unexpected Monetary Shock

The overall effect of an expected monetary shock in a country can be separated into direct and feedback effects. The direct effect can be decomposed into domestic effects and spill-over effects on the foreign country. The overall effect can be characterized by partially differentiating (41) and (43) with respect to $\mathcal{M}$. Denote by $\varepsilon_{w_0\mathcal{M}}$ and $\varepsilon_{w_0^*\mathcal{M}}$, respectively, the elasticities of equilibrium real wealth levels, w_0 and w_0^*, with respect to H's domestic monetary policy, $\mathcal{M}$; that is, for example, $\varepsilon_{w_0\mathcal{M}} = \frac{\mathcal{M}\partial\varphi}{\varphi\partial\mathcal{M}} + \frac{\mathcal{M}\partial w_0^A}{w_0^A\mathcal{M}}$ since $w_0 = \varphi \cdot w_0^A$. We adopt similar expressions to describe other elasticities with respect to $\mathcal{M}$.

4.1 Spill-Over Effects and Real Wealth Distribution

One of the most important features of our model with cross border nominal asset holdings is that, as explained below, an unexpected monetary shock in the home country has a disproportionately large spill-over effect on the foreign country if the size of the foreign country's real wealth is disproportionately small relative to that of the home country. Therefore, even if a monetary policy change has only a minor domestic effect, it has a significant impact on a foreign country with a much smaller real wealth.

This is because the partial default effect of a country's unexpected monetary shock gives rise to a redistribution of real wealth between the countries.

That is, by $H_0 + H_0^* = F_0 + F_0^* = 0$, (35) and (36) imply

$$w_0 + w_0^* = (1+\gamma)(\widetilde{y}_0 - \widetilde{g}_0 + \widetilde{y}_0^* - \widetilde{g}_0^*).$$

Since this relationship holds for any $\mathcal{M}$ and $\mathcal{M}^*$, the sum of real wealth levels, $w_0 + w_0^*$, does not respond to any change in a monetary policy factor, *i.e.*, the effect of an unexpected monetary shock is purely redistributive.. In consequence, the two countries' elasticities of real wealth with respect to a monetary shock must satisfy

$$\varepsilon_{w_0^*\mathcal{M}} = -\frac{w_0}{w_0^*} \cdot \varepsilon_{w_0\mathcal{M}}. \tag{47}$$

Therefore, even if the domestic effect of a change in a country's monetary policy is small, its effect on the foreign country may be significant if the home country's real wealth is sufficiently large relative to that of the foreign country.

4.2 Permanent Redistributive Effects

The second important feature of the effect of a monetary shock in our model is that an unexpected monetary shock has a permanent effect. By (28), the real interest rate r_t does not respond to a monetary shock. As a result, by (24) and (22), the future real wealth levels and present and future consumption levels of H change at the same rate as the real wealth level in the initial period, *i.e.*,

$$\varepsilon_{w_t\mathcal{M}} = \varepsilon_{c_t\mathcal{M}} = \varepsilon_{w_0\mathcal{M}} \text{ for } t = 0, 1, ... \tag{48}$$

Since a similar relationship holds for country F, by (47), it holds that

$$\varepsilon_{w_t^*\mathcal{M}} = \varepsilon_{c_t^*\mathcal{M}} = -\frac{w_0}{w_0^*} \cdot \varepsilon_{w_0\mathcal{M}} \text{ for } t = 0, 1, ... \tag{49}$$

The changes in real wealth levels affect the price levels of the respective countries. In the initial period, it follows from (33) and (34) that

$$\varepsilon_{p_0\mathcal{M}} = 1 - \varepsilon_{w_0\mathcal{M}} \text{ and } \varepsilon_{p_0^*\mathcal{M}} = \frac{w_0}{w_0^*} \cdot \varepsilon_{w_0\mathcal{M}}. \tag{50}$$

Since the foreign monetary policy, $\left\{M_t^{g^*}\right\}$, is kept unchanged, the effect on the foreign price level does not depend on time either; *i.e.*, since the counterpart of (30) for F implies that δ_t^* is kept unchanged, that of (31) implies

$$\varepsilon_{p_t^* \mathcal{M}} = \frac{w_0}{w_0^*} \cdot \varepsilon_{w_0 \mathcal{M}} \text{ for } t = 1, 2, ... \tag{51}$$

In contrast, the effect on H's future price path depends on the way in which H's money supply, $\{M_t^g\}$, will be changed in the future, as seen in the previous section.

4.3 Direct Effect: a Diagrammatic Exposition

In the real world, the nominal assets that are held internationally are denominated primarily in the U.S. dollar. Given this fact, it is of primary interest to investigate the case in which the initial cross-border nominal assets are denominated only in one of the two currencies. For this purpose, in this subsection, we first focus on the case of $F_0^* = 0$. In this case, as is shown in the equilibrium system developed in the previous section, a change in country F's price level creates no redistributive effect. Therefore, given $F_0^* = 0$, it suffices to focus on the case in which an unexpected monetary shock occurs in country H.

It follows from (41) and (42) that

$$\varepsilon_{w_0 \mathcal{M}} = -\frac{\frac{H_0}{p_0 \tilde{c}_0}}{1 - \frac{H_0}{p_0 \tilde{c}_0}}. \tag{52}$$

As this expression demonstrates, the direct effect depends on the sign pattern of initial cross-border holding. We first focus on the case in which the country that changes its monetary policy (H) has liabilities denominated in its own currency ($H_0 < 0$).

Case of $H_0 < 0$: In this case, by (52), $\varepsilon_{w_0 \mathcal{M}} > 0$. This demonstrates that the direct effect of a country's unexpected monetary expansion tends to increase that country's real wealth if the country has initial liabilities denominated in its own currency. The associated adjustments in international allocation and prices are illustrated in Figure 1. An unexpected monetary expansion of country H shifts H's money market clearing locus MM upwards, say to curve MM'. As Figure 1 shows, the excess supply of money created by this monetary expansion will be absorbed by an increase in H's price level (from P to Q). Since H has initial liabilities denominated in its own currency, this price increase entails a partial default on those liabilities. In consequence, real wealth is redistributed in favor of H; *i.e.*, in Figure 1, H's real wealth increases from W to V along curve RW while F's real wealth decreases from W* to V* along curve RW*. This change will raise F's price level P* to Q* along F's money market clearing locus MM* in Figure 1. In terms of elasticities, these responses can be summarized as follows.

$$\begin{cases} \varepsilon_{w_0\mathcal{M}} > 0 \\ \varepsilon_{p_0\mathcal{M}} = 1 - \varepsilon_{w_0\mathcal{M}} > 0 \\ \varepsilon_{w_0^*\mathcal{M}} = -\frac{w_0}{w_0^*}\varepsilon_{w_0\mathcal{M}} < 0 \\ \varepsilon_{p_0^*\mathcal{M}} = \frac{w_0}{w_0^*}\varepsilon_{w_0\mathcal{M}} > 0 \end{cases} \quad \text{if } H_0 < 0. \tag{53}$$

Since the exchange rate is $e_0 = p_0/p_0^*$, the effect on the exchange rate can be characterized by

$$\varepsilon_{e_0\mathcal{M}} = 1 - (1 + \frac{w_0}{w_0^*})\varepsilon_{w_0\mathcal{M}}. \tag{54}$$

This demonstrates that an unexpected monetary expansion of the home country may not depreciate that country's exchange rate if the home country holds nominal liabilities denominated in its own currency. In that case, as shown above, a monetary expansion may raise the foreign price level at a larger rate than the domestic price level if the home country's real wealth is sufficiently large relative to that of the foreign country. In that case, a domestic monetary expansion leads to an appreciation of the domestic currency.

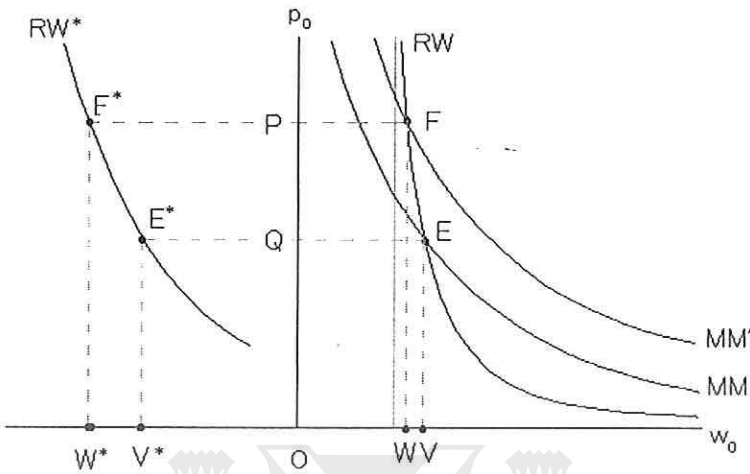

Figure 3 ($H_0 > 0$)

Case of $H_0 > 0$: In this case, as (52) shows, the sign of $\varepsilon_{w_0\mathcal{M}}$ is not determined by that of H_0. Given $F_0 = F_0^* = 0$, however, condition (46), which guarantees the existence of an equilibrium, implies $\frac{H_0}{p_0\tilde{c}_0} < \theta^* < 1$. Thus, by (52), $\varepsilon_{w_0\mathcal{M}} < 0$. This demonstrates that the direct effect of a country's unexpected monetary expansion tends to decrease that country's real wealth if the country has initial credits denominated in its own currency. The associated adjustments are illustrated in Figure 3. That is, country H's monetary expansion will raise the domestic price level (from Q to P in Figure 3) at a rate larger than the rate of monetary expansion. This price increase entails a partial default on the initial liabilities of F, thereby redistributing real wealth in favor of F. In consequence, H's real wealth falls from V to W while F's real wealth increases from V* to W* in Figure 3. The real wealth increase in F will reduce the price level of F along the MM* curve. In elasticity terms, these responses can be summarized as follows.

$$\begin{cases} \varepsilon_{w_0\mathcal{M}} < 0 \\ \varepsilon_{p_0\mathcal{M}} = 1 - \varepsilon_{w_0\mathcal{M}} > 1 \\ \varepsilon_{w_0^*\mathcal{M}} = -\frac{w_0}{w_0^*}\varepsilon_{w_0\mathcal{M}} > 0 \\ \varepsilon_{p_0^*\mathcal{M}} = \frac{w_0}{w_0^*}\varepsilon_{w_0\mathcal{M}} < 0 \end{cases} \quad \text{if } H_0 > 0. \tag{55}$$

Country H's exchange rate depreciates by more than the rate of price increase in H; *i.e.*,

$$\varepsilon_{e_0\mathcal{M}} = 1 - (1 + \frac{w_0}{w_0^*})\varepsilon_{w_0\mathcal{M}} > \varepsilon_{p_0\mathcal{M}} \text{ if } H_0 > 0. \tag{56}$$

4.4 Overall Effects

Given that, in the real world, cross-border nominal assets are denominated primarily in the U.S. dollar, the case of $H_0 \neq 0$ and $F_0^* \neq 0$ is of less practical importance than that in which either $F_0^* = 0$ or $H_0 = 0$ holds. However, it is not ignorable that bonds denominated in other major currencies than the U.S. dollar are also held internationally. For this reason, it is desirable to briefly examine the case of $H_0 \neq 0$ and $F_0^* \neq 0$.

As pointed out above, if $H_0 \neq 0$ and $F_0^* \neq 0$, then the foreign price change that is caused by the direct redistributive effect of an unexpected monetary shock in the home country creates a feedback effect, which further redistributes real wealth between the countries. The overall effect resulting from these effects, direct and feedback, is fairly complicated, as captured by the following expressions.

$$\varepsilon_{w_0\mathcal{M}} = -\frac{\frac{H_0}{p_0\widetilde{c}_0}}{1 - \frac{H_0}{p_0\widetilde{c}_0} - \frac{F_0^*}{p_0^*\widetilde{c}_0^*}} \tag{57}$$

and

$$\varepsilon_{p_0\mathcal{M}} = \frac{1 - \frac{F_0^*}{p_0^*\widetilde{c}_0^*}}{1 - \frac{H_0}{p_0\widetilde{c}_0} - \frac{F_0^*}{p_0^*\widetilde{c}_0^*}}. \tag{58}$$

These expressions demonstrate that the feedback effect does not outweigh the direct effect if and only if

$$\frac{H_0}{p_0\widetilde{c}_0} + \frac{F_0^*}{p_0^*\widetilde{c}_0^*} < 1. \tag{59}$$

In other words, if and only if (59) holds, the results summarized by (53) and (55) remain to hold, even if $F_0^* \neq 0$.[15] Condition (59) reveals that the feedback effect does not outweigh the direct effect if the volume of initial

[15] Given (59), (46) implies $\frac{F_0^*}{p_0^*\widetilde{c}_0^*} < \theta^*$ and $\frac{H_0}{p_0 \cdot \widetilde{c}_0} < \theta$, since $\theta > 0$ and $\theta^* > 0$ satisfy $\theta + \theta^* = 1$. These inequalities imply that (53) and (55) hold.

cross-border nominal assets, H_0 and F_0^*, is sufficiently small relative to the present values of resources for consumption, $p_0\widetilde{c}_0$ and $p_0^*\widetilde{c}_0^*$.[16] Real world evidence suggests that it is reasonable to assume that this condition, (59), is satisfied.

Whether or not the feedback effect augments the direct effect depends on patterns of cross-border holdings of nominal assets denominated in the two currencies. For the sake of discussion, take the case in which each country has the same net position on one country's currency denominated assets as on the other currency denominated assets (*i.e.*, $H_0F_0 = -H_0F_0^* > 0$). In this case, (57) and (58) demonstrate that the effect that a country's unexpected monetary shock creates is augmented by the feedback effect if that country has initial nominal liabilities (*i.e.*, $H_0 < 0$ and $F_0^* > 0$). If, instead, the country has initial nominal credits (*i.e.*, $H_0 > 0$ and $F_0^* < 0$), the feedback effect weakens the direct effect.

Another important implication that (57) has is that a country's monetary policy change has no redistributive effect unless that country's currency denominated assets are held across the border. That is, if $H_0 = 0$, $\varepsilon_{w_0\mathcal{M}} = 0$. However, as the above discussion indicates, if a country's currency denominated assets are held across the border in small volume, the effect of the country's monetary policy change may sometimes be augmented by a relatively large feedback effect created by the existence of a large volume of initial cross-border nominal assets denominated in foreign currencies.

[16]If condition (59) is not satisfied, an unexpected monetary shock can have all sorts of perverse effects on real wealth and price levels. The table below, for example, summarizes the relationship between parameter values and effects on the doemstic real wealth and price levels. Although those perverse effects are of theoretical interest, they may have little practical relevance because, in the real world, it appears unlikely that the volume of cross-border nominal assets is so large relative to the present value of consumption so that condition (59) fails to hold.

Domestic Price Effect (PE) and Wealth Effect (WE)

	$\frac{F_0^*}{p_0^*\widetilde{c}_0^*} < \theta$	$\theta < \frac{F_0^*}{p_0^*\widetilde{c}_0^*} < 1$	$1 < \frac{F_0^*}{p_0^*\widetilde{c}_0^*}$
$\frac{H_0}{p_0\widetilde{c}_0} < \theta^*$	Normal PE Normal WE	No Equilibrium	No Equilibrium
$\frac{H_0}{p_0\widetilde{c}_0} > \theta^*$	No Equilibrium	Perverse PE Perverse WE	Normal PE Perverse WE

5 Summary of Results and Real World Implications

In this study, we have analyzed the role of initial cross-border holdings of nominal assets in the determination of an equilibrium and the effect of an unexpected monetary shock in the presence of such assets. For this purpose, we have adopted the standard monetary model with two countries each of which is represented by a representative dynamically optimizing agent. Here, we first summarize the main findings from our analysis and then discuss some real world interpretations of our results.

1. The presence of initial cross-border holdings of nominal assets causes the neutrality of money to break down even in a standard monetary model in which money would otherwise be neutral. If cross-border nominal assets are held across borders, both real and nominal variables of countries become dependent on one another's foreign factors, in particular, their foreign monetary policies.

2. An unexpected change in a country's monetary policy has a redistributive effect on real wealth between that country and the rest of the world if nominal assets denominated in a country's currency are initially held across the border. This redistributive effect is a consequence of the fact that a rise in that country's price level facilitates a partial default on initial nominal debts denominated in that country's currency. Given the intertemporal optimization of agents, the redistribution of real wealth can be expected to have permanent effects. Our model is designed to capture those permanent effects.

3. If the initial cross-border assets are denominated only in the home country's currency, the home country's unexpected monetary expansion redistributes real wealth in favor of the country that holds initial liabilities denominated in the home currency. The home country's price level increases unambiguously. If the home country holds initial liabilities, however, the home country's price level does not rise at the same rate as the rate of monetary expansion while the foreign country's price level rises. If, instead, the foreign country holds initial liabilities, the home country's price level rises at a rate higher than the rate of monetary expansion while the foreign country's price level falls.

4. The magnitude of a spill-over effect on foreign real wealth and price levels depends positively on the relative size between domestic and foreign real wealth. If the domestic real wealth is disproportionately larger than the foreign real wealth, a monetary shock that has only a minor domestic effect has a significant effect on the foreign real wealth and price levels.

5. This disproportionate effect can cause several perverse responses of the exchange rate to an unexpected monetary shock. That is, if the foreign country's real wealth is disproportionately smaller than that of the home country, and if the home country has initial liabilities denominated in the home currency, the home country's monetary expansion may cause an appreciation, rather than a depreciation, of the home currency due to a large price increase in the foreign country. If, instead, the foreign country has liabilities denominated in the home currency, the foreign exchange rate may appreciate at a rate significantly larger than the rate of monetary expansion.

6. If not only domestic currency denominated bonds but also foreign currency denominated bonds are held across the border, the spill-over effect of a domestic monetary policy change can create a feedback effect on domestic variables. This is because the spill-over effect on the foreign price level creates another redistributive effect of real wealth due to the existence of cross-border assets denominated in the foreign currency. The feedback effect, however, does not outweigh the direct effect so long as the sum of the ratios between the initial nominal asset denominated in one country's currency and the present value of that country's consumptions for the countries is less than one, i.e. $H_0/p_0\widetilde{c}_0 + F_0^*/p_0^*\widetilde{c}_0^* < 1$.

One should be careful in using our results for explaining actual macroeconomic phenomena due to the simplicity of our model. Nevertheless, our results shed light on several real world phenomena, which have not been fully understood in the literature.

5.1 The Plaza Accord

In the mid 1980's, in the agreement known as the Plaza Accord, the G7 countries agreed to reduce the value of the US dollar *vis a vis* the rest of the G7 currencies. Within the G7 countries, the US is a net debtor and, by and large, its debt is denominated in US dollars. By identifying country H

with the US and country F with the other G7 countries, we may analyze the possible effect that the Plaza Accord might have had.

Since $e_0 = p_0/p_0^*$, any exchange rate target can be implemented by means of either domestic or foreign monetary policy. If $(1 + \frac{w_0}{w_0^*})\varepsilon_{w_0\mathcal{M}} < 1$, by (54), country H can depreciate its own currency, *i.e.*, increase e_0, by expanding its money supply. In order to implement this policy, in our model, country H must increase $\mathcal{M}$. Since $H_0 < 0$ and $F_0^* = 0$, (57) implies

$$0 < \varepsilon_{w_0\mathcal{M}} = \frac{-\frac{H_0}{p_0\tilde{c}_0}}{1 - \frac{H_0}{p_0\tilde{c}_0}} < 1.$$

Therefore, H's monetary expansion increases its own real wealth unambiguously while it decreases that of F by $\varepsilon_{w_0^*\mathcal{M}} = -\frac{w_0}{w_0^*}\varepsilon_{w_0\mathcal{M}}$. This redistribution of real wealth implies that consumption will increase in country H and fall in country F, shifting the home country's trade balance in the deficit direction not only in the present period but also in each future period. Thus, our model offers a potential explanation for the lack of responsiveness of the US trade deficit to the depreciation of the US dollar that followed the Plaza accord.

5.2 The Spillover Effect on Highly Indebted Developing Countries.

Many countries in Africa and Latin American have large foreign debts that are mostly denominated in US Dollars and, therefore, these economies are sensitive to US monetary policy[17]. Moreover, given the large disparity in the real wealth of the U.S. relative to these countries, the spillover effect of US policies on highly indebted developing economies can be quite large. In order to analyze such a case, we may think of country F as a highly indebted developing country and country H as the US.

If $H_0 > 0$ and $F_0 = 0$, the effect of the home country's monetary policy on the foreign country's wealth are given by the equation

$$\varepsilon_{w_0^*\mathcal{M}} = -\frac{w_0}{w_0^*}\varepsilon_{w_0\mathcal{M}} = \frac{w_0}{w_0^*}\frac{\frac{H_0}{p_0\tilde{c}_0}}{1 - \frac{H_0}{p_0\tilde{c}_0}} > 0.$$

[17]Strictly speaking, we should not consider the US, but a weighted average of lenders, as the home country. For simplicity we assume that the US is a representative lender.

The possible magnitude of the spillover effect of US monetary policy on developing countries can be illustrated by re-writing this equation in terms of readily available data. To this end, assume that the rate of growth of income is constant and government expenditure in both countries is a constant fraction of output, $1-\lambda$. Then the real interest rate will be constant and $\tilde{c}_0 = \lambda y_0 / r$. In addition, let μ be the average US future rate of money growth and write $\mathcal{M}$ as $\mathcal{M} = M_{-1}^{g}(1+\mu-\beta)$. Then, the elasticity of per capita wealth in a country with US Dollar denominated liabilities with respect to the rate of growth of the US money supply is

$$\varepsilon_{w_0^*\mu} = -r\,\frac{N^*}{N}\frac{\text{Debt}}{\lambda\ \text{GDP}^*}\left(\frac{\mu}{1+\mu-\beta}\right).$$

For illustrative purposes, assume government expenditures equal to 10% of output, a real interest rate of 3%, a ratio of populations equal of 1.74 (this corresponds to the populations of the US and Latin America) and a discount factor $\beta = .97$. Then, for a debt/income ratio of 60% and a US rate of growth of the money supply equal to 10%, $\varepsilon_{w_0^*\mu}$ equals -0.027. *An unexpected permanent fall in the rate of growth of the US money supply from* 10% *to* 5% *per year induces foreign consumption to fall by* 1.3% *permanently.* Moreover, if the ratio between the home and foreign income is 5 (again, this corresponds to the incomes of the US and Latin America), due to the per capita wealth disparity, the permanent fall in foreign wealth of 1.3% amounts to only 0.15% of US wealth.

Appendix

Derivation of (29): In order to derive this expression, let $\mu_t = M_{t+1}/M_t - 1$, and recall $\delta_t = 1 - \frac{p_t}{p_{t+1}}\frac{1}{1+r_t}$. It follows from these definitions that

$$\delta_t = 1 - \frac{1}{1+\mu_t}\cdot\frac{1}{1+r_t}\cdot\frac{m_{t+1}}{m_t}.$$

By using this expression, together with (22) and (24), (23) can be written as

$$m_t - \frac{1}{1+\mu_t}\cdot\frac{1}{1+r_t}\cdot m_{t+1} = \gamma c_t.$$

Thus, inductively,

$$m_t - \prod_{s=0}^{T-1}\left(\frac{1}{1+\mu_{t+s}}\cdot\frac{1}{1+r_{t+s}}\right)m_{t+T} \;=\; \gamma\sum_{s=0}^{T-1}\prod_{j=0}^{s-1}\left(\frac{1}{1+\mu_{t+s}}\cdot\frac{1}{1+r_{t+s}}\right)c_{t+s}$$

$$
\begin{aligned}
&= \gamma c_t \sum_{s=0}^{T-1} \beta^s \prod_{j=0}^{s-1} \left(\frac{1}{1+\mu_{t+s}} \right) \\
&= \frac{\gamma(1-\beta)}{1+\gamma} w_t \sum_{s=0}^{T-1} \beta^s \prod_{j=0}^{s-1} \left(\frac{1}{1+\mu_{t+s}} \right).
\end{aligned}
$$

Therefore, if $\prod_{j=0}^{s-1} \left(\frac{1}{1+\mu_{t+s}} \right)$ is uniformly bounded in s, the expression follows if the transversality condition $\lim_{T\to\infty} \prod_{s=0}^{T-1} \frac{1}{1+r_{t+s}} m_{t+T} = 0$ is satisfied. Consumer optimization implies

$$
\lim_{T\to\infty} \prod_{s=0}^{T} \left(\frac{1}{1+r_{t+s}} \right) \frac{A_{t+T+1}}{p_{t+T+1}} = \lim_{T\to\infty} \prod_{s=0}^{T} \left(\frac{1}{1+r_{t+s}} \right) \frac{A^*_{t+T+1}}{p^*_{t+T+1}} = 0.
$$

Since $\frac{A_{t+T+1}}{p_{t+T+1}}$ and $\frac{A^*_{t+T+1}}{p_{t+T+1}}$ can be re-written as

$$
\begin{aligned}
\frac{A_{t+T+1}}{p_{t+T+1}} &= (1+r_{t+T}) \left[(1-\delta_{t+T}) m_{t+T} + (b_{Ht+T} + b_{Ft+T}) \right] \\
\frac{A^*_{t+T+1}}{p_{t+T+1}} &= (1+r_{t+T}) \left[\left(1-\delta^*_{t+T}\right) m^*_{t+T} + \left(b^*_{Ht+T} + b^*_{Ft+T}\right) \right],
\end{aligned}
$$

where $b_{Ht} = B_{Ht}/p_t$ and $b_{Ft} = B_{FT}/p^*_t$, the transversality conditions on A_t and A^*_t are equivalent to

$$
\begin{aligned}
\lim_{T\to\infty} \prod_{s=0}^{T-1} \left(\frac{1}{1+r_{t+s}} \right) \left[(1-\delta_{t+T}) m_{t+T} + (b_{Ht+T} + b_{Ft+T}) \right] &= 0 \\
\lim_{T\to\infty} \prod_{s=0}^{T-1} \left(\frac{1}{1+r_{t+s}} \right) \left[\left(1-\delta^*_{t+T}\right) m^*_{t+T} + (b_{Ht+T} + b_{Ft+T}) \right] &= 0.
\end{aligned}
$$

Adding these last two equations and using the bond market equilibrium condition yields

$$
\begin{aligned}
\lim_{T\to\infty} \prod_{s=0}^{T-1} \left(\frac{1}{1+r_{t+s}} \right) \left(m_{t+T} + m^*_{t+T}\right) = {} & \lim_{T\to\infty} \prod_{s=0}^{T-1} \left(\frac{1}{1+r_{t+s}} \right) \cdot \\
& \left[\left(\delta_{t+T} m_{t+T} + \delta^*_{t+T} m^*_{t+T}\right) + \right. \\
& \left. \left(b^g_{Ht+T} + b^g_{Ft+T}\right) + \left(b^{g^*}_{Ht+T} + b^{g^*}_{Ft+T}\right) \right].
\end{aligned}
$$

Neumeyer, Pablo A. (1993), "Currencies and the Allocation of Risk: The Welfare Effects of a Monetary Union," manuscript, University of Southern California, 1993.

Obstfeld, M. (1981), "Capital Mobility and Devaluation in an Optimizing Model with Rational Expectations," *American Economic Review*, 71: 217-21.

Obstfeld, M. and K. Rogoff (1995), "Exchange Rate Dynamics Redux," *Journal of Political Economy,* 103 (June): 624-60.

Patinkin, D. (1965), *Money, Interest and Prices,* 2nd Ed., New York: Harper and Row.

Pigou, A. (1943), "The Classical Stationary State," *Economic Journal*, 53:343-51.

Rodriguez, Carlos Alfredo (1979), "Short- and Long-Run Effects of Monetary and Fiscal Policies under Flexible Exchange Rates and Perfect Capital Mobility," *American Economic Review*; v69 n1 (March), pp. 176-82.

Rose, A., and J. Yellen (1989), "Is there a J-Curve," *Journal of Monetary Economics*, 24:53-68.

Sidrauski, M. (1967), "Rational Choice and Patterns of Growth in a Monetary Economy," *American Economic Review*, 57 (2):534-44.

Stockman, A. (1980), "A Theory of Exchange Rate Determination," *Journal of Political Economy,* 88 (August), 673-98.

Svensson, L. E. O., "Trade in Nominal Assets: Monetary Policy, and Price Level and Exchange Risk," *Journal of International Economics*, February 1989, 26, 1-28.

Woodford, M. (1995), "Price Level Determinacy without Control of a Monetary Aggregate," *Carnegie-Rochester Conference Series on Public Policy,* 43 (December):1-46.