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A General Theory of Sovereign Debt Valuation

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Sovereign Debt Valuation

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Summary

This paper provides a new framework for the valuation of sovereign debt, when the risk of default is significant. We show that the cashflows of sovereign obligations are the equilibrium outcome of the strategic interaction between borrowers and lenders
Its main characteristics are:

i) Debt prices are a function of the payment history, because the market learns about the borrowers objective function through its payment record, .making this model the first one in which sovereign debt prices are endogenously determined by the credit history .

ii) The incentives for repayment are both reputation and penalties simultaneously. This contrasts with the literature on sovereign debt bargaining, where the authors chose either positive (market participation) or negative (punishment) incentives for repayment.

iii) This is done in the context of asymmetric information about the debtor government's resources level and strategic decisions (type).

iv) A main result of this paper, the Valuation Theorem, is the derivation of the (empirically well documented) negative effect that current and previous defaults have on the sovereign debt claim's value.

v) The set of a Perfect Bayesian Equilibria is characterized. The conditions for the uniqueness of equilibrium are stated. The conditions under which the equilibrium is either separating or pooling are characterized.

vi) A time series cross sectional analysis is performed, where payment behavior is the independent variable in a country specific variable intercept model. The model is shown to account for 86 % of the price variation in a sample of the major Latin American sovereign debtors , including Argentina, Brazil and Mexico, during the late 1980's

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1 Introduction

The problem of developing a pricing theory for sovereign debt has proved to be elusive when the risk of default is significant. The non occurrence of sovereign defaults in the more advanced economies since the 1930's, when several European countries defaulted on their payments, provides the basis for the usual practice in the financial literature of equating the default risk of sovereign debt to zero. Nevertheless, the increasing ratio of national debt to GNP in several industrial countries raises the possibility that investors might start to consider these obligations as subject to non-negligible default risk. The LDC's sovereign debt crises in the past decade provides an excellent example to study the effects of defaults on the pricing of sovereign debt. LDC's secondary market debt prices in the eighties show a pattern of strong deviations from face value. Previous theoretical work on the issue fails to explain these price movements. In particular, for some debtors there is a striking parallel (figures 1, 2, and 3) between debt market prices and countries payment behavior that suggests a strong pattern of influence of the claims' payment history on pricing. It is this fact which we seek to explain. To study this pricing relationship we construct a model in which there is uncertainty both about the level of foreign exchange resources and about the debtor's strategic behavior (type). We include both sources of uncertainty because most of the large price variations cannot be claimed to result solely from large movements in the countries' fundamentals, since in most cases, these fundamentals (such as aggregate product) change relatively little over the period during which the large price changes are observed.

In the next three graphs, figures 1, 2 and 3, we show the secondary market values of claims and the series of payments for a group of Latin American countries during the late 1980's. The horizontal axis corresponds to the years 1985 to 1990. On the vertical axis, the figures provide two sequences of values, for each country considered: One is the (yearly) average of secondary market prices for its claims, as a percentage of face value (lines with empty marks). The other one is the ratio of realized payments over realized payments plus arrears for each given year (lines with black marks). This last series provides a measure of the debtor's payment behavior. The empirical regularity that this paper explains is the remarkable parallel, for most observations, between the movements of the price values and the movements of the payment ratios. This empirical regularity is explained by the Valuation Theorem (section 4) that derives the effects of payment behavior on claims price.

Figure 1

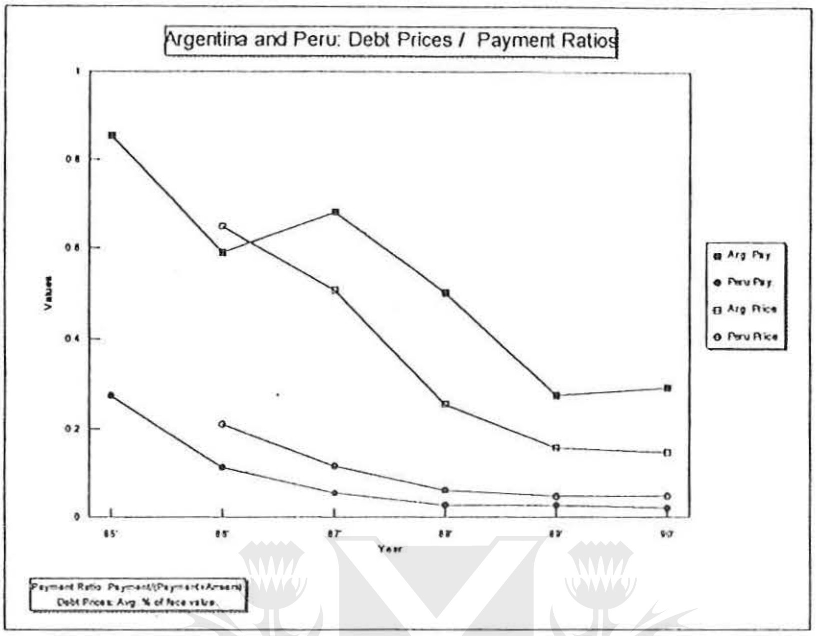


Figure 2

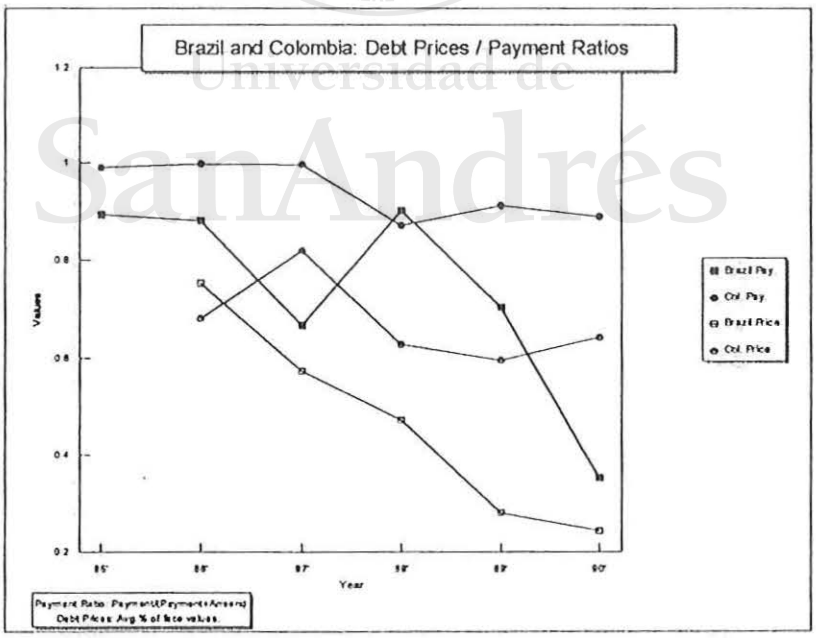
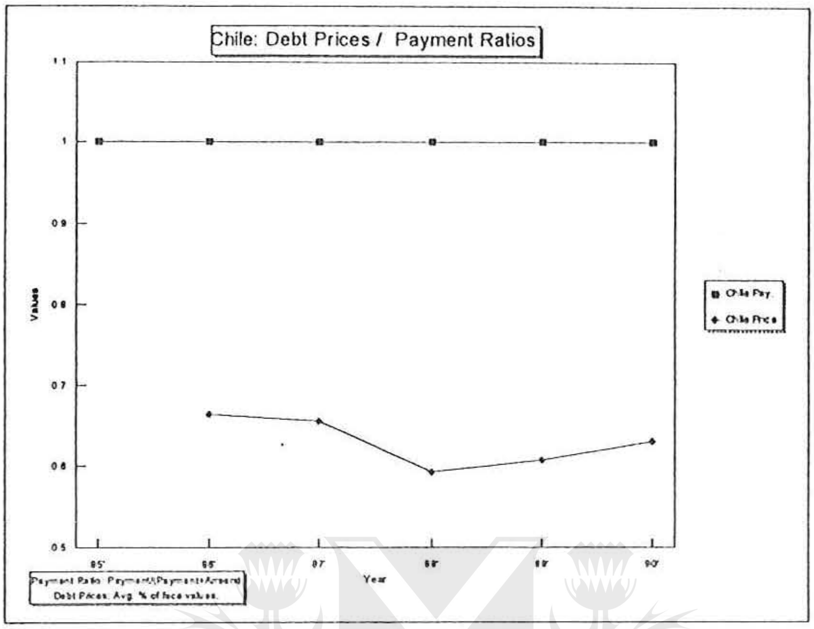


Figure 3



Our model explains the sovereign claim's price movements as arising from the learning process of market investors. We assume that there are two types of debtor governments, and the creditor does not know the government type when he makes the loan. At that time, the lender estimates the borrowers' type through a probability distribution, based on the past history of the country. We call μ_0 to the unconditional probability that the debtor is of type 1. The type 1 government is the one that will default if and only if it does not have sufficient resources; that is, only if it is experiencing the bad state of the world. The type 2 will similarly default when the resources are insufficient, but may also default for purely strategic reasons, in particular the interest to push up disposable income. The realization of the country's foreign exchange resources is assumed to be the private information of its government, allowing the type 2 government to make a strategically motivated non payment, which may be concealed as professed illiquidity. Non payments benefit the strategic governments since they increase current consumption, but they also diminish their reputation and may trigger punishments. Monitoring the state is costly for the creditor, and since monitoring may often verify that a bad state of resources actually did prevail, monitoring may not necessarily reveal the debtor's type. Therefore, the creditor's uncertainty about the debtor's government type may last for extended periods.

To study balance of payments crisis, Krugman [1979] assumes incomplete knowledge on the part of the investors about how much of its foreign resources the government is willing to use to defend a fixed exchange rate. Similarly, we assume imperfect information on the part of the lender about how much of its foreign exchange resources the government is willing to use to repay its debt.

If the country's type were known, its expected payments would be a well defined function of such exogenous stochastic processes as the country future net exports. The value of a loan to a known type of debtor could be determined independently of the payment history. Under the imperfect information scenario about the country's type that our paper introduces, and since the two types of governments have different payment behaviors, a sovereign debt claim's expected payments are a weighted average of the equilibrium payments for each of the debtor types. The weight coincides with the market assessment of the probability that the country is of type 1. This probability depends on the credit's history, and is revised every period following a bayesian rule, whose updating is endogenous to the payment policy of the country.

Our work is related to three other major literatures:

The applied game theory literature on sovereign debt bargaining:

The starting point of these models concerns the identification of the debtors incentives for repayment. Two approaches, "sticks" and "carrots", have been dominant. The models in these area have chosen either negative incentives, typically the threat to disrupt the country's trade flow (e.g. Bulow and Rogoff, [1989]) or positive incentives, typically the debtors access to international credit markets. To our knowledge, the model of this paper is the first one to allow for the two types of incentives, punishment and access to financial markets, to coexist simultaneously.

Bulow and Rogoff [1989] analyze the incentives resulting from punishment: They argue that a country can suffer severe losses following a default if its foreign creditors are able to seize a share of its commercial flows. Our model incorporates a punishment option because the threat of disrupting the country's economic activity is a significant bargaining chip for the creditors. Since the history of the eighties shows that, even for the countries with very low repayment ratios, direct seizure of goods did not happen, we recognize the need for a broader interpretation of the notion of punishment, as losses to the debtor countries welfare resulting from the creditor punishment actions. These would include trade sanctions, denial of trade credits, lobbying for the denial or toughening of lending conditions by international agencies, etc.

The other source of repayment incentives commonly quoted is access to the international capital markets by the debtor countries. Following Rubinstein's [1982] concept of a perfect equilibrium bargaining game, Fernandez and Rosenthal [1990] studied a multiperiod bargaining game in which the reason for repayment is a bonus that the debtor receives once its obligations are repaid in full. The bonus represents renewed access to international capital markets. Our model also allows a major role for market participation incentives; moreover, by introducing the debtor's reputation as an argument for his indirect utility function we model the debtor as caring about his credit rating at every period, rather than caring only about a positive payoff to be received at the end of the repayment period, as in Fernandez and Rosenthal [1990].

We incorporate the debtor's reputation as an argument for his utility because if the repayment period is long compared to the government's tenure in office, concern for the current reputation will be a much stronger enticement to repayment than a potential return to international financial markets once the debt is fully repaid. This is so because reputation affects the spread that the fiscal authorities pay in short term trade loans and in the domestic money markets: The LDC Debt Crisis in the eighties showed that distressed countries' governments continued to be active borrowers. When international long term loans were rationed, debtors continued to borrow at a substantial level in the domestic money markets and the international trade financing markets, and the risk premium that they paid over international rates such as LIBOR appears to be related to their credit standing (see Khor and Rojas Suarez [1991] for an empirical analysis of the Mexican case). It is immediate that debtor countries will care about their current credit standing, rather than just about the one they will have at the distant end of the repayment period for the distressed long term international loans.

In the previous models, a country's available resources for repayment were public knowledge, and the parameters of the repayment incentives were exogenous to its credit history. The market value of a claim, therefore, was not a function of the payment history, because this history did not affect either the expected resources, or the value of the country's' welfare losses due to disrupted trade (Bulow and Rogoff), or the value of the bonus after repayment (Fernandez and Rosenthal). Our work, by introducing the investors' uncertainty and subsequent learning process about the debtors strategic type is the first one to make debt prices history dependent.

The Industrial Organization literature:

The study of reputation in a context of strategic interaction was initiated by the pioneering work of Kreps and Wilson [1982] and Milgrom and Roberts [1982]. These authors study the effect of a monopolist's reputation on potential entrants. In their work, the incumbent's threat to act as a (seemingly irrational) predator has profound effects on the entrant's equilibrium behavior[2]. We have a different modeling goal than these authors, since our interest is in the dynamic effect of reputation on debt pricing, rather than characterizing the price wars of an oligopolistic market. Our work defines a reputation concept that is related to the one used in these papers because reputation represents the public's belief about the country's government type. The difference with the Milgrom and Roberts' concept is that in our paper the strategic debtor, in the context of imperfect information about his type, will look for the <u>reputation of being cooperative</u> (rather than a predator) as a means to enhance its creditworthiness. Furthermore, we investigate reputation effects in a game where there are two long lived opponents, rather than one long lived player facing many short lived ones.

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[2] In their words " Once the common knowledge assumption that accommodation is the best response to entry ...is relaxed... the lack of complete information gives rise to reputation possibilities.... Practicing predation now gives one a reputation as a predator which is valuable.." Milgrom and Roberts[1982] , pag 390.

Models of corporate debt valuation :

In the case of corporate debt, the moral hazard problem concerns the choice of investment projects by the debtor: Since the debt collateral is affected by the investment projects chosen, different investment decisions result in different debt values. In this context, D. Diamond [1989] analyzes the effect of reputation in a framework of imperfect information about the borrower. Reputation is seen to mitigate the borrower's incentive to choose excessively risky projects.
A major difference between sovereign debt and corporate debt is that in the sovereign case explicit collateral is usually nonexistent or only covers a minor fraction of the outstanding obligation. Furthermore government expenditures typically do not take the form of investments in assets that the creditor could seize through litigation. The moral hazard problem has therefore a different nature when the debt of a sovereign state is under consideration: Since collateral is not significant and seizing governments assets is costly, the debtor governments may have room to default on their debts based on strategic considerations. The reputation building mechanism in the sovereign debt case is therefore provided by the payment process itself, unlike the corporate debt case where reputation is obtained through the selection of safe investment projects: While the corporate debt value is endogenous to the investment decisions, we model the sovereign debt value as endogenous to the payment decisions.

An outline of the paper is as follows: Section 2 introduces the incomplete information model, including the players' payoff functions and the rule to update beliefs. In section 3 we show the existence and uniqueness of a perfect bayesian equilibrium for the game. The conditions under which there is a pooling equilibrium, or a separating equilibrium, are stated. In section 4, we prove the Valuation Theorem, that shows the negative effect of arrears on the sovereign claims' value. Section 5 discusses the time series cross sectional model that corroborates our theoretical work. Section 6 concludes.

2 The Model

2.1 Overview

We start with a loan contract in place, that stipulates coupon payments of c_t dollars in periods 1 through T and the repayment of the principal P in period T. We imagine this contract as having been issued in the past, at a time when the creditors were ignorant of the borrower's payment behavior type. The contract is also assumed to be in distress, meaning that some coupon payments have been missed in the past. Therefore, no new long term voluntary lending is currently received by the country, which hence has to rely on its own foreign currency resources to meet the coupon payments.

The countries' resources for repayment, S_t, consist of its export revenues, x_t, minus a minimum level of imports, m_t. We model this with the multinomial random variable S_t which can take the values: {0, c, 2c, 3c, ...,nc}, with $n \geq 1$.

The probability distribution of S_t is known by all players, but its current realization is only observed by the government. We allow for serial correlation in the realizations of S_tThis provision captures the possible tendency for the prices of traded goods to persist for more than one period at high, or low, levels. . Before making the payment decision, the debtors' administration observes the realization of S_t. This information is not shared with its creditors. The private information assumption means that the debtor government has proprietary knowledge about the level of required imports

Following Harsanyi (1967-68) we transform a game of incomplete information (in which one player, the lender, ignores the payoffs of his counterpart) into a game of imperfect information. In the transformed game, the lenders' incomplete information about the debtors' payoff function is converted into imperfect information about a move by nature, which would determine the debtor's type. In this framework the beliefs μ_t coincide with the (assessed) probability that this nature's move determines that the debtor is of type 1, and $1-\mu_t$ is the probability that nature's move provides the debtor a type 2.

There are several stages in each period of the game. The first one is the random realization of the level of resources:

For each period t, define:

Good State	$S_t \geq c$	(1)
Bad State	$S_t = 0.$	(2)

where S_t is the stochastic value of resources.

Actual Payments	a_t^c
Vector of payment payoffs	$\{0, c\}$
Required Imports:	$\bar{m}_t$
Imports' Consumption:	m_t

Since we assume that currency reserves are always null[3], we obtain the following condition, that equates exports earnings to imports plus payments to foreign creditors:

Balance of Payments Equilibrium: $x_t = m_t + a_t^c$ (3)

[3] Allowing the currency reserves to be variable would make this model less tractable, requiring the debtor to allocate exports earnings between imports consumption and reserves accumulation.

There are two types of governments: The type 1 or cooperative government honors the debt contract, and therefore pays the full coupon if the resources are in a good state. In bad states, he pays 0. The type 2 , or strategic government will choose its payment policy to balance the propensity to increase imports consumption against its interest to keep a positive reputation. Although we do not explicitly use the effect of creditworthiness on interest rates in this model, we assume that the debtor's reputation determines the premium over international interest rates that the country pays both on short term commercial loans and in the domestic money markets.[4]

We define the updated probability that the country is of type 1:

$$\mu_t:$$

The type 2 government utility function at period t is : $u(\mu_t, m_t)$, and it is strictly increasing in its two arguments, imports consumption and reputation. Because the resources' value S_t is stochastic and the probability of the bad state is strictly positive, both types of debtors will incur in partial non payments due to liquidity problems[5]. Moreover, as S_t is private information, the type 2 debtors will have the possibility of concealing strategic non-payment as professed illiquidity.

2.2 Full information example

If the country's government type were known, the claims' value would be exogenous to the credit history: A type 1 country will pay $s_t = c_t$ every time it has enough resources. In bad states, it pays 0. As the Probability of $(S_t \geq c_t)$ can be estimated independently of the credit history, then the risk neutral value of this loan, V_t^1 will be well defined.

The optimal strategy for a known type 2 country depends on the payoff function values: In case of non payment at period t, the lender might recover goods worth "b" from the debtor. This threat is credible (perfect) if the recovered wealth, b, exceeds in value the recovery costs , k, for the lender. If that is the case, repaying the coupon c is optimal for a known type 2 debtor if the value of wealth b exceeds in value the coupon repayment. Otherwise, the known type 2 debtor does not repay. Moreover, if the threat of action by the lender is not credible (b<k), the type 2 debtor will not repay its coupon. In this case, there would be no reputational concerns since the debtor is known to be of type 2. The value of a loan to a type 2 country is denoted V_t^2.

For a given contract and for each known type of debtor, the debt contract value at time t, V_{it} i=1,2 is a function of exogenous variables.

[4] Khor and Rojas Suarez [1991] have shown that the behavior of interest rates in Mexico was strongly influenced by international perceptions of Mexico's creditworthiness, as represented by the yield of its sovereign external debt.

[5] Defined as the problem faced by an economy, faced with a currently poor outcome for its currency resources while, at the same time, its foreign credit is rationed.

2.3.1 Incomplete information model

We construct a model with two players : A bank (the lender) and the debtor country government. There is uncertainty about the country's government type, which can either be strategic or non strategic one.

There are market beliefs $\mu(\theta|h^t)$ that influence the market value of the claim. h^t is the history of payments up to period t. At the beginning of the credit history, the known proportion of policy-makers of type 1 in the pool provides the initial value μ_0 = unconditional probability that the country is of type 1.

The market assessment, at time t of the probability that the government is of type 1, given the previous period assessment and the country's action at period t is given by:

$$\mu_{t-} = \mu_{t-}(\theta = 1) = \mu(\theta = 1|\mu_{t-1}, a_t^c) \qquad (4)$$

The market assessment at time t of the probability that the government is of type 1 , given the previous assessment and the two players actions in this period is given by:

$$\mu_t = \mu_t(\theta = 1) = \mu(\theta = 1|\mu_{t-1}, a_t^c, a_t^b) \qquad (5)$$

The lenders' actions influence μ_t since the discovery of either the occurrence of a bad state or a strategically motivated non payment convey information to the markets pricing of the sovereign claim.

<u>The credit value at time t</u> is equal to the discounted expected stream of payments:

$$V_t = \mu_t V_t^1 + (1 - \mu_t)V_t^2. \qquad (6)$$

where V_t^i is the value at time t of the claim for a type i government.

In the cooperative government case, the value V_t^1 is a function of exogenous variables, while for the strategic debtor, V_t^2 is given by the (discounted sum of) expected payments under the equilibrium strategies, which are found in section 3. At each period, the country's actions will trigger a bayesian updating of the market's beliefs, as will be discussed in section 2.3.3.

The country's government resources S_t have a publicly known distribution. Define:
Probability of a good state

$$P(S_t = Good|h^{t-1}) = P(S_t \geq c_t|h^{t-1}) = p_t. \quad . \qquad (7)$$

where c_t is the coupon payment due at period t and h^{t-1} is the history of payments up to period t-1. Notice that p_t is a function of the game's payment history: The occurrence of good (or bad) states can, in general, exhibit serial correlation. Nevertheless, because the true states of the world are not observable for the bank, it will base its inferences about p_t on a probability distribution over the vector of past histories of the world, conditional on the payment history. [6]

The banks probability distribution on the state of the resources, given the payments is :

$$P(S_t = Good|\ \ a_t^c = c)=1. \tag{8}$$

$$P(S_t = Good|\ \ a_t^c = 0) = P((S_t = Good) \cap (a_t^c = 0))/P(a_t^c = 0)= \tag{9}$$

$$(1-\mu_{t-1})(1-q_t)(p_t|h^{t-1})/[\mu_{t-1}(1-(p_t|h^{t-1})) + (1-\mu_{t-1})((1-(p_t|h^{t-1}) + (p_t|h^{t-1})(1-q_t))]$$

where h^{t-1} is the game's history of payments until period t-1.

The actions of the players will be denoted a_t^c for the country and a_t^b for the bank .

The action space for the debtor is:

$$a_t^c \in \{0, c_t\}$$

These two alternative payment levels reflect the possible realizations of foreign exchange resources.

The lender takes a decision on penalties, at every period. The punishment alternative includes any punishment action that the lender might take. They could include lobbying for trade sanctions, denials of trade credits, etc. Our results are not affected by the nature of these sanctions , they only depend on the realized losses for the debtor if these measures are applied, and the cost for the lender of enforcing them. The application of penalties triggers a mechanism that allows the debtor to seize part of the unpaid coupon, in case that the debtor is holding this value. This recovered value is denoted b in our payoff matrix. Nevertheless, these sanctions are costly to enforce: When punishing, the lender incurs a cost that we denote k. Moreover, if the coupon has actually been paid, the sanctions will not add any value to the creditor. And if the state of resources was indeed bad, the application of penalties cannot recover a value that was not in the debtors' hands in the first place.

The action space for the lender is:

$$a_t^b \in \{\text{ Penalize , not penalize }\}$$

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6 A coupon payment always signals a good state. A non payment will necessarily signal a bad state of the world if a pooling equilibrium exists. If the equilibrium is separating, a non payment will assign a positive probability to two different events. One is that the state is good and the debtor's type 2. The other is that the state is bad.

Period t information nodes

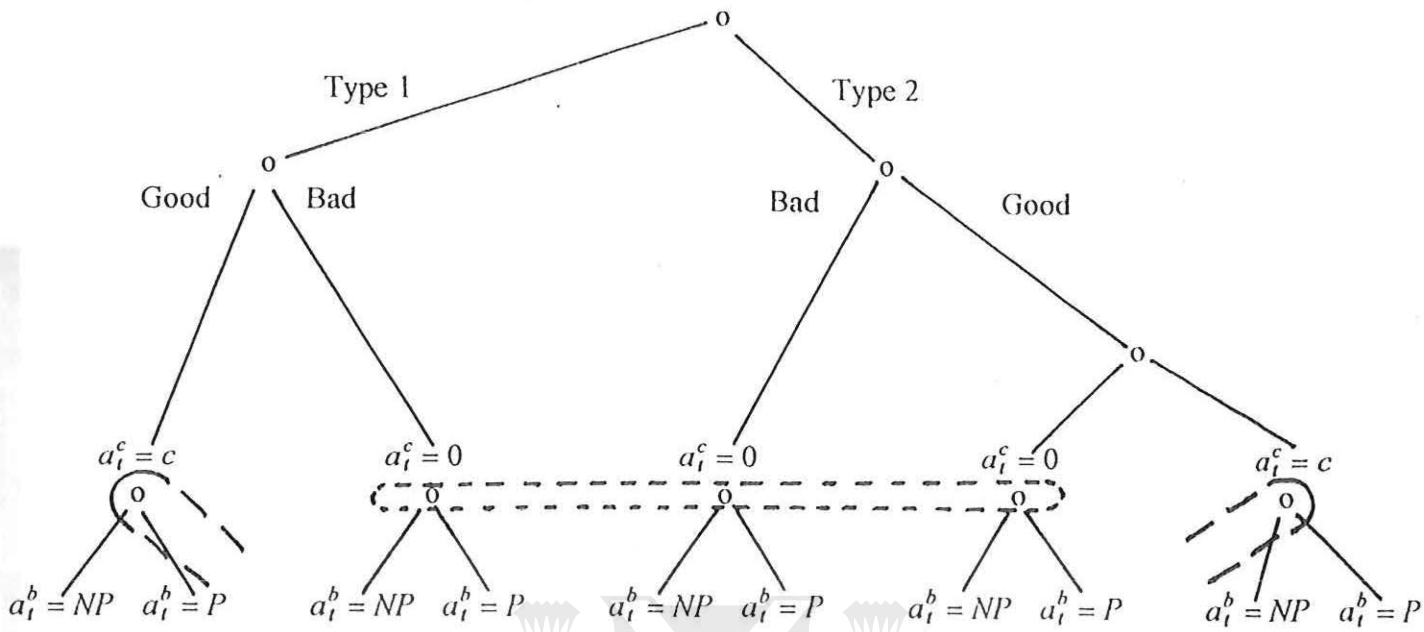

Figure 4: Period t payoffs

Good State

	$a_t^b = NP$	$a_t^b = P$
$a_t^c = c$	$[(\mu_t^1, x_t - c), c]$	$[(\mu_t^1, x_t - c), c - k]$
$a_t^c = 0$	$[(\mu_t^3, x_t), 0]$	$[(\mu_t^4, x_t - b), b - k]$

Bad State

	$a_t^b = NP$	$a_t^b = P$
$a_t^c = 0$	$[(\mu_t^3, x_t), 0]$	$[(\mu_t^2, x_t), -k]$

Where μ_{t-1}: Investor's assesment at t-1 that the policymaker is of type 1.
μ_t^i : i=1...4: The possible reputational outcomes, conditional on period t beliefs.
a_t^b : Lender decision on punishment.
a_t^c: Payments at period t.
b: Payoff to the lender for punishing if the state of resources is good
k: Cost that the lender incurs by punishing.

In these two tables, the first two values are the debtor's conditional reputation value and his imports, and the third is the lender's payoff at period t. To illustrate this, consider the case in which the resources are in the good state, the debtor pays the coupon worth c at period t, and there is no punishment action. Then, the arguments of the debtor's utility are his reputation, μ_t^1, and his imports consumption, $x_t - c$, and the lender's payoff is given by the payment that he receives, which is worth c.

2.3.3 Conditional Movements of Beliefs

At each period, after observing the borrowers' payments, μ_t, which is the markets' belief about the policy-maker type is updated following a bayesian rule. Its computation is shown as part of the proofs of Lemma 1, 2, 3 and 4. The μ_t^i , i=1, 2, 3, 4 are the possible values of reputation at the end of period t, conditional on the reputational value at the end of period t-1, μ_{t-1}. In what follows, the subindex t- refers to the moment after the lender's move and before the borrowers move. The subindex t denotes the time at which both agents have made their moves. From Figure 4, we observe that the period t players' payoffs, when the creditor decides to penalize the debtor's non payment, depend on whether the state of resources was good or bad. Therefore, the lender's decision to penalize will reveal the state of the resources, and may, in some cases, reveal the debtor's type, since only the type 2 defaults when the resources are in a good state.

There are two possible reputational outcomes of the debtors' moves:

The updated assessment that the policy-maker is of type 1 when full payment is made in period t is defined as:

$$\mu_{t-}^1 = \mu_{t-}(\theta = 1|\mu_{t-1}, a_t^c = c_t, F_t^b = c_t) = \mu_t(\theta = 1|\mu_{t-1}, a_t^c = c_t, F_t^b = c_t) = \mu_t^1 \qquad (10)$$

where F_t^b is the banks' payoff at period t. Here, the reputational outcomes at times t - and t are the same, since the lender does not have incentives to punish.

The updated assessment that the government is of type 1 when no payment is made at t is given by:

$$\mu_{t-}^2 = \mu_t(\theta = 1|\mu_{t-1}, a_t^c = 0) \qquad (11)$$

These updated beliefs input the bank's punishment decision at period t. If the country has not paid in period t, once the banks decision on sanctions is taken, and its outcome is observed, a new update of beliefs takes place:

The updated assessment that the government is of type 1 when no payment is made at period t, the banks decides to penalize and it receives a payoff of -k that corresponds to the state of resources being bad is given by:

$$\mu_t^2 = \mu_t(\theta = 1|\mu_{t-1}, a_t^c = 0, a_t^b = A, F_t^b = -k) \qquad (12)$$

The updated assessment that the government is of type 1 when no payment is made at period t, and the banks decides not to penalize is defined as:

$$\mu_t^3 = \mu_t(\theta = 1|\mu_{t-1}, a_t^c = 0, a_t^b = NA, F_t^b = 0) \qquad (13)$$

The updated assessment that the government is of type 1 when no payment is made at period t, the banks decides to penalize and receives the payoff b-k that corresponds to the good state of resources, which implies that the country has to be of type 2, is given by:

$$\mu_t^4 = \mu_t(\theta = 1 | \mu_{t-1}, a_t^c = 0, a_t^b = A, F_t^b = b - k) = 0 \qquad (14)$$

2.3.4 Endogenous Payoffs Values

Our model introduces a new game theoretic development: Since the updated beliefs, μ_t, are endogenous to the equilibrium strategies of the players, and this beliefs are an argument of the type 2 debtor's utility function, the elements of the payoff matrix become endogenous to the equilibrium strategies of the players. This defines a payoff object that is different from the usual payoff matrix, because the value of the elements of the usual payoff matrices are independent from the players' policies. Therefore, in our context, expected utility maximization implies maximizing expected value over the probability distribution on payoff elements that are, in itself, affected by the strategy chosen. Consequently, the mixing strategies operator is non linear.

3 Optimal Policies

3.1 Agents' strategies review

Type 1 government's payments are: $a_t^c = c_t$ if $S_t > c_t$ and
$a_t^c = 0$ if $S_t = 0$

Since the Type 1 debtor pays every time that he is in a good state, the interesting thing to study is the behavior of the Type 2 debtors.

Type 2 government's payments result from maximizing the discounted sum of expected utility. At any period, the utility obtained is an increasing function in the level of imports consumption and the current level of reputation : $u_t(\mu_t, m_t)$. From the Balance of Payments Equilibrium, Equation 3, we know that $m_t = x_t - a_t^c$. Therefore, the type 2 debtor's utility can be written $u_t(\mu_t, x_t - a_t^c)$.

At every period, the history of the game moves is summarized by the value μ_t, which is the country's current reputation, and can take values between zero and one. We will call the type 2 debtor strategies at period t, σ_t^c. We call q to the implied probability of payment by the type 2 player, when the state of resources is good. We will similarly refer to the lender strategy in period t, as σ_t^b. We call w to the implied probability of payment by the type 2 player, when the state of resources is good

The bank's continuation payoff , $EB(\sigma_t^b, \sigma_t^c; \mu_{t-1})$, as a function of the players strategies σ_t^b and σ_t^c consists of two parts. The first one is the expected payments at period t by the debtor, net of punishment costs for the lender, if incurred. The second one is the discounted expected flow of repayments, from period t+1 until period T, conditional on the new state vector μ_t. This new state vector is the one reached when strategies σ_t^b and σ_t^c are implemented, and the state vector μ_{t-1} has been inherited at period t.

The type 2 government's continuation payoff, $EC(\sigma_t^b, \sigma_t^c; \mu_{t-1})$, as a function of the players strategies σ_t^b and σ_t^c similarly consists of two parts. The first term is the expected utility of the debtor, at period t. The second one is the discounted expected stream of future utility from period t+1 until period T, conditional on the new state vector μ_t.

3.2 Perfect, Bayesian Equilibrium Definition:

Equilibrium is defined in this model as a set of strategies $\hat{\sigma}_t^c$ and $\hat{\sigma}_t^b$ such that the conditions P and B hold:

P) (Perfectness) $(\hat{\sigma}_t^c, \hat{\sigma}_t^b) \in ArgMax \;\; EC_t(\sigma_t^c, \sigma_t^b \;\; ; \mu_{t-1})$ subject to $\sigma_t^b \in ArgMax \;\; EB_t(\hat{\sigma}_t^c, \sigma_t^b \;\; ; \mu_{t-1})$

B) (Bayesian Updating) The market's belief on the government type is updated according to the Bayes' Rule, yielding on of the two possible beliefs values after the debtor's action, μ_{t-}^1 or μ_{t-}^2 and one of the four possible beliefs values, consistent with each equilibrium policy, after both the debtor and the lender have played, which are $\mu_t^1, \mu_t^2, \mu_t^3$ and μ_t^4. All of this beliefs values are conditional on the value received at the beginning of the period t, μ_{t-1}.

We analyze the equilibrium strategies of the lender and the Type 2 debtor . For the Type 1 debtor its equilibrium strategy is trivial, since it always pays if and only if it has resources.. Moreover we consider the debtors strategies when the state of resources is good, since the other case is also trivial, because they cannot pay when they do not have the resources to do so.

3.3 Payoff values relationships:

Before characterizing the game's equilibria, we observe the following relationships among the debtor's utility values:

For a given equilibrium strategy,

$$u(\mu_t^4, x_t - b) < u(\mu_t^3, x_t) \qquad (15)$$

because b>0 and $\mu^4 \geq \mu^3$

Also, if q<1,

$$u(\mu^2, x_t) > u(\mu^3, x_t) \qquad (16)$$

because $\mu^2 > \mu^3$

3.4 Proposition: No Pooling Equilibrium

No Pooling equilibrium can arise, at any period t.

Proof: Suppose, by contradiction, that a pooling equilibrium with q=1 existed at period t. Therefore, if a non payment occurred, the lender would interpret this default as necessarily coming from a bad realization of resources. Consequently, there would be no negative reputational effect ($\mu^3 = \mu^1$) and also no incentive to punish the default. Under this scenario, the debtor has incentives to deviate, defaulting in his current payment, what shows that q=1 cannot be a Nash Equilibrium

We will prove by rolling induction on the number of periods remaining before the last one the following theorem:

3.4 Equilibrium Characterization Theorem:

i) At each period t, either a Mixed Strategy Perfect Bayesian Equilibrium or a Separating Perfect Bayesian Equilibrium may arise.

ii) The equilibrium is unique if the country's continuation payoff $M^c(\mu_t)$ is increasing in μ_t.

iii) For any period t and any value μ_{t-1}, the following value functions are well defined (i.e. they adopt a unique value) under the equilibrium strategies , from period t on:

$$M^c(\mu_{t-1}) = Max_{\sigma^c,\sigma^b} EC_t(\sigma_t^c, \sigma_t^b \;\; ; \mu_{t-1}) \quad \text{subject to } \sigma_t^b \in ArgMax \quad EB_t(\hat{\sigma}_t^c, \sigma_t^b \;\; ; \mu_{t-1})$$

$$M^b(\mu_{t-1}) = Max_{\sigma^b} \quad EB_t(\hat{\sigma}_t^c, \sigma_t^b; \mu_{t-1}).$$

Next, we will characterize the Set of Perfect Bayesian Equilibria. We will do this by backward induction: First we study the set of equilibria for the last period of the game, T, and then, assuming that the equilibrium policies are defined from period t+1 to T, we characterize the set of equilibria for period t.

Period T:
We first discuss the lender's actions at the last period: Conditional on the coupon payment at T, the lender compares the outcomes under no action and action, c and c-s respectively. Therefore, the lender always chooses not to punish when payment has been received.

If no payment is made at period T, the lender's expected payoff for penalizing is:

$\mu_{T-}^2(-k) + (1-\mu_{T-}^2)[(1-p)(-k) + p(1-q)(b-k)]/[1-p+p(1-q)]$, or equivalently,

$$\mu^2_{T-}(-k) + (1-\mu^2_{T-})([-k+p(1-q)]/[1-p+p(1-q)])b \tag{18}$$

and the payoff for not punishing is zero. Therefore, conditional on non payment at T, the lender chooses his punishment policy, $w \in [0,1]$, to maximize the following expression:

$$w\{-k+b \quad [(1-\mu^2_{T-})p(1-q)]/[1-p+p(1-q)] \quad \} \tag{19}$$

The debtor chooses his policy to maximize expected utility. The perfectness condition requires that the debtor chooses the strategy that maximizes his expected utility, taking into account the lender's optimal strategy and also considering his (the debtor's)action's influence on the other player's moves. The debtor accomplishes this by:

a) Incorporating the effects of coupon payment, or non payment, on the bayesian updating of beliefs, that will become either μ^1_{T-} or μ^2_{T-}.

b) Incorporating the lender's optimal strategy in his (the debtor's) expected utility maximization

To characterize the set of Nash Equilibria at period T, we analyze which values of the debtor repayment policy q can make part of an equilibrium pair. We have just seen that q=1, Pooling, is not a possibility.

Mixed Strategy Equilibrium:

The necessary and sufficient conditions for the existence of an equilibrium in nondegenerate mixed strategies at period T are:

Under $(\hat{q}, \hat{w})$,

$$u(\mu^3_T, x_T) \geq u(\mu^1_T, x_T - c) \geq u(\mu^4_T, x_T - b) \tag{21}$$

In this case, the strategies σ^c and $\sigma^b$are characterized by the values $(\hat{q}, \hat{w})$, where $\hat{q}$ is the probabilty value that makes the equation (18) equal to zero, and $\hat{w}$ is the probability value that makes the following equation (22) equal to zero:

$$u(\mu^1_T, x_T - c) - wu(\mu^4_T, x_T - b) - (1-w)u(\mu^3_T, x_T) \tag{22}$$

Proof: Trivial, since each player randomizes between pure strategies if and only if he is indifferent among the outcomes of these pure strategies.

Separating Equilibrium

The necessary and sufficient condition for the existence of a separating equilibrium in which the Type 2 debtor never repays at period T is:

Under (q=0,w=1),

$$u(\mu_T^1, x_T - c) < u(\mu_T^4, x_T - b) \qquad (23)$$

Proof: This says that (q=0, w=1) will be the equilibrium strategy if and only if the debtor extracts more utility by not repaying, which is exactly the rationale for the pure strategy separating equilibrium.

Uniqueness:

We prove below that the mixed strategy equilibrium and the separating equilibrium cannot arise simultaneously:

Proposition : The conditions (21) and (23) cannot hold simultaneously.

Proof: Suppose that (21) holds. In particular, there exists an equilibrium repayment policy, $0 < \hat{q} < 1$ such that

$$u(\mu_T^1(\hat{q}, \hat{w}), x_T - c) \geq u(\mu_T^4(\hat{q}, \hat{w}), x_T - b).$$

But $\mu^1(q=0, w=1) > \mu^1(\hat{q}, \hat{w})$, since the positive reputational effect of a payment is stronger in the separating equilibrium. Therefore,

$$u(\mu^1(q=0, w=1), x_T - c) > u(\mu^1(\hat{q}, \hat{w}), x_T - c) \geq u(\mu_T^4(\hat{q}, \hat{w}), x_T - b) = u(\mu_T^4(q=0, w=1), x_T - b)$$

where the last identity appears because μ^4 is zero, independently of the player's strategy. Consequently, condition (23) cannot hold simultaneously with condition (21).

We have proved above that either a non-degenerate mixed strategy equilibrium or a pure strategy separating equilibrium can arise at period T, and that the two equilibria cannot coexist. Therefore, the expected payoffs under equilibrium policies define the value functions,

$M^c(\mu_{T-1}) = Max_{\sigma^c,\sigma^b} EC_T(\sigma_T^c, \sigma_T^b \;\; ; \mu_{T-1})$ subject to $\sigma_T^b \in ArgMax \quad EB_T(\hat{\sigma}_T^c, \sigma_T^b \;\; ; \mu_{T-1})$
$M^b(\mu_{T-1}) = Max_{\sigma^b} \quad EB_t(\hat{\sigma}_T^c, \sigma_T^b; \mu_{T-1})$, which proves iii) for period T.

Period t:

Assuming that the equilibrium policies are defined from period t+1 on, we will characterize the set of equilibria at period t. Conditional on coupon payment at t, the lender's continuation outcomes are: $c-k+M(\mu_t^1)$ for penalizing and $c+M(\mu_t^1)$ for not punishing. Therefore he always chooses not acting after payments.

If no coupon payment is made at t, the lender's expected outcome for punishing is:

$$\mu_{t-}[(-k)+M^b(\mu_t^2)]+(1-\mu_{t-})[p(1-q)((b-k)+M^b(\mu_t^4))+(1-p)(-k+M^b(\mu_t^2))]/[1-p+p(1-q)]$$

$$-k+\mu_{t-}[M^b(\mu_t^2)]+(1-\mu_{t-})[p(1-q)(b+M^b(\mu_t^4))+(1-p)(M^b(\mu_t^2))]/[1-p+p(1-q)] \qquad (24)$$

Since the expected payoff for not acting is $M^b(\mu_t^3)$, the lender's chooses his punishment policy $w\in[0,1]$ to maximize:

$$w\{-k+\mu_{t-}M^b(\mu_t^2)+(1-\mu_{t-})[p(1-q)(b+M^b(\mu_t^4))+(1-p)M^b(\mu_t^2)]/[1-p+p(1-q)]-M^b(\mu_t^3)\}$$

As we discussed in the proof for period T, the debtor chooses his strategy to maximize expected utility, incorporating the effects of his payments to update beliefs, and also incorporating the lender's optimal strategy in his expected utility maximization.
We will characterize the different equilibria that can arise at period t:

Mixed Strategy Equilibrium:

The necessary and sufficient conditions for the existence of an equilibrium in nondegenerate mixed strategies at period T are:

Under $(\hat{q},\hat{w})$,

$$u(\mu_t^3,x_t)+M^c(\mu_t^4)\geq u(\mu_t^1,x_t-c)+M^c(\mu_t^1)\geq u(\mu_t^4,x_t-b)+M^c(\mu_t^4) \qquad (25)$$

In this case, the strategies σ^c and σ^b are characterized by the values $(\hat{q},\hat{w})$,where $\hat{q}$ is the probabilty value that makes the equation (24) equal to zero, and $\hat{w}$ is the probability value that makes the following equation (26) equal to zero:

$$(\mu_T^1,x_T-c)+M^c(\mu_t^1)-w[u(\mu_T^4,x_T-b)+M^c(\mu_t^4)]-(1-w)[u(\mu_T^3,x_T)+M^c(\mu_t^3)] \qquad (26)$$

Proof: As in period T, a player randomizes between pure strategies if and only if he is indifferent between these pure strategies

Separating Equilibrium:
The necessary and sufficient conditions for the existence of a Separating Equilibrium in which the Type 2 debtor does not repay in period t are:
Under (q=0, w=1),

$$u(\mu_t^1, x_t - c) + M^c(\mu_t^1) < u(\mu_t^4, x_t - b) + M^c(\mu_t^4)$$

The proof is straightforward, since this is exactly the condition under which the debtor is better off by not repaying.

Uniqueness:

The mixed strategy equilibrium and the separating equilibrium cannot arise simultaneously if the debtors continuation payoff, $M^c(\mu_t)$, is increasing in the value of reputation.
The proof is similar to the one for period T.

The expected payoffs under equilibrium policies define the value functions,

$$M^c(\mu_{t-1}) = Max_{\sigma^c,\sigma^b} EC_t(\sigma_t^c, \sigma_t^b \;;\mu_{t-1}) \text{ subject to } \sigma_t^b \in ArgMax \;\; EB_t(\hat{\sigma}_t^c, \sigma_t^b \;;\mu_{t-1})$$

$$M^b(\mu_{t-1}) = Max_{\sigma^b} \;\; EB_t(\hat{\sigma}_t^c, \sigma_t^b; \mu_{t-1}).$$, which proves iii) for period t.

4 Equilibrium properties:

In the following section we will characterize the properties of the equilibrium found:

4.1 Valuation Properties

In this section, we characterize the dynamic effects of payment behavior on the debt's price.

Lemma 1:

μ_t^1, the updated probability of the country being of type 1 when full payment is made at period t, is larger than or equal to μ_{t-1}, the market belief (probability) on the country being of type 1 before period t.
Proof: By Bayes' rule,

$$\mu_t^1 = [\mu_{t-1}p_t]/[\mu_{t-1}p_t + (1-\mu_{t-1})p_t q_t]$$

$$q_t \leq 1 \Rightarrow [\mu_{t-1}p_t + (1-\mu_{t-1})p_t q_t] \leq p_t \text{ then } \mu_t^1 \geq \mu_{t-1}.$$

In what follows, assume that the probability assessment on the country being of Type 1, before the start of period t, μ_{t-1}, is smaller than one.

Lemma 2:

When no payment is made at t, and the lender's actions reveal that a bad state has happened at t, there is no change in the markets belief about the government type:

$$\mu_t^2 = \mu_{t-1}$$

Proof:

$$\mu_t^2 = \mu(\theta = 1 | \mu_{t-1}, a_t^c = 0, a_t^b = A, bad \quad state) =$$

$$\mu_t(\theta = 1 \cap a_t^c = 0 \cap a_t^b = A \cap bad \quad state)/\mu_t(a_t^c = 0 \cap a_t^b = A \cap bad \quad state) =$$

$$\mu_{t-1}(1-p)w_t \quad /\mu_{t-1}(1-p)w_t + (1-\mu_{t-1})(1-p)w_t \quad = \mu_{t-1}$$

Lemma 3:

The updated probability (market's belief) of the country being of Type 1 when no payment is made at t and the lender decides not to penalize, μ_t^3, is no larger than the probability assessment that the country was of type 1, before period t moves, μ_{t-1}. If the equilibrium is separating, then μ_t^3 is strictly smaller than μ_{t-1}.

Proof: We defined $\mu_t^3 = \mu_t(\theta = 1 | \mu_{t-1}, a_t^c = NA) =$

$$\mu_t(\theta = 1 \cap a_t^c = 0 \cap a_t^b = NA)/\mu_t(a_t^c = 0 \cap a_t^b = NA) = H_1/H_2$$

where the numerator is

$$H_1 = \mu_{t-1}[(1-p_t)(1-w_t)] \qquad \text{and the denominator is}$$

$$H_2 = \mu_{t-1}[(1-p_t)(1-w_t)] + [1-\mu_{t-1}][(1-p_t)(1-w_t) + p_t(1-q_t)(1-w_t)]$$

This is equivalent to

$$H_2 = (1-p_t)(1-w_t) + (1-\mu_{t-1})p_t(1-q)(1-w_t)$$

Therefore

$$H_1/H_2 = \mu_t^3 \leq \mu_{t-1} \quad \text{because} \quad (1-\mu_{t-1})p_t(1-q)(1-w_t) \geq 0$$

Moreover, if the equilibrium is separating,

$$q_t = 0 < 1, \quad then \quad \mu_t^3 < \mu_{t-1}.$$

Lemma 4:

The market assessment on the administration type μ_t^4 will be zero if a non payment happens and the state of the economy is shown to be good following the creditors action.

Proof: Follows from $\mu(\theta = 1, a_t^c = 0, A, good\ state) = 0$

4.2 Equilibrium Valuation : The equilibrium market valuation of the claim at (the end of) period t, W_t, is the discounted sum of expected payments, conditional on the updated market beliefs about the government type :

$$W_t = \mu_t V_t^1 + (1 - \mu_t) V_t^2$$

Below, we define the four values that the sovereign claim can take, conditional on the value obtained at the end of period t-1, W_{t-1}:

Value after c_t is paid

$$W_t^1 = \mu_t^1 V_t^1 + (1 - \mu_t^1) V_t^2;$$

Value if c_t is not paid and the lender penalizes, receiving a payoff that corresponds to a bad state of resources:

$$W_t^2 = \mu_t^1 V_t^2 + (1 - \mu_t^2) V_t^2$$

Value if c_t is not paid and the lender does not penalize:

$$W_t^3 = \mu_t^1 V_t^2 + (1 - \mu_t^2) V_t^2$$

Value if c_t is not paid and the lender penalizes, receiving a payoff that corresponds to a good state of resources.

$$W_t^4 = \mu_t^1 V_t^2 + (1 - \mu_t^2) V_t^2$$

The following theorem proves the monotonic relationship between the sovereign claim's values, conditional on the value received at the end of the previous period. In particular, it shows that the value after the coupon payment takes place, W_t^1, provides the upper bound for the conditional values of the claim. It also proves that W_t^4, the claim's value after a coupon non payment is followed by the lender's penalty and its discovery that a good state of resources held, provides the lower bound for the conditional values of the claim.

Valuation Theorem

i) After a coupon payment takes place at period t, the equilibrium valuation, W_t^1 is no smaller than the equilibrium valuation after the coupon is not paid. If the equilibrium is a separating one, then the equilibrium valuation is strictly larger when the coupon is paid.

ii) If the payment does not take place at t and the lender's punishing action reveals that a bad state has occurred, the valuation, W_t^2 is no smaller than the one when no payment occurs and the bank does not apply penalties, W_t^3.

iii) The valuation after no payment by the debtor and no action by the lender at period t, W_t^3 is no smaller than the one when non payment occurs at t, the bank acts and finds out that a good state had prevailed, W_t^4.

Proof i) Just observe that $W_t^1 \geq W_t^2$ because $\mu_t^1 \geq \mu_t^2$ by combining Lemma 1 and Lemma 2. Also, using Lemma 1 and Lemma 2 we check that $W_t^1 > W_t^2$ holds when $q_t = 0$.

Proof ii) Similar to i) and using Lemma 2 and Lemma 3, shows that $W_t^2 \geq W_t^3$ because $\mu_t^2 \geq \mu_t^3$

Proof iii) Using Lemma 3 and Lemma 4, shows that $W_t^3 \geq W_t^4$ because $\mu_t^3 \geq \mu_t^4$.

5 Empirical Analysis

To corroborate the hypothesis of price dependence on payment history, we performed a time series cross sectional study for several large sovereign debtors. The proposed model is $P_{it} = \alpha_i + \beta d_{it}$, where i = 1...,6 is the country index, t denotes the time (year), the P_{it} are the average debt's secondary market prices, as a percentage of its face values, and the d_{it} are the payment behavior of these countries, as measured by the yearly ratio of arrears in long term debt to arrears plus repayments of long term debt. The data comes from the International Monetary Fund and the Nederlansche Middenstendsbank N. V. The group of countries considered includes the three largest LDC debtors, Argentina, Brazil and Mexico, and also Colombia, Chile and Peru.

The two data series analyzed range from 1986, which is the first year at which reliable data on secondary market debt prices is available, to 1990, because from that year on most of these loans were exchanged for new issued bonds under the provisions of the Brady Initiative. The data on arrears can only be obtained as a yearly frequency: As a result, there is a reduced number of observations in our sample.

To account for country-specific effects, a time series cross sectional model with individual specific, variable intercepts was computed. As a result, a positive correlation coefficient of 0.847 for the payment behavior variable was obtained.

An F-test was performed, to test the hypothesis that all coefficients are zero. As a result, the hypothesis that not all coefficients are zero cannot be rejected at either a 5 percent or a 1 percent confidence level.

The t-test performed, shows that with a confidence level of a 1 percent, the coefficient of the independent variable, payment ratios, is significantly different from zero.

The r-squared value obtained is 0.864, showing that a substantial part of the price variations are explained by the proposed model.

When the dummy variables are constrained, the regression coefficient for the dependent variable is 0.55. In this case, the model loses some of its predictive power, resulting in an r-squared coefficient of 0.762.

6 Conclusion

This paper presents a bayesian reputational model to study the country risk, or intrinsic uncertainty in sovereign debt.

The set of of a bayesian equilibria is characterized. . We show how the claims' prices will incorporate the information provided by past payment record , a theoretical result that replicates the negative impact of defaults on the sovereign claim prices.

We also provide the conditions under which the lender will be able to enforce the coupon payments, i.e. will force the strategic government to behave as a non-strategic one.

Our empirical analysis shows that payment behavior accounts for most of the price variations that took place in the sovereign's debt claims during the late 1980's..

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