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Hold-up problem in multitask ventures

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Hold-up Problem in Multitask Ventures

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Abstract

Two tasks need to be completed in order to generate a surplus. The feasibility and difficulty of the tasks is ex ante unknown. This paper studies the optimal way to tackle these tasks and how this will differ from the way two partners with task specific skills and no commitment would organize the work. The no commitment solution might be associated with higher probability of partial success.

1 Introduction

Some enterprises require solving multiple problems to be successful. As an example, consider an electric car company. To be successful the company might need to first solve a technical problem (say something related to the battery) as well as a marketing problem (how to sell electric cars to average consumers). A scientific paper, on the other hand, might require developing a convincing model and applying an appropriate identification strategy in order to be accepted for publication in a general interest journal. For these 'multi-task' ventures, it is natural to think in terms of partnerships among individuals with complementary skills: an engineer and a marketing specialist; a theorist and an econometrician.

This paper studies a model of search with complementary tasks: Two successes are required to generate some fix surplus. The order of the successes is not determinant. Moreover, we consider a setting with uncertainty about the task feasibility

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and difficulty. An unfeasible task will never be completed, while for a feasible one there is uncertainty about its difficulty (i.e. how long it will take to solve it).

In a dynamic and uncertain setting, the incentives to stay in the relationship and contribute to the joint venture might change over time. Renegotiation is of central importance in explaining how the current and future revenues will be shared among the partners: once an individual solves his task, all the effort he put into it is sunk. This lowers his negotiation power vis-a-vis his not-yet-successful partner, who would request a higher share of the future pie to continue putting effort.

When the tasks are symmetric, the first best involves solving the tasks sequentially: only start to work on the second task once the first one is solved. This result is formally presented as Proposition 1. When we consider task-specific partnerships, the first best cannot be achieved without commitment. The reason is a clear hold-up problem: To achieve the first best, one of the partners should work in the first task in the sequence. Once this task is solved then the cost of the effort is sunk and will not be considered in the negotiation of how to split the surplus. The second partner cannot commit not to renegotiate and therefore the first one would prefer to quit earlier than what would be optimal. This result is formally presented as Proposition 2.

The partners can always, however, work in simultaneous. Proposition 3 shows that there is always an equilibrium in which the partners work only when the other partner is also working. I compare this equilibrium with the first best and find that for some parameters the probability of partial success for partnerships is larger than that of the first best. The reason is that before any success the first best requires to explore the less promising task. In the partnership equilibrium both tasks are explored at the same time, what brings the probability of success up. This result contrasts with the classical hold-up models, which feature underinvestment and therefore worst average performance.

Arrow [1962] and Awaya and Krishna [2019] provide explanations for the apparent phenomenon that innovation happens more often in challengers rather than incumbent firms. This paper gives another possible explanation: established firms can pay wages to their employees to compensate them for their effort and achieve the first best. Startups, on the other hand, usually use shares and options as incentives: contracts contingent on eventual revenues. The inability to commit can lead to more partial successes happening in startups, even though these are less profitable.

Finally, I compare the partnership with the case of a decentralized relationship. The conceptual difference lies in the timing of negotiation: In the decentralized relationship the tasks are carried away independently and negotiation happens ex-post, only when both tasks have been solved. In the partnership, the negotiation is dynamic and evolves with the arrival of new information.

A plausible concern is that even with the ability to write a contract and commit to a contingent split of the surplus, partners might find it profitable to inflate the difficulty of their assigned tasks by hiding early successes to get, in this way, a higher share of the revenues. It turns out that a simple sufficient condition guarantees that the simultaneous equilibrium is achievable even when successes are voluntarily disclosed. This condition is satisfied when the cost of effort is relatively large.

The paper develops as follows: Section 2 introduces the first best i.e. the individual decision maker problem. In Section 3 the different strategic settings are discussed: ex-ante, ex-post and continuous bargaining. Finally, we introduce asymmetric information in Section 4, considering specifically voluntary disclosures of successes. Section 5 concludes.

1.1 Related Literature

This paper introduces perfect complements to the search literature. Klabjan et al. [2014] considers a DM who can learn about different attributes of a product before making a decision. The payoff of the DM is linear in the attributes. In contrast, this paper considers a unit payoff when both tasks are completed. This can be also be rewritten as the payoff being the product of two underlying attributes and some action, and an uncertain time required to learn about these attributes.

The first best problem in this paper can be seen as a multi-armed model with dependent arms. Gittins [1979] and Weitzman [1979] are the classical references. As it was pointed out in the introduction, the ability to renegotiate generates a clear hold-up problem. The hold-up problem was extensively studied in the industrial organization context since the seminal contributions of Klein et al. [1987] and Rogerson [1984].

There are recent papers that study multistage projects, where the order is fixed. Toxvaerd [2006] and Green and Taylor [2016] consider multistage project where the order of the tasks is fixed and known. The set up is related to the experimentation literature with Poisson success arrivals. In most of these papers the effort choice is unobservable. Bonatti and Hörner [2011] consider a model with collective projects but where the efforts of the agents are substitutes instead of complements. The most relevant issue is therefore free-riding.

Finally, there is a recent set of papers that consider the optimal dynamic information acquisition before decision making. Among these some relevant papers are Ke and Villas-Boas [2019], Che and Mierendorff [2017] and Zhong [2018].

2 Individual Decision Maker (First Best)

There are two tasks, labeled with $i \in \{A, B\}$. A fixed surplus of 1 is generated when there is a success in both tasks. Feasibility of tasks i is denoted $\theta_i \in \{0, 1\}$. Assume that the feasibility of tasks are independent with prior probability p_i. At each instant the DM chooses which tasks, if any, to explore $a_i \in \{0, 1\}$. Notice that by exploring only one task you don't learn anything about the feasibility of the other task.

The rate of success arrival for task i is $a_i \lambda_i \theta_i$. This structure gives us a decreasing unconditional hazard rate $\lambda_i p_{i,t}$ where $p_{i,t}$ is the posterior probability of the task being feasible using Bayes' rule.[1] The flow cost of working on tasks i is $c_i > 0$.

The fundamental question is what is the optimal way to tackle these tasks. Once one of the tasks i is solved, the continuation must be the solution to the single-task case: explore the unsolved task j until the unconditional hazard rate falls below c_j. The interesting question is what is the optimal thing to do before the first success. In the extreme cases you have *multitasking*, where you try to solve both tasks at the same time, and *sequential*, where you stick to one task and only jump to the other one after a success. So far, there is no reason why the optimum should be one of these two extreme cases.

[1] The solution for generic decreasing hazard rate function is left for future research.

2.1 Simple discrete example

Consider a discrete version of this model where λ_i is now interpreted as the probability of success at each period of time. Moreover consider the specific case where $\lambda_A = p_B = 1$ and $\lambda_B, p_A < 1$. That is the feasibility of task A is uncertain, but not its difficulty given feasibility: if task A is feasible, it will be solved the first time the agent tries to solve it. This makes learning very fast for task A: after one try you can conclude whether the task is feasible or not. For task B the feasibility is guaranteed, so there is no learning. However, there is uncertainty about how long it would take to solve the task. Assume that task B is worth trying to solve when A was already solved ($\lambda_B > c_B$) and that task A is sufficiently likely to be feasible $p_A > \frac{\lambda_B c_B}{(\lambda_B - c_B)}$.[2]

It will never be optimal to try task A more than once. Also, the conditions guarantee that it will be optimal to abandon only after learning that task A is not feasible. Therefore, the optimal strategy can be characterized by the time task A is explored. Consider the strategy that tries task B for n times before trying strategy A. Notice that there is no learning after a failure in B, so quitting before trying task A is never optimal. If tasks A is feasible, the ex-post payoff is $1 - c_A - c_B\tau$, where τ is the ex-post realization of the first success of task B (we call this the 'realized difficulty' of task B). If task A is not feasible, the payoff for the DM is $-c_A - c_B \min\{\tau, n\}$. n only affects the payoff by decreasing it when the task A is unfeasible and τ large. The optimal n is clearly 0, since $p_A < 0$ and all $\tau \in \mathbb{N}$ have positive ex-ante probability.

The optimal strategy is therefore to try task A, if failure then quit. If success then continue to task B until success.

2.2 Symmetric tasks in continuous time

Going back to continuous time, consider a symmetric setting, that is take $p_A = p_B = p_0$, $\lambda_A = \lambda_B = \lambda$ and $c_A = c_B = c$.

The set of pure strategies is the set of functions specifying at each point in time what to do as a function of history, i.e. past actions and outcomes. For the individual decision problem, consider only strategies that depend on past outcomes.

[2]This last condition guarantees that the DM will not immediately abandon.

Moreover, after a success the optimal continuation is the solution to a single-task problem: continue to explore the uncompleted task until the beliefs about its feasibility reach c/λ. We define $\bar{t}$ as the threshold time after which the belief c/λ is reached:

$$\bar{t} = \frac{1}{\lambda} \log \left(\frac{p_0}{1-p_0} \frac{1-c/\lambda}{c/\lambda} \right). \tag{1}$$

Any strategy that satisfies this second stage optimality can be characterized by the behavior before the first success. Let $s_i : \mathbb{R}_+ \to \{0,1\}$ be the function specifying for each time t whether task i is being explored, given no success so far. Moreover, consider without loss of optimality strategies that never leave gaps where the agent doesn't do anything: Let $\tilde{S}$ be the set of all pairs of functions (s_A, s_B) that satisfy second-stage optimality and such that $s_A(t) = s_B(t) = 0$ implies $s_A(t') = s_B(t') = 0$ for all $t' > t$.

For $s \in \tilde{S}$, Lemma 1 shows that the ex-post payoff $\hat{\pi}(s,\tau)$ is pinned down by the stopping times $t_s = (t_A(s), t_B(s))$ defined as

$$t_i(s) := \int_0^\infty s_i(t)\, dt$$

Lemma 1. Let $s, s' \in \tilde{S}$ two strategies with the same stopping times ($t_s = t_{s'}$) and $\tau \in \bar{\mathbb{R}}^2_+$ the realized difficulty of the tasks, then the ex-post payoff is the same for both strategies, i.e.

$$\hat{\pi}(s,\tau) = \hat{\pi}(s',\tau)$$

Proof. The proof is simply noticing that for a strategy and a success time pair $\tau \in \bar{\mathbb{R}}^2_+$, we can write the ex-post payoff function $\hat{\pi}$ as

$$\hat{\pi}(s,\tau) = \begin{cases} 1 - c\tau & \tau \leqslant (\bar{t},\bar{t}) \text{ and } \neg(\tau \gg t_s) \\ -ct_s & \tau \gg t_s \\ -c(\bar{t} + \min\{\tau_A, \tau_B\}) & \neg(\tau \leqslant (\bar{t},\bar{t})) \text{ and } \neg(\tau \gg t_s) \end{cases} \tag{2}$$

□

Finding the optimal strategy is equivalent to finding the optimal stopping point $t_s \in \mathbb{R}^2_+$. This result is very useful because tells us that the only payoff relevant decision is the stopping point and equivalently, the 'path' that leads to that point is irrelevant. Therefore, we can define a function $\pi(t,\tau) := \hat{\pi}(s,\tau)$ for some s with $t_s = t$. We will use $V(t) := E(\pi(t,\tau))$ for the ex-ante expected value given a stopping time.

Let $p(t)$ be the posterior function after time t exploring a task unsuccessfully,

$$p(t) := \frac{p_0 e^{-\lambda t}}{1 - p_0 + p_0 e^{-\lambda t}}. \tag{3}$$

It will be useful to define the second stage payoff $v(t)$

$$v(t) := p(t) \int_0^{\bar{t}-t} \lambda e^{-\lambda \tilde{t}} [1 + c(\bar{t} - \tilde{t})] \, d\tilde{t} - c(\bar{t} - t) \tag{4}$$

By Lemma 1, the problem for the individual decision maker boils down to choosing a stopping time that maximizes the expected payoff:

$$\max_{t_s \in \mathbb{R}^2_+} V(t_s) \tag{5}$$

Definition 1. We say that a strategy $s \in \tilde{S}$ is *sequential* if $\neg(t_s \gg 0)$. We say that a strategy $s \in \tilde{S}$ is *simultaneous* if t_s is in the 45 degree line ($t_s = me$ for some $m \in \mathbb{R}_+$).

The following Lemma gives three possible candidates for the solution to the problem.

Lemma 2. The optimum stopping time (the one that solves Equation (5)) is one of the following candidates:

- $(\hat{t}, \hat{t})$ where $\hat{t}$ is uniquely defined by $p(\hat{t})v(\hat{t}) = c/\lambda$.
- $(0, t^*)$ or $(t^*, 0)$ where t^* is uniquely defined by $p(t^*)v(p_0) = c/\lambda$.

Intuitively, the point $(\hat{t}, \hat{t})$ is the optimal stopping when the individual is forced to work always on both tasks at the same time (multitasking) while the corner

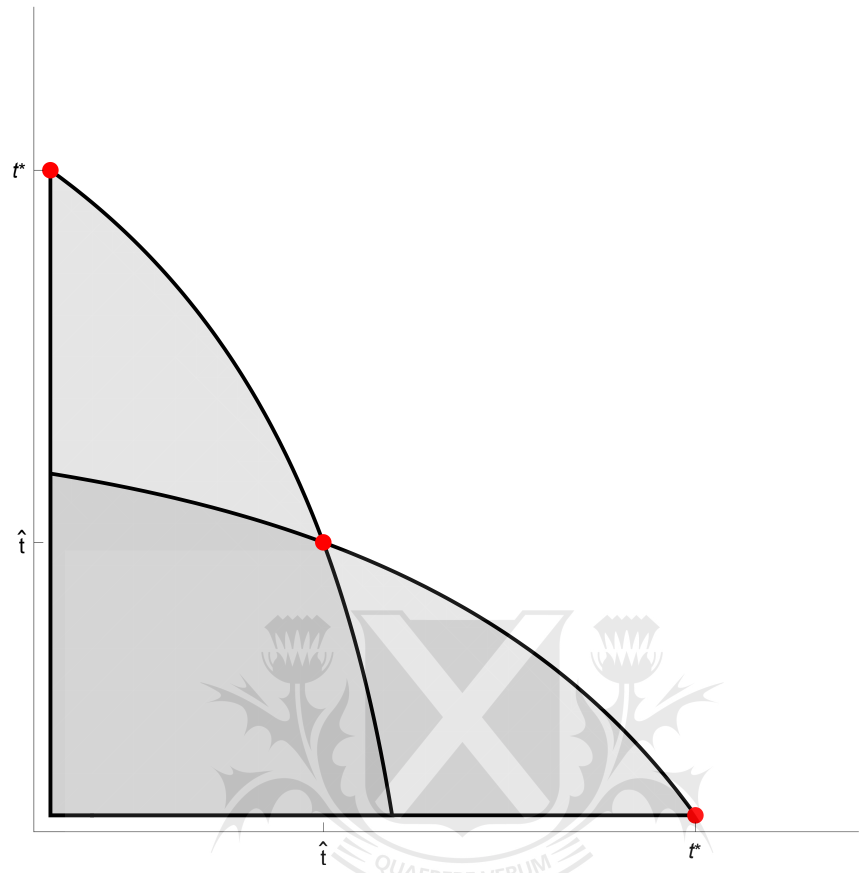


Figure 1: Regions with positive continuation value.
The candidates to the optimal stopping point are
the three red dots.

points are the optimal stopping points when the order of the tasks is given (sequential). We can see the relationship between these times in Figure 2. The thick black curve represents the function $c/\lambda p(t)$. The dashed line represents the function $v(t)$ so these two intersect at $\hat{t}$. $c/\lambda p(t)$ reaches value equal to 1 at $\bar{t}$. Finally, the dotted line is $v(0)$, intersecting $c/\lambda p(t)$ at t^*. Here is a sketch of the proof:[3]

Proof.
- To simplify notation consider the unconditional hazard rate $h(t) = \lambda p(t)$.

[3]For this proof we don't need the extra structure given by the feasibility and constant conditional hazard rate. Any decreasing unconditional hazard rate where the functions cross once from above as in Figure 1 will suffice.

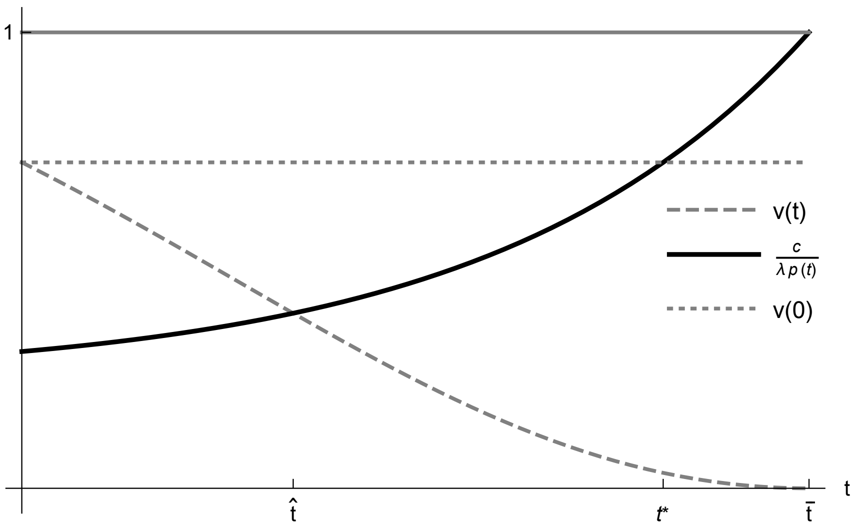


Figure 2: Different relevant times.

- Take the function $f : \mathbb{R}_+ \to \mathbb{R}_+$ defined by $f(t) = v^{-1}\left(\frac{c}{h(t)}\right)$ and its inverse $f^{-1}(t) = h^{-1}\left(\frac{c}{v(t)}\right)$. These two functions define the boundaries of the regions with positive myopic continuation value as in Figure 1. f and f^{-1} cross each other once at $(\hat{p}, \hat{p})$ by symmetry and concavity of f. Moreover the zero of f is larger than the zero of f^{-1}.

- The optimal stopping point has to be such that one of the myopic payoffs is negative and the other one is zero, i.e. in the boundary of the shaded area in Figure 1. The intuition is that strictly outside the shaded area the individual's last exploration second was not optimal independently of what he was doing: he could have done better but quitting earlier. Strictly inside the shaded area the individual quit too early. There were continuation strategies with strictly positive continuation value in at least some direction.

- Finally, if at the stopping point the myopic payoff of a task is negative, then the cumulative time spent exploring that task should have been zero. Remember that all paths lead to the same payoff (by Lemma 1) but a path that ends up with the negative myopic exploration has to be suboptimal.

- This last insight discards all points in the boundaries except for the corners and the point in the 45 degree line (the red points in Figure 1).

□

Proposition 1 concludes our characterization of the first best for the symmetric case. $(\hat{t}, \hat{t})$ is a saddle point of the function $V(t)$, and therefore the two optimal points are the corners that give the same value by symmetry. We prove a stronger result in Lemma 3: every interior stopping time t gives lower value than a point in the axes.

Lemma 3. Let $t \gg 0$, then $V(t) < \max\{V((t_A + t_B, 0)), V((\bar{t}, 0))\}$

A formal proof of this lemma can be found in Appendix A.1. For an intuition consider a $t \gg 0$ with $t_A > t_B$ and $t_A + t_B < \bar{t}$ as in Figure 3. We will show that in this case $V(t) < V(t')$ where $t' = (t_A + t_B, 0)$. As we argued in Lemma 1, the path is irrelevant. In particular, both strategies could start exploring task A for a period t_A. The remaining question is what is optimal after time t_A with no success. Strategy t switches to explore task B for time t_B but t' continues exploring A for the same time t_B.

The proof considers the ex-post realizations of τ that would generate different payoffs for the two strategies (shaded areas in Figure 3). The lightly shaded areas cancel out (part $\mathcal{D}$ in Appendix A.1): if both tasks are feasible, then the past is irrelevant and the continuations give symmetric payoffs. The dark areas don't cancel out, however, since these include the possibility that one of the tasks is not feasible. If the continuation for t' was to stop at $\bar{t} - t_A$ after a success in task A, the payoffs will be the same but with lower probabilities for the vertical dark rectangle, what gives an advantage to strategy t'. The strategy with the optimal continuation does, of course, even better. (Part $\mathcal{E}$ in Appendix A.1.)

This result immediately gives us the solution to the individual decision maker problem from Equation (5). The result is formally presented in the following proposition:

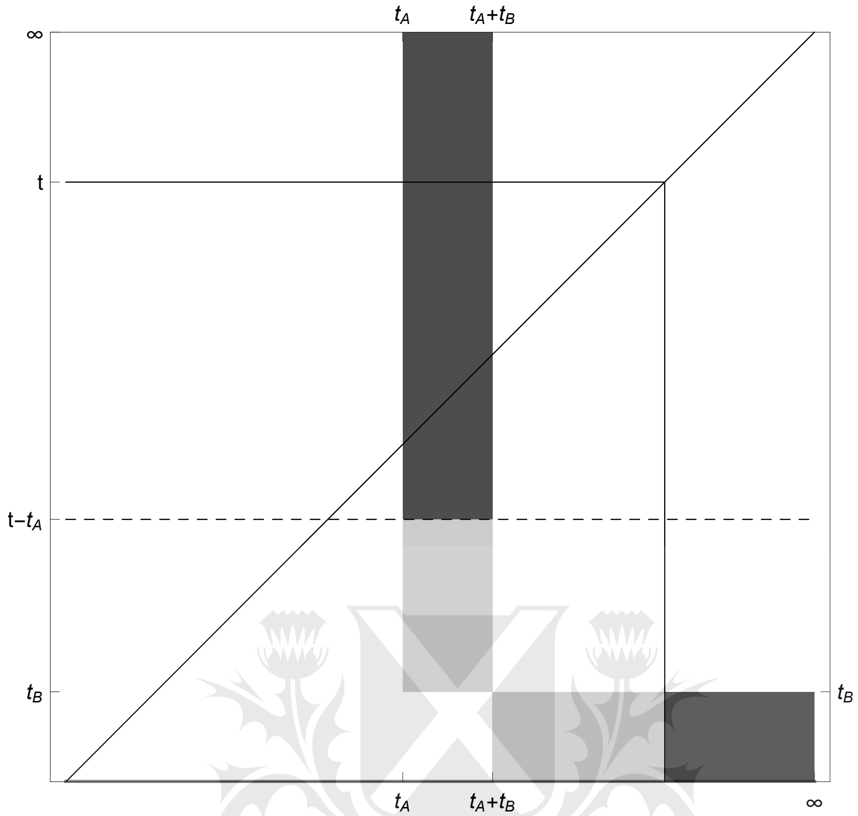


Figure 3: Shaded areas where the two strategies lead to different ex-post payoffs. The lightly shaded areas correspond to part $\mathcal{D}$ and cancel each other. The dark gray areas correspond to part $\mathcal{E}$.

Proposition 1. The optimal exploration is sequential. Moreover,

$$\arg\max_{t} V(t) = \{(0, t^*), (t^*, 0)\}$$

.

2.3 Extentions: asymmetric prior and discounting

The result in Proposition 1 can be extended to settings with asymmetric prior ($p_A \neq p_B$) since the solution to the asymmetric problem is equivalent to the continuation problem of a setting with symmetric prior. Consider for example $p_A < p_B$.

For the symmetric case where $p_0 = p_B$ one solution is sequential starting with task A. Consider the history where task A is unsuccessfully explored for time $t' = \frac{1}{\lambda}\log\left(\frac{p_B}{1-p_B}\frac{1-p_A}{p_A}\right)$, then the posterior $p_A(t')$ is exactly p_A, and the continuation play of continuing exploring the same task has to be optimal. The optimal exploration involves only exploring sequentially, starting with the task with more pessimistic prior. Formally,

Corollary 1. With $p_A < p_B$, the solution to the problem is simply $(t_A^*, 0)$ where t_A^* solves:

$$p_A(t_A^*)\, v(p_B) = c/\lambda.$$

The whole proof follows through if we add discounting, provided that the decision maker cannot speed up innovation by exploring both tasks at the same time. If he could do so, simultaneous exploration has the advantage of a higher payoff for a given realization of τ and the result could be inverted. If, on the other hand, we restrict the agent to explore one task at a time the solution preserves the same qualitative features as in the no discounting case.

The reason is that the discount only affects incentives by lowering the second stage payoff, represented by the function v. With discounting we can redefine v as:

$$v(t,\delta) := p(t)\int_0^{\bar{t}-t} \lambda e^{-(\lambda+r)\tilde{t}} - (1-p(t)+p(t)e^{-\lambda\tilde{t}})e^{-r\tilde{t}}c\, d\tilde{t}$$

The incentives previous to the first success are myopic: you want to continue exploring one task if and only if it is myopically profitable to do so. This gives us back Lemmas 1 and 2.

In the next section we explore strategic settings with task-specific individuals. The cases of single decision maker with asymmetric conditional hazard rate and costs are yet to be explored.

3 Task-specific decision makers

In this section we study different strategic settings. The common theme is that there are two task-specific players, i.e. two agents with the ability to explore one of the two tasks. There is no private information: all actions and outcomes (successes) are publicly observed.

Moreover, we maintain the fact that transfers are not allowed. The only thing the players can do is split the surplus, only possible after both tasks are completed. The differences among the setting will lay on the ability of the players to commit to a contingent split of the generated revenues. In the decentralized relationship the players explore independently and only after both successes they negotiate how to split the surplus. In the partnership negotiation happens within each instant: the players negotiate who will work and how to split the surplus. However, they cannot commit to not renegotiate in the future. In the contract with commitment, the players can sign a binding contract at the beginning of the relationship.

3.1 Decentralized relationship

Two players independently explore the two tasks. The negotiation happens ex-post: only after they both succeed they negotiate how to split the unit surplus. The symmetric Nash bargaining solution tells us that the payoff will be split in half (the outside option is zero for both players since the scrap value) $1/2$ for each, independently of the history, i.e. the realized difficulty of each task.

Any equilibrium of this game must feature under exploration: an abandonment point t in the interior of the shaded area of Figure 1. The reason is the positive externalities: a player cannot appropriate all the value generated by exploring his task. This is the closest setting to the classical hold-up problem.

3.2 Contract with commitment

If the two partners could negotiate a (potentially random) contract at time zero and there were no information asymmetries, the first best could be achieved. The optimal contract could take the following form: First, randomize who will start exploring. Whoever is selected first must explore until t^* and is promised all the value in the relationship. The other player only explores after the first one completes the task and gets pay exactly what leaves him indifferent between continuing exploring or quitting right away.

Notice that the share of the pie for the player that explores second will have to be increasing in the difficulty of his task. This means that, if he could choose, he would prefer to get a late success. We will analyze the incentives to delay disclosure of successes in Section 4.

3.3 Partnership

The most interesting situation is when the players can negotiate in every period. In a discretized version this would mean that at the beginning of each period the players negotiate à la Nash which tasks are going to be explored on that period and how are revenues going to be split in case there are any. After the first success, the player who succeeded can decide to walk away and both players get zero. The player that did not succeed on the other hand has a higher outside option: if we walks away he saves the cost of effort. We don't make any claims about the bargaining power in the second stage, just that it is interior. Let's parametrize by $\alpha \in (0,1)$ the relative bargaining power of the individual that did not succeeded yet.

For this project we take a holistic approach and take the discrete version to the limit. The player that has not succeeded yet gets therefore a fraction α of the remaining value. In case of success his payoff is $\alpha\left(1+\frac{c}{\lambda p(\tau_2)}\right)$. Notice that the payoffs do not depend on the first players time of success. The player that acquires information first will only retain $(1-\alpha)$ of the value, independently of his disclosure time. Therefore, the first best cannot be achieved in a partnership: it would require one of the players to put costly effort for t^*, even though he will receive half of the value. Close to t^* effort is not worth it and the partner would prefer to quit.

Proposition 2. The first best cannot be achieved by a partnership.

The point $(\hat{t}, \hat{t})$, on the other hand, can always be achieved as an equilibrium. Consider the following strategy for player B: Start working and keep working as long as the player A is working or until $\hat{t}$. If the other player stops before $\hat{t}$ stop immediately and only resume when he starts working again. If any of the players succeeds, the continuation payoffs and strategies are the ones stipulated by Nash bargaining. Its easy to check that this strategy is a best response against itself, as long as the myopic payoff of the players remains positive. A player receives:

$$\lambda p(t)\alpha v(t) + \lambda p(t)(1-\alpha)v(t)$$

So it makes sense to work as long as this expression is larger than c, that is until $\lambda p(t)v(t) = c$. That is exactly until $\hat{t}$. We call this a *partnership equilibrium*.

Proposition 3. A simultaneous partnership equilibrium exists for every α.

We showed in the proof of Proposition 1 that the partnership equilibrium was not efficient. It can, however, generate more value than the best sequential incentive compatible path. Interestingly, the partnership equilibrium can induce a higher probability of partial success than the first best. As we can see in Figure 4b for small enough costs the partnership equilibrium is associated with higher probability of reaching at least one success. The reason is that in the first best the task with lowest probability of success is the one being explored, while in the simultaneous partnership equilibrium both tasks are explored at the same time. For some parameters, moreover, the total amount of time exploring before abandonment is larger in the simoultaneous parnership equilibrium than in the first best ($2\hat{p} > p^*$).

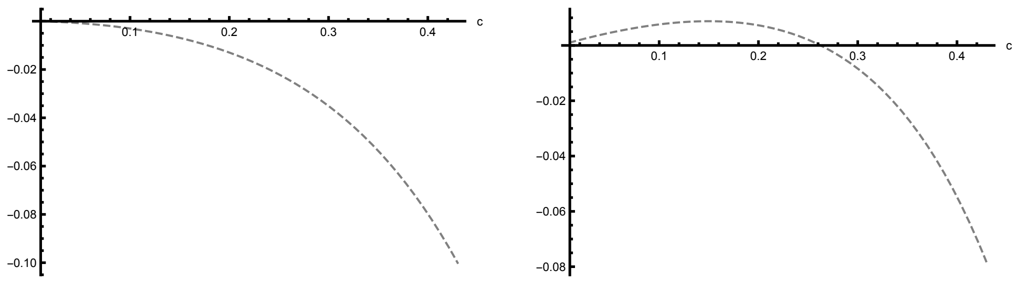
As we can see in Figure 4a, the probability of a success in both tasks is, however, lower compared to the first best for any c.[4]

4 Voluntary disclosure

Consider again partners with task specific skills. We are going to allow them to write a contract specifying how to split the surplus but now they can hide successes. The reason why they might want to hide a success is that the contract might pay them more for later breakthroughs. We ask the question of whether the simultaneous partnership equilibrium is achievable with a contract when successes are self-reported.

Since there is no moral hazard, we keep the assumption that effort is observable. To delay the disclosure of a success a partner has to keep paying the cost. Moreover,

[4]This is not a general result. A general proof of this result or conditions under which is true or not is left for future research.



(a) Joint success

(b) At least one success

Figure 4: Difference between probabilities of success in simultaneous partnership equilibrium vs first best for $p_0 = 0.9$ and $\lambda = 1$.

we are going to focus on renegotiation proof contracts: the players cannot commit to leave value on the table. Consider the following contract:

$$q_2(\tau_1, \tau_2) = 1 - c(\bar{t} - \tau_1) \tag{6}$$

Where τ_1 and τ_2 are the times of the first and second success and q_2 is the share of the pie that the second player to succeed gets. The first player to succeed gets the residual: $q_1(\tau_1, \tau_2) = c(\bar{t} - \tau_1)$.

This can be interpreted as the second player getting a fixed bonus $1 - c\bar{t}$ plus compensation for his cost $c\tau_i$, and the first player being the residual claimant. This contract pays the second player the minimum he will receive in any renegotiation-proof mechanism. He will have incentives for immediate disclosure since the marginal revenue c is equal to the marginal cost. A sufficient condition for him to put effort along the way is that $p_0 \in [c/\lambda, 0.5]$[5].

Now all rest to study is the incentives of the first player to succeed. When he succeeds at time τ_1, the expected payoff of immediate disclosure is

$$E[c(\bar{t} - \tau_2) | \tau_2 > \tau_1]$$

[5]The necessary and sufficient condition is that $f(p_0) \leqslant f(\frac{c}{\lambda})$ where $f(x) = \frac{1}{x} + \log(\frac{x}{1-x})$. f is decreasing on $(0, 0.5)$ and increasing on $(0.5, 1)$. So the necessary and sufficient condition is $p_0 \in [a, b]$ where a and b are the two roots of $f(x) = f(c/\lambda)$. A sufficient condition is the one given.

Proof: $c/\lambda p_t$ is convex in t: the second derivative is $-\frac{c}{p_t^2}\frac{\partial p_t}{\partial t} > 0$. So, all need to prove is that at time zero the incentives are correct: $c/\lambda p_0 \leqslant 1 - c\hat{t}$. This is equivalent to $f(p_0) \leqslant f(c/\lambda)$.

At $\hat{t}$, the player would like to disclose his success, since otherwise the other player will stop (what brings a payoff of zero). Before $\hat{t}$, the marginal incentive to delay disclosure relies on the event that the other player succeeds and therefore getting a higher payoff as a observably second player. The cost is having to keep exerting effort for the period in which the player pretends to not have succeeded.

Here we compute the marginal net benefit of delayed disclosure at time t. A necessary and sufficient condition for no delay is that this marginals are not positive for all times before $\hat{t}$.

We want for all t

$$p(t)\left[\int_0^{\Delta} \lambda e^{-\lambda\tau}[1-c(\bar{t}-t-\tau)-c\tau]\,d\tau + \int_{\Delta}^{\bar{t}-t} \lambda e^{-\lambda\tau} c(\bar{t}-t-\tau)\,d\tau\right] - (1-p(t)+p(t)e^{-\lambda\Delta})c\Delta$$

to be maximized at $\Delta = 0$ among all $\Delta \leqslant \hat{t}-t$. A necessary and sufficient condition is the partial at zero to be non-positive for all t.

Taking partial and evaluating at $\Delta = 0$,

$$\lambda p(t)[1-2c(\bar{t}-t)] - c \leqslant 0 \qquad \forall t < t_0$$

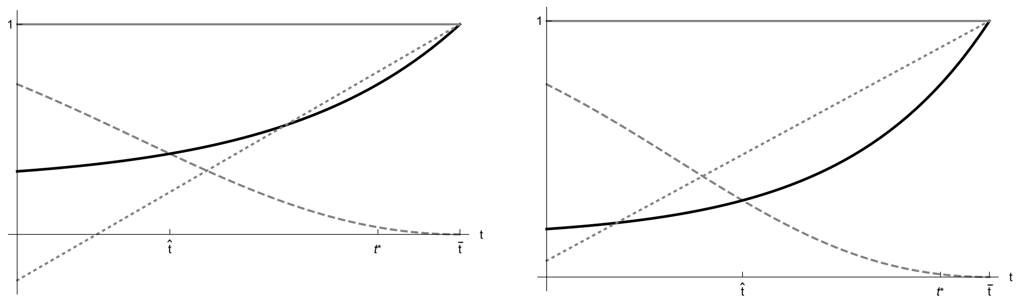
This is equivalent to

$$1-2c(\bar{t}-t) \leqslant \frac{c}{\lambda p(t)} \qquad \forall t < t_0$$

$\frac{c}{\lambda p(t)}$ is convex and both left and right hand side are equal to 1 for $t = \bar{t}$. So a necessary and sufficient condition is that at $\hat{t}$ the inequality holds.

$$1-2c(\bar{t}-t) \leqslant \frac{c}{\lambda p(\hat{t})} \tag{7}$$

In Figure 5 we can see examples of the condition being satisfied and violated. As in Figure 2, the thick black line represents the function $c/\lambda p(t)$ and the dashed gray function is the function $v(t)$, so these two intesect at $\hat{t}$. In the left picture the dotted function $1-2c(\bar{t}-t)$ is below the intersection so the condition is satisfied and therefore immediate disclosure is compatible with optimal behavior of the players. In the right picture the dotted function is above the intersection, so there are incentives to delay disclosure of success.



(a) Immediate disclosure with $p_0 = 0.8$, $\lambda = 1$, and $c = 0.24$

(b) Delay in disclosure with $p_0 = 0.8$, $\lambda = 1$, and $c = 0.15$

Figure 5: Sufficient conditions for immediate disclosure

Proposition 4. If eq. (7) holds, the partnership equilibrium is achievable with voluntary disclosure of successes.

5 Conclusion

This paper provides an answer to a fundamental question: how to optimally tackle tasks that are perfect complements when there is uncertainty about their feasibility and difficulty. In the independent and symmetric case the tasks should be completed sequentially. The more general question for the case with less structure in the arrival rate and asymmetry is left for future research.

Incorporating in the analysis the incentives for partners with complementary skills, I find that if the partners cannot commit to a contingent split of the surplus or if the task completion is not publicly observable then the first best is not achievable. An equilibrium where both partners work simultaneously is always achievable in the no-commitment case and achievable for some parameters in the voluntary disclosure case.

This equilibrium has some desirable properties, but it is not unique. What can be said about the set of all equilibria or sets for relevant refinements is an important step forward. This paper does not provide any insights for the case of voluntary disclosure when the sufficient condition for simultaneous equilibrium is not satisfied.

A Proofs

A.1 Proof of Lemma 3

Proof. We just need to prove that $(\hat{t},\hat{t})$ is not optimal. That there is an element in S that dominates it. We are going to prove a more general claim: Grab $t := (t_A, t_B)$ with $t_A > t_B$ and $t' := (t_A + t_B, 0)$. We will see that $V(t') - V(t) > 0$.

$$V(t') - V(t) = \int [\pi(t',\tau) - \pi(t,\tau)]\, dF(\tau)$$

Notice that $\pi(t',\tau) = \pi(t,\tau)$ for $\tau \geqslant t$, $\tau \leqslant (t_A + t_B, t_B)$ and $\tau_A \leqslant t_A$. So we only need to consider $\tau \in [t_A, t_A + t_B] \times [t_B \times \infty]$ and $\tau \in [t_A + t_B, \infty] \times [0, t_B]$.

$$V(t')-V(t) = \int [\pi(t',\tau)-\pi(t,\tau)]\mathbb{1}_{\tau\in D\cup E}\, dF(\tau) - \int [\pi(t,\tau)-\pi(t',\tau)]\mathbb{1}_{\tau\in D'\cup E'}\, dF(\tau)$$

We divide the proof in two:

$$\int [\pi(t',\tau) - \pi(t,\tau)]\mathbb{1}_{\tau\in D}\, dF(\tau) = \int [\pi(t,\tau) - \pi(t',\tau)]\mathbb{1}_{\tau\in D'}\, dF(\tau) \qquad (\mathcal{D})$$

$$\int [\pi(t',\tau) - \pi(t,\tau)]\mathbb{1}_{\tau\in E}\, dF(\tau) > \int [\pi(t,\tau) - \pi(t',\tau)]\mathbb{1}_{\tau\in E'}\, dF(\tau) \qquad (\mathcal{E})$$

Part $\mathcal{D}$: Lets do the following change of variables: $(x,y) \in [0,t_B] \times [0, \bar{t} - t_1 - t_2]$. Then all elements in D can be written as $t + (x,y)$ and the elements of D' as $t' + (y,x)$

$$\int [\pi(t',\tau) - \pi(t,\tau)]\mathbb{1}_{\tau\in D}\, dF(\tau) =$$
$$\int_0^{t_B}\int_0^{\bar{t}-t_1-t_2} p_0^2\lambda^2 e^{-\lambda(t_A+x)} e^{-\lambda(t_B+y)} [\underbrace{\pi(t', t + (x,y))}_{1-c(t_A+t_B+x+y)} - \underbrace{\pi(t, t + (x,y))}_{-c(t_A+t_B)}]\, dy\, dx$$

$$\int [\pi(t,\tau) - \pi(t',\tau)]\mathbb{1}_{\tau\in D'}\, dF(\tau) =$$
$$\int_0^{t_B}\int_0^{\bar{t}-t_1-t_2} p_0^2\lambda^2 e^{-\lambda(t_A+t_B+y)} e^{-\lambda x} [\underbrace{\pi(t, t' + (y,x))}_{1-c(t_A+t_B+x+y)} - \underbrace{\pi(t', t' + (y,x))}_{-c(t_A+t_B)}]\, dy\, dx$$

Part $\mathcal{E}$: Now consider $(x,y) \in [0,t_B] \times [0,\infty]$ so that all elements in E can be written as $(t_A, \bar{t} - t_A) + (x,y)$ and the elements in E' as $(\bar{t}, 0) + (y,x)$.

$$\int [\pi(t',\tau) - \pi(t,\tau)]\mathbb{1}_{\tau\in E}\, dF(\tau) \geqslant \int [\hat{\pi}(t,\tau) - \pi(t,\tau)]\mathbb{1}_{\tau\in E}\, dF(\tau) =$$

$$-\int_0^{t_B} p_0 \lambda e^{-\lambda(t_A+x)} \left[(1-p_0) + p_0 e^{-\lambda \bar{t}}\right] c(\bar{t} + x - t_A - t_B)$$

Where $\hat{v}$ is the value of a strategy that suboptimally quits trying to solve task B at $\bar{t} - t_A$ when $\tau_A \in (t_A, t_A + t_B)$. The inequality comes from the fact that by definition of $\bar{t}$ the agent wants to continue exploring task B at $\bar{t} - t_A$ when in this situation:

$$\begin{aligned} E[\pi(t', \tau)|\tau \in E] &= \frac{1}{Pr(\tau \in E)} \int \pi(t', \tau) \mathbb{1}_{\tau \in E} \, dF(\tau) \\ &\geqslant E[\hat{\pi}(t, \tau)|\tau \in E] \\ &:= \frac{1}{Pr(\tau \in E)} \int \hat{\pi}(t, \tau) \mathbb{1}_{\tau \in E} \, dF(\tau) \end{aligned}$$

On the other hand,

$$\begin{aligned} &\int [\pi(t, \tau) - \pi(t', \tau)] \mathbb{1}_{\tau \in E'} \, dF(\tau) = \\ &-\int_0^{t_B} p_0 \lambda e^{-\lambda x} \left[(1-p_0) + p_0 e^{-\lambda \bar{t}}\right] c(\bar{t} + x - t_A - t_B) \end{aligned}$$

Subtracting side by side,

$$\begin{aligned} &\int [\pi(t', \tau) - \pi(t, \tau)] \mathbb{1}_{\tau \in E} \, dF(\tau) - \int [\pi(t, \tau) - \pi(t', \tau)] \mathbb{1}_{\tau \in E'} \, dF(\tau) \geqslant \\ &\int_0^{t_2} p_0 \lambda e^{-\lambda x} c(\bar{t} + x - t_1 - t_2)(1-p_0)(1 - e^{-\lambda t_1}) \, dx > 0 \end{aligned}$$

$\square$

A.2 Proof of Proposition 1

Given the candidates given by Lemma 2 and that Lemma 3 rules out the interior point $(\hat{t}, \hat{t})$, the corners that give the same value by symmetry achieve the maximum value.

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