THE OPTIMAL INFLATION TAX IN AN ECONOMY WITH A LARGE UNDERGROUND SECTOR

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SUMMARY

On this paper we show how the government can optimally use inflation to tax the underground sector of the economy. Modeling the underground economy as markets that avoid paying ordinary consumption taxes, we show how a positive inflation rate can reduce the welfare loss generated by the tax system. In particular, we show that if there are markets in the underground sector in which transactions are carried on using cash, the optimal inflation rate is positive. Preliminary simulation work indicates that the larger the cash\underground sector relative to the credit\official sector, the larger is the optimal rate of inflation. Thus, countries with large underground sectors -like most latin american ones- should not follow a zero inflation policy. We also show that the welfare loss of following a zero inflation rate can be large if the underground sector is important.

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1. Introduction

The purpose of this paper is to study the optimal combination of inflation and excise taxes to finance a given government expenditure in an economy where a large underground sector makes it impossible to apply ordinary excise taxes to all consumption goods.

The explicit consideration of inflation as a revenue source for the government is not new in the public finance literature. Phelps (1973), using a model where liquidity enters the utility function, pointed out that if lump-sum taxes are not available, the optimal inflation rate should be related to price elasticities in a Ramsey fashion. So, under certain regularity assumptions concerning de second derivatives of the utility function, the optimal rate of inflation would be higher than Friedman's rule. However, Lucas and Stokey (1983) showed that in a cash-in-advance economy, Friedman's rule would still be optimal even if only distorting taxes were available, provided that the government can tax differently cash and credit goods. The difference is that in their model, liquidity is not an additional good, but rather the means to acquire goods. Thus, a positive inflation rate is an additional tax on those goods.

In this paper we assume that there is a large number of markets which are underground, in the sense that the government cannot tax the goods traded in those markets. The model we present shares with the one in Lucas and Stokey the property that liquidity is not a good but rather the means to acquire goods. The main point of departure is that the subset of goods the government cannot tax includes not only leisure as in Lucas and Stokey but also goods that are bought using cash. Thus, inflation can be indirectly used to tax those goods. The idea of using inflation to tax the underground economy has been already used in an independent work by Canzonery and Rogers(1990). In a two country model, they assume that one country has an underground sector and the other does not. Thus, optimality requires following Friedman's rule in one country and a positive inflation in the other country[2]. However, they do not really solve for a Ramsey problem, because they can reproduce lump-sum taxation in both countries. They model the underground sector as a different good in the utility function, and, as they are not interested in the optimal combination of inflation and consumption tax, there is no interesting trade off between them in their model.

The paper proceeds as follows. In section 2 we briefly discuss the literature on underground economies to motivate the proposed model. In section 3 we present the model and study efficient allocations. We also study optimal tax structures under alternative assumptions regarding the fiscal technologies available. Section 4 discusses some simulation exercises and a final section contains some concluding remarks.

2. Characteristics of underground economies.

During the last decade, there has been a great deal of attention dedicated to underground economies in the literature (see Tanzi(1982), Feige(1986), Dallago(1990), Harding and Jenkings(1989)). All these studies provide with a good set of definitions, estimation methods and estimates of underground economies for a wide set of countries.

There is not a single definition of the underground economy, since there

[2] Their interest is to explain why it may be optimal for two different countries to have different inflation rates. They are not interested in looking at the interaction between the inflation rate and the consumption tax and the optimal fiscal problem is a trivial question in their model.

are many different but related concepts attached to it. The term is equally used to refer to illegal, unmeasured or unreported activities, depending on the interest of the researcher. As we are interested on the public finance aspect of the phenomenon, we define the underground sector of an economy as all income generating activities which do not compile with the tax obligations. Thus, the object we are interested in includes all illegal activities (like drugs and prostitution) and all legal activities that do not pay the corresponding taxes to the fiscal authority.

Many indirect estimation methods have been proposed in the literature, each of them stressing a particular feature of the underground sector. The estimates vary considerably as different methods are used. For instance, for the US economy, estimates for the late seventies vary from 4 % of GNP (Park(1979)) to 27 % of GNP (Feige (1980)), and for Italy, it varies from 7,5 % of GNP (Contini (1979)) to 30,8 % of GNP (Saba (1980)). The only direct study of the underground sector we know of, was done by de Soto(1987) for Peru. By direct study, we mean an estimate based on street interviews, carried on for an extended period of time, with intensive questioning (census type of work) for a few key sectors of the economy. He found that the income generated on the underground sector accounts for 38,9 % of registered GNP.

Several key patterns of the underground sector can be obtained from the studies, and some of them are particularly interesting for our purposes.

The first one, is that the range of goods that are traded in the underground sector is wide and different for each country. For instance, in Austria, underground activity is concentrated in retail trade, restaurants and hotels, in Germany is concentrated in construction and car repair, in Spain is concentrated in construction, textiles and shoe and leather products, while in the US is concentrated on home repairs and additions, food, child care and domestic service (Skolka(1987)). In addition, illegal activities contribute with a small percentage of the income generated at the underground sector. Feige(1986) estimates that unreported income from illegal activities is less that 15 % of total unreported income for the US in 1980. White (1987) reports an estimate slightly more than 10 % for 1981. These evidence shows that the underground economy is not characterized by a specific good or set of goods, but rather by all those goods in which evasion is easy and the probability of been discovered is very small.

The second one, is that among the causes of the underground sector are the high burden of taxation, regulation and tax morality. However, the most important one is the enforcement ability of the fiscal authority, given the institutional constraints it faces, and the technological constraints that the tax evaders face. As anecdotic evidence, it is illustrative to mention the existence of whole buildings in Napies, Italy, occupied by a large number of small underground textile factories, which can be quickly hidden when the fiscal inspectors arrive, or the existence of factories in Peru, that divide their workers in small groups in different places, to avoid detection.[3]

The third one is that both the formal and the underground sector use cash as well as other mechanisms in exchanges (see Feige(1987)), while there exists a consensus that the underground sector is cash intensive.

Given the mentioned facts, our model does not identify the underground

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[3] Skolka (1987) makes a more extensive argument about the importance of technological and institutional constraints that cause the underground markets and provides more anecdotic evidence. Del Boca y Forte(1982) provide a detailed discussion on the italian labor market, explaining why the technological characteristics of italian exporting industries and the institutional restrictions make attractive clandestine employment.

sector as a specific set of goods in the utility function, as it has been done in the literature (See Canzonery and Rogers(1989)). Rather, the underground markets exist because of technological constraints the government faces, which make impossible tax collection. Thus, in our model, all goods will be identical from the viewpoint of preferences and technology. We will assume that there is a large subset of markets that the government cannot reach[4], and that subset will be identified with the underground sector. We will also impose a cash-in-advance constraint in consumers optimal problem, such that there will be cash and credit goods in both the underground and official sectors of the economy.

3. The Model

We assume that there is a representative consumer with preferences over leisure and a continuum of goods indexed by the interval [0,1]. We assume that the utility function is of the form

$$(1) \quad W\,(c(z)_t,\ n_t) = \sum_{t=0}^{\infty} \beta^t \{ [\textstyle\int_0^1 U(c(z)_t)dz] - V(n_t) \}$$

where U is increasing and concave, and V is decreasing and convex; $c(z)_t$ is consumption of the good c(z) at time t, and n_t is time dedicated to labor at time t. We assume that the goods are produced using only leisure, by the linear production function

$$(2) \quad n_t = g_t + \textstyle\int_0^1 c(z)_t\, dz$$

where g_t is total government consumption at time t.

We can define then a Pareto optimum allocation as sequences $\{c(z)_t, n_t\}$

4 For a more stylized model that generates underground markets and monetary equilibria by imposing location constraints on agents and on the government, see Nicolini (1991).

from t=0 to infinity, that maximize (1) subject to (2). The first order conditions of this problem are

$$(3) \quad U'(c(z)_t) = V'(n_t) \qquad \text{for all } z \in [0,1] \text{ and all } t.$$

Thus, consumption must be constant across z's at the level where the marginal utility of consumption is equal to the marginal utility of leisure (the negative of the marginal desutility of labor). Together with condition (2), the solution can be obtained. The solution is standard. The marginal rate of substitution between any two consumption goods and between any consumption good and leisure must be equal to the marginal rate of transformation. Because of the linear technology assumed, all marginal rates of transformation are equal to one. As it is well known in the literature, this allocation can be implemented as a competitive equilibrium if lump sum taxes are available.

3.1 Competitive equilibrium with taxes

Now, we assume that the government cannot use lump sum taxation. Instead we assume that only consumption taxes are available, and that leisure cannot be taxed. Government expenditure cannot be financed without distorting the economy.

In addition, as we want to introduce money into the model, we assume that some of the goods can only be traded using cash. So, we follow Lucas and Stokey (1983) and impose a cash-in-advance constraint to the maximum problem of the consumer. We partition the unit interval in the following way

$$[0,1] = [0,a] \cup (a,a+b] \cup (a+b,a+b+c] \cup (a+b+c,1]$$

Obviously, a, b and c are positive real numbers such that their sum is lower than one. We will assume that all goods in the interval (a, a+b+c] must be traded using cash, from now on, cash goods. The reason way we separate the unit interval in four intervals will soon become evident.

We also assume that the government prints bonds, i.e., obligations which pay a nominal interest R. This gives the government the possibility to run deficits or surpluses at particular periods.

Thus, the maximum problem of the consumer will be to maximize the utility function in (1) subject to the constraints

(4) $M_{t+1}+B_{t+1}+\int_0^1 p(z)_t . c(z)_t . (1+\tau(z)_t)dz = p_t . n_t + M_t + B_t(1+R_t)$

(5) $Mt \geq \int_a^{a+b+c} [p(z)_t . c(z)_t]\, dz$

where $p(z)_t$ is the money price of good z at time t and p_t is the money price of labor at t. Now, we are ready to define a competitive equilibrium allocation

<u>Definition</u>: A competitive equilibrium given government expenditures, taxes and government bonds is a set of sequences {c(z)t, nt, pt, Rt, Mt } such that quantities maximize (1) subject to (4) and (5) and such that market clearing condition (2) is satisfied.

Before solving for consumer's problem, note that given the technology, it must be true that at any equilibrium

(6) $p(z)t = pt$ for all z E [0,1]

In addition, the first order conditions of consumer's problem are

(7) $U'(c(z)_t) = V'(n_t).(1+ T(z)_t + D(z).R_t)$ all t, $z \in [0,1]$

where D(z) is a dummy variable equal to one when $z \in [a, a+b+c]$, (cash goods) and equal to zero otherwise (credit goods).

Equations (4) to (7), plus the life-time budget constraint of the consumers

$$(8) \quad \sum_{t=0}^{\infty} Q_t . n_t = \sum_{t=0}^{\infty} Q_t . \int_0^1 c(z)_t . (1+T(z)_t + D(z).R_t)\, dz$$

and market clearing condition (2) characterize the competitive equilibrium. Q_t in equation (8) is the inverse of the interest rate from zero to t. The life-time budget constraint is constructed from the sequence of one period budget constraints and a no Ponzi game condition on government debt.

As it is clear from the equilibrium conditions, equilibrium quantities depend on fiscal (the function $\tau(z)$) and monetary policies (the nominal interest rate R). Note that in this cash-in-advance model, at the stationary equilibrium, the nominal interest rate is the sum of the market determined real interest rate, and the inflation rate, which is a function of the growth rate of the money supply. So, the government can choose the nominal interest rate by choosing the appropriate rate of money growth. From now on, we will assume that the control variable for the government is the nominal interest rate and we will solve for the optimal interest rate. The supporting inflation rate can be obtained from Fisher's equation.

Different assumptions about feasability of government policy will lead to different allocations. We will study now several alternative situations.

3.2 Some goods cannot be taxed. No cash goods.

Now, we assume that there are some markets the government cannot tax. We identify those markets with the goods in the interval $[0, a+b]$. Thus, under the assumption that $a+b > 0$, there exists an underground sector in this economy. In this case, we assume that the government can choose a function $T(z)$, for all z in [a+b, 1], and the tax rate must be zero for all z in [0, a+b). In addition, we will assume that there are no cash goods, i.e., $b = c = 0$.

The equilibrium conditions for this case are

$$(9a)\quad U'(c(z)_t) = V'(n_t).(1+T(z)_t) \qquad \text{all } t,\ z \in [a+b,1]$$

$$(9b)\quad U'(c(z)_t) = V'(n_t) \qquad \text{all } t,\ z \in [0,a+b)$$

$$(10)\quad n_t = \int_0^1 c(z)_t dz + g_t \qquad \text{all } t$$

$$(11)\quad \sum_{t=0}^{\infty} Q_t.n_t = \sum_{t=0}^{\infty} Q_t.\int_0^1 c(z)_t.(1+T(z)_t)\ dz$$

$$(12)\quad \beta^t.V'(n_t)/V'(n_0) = Q_t \qquad \text{all } t$$

As it is clear from conditions (9) to (12), the allocation will depend on the fiscal policy, i.e., on the chosen function T(z). A welfare maximizer government, will choose that function T to maximize the utility function of the representative consumer, subject to the constraint that the allocation is a competitive equilibrium. Thus, it has to maximize (1), subject to (9) to (12).

We can follow Lucas and Stokey(1983) and reduce the dimension of the problem by eliminating prices from equilibrium conditions. In this case, we can use (9a) and (12) to eliminate taxes and the real interest rate. Thus, (11) can be written as

$$(13)\ \sum_{t=0}^{\infty} \beta^t.[V'(n_t).n_t - \int c(z)_t.U'(c(z)_t)\ dz] = 0$$

In this way, the optimal problem of the government is to maximize (1) subject to (9b), (10) and (13).

We assume (as in Lucas and Stokey) that there is a unique interior maximum. Then, if we let ω, ε and λ be the lagrange multipliers associated with constraints (9b), (10) and (13) respectively, the first order conditions of the optimal policy problem are

(14a) $U'(c(z)_t)(1+\lambda) + U''(c(z)_t)\ \lambda\ c(z)_t = \varepsilon_t$ all t, $z \in [a+b, 1]$

(14b) $U'(c(z)_t)(1+\lambda) + U''(c(z)_t)\ [\lambda c(z)_t + \omega_t] = \varepsilon_t$ all t, $z \in [0, a+b)$

(15) $V'(n_t)(1+\lambda) + V''(n_t)\ [\lambda n_t + \omega_t] = \varepsilon_t$ all t.

Condition (12a) (or (12b)) is exactly the same for all $z \in [a+b, 1]$ (or $z \in [0, a+b)$) at time t, which means that the optimal solution implies the same quantities of all $c(z)$, $z \in [a+b, 1]$ (or $z \in [0, a+b)$) at any given period. Thus, the optimal policy requires the same tax rate for all consumption goods for any time period. Thus, the optimal tax function is

$$T(z)_t = \begin{cases} \tau_t & \text{if } z \in [a+b, 1] \\ 0 & \text{if } z \in [0, a+b) \end{cases}$$

which implies that the optimal policy is taxing all goods you can at the same rate. The size of the tax rate will depend on the amount of revenue that must be raised. Note that the multiplier ω_t is a measure of the marginal welfare cost of the underground sector.

Note that this result is consistent with the typical Ramsey problem. It is well known that the optimal policy requires equal percentage changes in the equilibrium quantities of all taxable goods. As in the equilibrium without government expenditures (the Pareto solution) consumption was equal across z's, the optimal policy also requires equal consumption quantities across z's.

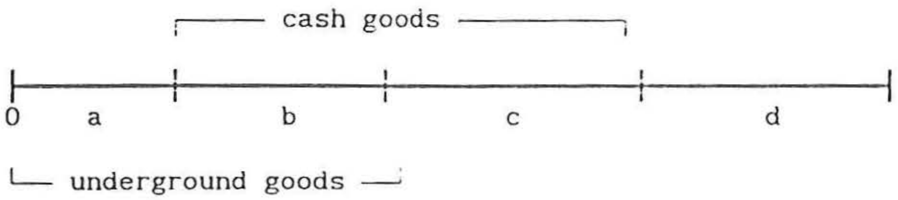
Some things are worth mentioning regarding this solution. First, note that the larger the underground sector, the smaller the official sector and thus the larger the tax rate. Thus, the larger the underground sector, the stronger the tax presume on the official sector. Second, note that the larger the underground sector, the lower the welfare level of the representative agent. Compare two economies that are identical but one has a larger underground sector. The optimal policy of this one is feasible for the one with the smaller underground sector. However, as it is not optimal, it must have a higher welfare level.

3.3. Introducing cash goods.

In this case, we assume that there are both cash and credit goods in both the underground and the official sector, i.e., we partition the unit interval in four disjoint non empty intervals. This is equivalent to assume that a, b, c and d are strictly positive.

Note that the cash-in-advance constraint (5) affects only the goods in the interval (a,a+b+c]; thus the cash goods are the ones on that interval. So, as depicted in figure 1, we have the unit interval divided in the following four categories.

Figure 1



a) Credit/underground goods [0,a]

b) Cash/underground goods (a,a+b]

c) Cash/official goods (a+b,a+b+c]

d) Credit/official goods (a+b+c,1]

In equilibrium, no a-type good is taxed; all b-type goods bear the inflation tax; all c-type goods bear the consumption tax and the inflation tax and, finally, all c-type goods bear only the consumption tax.

The optimal problem of the consumer is to maximize utility subject to the cash-in-advance constraint (5) and the budget constraint (8).

Let x be the generic element of the goods in group a), and y, v, w the ones for groups b), c), d). Then, the first order conditions for the consumer's problem are

$$(16)\quad U'(y_t)/U'(x_t) = (1+R_t) \quad \text{all } t,\ x \in [0,a),\ y \in (a, a+b]$$

$$(17)\quad U'(v_t)/U'(x_t) = (1+R_t+T(v)_t) \quad \text{all } t,\ x \in [0,a),\ v \in (a+b, a+b+c]$$

$$(18)\quad U'(w_t)/U'(x_t) = (1+T(w)_t) \quad \text{all } t,\ x \in [0,a),\ w \in (a+b, 1]$$

$$(19)\quad V'(n_t)/U'(c(x)_t) = 1 \quad \text{all } t,\ x \text{ in } [0,a)$$

$$(20)\quad V'(n_t)/\beta . V'(n_{t+1}) = (1+R_t)/(1+\pi_t) \quad \text{all } t$$

where π_t is the inflation rate. A competitive equilibrium allocation must satisfy these first order conditions, the two restrictions on consumer's problem and market clearing conditions.

In addition, in any equilibrium, it must be true that the nominal interest rate must be greater or equal to zero; otherwise, consumers can make arbitrarily large profits by holding arbitrarily large quantities of money. Thus, the last equilibrium condition is

$$(21a)\quad (1+R_t) \geq 1$$

which, using competitive equilibrium conditions can be written as

$$(21b)\quad U'(y_t) - U'(x_t) \geq 0$$

The optimum problem of the government, is to maximize the utility function of the consumer subject to (16) to (20), the budget constraint (4), condition (5), market clearing condition (2) and condition (21b). As before, we can eliminate all prices (πt, Rt and the taxes) and simplify the problem. If we use (16),(18),(20) and (5) in the budget constraint (4), we also obtain equation (11). So, the optimal problem is reduced to maximize (1) subject to (11), (19), resources constraint (2) and condition (21b). We can thus obtain the optimal quantities and, using equilibrium conditions, we can obtain the function $\tau(z)$ and the interest rate that sustain the optimal quantities. Up to this point, the only restriction we made on fiscal policy is that the tax rate on the goods traded at the underground sector is zero, i.e., that $\tau(z) = 0$ for $z \in [0, a+b)$. Thus, we had been working under the assumption that the fiscal authority can choose any tax rate for every single good in the official sector. This may not be a reasonable assumption. In particular, it implies that the government can discriminate between cash and credit goods at the official sector, and there are reasons to believe that it is not a reasonable assumption. The distinction between cash and credit goods, as it is found in the literature, is a very subtle one, (see Lucas and Stokey (1983)) not necessarily based on physical characteristics of the goods, but on the credit technologies of consumers which may be unobservable for the government.

We proceed in the following way. First, we assume that the government can perfectly discriminate among goods at the official sector, and we study the implications on the optimal nominal interest rate. Then, we solve the problem under the (we think more realistic) assumption that the government cannot discriminate among goods and can only choose a single consumption tax rate levied on all goods at the official sector, and the nominal interest rate. We show that the results critically depends on this assumption[5].

3.3.1. The Government Can Discriminate Among Goods.

In this case, we have no further restrictions on government's problem. With regard to the non-negativity constraint on nominal interest rate, we will solve the problem assuming that it is not binding (i.e., $\theta = 0$). Then, we will check that at the optimum the restriction holds.

If we let λ, ω, and ε be the respective lagrange multipliers, the first order conditions with respect to consumption goods will be

$$(22)\quad U'(c(z)_t).(1+\lambda) + U''(c(z)_t)\,\lambda\, c(z)_t = \varepsilon_t \qquad \text{all } t,\ z \in (a,1]$$

$$(23)\quad U'(c(z)_t).(1+\lambda) + U''(c(z)_t)(\lambda\, c(z)_t + \omega_t) = \varepsilon_t \qquad \text{all } t,\ z \in [0,a]$$

Again, it is clear that the optimal solution requires equal consumption for all goods in (a,1], i.e., all goods except for credit goods in the underground sector. The (unique) tax structure which supports this allocation is letting the nominal interest rate equal to the tax rate on credit/official goods[6], and the tax rate on cash/official goods equal to zero. In this way, you put the same tax rate to all goods you can (all but the credit goods in the underground sector). The level of the tax rate will depend on the required revenue. Note that the optimal nominal interest rate is equal to the tax rate, independently of any of the parameters of the

[5] Even though they seem two extremes, they are not in the context of this model. As the optimal solution is the same tax rate to all goods, the restriction is only important to the extent that cash and credit goods cannot be discriminated. The government will want to tax equally all credit goods and equally (though at a different rate) all cash goods.

[6] Note that if government revenues must be positive, $R_t > 0$ and constraint (21b) holds.

model. The fact that there are cash goods implies that the government has the ability to tax a broader set of goods, and therefore, to improve welfare. This is so, because we assume that the government can discriminate cash and credit goods from the fiscal policy point of view.

Note that the optimal allocation of this economy is identical to an economy without cash goods, but with an underground sector of [0,a). Or, if all underground goods were cash goods, the equilibrium is the one we would obtain if there were no underground goods[7] and only consumption tax. This is a very interesting result, because it shows that imposing a cash-in-advance constraint on consumers increases (in a weak sense) welfare rather than decreasing it (in a weak sense) like most cash-in-advance models found in the literature. Thus, this result (which can also be found in Canzonery and Rogers(1990)) could explain why legal restrictions theories of money can be based on a welfare maximizer government.[8]

3.3.2 <u>The Government Cannot Discriminate Among Goods.</u>

In this section, we consider the (we think, more realistic) case in which the government cannot discriminate among goods, and thus, it must levy the same tax rate to all consumption goods of the official sector. This would be the case of a general consumption tax. Thus, the fiscal policy consists on two numbers, one for the consumption tax, and one for the nominal interest rate. Thus, in equilibrium, there will be only four different types of goods, the ones called x, y, v and w in page 6. The equilibrium conditions will be

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[7] This is a very similar outcome to the one in Canzonery and Rogers(1990)

[8] Again, Nicolini (1991) provides a more stylized model where the government finds optimal to impose legal restrictions on the bonds it issues so money does not go out of circulation and welfare is improved.

the same than in page 6, but with the additional restriction that $\tau(w)_t = \tau(v)_t$ for all v,w.

In defining the optimal problem of the government, we will consider explicitly that there are only these four types of goods. In this case, the utility function becomes

$$(24)\quad W = \sum_{t=0}^{\infty} \beta^{t} . \{a.U(x_t)+b.U(y_t)+c.U(v_t)+d.U(w_t) - V(n_t)\}$$

where d = 1-a-b-c . The constraints of this problem, once we acknowledge that there are only four type of goods are

$$(25)\quad \sum_{t=0}^{\infty} \beta^{t}. [a\ x_t U'(x_t)+b\ y_t U'(y_t)+c\ v_t U'(v_t)+d\ w_t U'(w_t) - n_t V'(n_t)] \geq 0$$

$$(26)\quad a\ x_t + b\ y_t + c\ v_t + d\ w_t + g_t = n_t$$

$$(27)\quad U'(x_t) = V'(n_t)$$

$$(28)\quad U'(x_t) - U'(y_t) \geq 0$$

$$(29)\quad U'(x_t) + U'(v_t) - U'(y_t) - U'(w_t) = 0$$

where the last restriction implies that the government must levy the same tax rate to cash and credit goods in the official sector. Thus, once the tax on w's (τ) and the tax on y's (R) are chosen, the tax on v's is not a free variable (it is equal to τ + R). This last restriction is what makes this problem different than the last one, and introduces a very interesting trade-off between inflation tax and consumption tax. The objective of the government (given the implicit assumption we made about elasticities by using a separable and symmetric utility function) is to put the same tax rate on all those goods that can be taxed. In the previous case, it could do it, by putting different tax rates to cash and credit goods. But now, it cannot. If inflation is used to tax the cash goods in the underground sector, then the cash goods of the official sector will be taxed at a higher rate than the credit goods of the official sector. So, in this case, there is a more interesting trade-off when inflation is used to tax the underground sector. As you increase the tax on the underground sector, reducing the distortion between the official and the underground sectors, you increase the distortion between cash and credit goods in the official sector. And in this case, as it will be shown, the relative importance of the sectors will play a crucial role on the optimal rate of inflation.

If we let λ, ε, ω, θ and ϕ be the respective lagrange multipliers, the lagrangian is

$$L = \sum_{t=0}^{\infty} \beta^{t} [a\, U(x_t) + b\, U(y_t) + c\, U(v_t) + d\, U(w_t) - V(n_t)]$$

$$+ \lambda \sum_{t=0}^{\infty} \beta [ax_t U(x_t) + by_t U(y_t) + cv_t U(v_t) - dw_t U(w_t) - n_t V(n_t)]$$

$$+ \sum_{t=0}^{\infty} \phi_t \; [U(x_t) - U(y_t) + U(v_t) - U(w_t)]$$

$$+ \sum_{t=0}^{\infty} \theta_t \; [U(y_t) - U(x_t)]$$

$$+ \sum_{t=0}^{\infty} \omega_t \; [V'(n_t) - U(x_t)]$$

$$+ \sum_{t=0}^{\infty} \varepsilon_t \; [n_t - (ax_t + by_t + cv_t + dw_t + g_t)]$$

and the first order conditions are

$$(30) \quad U'(x_t).(1+\lambda) + U''(x_t).[\lambda . x_t + (\phi_t - \theta_t - \omega_t)/a] = \varepsilon_t$$

$$(31)\ U'(y_t).(1+\lambda) + U''(y_t).[\ \lambda.y_t + (\theta_t - \phi_t)/b\ ] = \varepsilon_t$$

$$(32)\ U'(v_t).(1+\lambda) + U''(v_t).[\ \lambda.v_t + \phi_t/c\ ] = \varepsilon_t$$

$$(33)\ U'(w_t).(1+\lambda) + U''(w_t).[\ \lambda.w_t - \phi_t/d\ ] = \varepsilon_t$$

$$(34)\ V'(n_t).(1+\lambda) + V''(n_t).[\ \lambda.n_t - \omega_t\ ] = \varepsilon_t$$

To find a solution, one can solve equations (26) to (34) for consumption quantities, work effort and all time indexed multipliers as functions of λ. Then, the value of λ is obtained from equation (25).

Our focus is on the optimal mix between consumption taxes and the nominal interest rate, rather than on the evolution of taxes over time[9]. Thus, in order to simplify the analysis we will assume that government expenditures are constant over time. In this case, the set of equations (25) to (34) is the same for every t, and the optimal quantities and prices constant through time. Thus, from now on, we get rid of time subscripts.

In order to study the optimum quantity of money problem, or what is the same, the optimal nominal interest rate, the multiplier θ will be of great use. Note that the Khun-Tucker conditions imply

$$\theta \geq 0\ ,\quad U'(x) - U'(y) \geq 0\ ,\quad [U'(x) - U'(y)].\theta = 0$$

We will show that $[U'(x) - U'(y)] > 0$ and $\theta = 0$ if government expenditures are positive.

PROPOSITION: If government expenditures are strictly positive and there exists an underground sector which uses cash in transactions (i.e., $b > 0$), then the optimal nominal interest rate is strictly positive.

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[9] The temporal structure of the model is the same as in Lucas and Stockey (1983), so no new results regarding temporal issues will arise.

Before proving this proposition, we will derive several useful results from the optimal conditions, assuming that Friedman's rule is optimal. From competitive equilibrium conditions (16) to (18)

$$(35) \quad U'(x) = U'(y) \Rightarrow x = y$$

$$(36) \quad U'(v) = U'(w) \Rightarrow v = w$$

Thus, using (36) and optimal conditions (32) and (33)

$$(37) \quad \phi = 0.$$

Using (37), (30) and (31),

$$(38) \quad \theta = - \frac{b}{(a+b)} \omega$$

Now, we use (30), (34), (37) and (38) to obtain

$$(39) \quad \lambda = \frac{[U''(x).x - V''(n).n]}{[V''(n).(a+b) - U''(x)]} . (\theta / b)$$

Now, multiplying (30), (31), (32), (33) and (34) for x, y, v, w, and -n respectively, adding up, and using (35) to (39), it is possible to obtain[10]

$$(40) \quad \lambda Q = - \varepsilon g$$

where

$$Q = x^2(a+b)U''(x)+v^2(c+d)U''(v)-n^2V''(n) + (a+b)\frac{[x.U''(x) - n.V''(n)]^2}{[V''(n)(a+b)- U''(x)]}$$

after some tedious algebra, it can be shown that

$$Q = v^2(c+d)U''(v) + \frac{U''(x)V''(n)[n -ax-by]^2}{[V''(n)(a+b) - U''(x)]}$$

---

[10] This procedure is similar to the one used by Lucas and Stockey (1983)

and note $Q < 0$ because $V'' > 0$ and $U'' < 0$. With these results, the proof of the proposition is straightforward.

Proof: Assume that the optimal nominal interest rate is zero. Then, conditions (35) to (40) hold. As $g > 0$, condition (40) implies that $\lambda > 0$. Thus, condition (39) implies that $\theta < 0$, which contradicts the Khun-Tucker conditions. As the nominal interest rate cannot be negative, it must be positive.

The result would be quite different if $g < 0$, i.e., if the government must make transfers in a distorting way. In that case λ would be negative and θ positive which implies that the restriction would be binding. The intuition behind this result is clear. If the government must make net transfers, it must be overall subsidizing. In order to reduce the welfare loss, it should spread distortions over goods. But the nominal interest rate cannot be negative, so all the transfer must be in the form of a consumption subsidy. On the contrary, if government expenditures are positive, it is possible to further spread distortions among goods by means of a positive inflation tax.

So far we have established that the optimal nominal interest rate is positive if there are underground markets which use cash in transactions. Note that if there are no cash\underground goods, (i.e., $b = 0$) then there will be no y-type good and the optimal policy would be Friedman's rule together with a constant consumption tax rate across goods.

That b is a very important parameter to determine the optimal nominal interest rate becomes also evident if we assume that $b = d$, i.e., that the size of the cash/underground sector is the same as the size of the credit/official sector. In this case, equations (32) and (34) are identical (remember that θ =0), which means that the optimal values of y and v are the same. But this means that the tax rate and the nominal interest rate are the same. Which is the intuition of this result?. Consider the case where the interest rate is zero, and all expenditures are financed by consumption tax. Then, there is no distortion between cash and credit goods at the official sector, but a large distortion between cash goods at the official and the underground sectors. On the other extreme, assume that all the expenditure is financed with inflation. There will be no distortion between y and v but a large distortion between v and w. Thus, by switching from one way of raising revenue to another, you decide where you put the distortion. The government will prefer to put the higher distortion in the smaller sector. If both sectors have the same size, you distribute the distortion equally, which is the result mentioned above.

4. Simulation exercise.

In this section we present numerical solutions for some parameter values in order to get an idea of the quantitative relevance of the question posed in the paper. In particular, we want to isolate the parameters that influence in a systematic way the optimal nominal interest rate.

We solved the model assuming that the utility function is of the form

$$U(c) = (1-\sigma)^{-1} \cdot c^{(1-\sigma)}, \quad V(n) = n$$

Thus, the parameters of the model are a, b, c, d, g and σ. The results of the simulations are in figures I to III.

Figure I plots the optimal nominal interest rate as a function of the relative size of the underground/cash sector, b. For any single curve in figure I, σ, g, a and c are constant, and we vary b and d such that $\Delta b = -\Delta d$, such that the linear restriction between a, b, c and d is

OPTIMAL NOMINAL INTEREST RATE

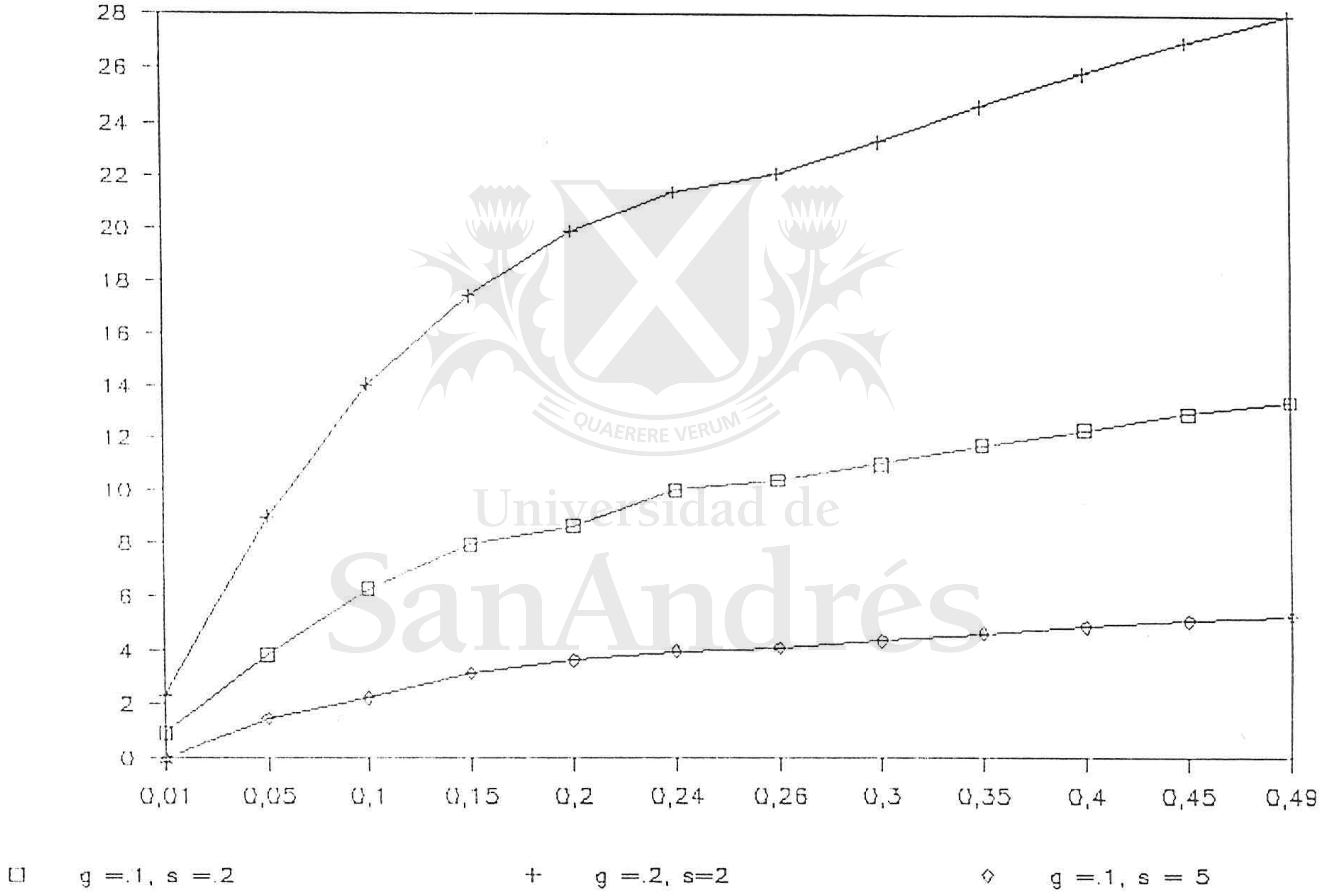

FIGURE I

satisfied. There are three curves in figure I, corresponding to different values of g and σ. The curve in the middle corresponds to the benchmark case ($a = c = .25$, $g = .1$, $\sigma = 2.1$). If we solve the same model but with a higher g ($g = .2$), then we obtain the expected result, a higher interest rate for all values of b. If we solve the model with a higher σ (which implies a lower price elasticity), we also obtain the expected result, a lower interest rate for all values of b.

Note that as the system of equations (36)-(43), is symmetric between y and w, it is possible to read the optimal consumption tax from the same graph, if we put d rather than b in the horizontal axis. Thus, even though the level of the inflation tax depends on elasticities and on the level of government expenditure, a clear pattern emerges from the graph; the bigger the underground/cash sector relative to the official/credit sector, the higher the inflation tax relative to the consumption tax.

Figure II is an attempt to measure the sensitivity of the optimal inflation tax with respect to the relative importance of the official/cash sector (good v) with respect to the underground/credit sector (good x). We made a very similar exercise, but fixing b and d and varying a and c such that $\Delta a = -\Delta c$. We plot the optimal consumption tax rate and the optimal inflation tax rate as a function of the size of the underground/credit sector (goods a). The benchmark case (figure IIa) is $b = .2$, $d = .3$, $s = 2$ and $g = .1$. We also solve it for the case of $g = .2$ (figure IIb). Finally, we also plot the ratio between the optimal consumption tax and the optimal inflation tax, but multiplied by ten, so it is easier to observe its fluctuations. In both cases we obtain the same pattern. As a increases, both taxes increase, which is reasonable because the overall tax base is reduced. But when g is higher, both taxes are

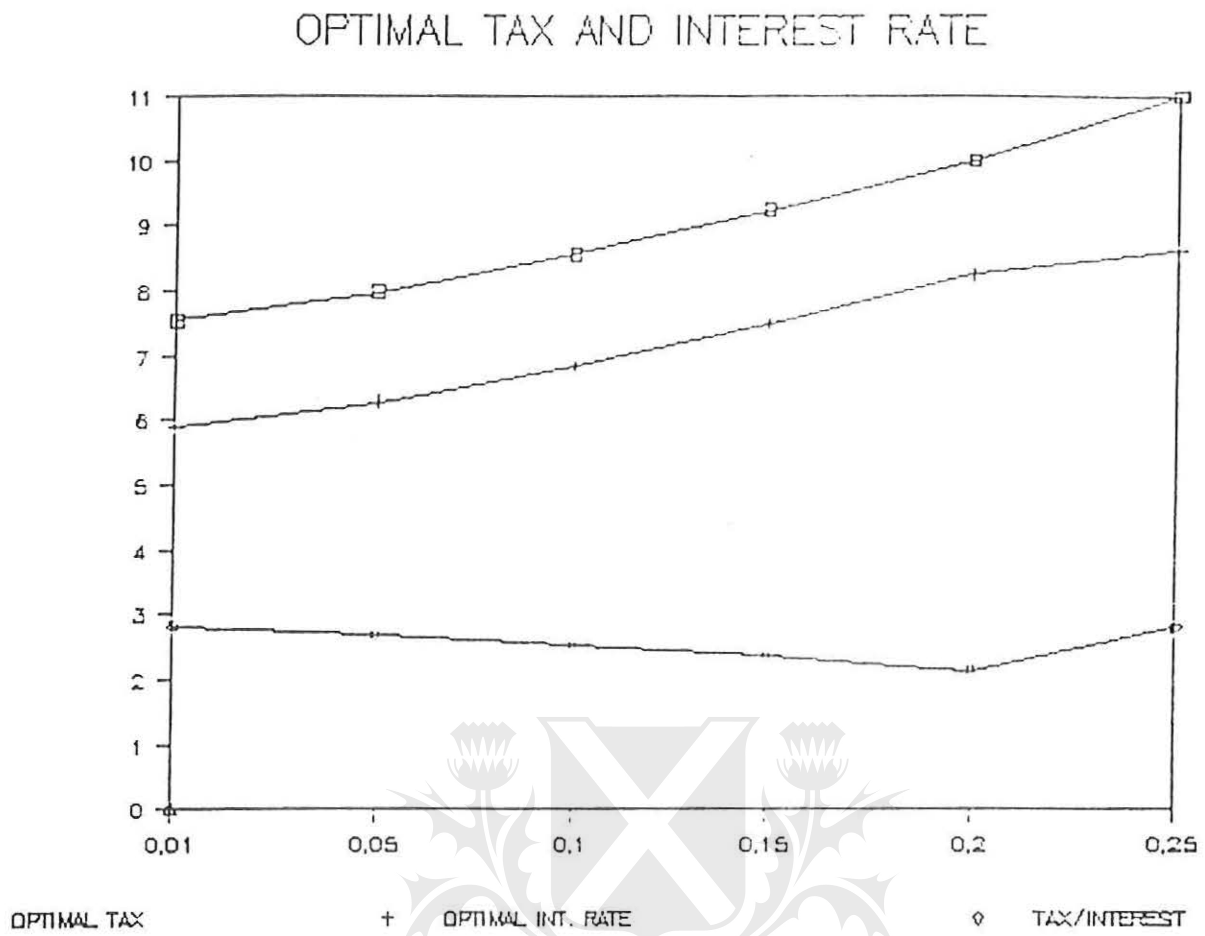

FIGURE II . a

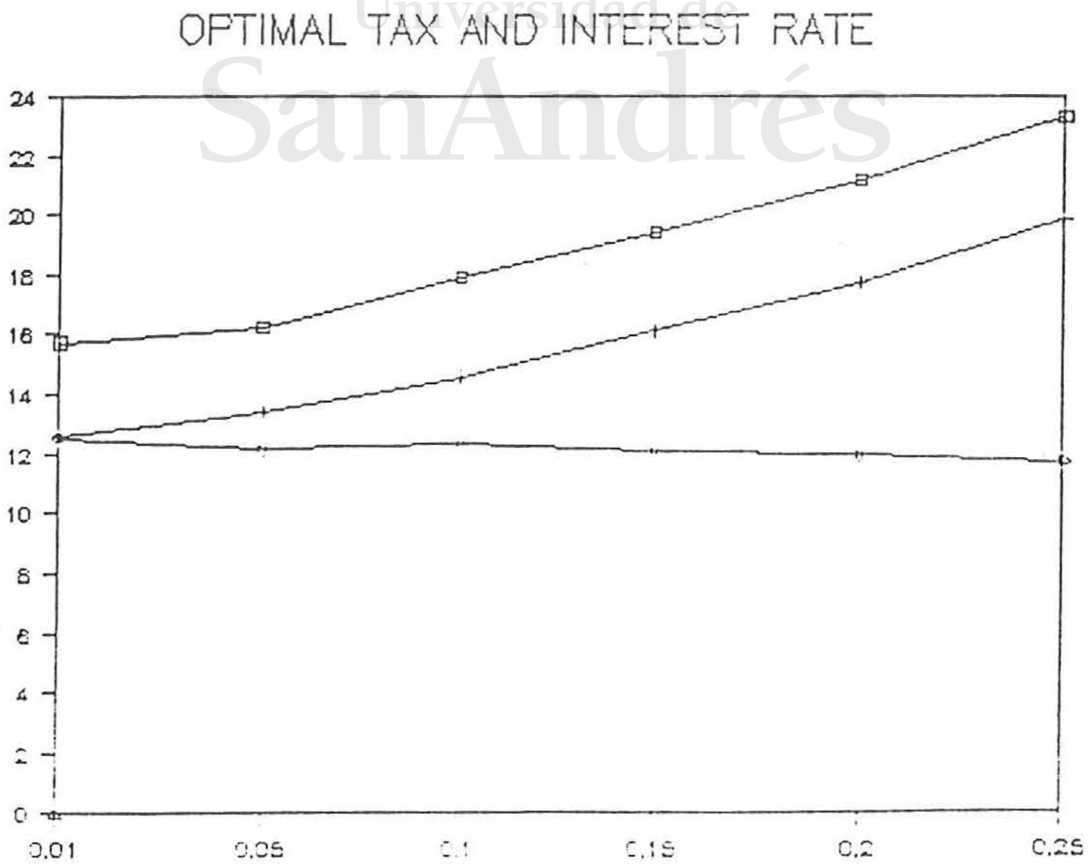

FIGURE II . b

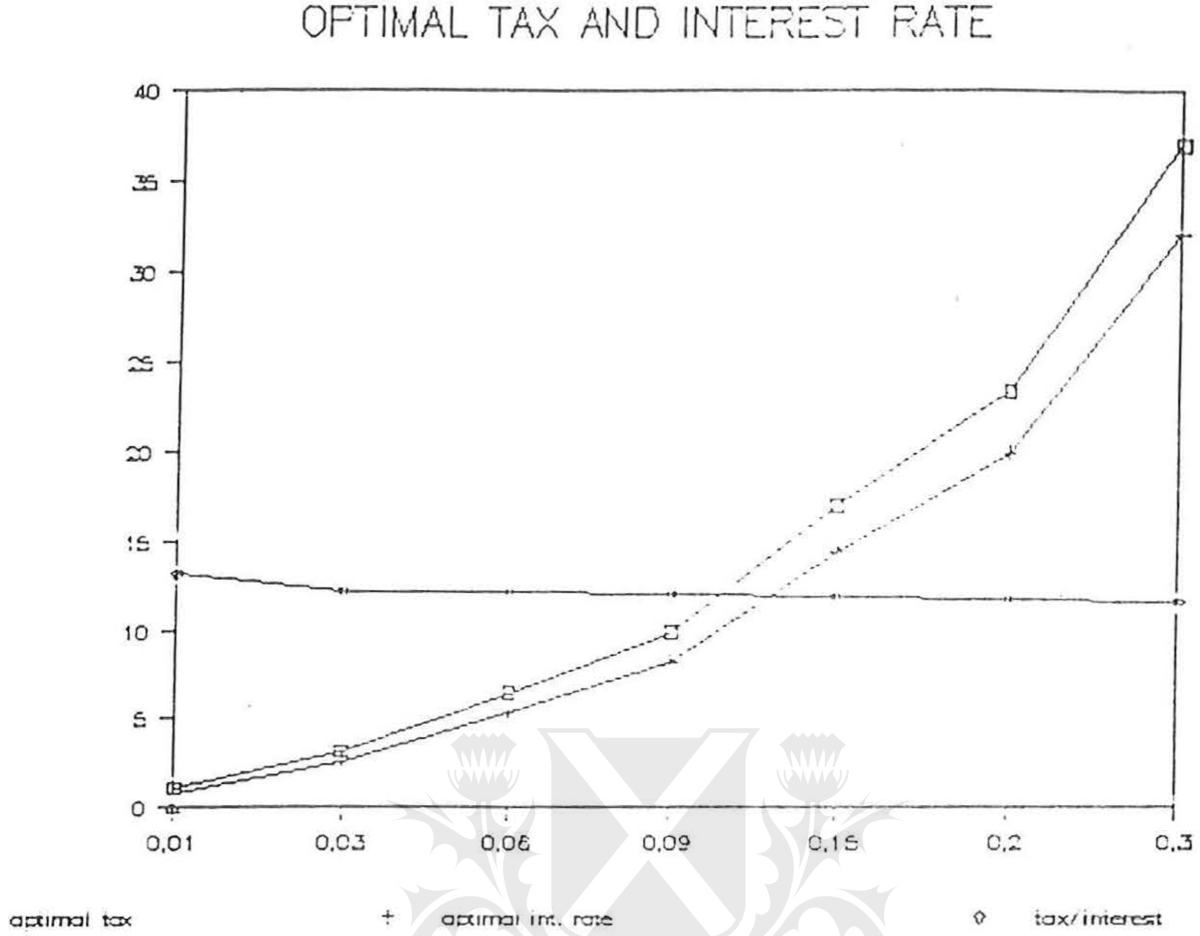

FIGURE III . a

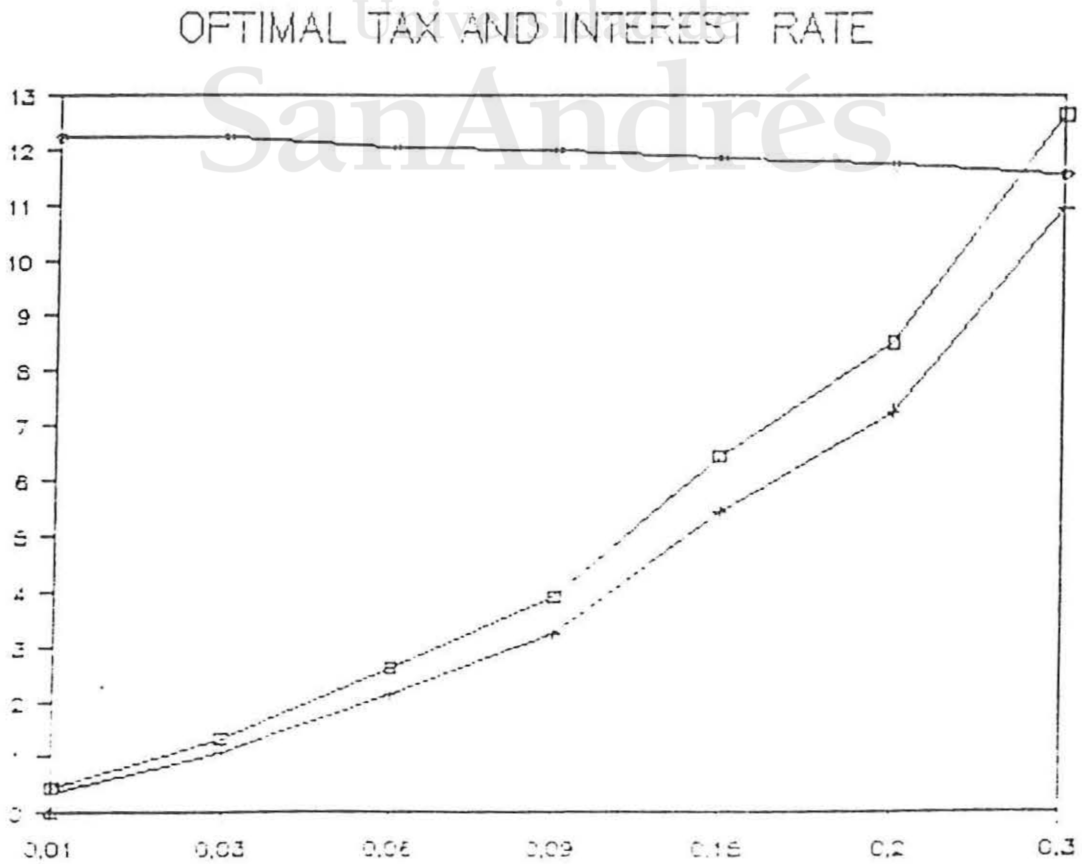

FIGURE III . b

higher. In none of the two cases we observe the ratio between the two taxes being very sensitive to changes in a.

In figure III, we want to measure the effect of changes in government expenditures. The benchmark case is a = .25, b = .2, c = .25, d =.3 and s = 2. We also solve the model assuming σ = 5 (figure IIIb). As before, we plot the optimal consumption tax, the optimal inflation tax and the ratio between the two. Again, the ratio has been multiplied by ten, for a better appreciation of its fluctuations. In both graphs we obtain the same expected pattern. As government expenditures increase, both taxes increase, with the ration between the two showing a very small downward trend.

The main conclusion we obtain from this preliminary calculations is that the key parameter for the determination of the importance of the inflation tax relative to the consumption tax is the size of the official/credit sector relative to the size of the underground/cash sector. Varying the other parameters of the model does not affect that result in a sensible way.

5. Conclusions.

We developed a model to study the optimal rate of inflation in an economy with a large underground sector. We showed that if there are markets which belong to the underground sector in which transactions are carried on using cash, then the optimal rate of inflation is higher than the one implied by Friedman's rule. The basic idea of the model is that inflation is an indirect way of taxing the underground sector.

This model implies that, in general, the determination of the optimal rate of inflation should not be addressed independently of the level of government expenditures. More generally, it implies that the discussion about the appropriate inflation rate should be one piece of a broader discussion which includes other taxes, government expenditures and debt.

These conclusions are similar to the ones reached by Phelps and that we discussed in the introduction. However, we draw the same conclusions from very different models. He considered liquidity as a good that should be taxed as any other good. In our model liquidity is not a good but rather the means to acquire goods. As the government faces restrictions and cannot directly tax some goods, inflation can be used to overcome, at least partially, those restrictions.

Some theoretical results and preliminary simulation work indicate that the key variable to determine the optimal inflation rate is the size of the cash/underground sector of the economy relative to the size of the credit/official sector. The optimal consumption tax rate relative to the optimal nominal interest rate does not appear to be sensitive to changes in the other parameters of the model.

REFERENCES: 1) Canzonery and Rogers. "Is the European Community an optimal currency area?" Working Paper. Georgetown University. (1989)

2) Contini, B. "The irregular economy of Italy: a survey of contributors", in Feige "The underground economies" (1989).

3) Dallago, B. "The irregular economy". (1990)

4) Feige, E. "The underground economies". (1989)

5) ———, —"The meaning and measurement of the underground economy" in Feige, E. "The underground economies" (1989).

6) Hardings, P. and R. Jenkings "The myth of the hidden economy", (1989)

7) Lucas R. and N. Stockey "Optimal fiscal and monetary policy in an economy without capital" Journal of Monetary Economics. (1983)

8) Nicolini J. Unpublished dissertation. University of Chicago. (1991)

9) Park, T. "Reconciliation between personal income and taxable income". Mimeograph. Bureau of Economic Analysis. (1979)

10) Phelps E. "Inflation in the Theory of Public Finance" Swedish Journal of Economics. (1973)

11) Skolka, J. "A few facts about the hidden economy", in Allesandrini and Dallago "The unofficial economy" (1987).

12) Soto H. "El otro sendero" Editorial Sudamericana. (1987)

13) Tanzi V. "The underground economy in the United States and abroad". (1982).

14) White, A. "The nature and extent of unrecorded activity: a survey concentrating on US research" in Alessandrini and Dallago "The unofficial economy" (1987).

15) Woodford, M. "The optimum quantity of money". University of Chicago. Prepared for Friedman, B. and F. H. Hahn, eds., Handbook of Monetary Economics.