Universidad de San Andrés

Departamento de Economía

Maestría en Economía

The role of information on the efficiency of Democratic Representative Institutions

Gonzalo ROMERO VILLANUEVA

DNI 41.558.973

Mentora: Lucía QUESADA

Buenos Aires, Argentina

31 de marzo, 2023

Tesis de Maestría en Economía de

Gonzalo ROMERO VILLANUEVA

"El rol de la información en la eficiencia de las Instituciones Democráticas Representativas"

Resumen

¿Cómo afecta la información a las Instituciones Democráticas Representativas? Propongo un modelo de competencia espacial circular en tres etapas que captura los procesos electorales y lleva el análisis a un nuevo nivel, que es la capacidad de los parlamentos o legislaturas para generar consenso. Estas instituciones se entienden como tecnologías que transforman los resultados de los procesos electorales en consenso, que es una característica fundamental de los sistemas democráticos, y están afectadas por un factor de información. Utilizando un escenario simple, argumento que el crecimiento de la información disponible para los votantes puede reducir la capacidad de los partidos políticos de representar rangos amplios de preferencias y aumentar el número de equilibrio de partidos, lo cual tiene un impacto negativo en el consenso. Este efecto es menor cuando los políticos tienen una preferencia por el consenso. También exploro cómo se podría entender la participación de los votantes en este contexto y cómo la información puede afectar a la participación electoral al aumentar el costo de moverse por el círculo de la agenda y potencialmente aumentar el número de equilibrio de partidos. Mi modelo proporciona un marco para la investigación empírica, ya que permite calibrar factores para verificar las predicciones teóricas utilizando datos reales. Al llamar la atención sobre la compleja relación entre la información y el funcionamiento de las Instituciones Democráticas Representativas, espero contribuir a una mejor comprensión del papel que juega la información en los sistemas democráticos.

Palabras clave: Información Asimétrica, Salop, Participación Electoral, Instituciones Democráticas, Consenso

"The role of information on the efficiency of Democratic Representative Institutions"

Abstract

How does information affect Democratic Representative Institutions? I propose a three stage model of circular spatial competition that captures electoral processes and takes the analysis to a new level, which is the capability of parliaments or legislatures to generate consensus. These institutions are understood as technologies that transform the results from electoral processes into consensus, which is a key feature of democratic systems, and they are affected by an information factor. Using a simple scenario, I argue that the growth of available information for voters can make it harder for political parties to represent a broader range of preferences and rise the equilibrium number of parties, which has a negative impact on consensus. This effect is smaller when politicians have a preference for consensus. I also explore how voter participation could be understood in this setup and how information can affect voter turnout by rising the cost of moving through the agenda circle and potentially rise the equilibrium number of parties. My model provides a framework for empirical research, allowing for factor calibration in order to test theoretical predictions using real data. By drawing attention to the complex relationship between information and the functioning of Democratic Representative Institutions, I expect to contribute to a better understanding of the role that information plays in democratic systems.

Keywords: Asymmetric Information, Salop, Voter Participation, Voter Turnout, Democratic Institutions, Consensus

Códigos JEL: C72, D62, D72, D82

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1 Introduction

Democratic representation is a widely used form of political organization. Recent data identifies more than half of the world's countries as democratic (Roser, 2013; Boix et al., 2018) [1] [2]. These institutions, however different, are often based on or inspired by systems that were created hundreds of years ago. Technology has advanced rapidly during that time, especially information and communication technologies (ICTs). In this paper, I explore how the growing availability of information affects the efficiency of representative institutions in generating consensus. Specifically, I argue that higher levels of information available to voters about the actions and decisions of their representatives may negatively impact the way democratic institutions channel different interests into consensus.

In their origins, representative institutions had a direct social anchor, especially at a local level. For example, in the late eighteenth century in the United States, Legislatures channeled social demands under direct pressure from the people they represented (Gargarella, 1996) [3]. With the advance of time and the growth of cities, society's pressure over politics loosened. As a result, the distance between voters and their representatives, whether at the local, regional or national levels, grew as well. What candidates and politicians did in fulfillment of the role which they had been elected or were aspiring to was not always accessible to the public. This dynamic allowed political actors to negotiate and agree on projects and policies with a certain degree of freedom. It is important to note that these negotiations could imply that, at some point of the process or agreements, representatives had to act or vote against their voters' interests.

Advances in ICTs and the recent emerge of mass media, together with a wider and increasing access to them, broke the foundations on which these negotiations relied. In 2000, only 6.73% of the world's population had access to the internet. This proportion grew to 28.93% by 2010 and 59.94% by 2020, and it is especially widespread in wealthy countries (Roser et al., 2015) [4]. Through social networks, voters can now learn virtually everything about politicians: their origins, ideas, decisions, lifestyles, and customs, among others things. In addition, and more importantly, legislative activity is generally broadcast and, in consequence, known. The instances of deliberation and decision making, which used to remain private, are now public both in what happens and in terms of what each representative says and does. Citizens now have greater access to information about their representatives and political processes, and they also have better tools to channel social demands, which allows for greater transparency and understanding of people's preferences.

In this sense, the digital era provides a new vehicle for social organizations and movements (Hara & Huang, 2011) [5], which might affect political processes (Rohlinger, Bunnage & Klein, 2014) [6] and even result in direct responses by many governments (Tufekci, 2014) [7]. However, this might also erode the capability of representative institutions to channel different interests and generate consensus, particularly when the interests of different groups conflict. Voters do not tolerate their representatives acting against their ideas, even when that would imply the feasibility of projects that could favor them afterwards. Being able to make agreements that ultimately favor everyone, which is at the heart of the functioning of politics, is increasingly difficult in a hyper-connected world.

This limits possible consensus, since political parties are forced to keep certain basic principles and structures firmly and are punished by voters when they deviate from them.

It has been recorded how partisanship and polarization increased in congresses in different parts of the world, and in particular in the United States[1]. Battaglini et al. (2020) [14] show that social connections among legislators in the U.S. Congress have fallen at both intra-party and inter-party levels, although parliamentary activity has remained constant using machine learning techniques. At the same time, they show the growing importance of shocks at the partisan level over the individual level. This, in turn, suggests that political parties are behaving in a more homogeneous way. As a result, there might be growing space for new parties to compete by representing heterogeneous preferences.

In general, economist have studied the efficiency of political institutions as a means of aggregating information, understood as voter preferences. The theoretical framework that began with Condorcet (1785) and was later formally analysed by Hotelling (1929) [15], Downs (1957) [16] and Black (1958) [17] understands that voting rules may lead to different results according to different conditions. More recently, Caillaud and Tirole (1997) [18] study how political parties work as an instance of internal information aggregation that allows the electorate to behave as if individuals were more homogeneous. A similar argument is made by Fitts (1990) [19], who argues that the value of political parties depends on the ability to limit information. In this way, choices can be reduced and inter-group dialogue can be encouraged, as well as rationality in decision making can be improved. In a deeper sense, he argues that less information can help overcome collective action problems when different groups pursue their own interests, especially when they have a stake in government.

In this context, it is of utmost importance to understand the factors or conditions that may affect the capability of Democratic Representative Institutions to organize and channel societies' interests in order to generate action through consensus, which determines their efficiency. The hypothesis of this paper is that voters having more information about the actions and decisions taken by politicians may erode political parties' capability to function as institutions that homogenize preferences, and foster polarization. This is because information increases the degree of association between individuals and their representatives.

I argue that, because of this, modern Democratic Representative Institutions may struggle with generating social consensus. To illustrate the argument, I develop a mathematical model of spatial competition between political parties where the equilibrium depends on an information factor.

In Downsian models, voters are distributed along a line segment that represents the agenda space. This setting allows for variations that generate different outcome predictions. These are generally well suited for analysis regarding limited political parties involved, but lead to more concentrated solutions around the median voter, especially when free movement is allowed at a party level. For the purpose of this paper, a circular city model for the aggregation of preferences is more suitable,

[1] On partisanship, see Rohde (1991) [8], Snyder & Grosecles (1996) [9], Cox and Poole (2002) [10]. On polarization, see Shor & McCarty (2011) [11], McCarty (2015) [12] and McCarty & Poole & Rosenthal (2006) [13].

because it allows for an easier study of settings where many parties can compete simultaneously.

The literature often assumes that political parties have to commit to a certain platform or agenda during elections, and that they stay true to it during office. However, this is not necessarily true. This paper seeks to overcome this view, understanding that efficiency should not be considered merely at the level of electoral rules and political parties, but also at the level of Democratic Representative Institutions, which function as arenas of negotiation in which the actors resulting from information aggregation processes must represent the preferences of their voters. I will build on the basis of Peeters, Saran & Yüksel (2016)'s two-stage circular city spatial competition model [20] by incorporating a new stage into the game setup where elected representatives have to negotiate within parliaments or legislatures and decide how flexible they are regarding the ideas they have been voted to represent. This will be affected by the capability of Democratic Representative Institutions to spur consensus and the level of information available to voters.

The purpose of the setup is to draw academic attention towards the institutions themselves as technologies that could be improved or adapted, focusing on social consensus as a desired quality of democratic systems. In no way is this a criticism of democracy per se, but rather a warning about the current functioning of institutions and the way in which preferences are represented through them in order to channel public actions and decisions. It is utmost necessary that people trust them as a vehicle for the aggregation of social preferences and their capability to fulfill their objectives.

2 Information as a determinant of consensus

In the context of this paper, political parties are defined by the agendas chosen simultaneously by politicians trying to maximize votes. They are internally homogeneous, but they do not necessarily have to commit to their policy platform once elected. This follows literature on moral hazard by understanding electoral platforms as political commitments which may or may not be credible. When information asymmetries are high, voters cannot know how their representatives actually act after the elections, but are affected by their decisions. However, unlike typical moral hazard issues, the externalities associated with this asymmetry could be positive for voters.

Voters choose according to their own preferences considering both their resemblance to their candidates and the level of consensus resulting from Democratic Representative Institutions. This is supposed to be known beforehand by voters in order to simplify the analysis. The information factor ponders how relevant each component is. As it grows, resemblance becomes more relevant because the actual actions and decisions of representatives are better known.

The purpose of Democratic Representative Institutions is understood to be the generation of consensus among societies given differences in voter preferences based on views, likes or interests. These systems act like technologies that transform the electoral results of voter preferences and political parties into consensus and they are affected by the level of information available to voters.

2.1 Game and consensus

I build on the two-stage circular city spatial electoral competition model set by Peeters, Saran & Yüksel (2016) [20] and add a third-stage that reflects actions of elected representatives after the elections.

In this model, there is a finite set of politicians I. The set of agendas A is the circumference of a circle of unit length and agendas are denoted by a, whilst voters distribute uniformly on A. Voter preferences are identified by their location on A. The first stage is defined as follows:

Stage 1. *The I politicians simultaneously choose the agenda each supports within the circle. I assume they can only play pure strategies, and A is the set of strategies for each party. Politicians' strategy profiles are denoted by s.*

Political parties P are partitions among which politicians choose the same strategy profile s and consequently support the same agenda a. Therefore, $\Psi(s)$ are the set of political parties where each is defined by the unique agenda a supported by the politicians that conform them, so $s_j = a$ for all $j \in P$ and all $s_j \neq a$ for $j \notin P$.

Stage 2. *Voters choose between political parties $\Psi(s)$ resulting from Stage 1 according to their preferences Ω and cast their votes. It is assumed they can only play pure strategies. As a result, their strategy profile is a mapping $v_s : A \to \Psi(s)$ such that the voter located at agenda a votes for the political party $v_s(a)$.*

Given s and v_s resulting from Stages 1 and 2, the number of votes a political party $P \in \Psi(s)$ receives determines its weight $w_P(s, v_s)$, which is always positive and such that $\sum_{P \in \Psi(s)} w_P(s, v_s) \leq 1$. Voting rules ρ determine the resulting power $\rho_P(w(s, v_s))$ for each party $P \in \Psi(s)$ as a function of the distribution of weights $w_P(s, v_s)$. As the purpose of this paper is not to analyze how voting rules may affect results, but rather draw focus on the relevance of information affecting political consensus, I can restrict attention to plurality rule as it is done in Peeters, Saran & Yüksel (2016) [20]. In this setup, power is distributed equally among parties with maximum weight (i.e., parties that received the highest number of votes), whilst any other party receives zero power. Therefore, a party is effective under strategy profiles (s, v_s) if it has positive power $\rho_P(w(s, v_s)) > 0$.

The information factor Θ ponders how well the actions and decisions of political parties are known by voters and $Y(\rho_P(w(s, v_s)), \Theta)$ is the resulting consensus derived from the system. Hence, the set Y contains all possible consensus that could be achieved by political parties.

Stage 3. *Given the resulting set of political parties $\Psi(s)$ and their power $\rho_P(w(s, v_s))$, consensus $Y(\rho_P(w(s, v_s)), \Theta)$ is obtained through Democratic Representative Institutions R, according to the information factor Θ.*

Political parties' power is shared equally by politicians that conform them, who receive utility according to their individual power. I incorporate a fixed cost F for establishing or sustaining a political party, which is born for equally among member politicians and pondered by the information factor Θ. This last relation aims to capture how better information technologies allow for a greater and faster diffusion parties' agendas, thus favoring their establishment and persistence in the electoral competition. Given that politicians are also members of society, the resulting consensus achieved affects them as well. Therefore, their utility is:

$$u_j(s, v_s) = \frac{\left(\rho_P(w(s, v_s)) - \dfrac{F}{\Theta}\right) Y\left(\rho_P(w(s, v_s)), \Theta\right)}{J}$$

Where $j \in P \in \Psi(s)$ and J is the amount of politicians who chose the same agenda a and conform P. Voters receive utility according to resulting consensus Y and by how far the political party they choose is from their location a. Voter located at a and voting for party P receives utility $u_i(Y, \Theta, a(P), a)$ where $a(P)$ is the agenda supported by the political party they vote for and u is a continuous function in all its arguments. Hence, $u(Y, \Theta, a(P), a) < u(Y, \Theta, a(P'), a)$ if and only if resulting consensus remains the same and $a(P)$ is located at a greater distance than $a(P')$, or if moving from choosing one political party to another results in greater consensus that overcompensates the costs from moving.

Each voter's location on A represents its most-preferred agenda. Utility decreases continuously in the distance between their location and the agenda supported by the political party they choose. They consider effective political parties' agendas and choose according to their expected utility.

I define efficiency in terms of Democratic Representative Systems as their capability to generate consensus. This is because the level of consensus achieved is related with the satisfaction of social preferences. I can state that:

Definition 1. *A Democratic Representative System R is more efficient than R' if it can generate more consensus Y given a set of politicians I, a set of agendas A, voter preferences Ω and the information level Θ. Formally, if:*

$$Y(R, I, A, \Omega, \Theta) > Y(R', I, A, \Omega, \Theta)$$

There are many possible variations within this framework. I could consider changes at the level of voter or politicians' preferences, voting rules, general behavior (sincere or strategic voting, defections, etc.), Democratic Representative Systems and many other aspects. I will focus on the case of sincere voting. First, I will present a basic model in which politicians' only concern is individual power, in consonance with standard literature that follows the Downsian approach [16]. I will then expand it to the case in which politicians also care about resulting consensus. Afterwards, I will show how voter participation can be understood in this model and explore how information can potentially affect voter turnout.

2.2 Model

Politicians choose their agendas and, by doing so, they also choose the political parties they will conform. Starting a new political party has a fixed cost F, which is pondered by the information factor Θ, and this will be the only entry cost at this level. There are enough politicians so any number of political parties can be a Nash equilibrium, i.e. $I > 2\Psi(s)^2$, where $\Psi(s) \geq 2$, and there is always the possibility for a new political party to enter the electoral competition. Following the Downsian approach[16], politicians' only concern is individual power and they do not receive utility from resulting consensus in this setup.

Let voter preferences be represented by the following function:

$$u_i(Y(\Theta, \Psi(s)), \Theta, d) = Y(\Theta, \Psi(s)) - \Theta d$$

The cost of moving distance $d = |a - a(P)|$, is pondered by an informational factor Θ that reflects the level of information available for voters regarding politicians' actions and decisions. The resulting consensus $Y(\Theta, \Psi(s))$ is determined by both voters and politicians' strategies, and it is known by them. I will assume that all voters must cast their votes, which implies that $\sum_{P \in \Psi(s)} w_P(s, v_s) = 1$, but note that this setup also allows for voters choosing not to vote when $Y(\Theta, \Psi(s)) < \Theta d$.

Political parties group politicians who maximize their individual power which is given by the party's power $\rho_P(w(s, v_s))$, which depends on its weight given by the amount of votes they can capture according to their strategy s and voters' strategy profile v_s minus a cost of entry F pondered by the information factor Θ, and can only locate themselves in one point of the circle. Politicians share power equally among political parties and their utility function is:

$$u_j(s, v_s, F, \Theta) = \frac{\left(\rho_P(w(s, v_s)) - \dfrac{F}{\Theta}\right)}{J}$$

Where J denotes the number of politicians j that choose the same agenda a and conform a political party P. Finally, Democratic Representative Institutions act like technologies that transform the resulting political parties of the election $\Psi(s)$ into consensus, pondered by the information factor Θ:

$$Y(\Theta, \Psi(s)) = \frac{y^{\frac{1}{(1+\Theta)}}}{\Psi(s)}$$

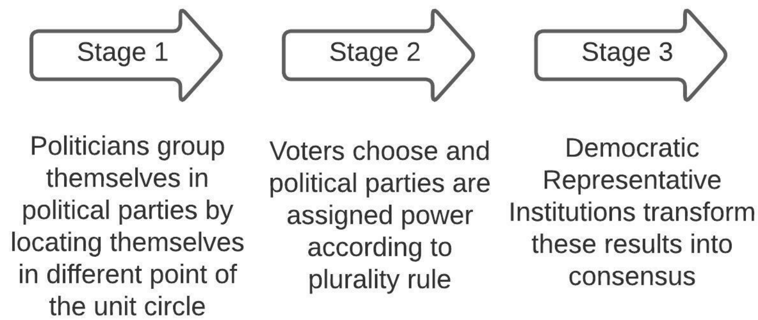
Where y is an adjustable parameter that represents the maximum level of consensus possible, and $Y(\Theta, \Psi(s))$ is decreasing on Θ. The parameter y does not affect the general results of the model, but could be used for calibration. The capability of Democratic Representative Institutions to generate consensus is decreasing in the amount of political parties $\Psi(s)$, because it becomes more difficult when more interests have to be appeased.

The information factor affects the model in three different ways. The first is regarding voters' choices, by pondering the cost of choosing a political party that supports agendas different from the one represented by their location on the circle. The second is by allowing faster diffusion and establishment of new political parties reducing the effective cost of deviation for politicians. The third

[2] This implies that politicians' incentives to deviate from a set of political parties and create a new singleton one are higher when there are at least three politicians in the most crowded party, which always contains the highest incentives to deviation since power is divided equally among politicians conforming each party. This assumption is not strictly necessary. However, in this setup, if the most crowded party has two politicians, there is no difference in the equilibrium conditions when there are two or three political parties. As I am interested in providing a framework where the number of political parties is endogenous and affected by the information factor, I rule out this scenario.

is regarding the functioning of Democratic Representative Institutions, by affecting their capability to generate consensus. Note that, in this setup, the third stage of the model does not affect the game's outcomes. This is because even though voters are concerned about the level of consensus Y, they have no impact on it since politicians' location choice determines the stages' development.

In sum, the timing of decisions in the model is the following:



I can show the following results[3]:

Proposition 1. *In equilibrium, all political parties have the same power determined by their weight given by the number of votes they receive, which is:*

$$\rho_P(w(s,v_s)) = w(s,v_s) = \frac{1}{\Psi(s)} \tag{1}$$

And:

Proposition 2. *In equilibrium, the number of politicians among political parties varies by at most one and the incentive to start a new political party is always greater for parties that are more crowded.*

A politician chooses to keep its choice of agenda a and its consequential political party instead of deviating and starting a new one if:

$$u_j(s,v_s,F) = \frac{\left(\rho_P(w(s,v_s)) - \dfrac{F}{\Theta}\right)}{J} \geq \left(\rho_{P'}(w(s',v_s)) - \frac{F}{\Theta}\right) = u'_j(s',v_s,F)$$

Where $\rho_P(w(s,v_s)) = w(s,v_s) = \dfrac{1}{\Psi(s)}$ and $\rho_{P'}(w(s',v_s)) = w(s',v_s) = \dfrac{1}{\Psi(s)+1}$. Alternatively, politicians have incentives to stray from current political parties and create new ones if the previous condition does not hold. Then:

Proposition 3. *A certain number of political parties $\Psi(s)$ can be an equilibrium if the following condition holds for every $j \in I$:*

$$\frac{F}{\Theta} \geq \left(\frac{1}{\Psi(s)+1} - \frac{1}{\Psi(s)J}\right)\frac{J}{J-1} \tag{2}$$

[3] All proofs are in the Appendix.

Only higher number of parties can be an equilibrium when Θ grows relative to F or when the number of politicians grows, and there is always a number of political parties consistent with a given relation between F and Θ and a given number of politicians J.

The term $\left(\frac{1}{\Psi(s)+1} - \frac{1}{\Psi(s)J}\right)$ is the difference between politician j's initial individual power $\frac{\rho_P(w(s,v_s))}{J}$ and the potential one if he chooses to start a new political party $\rho_{P'}(w(s',v_s))$. The right-hand side of (2) is decreasing in the amount of political parties $\Psi(s)$ and growing in the amount of politicians J. This is how the number of political parties in the model becomes endogenous. Since both sides of the relation are always positive, this implies that for a given level of information Θ and amount of politicians in the most crowded party $\hat{J}$, there is always a number of political parties $\Psi(s)$ such that the condition holds. This way, the level of consensus $Y(\Theta, \Psi(s))$ is affected negatively by the level of information both directly through the functioning of Democratic Representative Institutions and indirectly through the equilibrium amount of political parties, which rises when Θ grows.

Finally, note that this setup allows for multiple equilibria, since once a certain number of political parties Ψ is high enough that Proposition 3 holds, it will also hold for any $\Psi' > \Psi$. However, there's always a greater level of consensus $Y(\Theta, \Psi(s))$ when there are fewer parties competing. Hence:

Definition 2. *The social optimum equilibrium number of political parties is:*

$$\Psi^*(s) = \{\min \Psi(s) \in \mathbb{N} : \frac{F}{\Theta} \geq \left(\frac{1}{\Psi(s)+1} - \frac{1}{\Psi(s)J}\right)\frac{J}{J-1} \;\forall\; \Psi(s) \geq 2, J \in \mathbb{N}, F \in \mathbb{R}_+, \Theta \in \mathbb{R}_+\}$$

And thus the equilibrium is now reduced to the case in which the number of political parties is lower.

2.2.1 Example

The purpose of this subsection is to show how the model results in different social optimum equilibrium number of political parties according to the relation between the information factor and the fixed cost. Consider the simplest scenario in which all parties are conformed by two or three politicians. Lets also limit the example by limiting the possible amount of politicians $I \leq 17$ and keep $\Psi \leq 6$. As a result:

Proposition 4. *When $3\Psi(s) \geq I > 2\Psi(s)$ and $I \leq 17$, the social optimum equilibrium number of political parties Ψ depends on the relation between the fixed cost F and the information factor Θ in the following way:*

i $\Psi = 2$ *if* $4F \geq \Theta$

ii $\Psi = 3$ *if* $\frac{24}{5}F \geq \Theta > 4F$

iii $\Psi = 4$ *if* $\frac{40}{7}F \geq \Theta > \frac{24}{5}F$

iv $\Psi = 5$ *if* $\dfrac{20}{3}F \geq \Theta > \dfrac{40}{7}F$

v $\Psi = 6$ *if* $\Theta > \dfrac{20}{3}F$

And thus the relation captures the prospect that higher information levels imply a higher number of political parties competing electorally in equilibrium, which in turn makes consensus more difficult. Therefore, the subgame perfect Nash equilibrium is defined by politicians and voters' strategies s and v_s and results in an amount of political parties $\Psi(s)$ and a level of consensus $Y(\Theta, \Psi(s))$ according to the relation between the information factor Θ and the fixed cost of starting a new political party F.

2.3 Preference for consensus

In the simplified setup described before, politicians did not receive utility from the resulting consensus. In this subsection, I modify the model by assuming that politicians do take it into consideration. All other characteristics of the game remain unchanged. Politicians' utility function is:

$$u_j(s, v_s, F, Y) = \left(\rho_P(w(s, v_s)) - \frac{F}{\Theta}\right)\frac{Y}{J}$$

The level of consensus Y now affects politicians in relation to the utility derived from their political activity. As before, Democratic Representative Institutions transform decisions into consensus according to the following function:

$$Y(\Theta, \Psi(s)) = \frac{y^{\overline{\frac{1}{(1+\Theta)}}}}{\Psi(s)}$$

The capability of Democratic Representative Institutions to create consensus is decreasing in the amount of political parties $\Psi(s)$, which all have equal power. Politicians' utility can be rewritten as:

$$u_j(s, v_s, F, \Theta, \Psi(s)) = \left(\rho_P(w(s, v_s)) - \frac{F}{\Theta}\right)\frac{y^{\overline{\frac{1}{(1+\Theta)}}}}{(J\Psi(s))}$$

The information factor now affects the model in a new way, which is by affecting politicians' location decision given their new preferences. Every political party receives the same power as before $\rho_P(w(s, v_s)) = w(s, v_s) = \dfrac{1}{\Psi(s)}$. In equilibrium, the amount of politicians within political parties varies by at most one and the incentive to start a new political party is still greater for parties that are more crowded.

Politicians' decision to keep their choice of agenda a and the political party identified by it or deviating and starting a new one is now determined by:

$$u_j(s, v_s, F, \Theta, \Psi(s)) = \left(\rho_P(w(s, v_s)) - \frac{F}{\Theta}\right)\frac{y^{\overline{\frac{1}{(1+\Theta)}}}}{(J\Psi(s))} \geq \left(\rho_{P'}(w(s', v_s)) - \frac{F}{\Theta}\right)\frac{y^{\overline{\frac{1}{(1+\Theta)}}}}{\Psi(s)+1} = u_j'(s', v_s, F)$$

Alternatively, politicians have incentives to stray from current political parties and create new ones if the previous condition does not hold. Then:

Proposition 5. *A certain number of political parties $\Psi(s)$ can be an equilibrium under the preference* *for consensus setup if the following condition holds:*

$$\frac{F}{\Theta} \geq \frac{\Psi(s)^2 J - (\Psi(s)+1)^2}{(\Psi(s)+1)\Psi(s)(J\Psi(s) - (\Psi(s)+1))} \tag{3}$$

In the previous simple setup, in order for the equilibrium condition for political parties to grow progressively according to the relation between the information factor and the fixed cost, I had to assume that $I > 2\Psi(s)$, so the most crowded party was always conformed by at least three politicians. Now, I need to assume that $I > 3\Psi(s)$ in order to maintain the progressive relation[4].

As both sides of the equation are always positive and the right-hand side is decreasing in the amount of political parties $\Psi(s)$ and growing in the amount of politicians J, there is always a certain combination of factors that can be an equilibrium. Also, the previous relation stands where only a higher number of political parties can be an equilibrium when the amount of politicians is higher or the information factor grows in terms of the fixed cost. The level of consensus $Y(\Theta, \Psi(s))$ is now a determinant of the equilibrium amount of political parties by affecting politicians' strategies, and it is also affected directly and indirectly by the information factor.

As before, there can also be multiple equilibria and the level of consensus is greater when there are fewer parties competing. Therefore:

Definition 3. *Under the preference for consensus setup, the social optimum equilibrium number of political parties is:*

$$\Psi^*(s) = \{\min \Psi(s) \in \mathbb{N} : \frac{F}{\Theta} \geq \frac{\Psi(s)^2 J - (\Psi(s)+1)^2}{(\Psi(s)+1)\Psi(s)(J\Psi(s) - (\Psi(s)+1))} \;\; \forall \;\; \Psi(s) \geq 2, J \in \mathbb{N}, \\ F \in \mathbb{R}_+, \Theta \in \mathbb{R}_+\}$$

Note that condition (3) is always less strong than (2), implying that when politicians have a preference for consensus, they have less incentives to deviate and start a new political party. Therefore, the social optimum equilibrium number of political parties is generally lower under this setup.

2.4 Voter participation

On the previous sections, it was implicitly assumed that all voters were forced to cast their votes. However, this is not necessarily true. The model allows for voters to choose whether to vote or not, in which case they receive a standard level of utility $Z = Y(\Theta, \Psi(s)) - C$, where C captures the cost of not voting, which is always positive[5], and $\sum_{P \in \Psi(s)} w_P(s, v_s) < 1$. Formally, voters choose to cast their vote if:

[4]In the new scenario, when the number of politicians is low enough (so the benefits of staying in a given political party are divided among less individuals), the equilibrium with two political parties is stable under a broader range of relations between the information factor and the fixed cost than the one with three. Note that there is still always a number of political parties that could be an equilibrium under any relation between them. However, as the intention is to capture a dynamic where the number of parties grows according to this relation, a higher number of politicians have to compete in this model.

[5]This could capture two effects. The first is that voters like casting their votes, which leads to them feeling that they influenced in the results of the system. The second is a negative economic or social punishment towards those who do not participate in elections.

$$u_i(Y(\Theta,\Psi(s)),\Theta,d) = Y(\Theta,\Psi(s)) - \Theta d \geq Z$$

For simplicity, in this setup I assume that political parties must be equally distanced. Maximal differentiation implies that voters who are farthest from their nearest political party are those in the middle of two of them, implying that $d \leq \frac{1}{2\Psi(s)}$, where $\Psi(s)$ is the equilibrium number of political parties resulting from the game. Therefore, the threshold for participation is set when:

$$d \leq \frac{C}{\Theta}$$

This equation captures two effects. Higher levels of information Θ and higher utility received for not voting $Z = Y(\Theta,\Psi(s)) - C$ (i.e. a lower cost of not voting) reduce the distance threshold for participation. In order for some voters not to cast their votes, the threshold has to be low enough for the restriction to be active. Formally, this happens when $\frac{C}{\Theta} \leq \frac{1}{2\Psi(s)}$. When more political parties compete in equilibrium, voters can support agendas that are closer to them and thus the distance threshold can be lower and remain inactive. As a result:

Proposition 6. *In the subgame perfect Nash equilibrium, the proportion of people who vote is:*

$$t = \min\left[1, \Psi(s)\left(\frac{C}{\Theta}\right)^2\right] \tag{4}$$

Thus, when the threshold for participation is active, each political party receives a total of $\left(\frac{C}{\Theta}\right)^2$ votes. Note that, in this setup, the capability of Democratic Representative Institutions to generate consensus and drive action $Y(\Theta,\Psi(s))$ does not affect the distance threshold. The literature is divided on whether people are more politically active when they are happier with political institutions[6]. Different definitions of Z can easily capture different effects. However, the distance threshold does affect politicians' incentives and, consequently, the equilibrium number of political parties and the system's resulting consensus. Using the simple model in which politicians do not have a preference for consensus in order to simplify the analysis, I can show the following result:

Proposition 7. *A certain number of political parties $\Psi(s)$ can be an equilibrium under the voter participation setup if one of the following conditions hold for every $j \in I$:*

i

$$\frac{F}{\Theta} \geq \left(\frac{1}{\Psi(s)+1} - \frac{1}{\Psi(s)J}\right)\frac{J}{J-1}, \quad \textit{and} \quad \frac{C}{\Theta} > \frac{1}{2\Psi(s)} \tag{5}$$

ii

$$\frac{F}{\Theta} \geq \left(\frac{1}{\Psi(s)+1} - \frac{\left(\frac{C}{\Theta}\right)^2}{J}\right)\frac{J}{J-1}, \quad \textit{and} \quad \frac{1}{2\left(\Psi(s)+1\right)} \leq \frac{C}{\Theta} \leq \frac{1}{2\Psi(s)} \tag{6}$$

[6] The direction of the effect of satisfaction in democracy and voter turnout is disputed (See Pacek, Pop-Eleches & Tucker, 2009 and Ezrow & Xesonakis, 2016)[21][22].

iii

$$\left(\frac{C}{\Theta}\right)^2 \leq \frac{F}{\Theta}, \quad \textit{and} \quad \frac{C}{\Theta} \leq \frac{1}{2\left(\Psi(s)+1\right)} \tag{7}$$

Under the voter participation setup, politicians can now face three different scenarios with different incentives. On the first one, the distance threshold for participation is inactive, and politicians' problem is the same as in the original model. On the second case, the restriction is active, but it would become inactive if a new political party were to enter the competition. In the third scenario, the restriction is strong enough to be active even if politicians wanted to create a new political party, in which case any number of political parties Ψ could be an equilibrium if the cost of starting a new political party is greater than the individual power resulting from doing so.

Once again, there can be multiple equilibria in this scenario, and the consensus level is still greater when there are fewer political parties competing, since Democratic Representative Institutions' technology remains unchanged. Then:

Definition 4. *Under the voter participation setup, the social optimum equilibrium number of political parties is:*

$$\Psi^*(s) = \{\min \Psi(s) \in \mathbb{N} : \begin{cases} \dfrac{F}{\Theta} \geq \left(\dfrac{1}{\Psi(s)+1} - \dfrac{1}{\Psi(s)J}\right)\dfrac{J}{J-1}, & \textit{and} \quad \dfrac{C}{\Theta} > \dfrac{1}{2\Psi(s)} \\ \dfrac{F}{\Theta} \geq \left(\dfrac{1}{\Psi(s)+1} - \dfrac{\left(\frac{C}{\Theta}\right)^2}{J}\right)\dfrac{J}{J-1}, & \textit{and} \quad \dfrac{1}{2\left(\Psi(s)+1\right)} \leq \dfrac{C}{\Theta} \leq \dfrac{1}{2\Psi(s)} \\ \left(\dfrac{C}{\Theta}\right)^2 \leq \dfrac{F}{\Theta}, \quad \textit{and} \quad \dfrac{C}{\Theta} \leq \dfrac{1}{2\left(\Psi(s)+1\right)} \end{cases}$$

$$\forall \ \Psi(s) \geq 2, J \in \mathbb{N}, F \in \mathbb{R}_+, \Theta \in \mathbb{R}_+\}$$

The first condition (5) is the same as (2), and the equilibrium amount of political parties is unchanged. When the restriction is active and the condition is (6), politicians have greater incentives to deviate and start a new political party, and the social equilibrium number of parties is generally higher. Finally, under (7), any number of political parties could be an equilibrium, and the social optimum is $\Psi(s) = 2$ which is the least possible.

3 Conclusions and final remarks

As information and communication technologies become more advanced and widespread, their impact on different aspects of our lives grows as well. This may not be limited to our daily affairs, but rather have a deep effect on the institutions that shape and sustain societies. The purpose of this paper is to rise awareness about how information might be affecting the functioning of Democratic Representative

Institutions and suggest a plausible way in which it might be doing so through their capability to generate consensus.

The proposed framework is based on a three stage game in which voters, politicians and political parties interact in a model that allows for possible variations among voting rules or Democratic Representative Institutions, which are defined as technologies that generate consensus according to the results of elections. In the proposed setup, information has a negative impact on consensus by increasing the level of association between voters and politicians. This makes it more difficult for political parties to represent a broader spectrum of voter preferences and hinders agreements between different parties because it reduces politicians' liberty to act contrary to their voters' ideas. However, this interpretation is not necessarily universal or true. It is possible that growing information is always positive, by allowing for greater transparency and accountability within political processes, as well as for a better understanding and representation of voter preferences. Alternatively, it may be positive until a certain threshold, beyond which excessive information could limit politicians' actions. Changes in the Democratic Representative Institutions technology can capture different effects, allowing for information to act as an enhancing factor, or assuming an inverted U-shape curve according to its level.

Further expansions could also help capture other dynamics that play relevant roles in politics. In this paper, two of them are examined. The first is the case in which politicians have a preference for consensus, which leads to a lower number of political parties in equilibrium and rises consensus. The second is the case in which voter participation is optional, which may result in a greater number of parties in equilibrium and reduce consensus.

The model allows for possible calibration in order to test its predictions in reality. One measure of consensus could be the number of bills approved with votes from both the ruling party or first minorities and the opposition or second minorities. Further research is necessary in order to conclude about the real direction of the effects. The model itself should be redefined according to the resulting data, as different effects could be captured with different variations. For instance, the proposed setup only assigns power to parties who receive the most votes. There could be a scenario with more parties but with power divided unequally among them, leading to a better representation of social preferences through elections and more consensus if few parties concentrate most of the power. Alternatively, allowing for strategic voting could give voters a more active role in the determination of the results of the game, by forcing politicians to take their behavior into consideration when they compete and locate themselves throughout the circle, leading to equilibrium with fewer number of political parties, as they will be more concerned by the resulting consensus.

It is my hope that this paper drives more academic attention towards the economic relations and dynamics involved in parliaments and legislatures, where the results of democratic elections are determined in terms of real social consensus, which is one of the desired characteristics of democratic systems.

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Appendix: Proofs

Proof: Proposition 1

Plurality rule implies that any political party P characterized by agenda a that receives less votes than the maximum amount received by any other party P' characterized by agenda a' is not effective and has zero power. Formally, if $w_P(s, v_s) < \max_{P' \in \Psi(s)} w_{P'}(s, v_s)$, then $\rho_P(w(s, v_s)) = 0$. Assume a strategy profile (s, v_s) in which politician j supports agenda a and receives a utility $u_j(s, v_s, F) = -\dfrac{F}{\Theta J}$. Under strategy profile (s', v_s), where politician j supported agenda a' instead of a, his utility $u'_j(s', v_s, F) = \dfrac{\left(\rho'_P(w(s', v_s) - \dfrac{F}{\Theta}\right)}{(J'+1)} > u_j(s, v_s, F)$. Therefore, in equilibrium all parties must have the same weight and in consequence the same power, and there cannot be parties with zero power.

Furthermore, since all voters must cast their votes, $\sum_{P \in \Psi(s)} w_P(s, v_s) = 1$ and all parties have the same weight, in equilibrium it must be true that $\rho_P(w(s, v_s)) = w_P(s, v_s) = \dfrac{1}{\Psi(s)}$ for every $P \in \Psi(s)$

Proof: Proposition 2

Proposition 2 states that "in equilibrium, the amount of politicians among political parties varies by at most one and the incentive to start a new political party is always greater for parties that are more crowded".

The first implication can be proved by noting that politicians always have incentives to move towards less crowded parties given a set of political parties $\Psi(s)$. Formally, $max_{P,P' \in \Psi(s)} J - J' \leq 1$. Suppose that within a set of political parties $\Psi(s)$, there is a difference of $J - J' = k > 1$ politicians. Then, politician j who chose agenda a that defines political party P could have chosen agenda a' that defines political party P' and:

$$u'_j = \frac{\dfrac{1}{\Psi(s)} - \dfrac{F}{\Theta}}{J'+1} > \frac{\dfrac{1}{\Psi(s)} - \dfrac{F}{\Theta}}{J} = u_j$$

The second implication can be proved by noting that, given a set of political parties $\Psi(s)$, politicians j and j' do not have incentives to deviate from political parties P and P', where $J > J'$, when:

$$u_j(s, v_s, F) - u_j^d(s^d, v_s, F) \geq 0$$

$$\frac{\rho_P(w(s, v_s)) - \dfrac{F}{\Theta}}{J} - (w(s^d, v_s) - \frac{F}{\Theta}) \geq 0$$

Since $u_j^d(s^d, v_s, F) = \left(w(s^d, v_s) - \dfrac{F}{\Theta}\right)$ is the same for both politicians j and j', $J > J'$ and $\rho_P(w(s, v_s)) = \rho_{P'}(w(s', v_s))$:

$$\frac{\rho_P(w(s, v_s)) - \dfrac{F}{\Theta}}{J} < \frac{\rho_{P'}(w(s', v_s)) - \dfrac{F}{\Theta}}{J'}$$

And thus the above condition will not be met first for politician j belonging to the most crowded party P. Another way to see this is that if politician j' has incentives to deviate and create a new political party, then politician j from the most crowded party P also has incentives to deviate. However, politician j having incentives to do the same does not imply the same for politician j'

Proof: Proposition 3

In section 2.2, incentives for creating a new political party are studied, and the equilibrium number of political parties is presented according to the relation between the information factor Θ and fixed costs F. There are always at least two political parties. A politician in the most crowded party keeps its strategy if:

$$u_j(s, v_s, F) = \frac{\left(\rho_P(w(s, v_s)) - \frac{F}{\Theta}\right)}{J} \geq u'_j(s', v_s, F) = \left(\rho_{P'}(w(s', v_s)) - \frac{F}{\Theta}\right)$$

Rearranging I get to:

$$\frac{F}{\Theta}\left(1 - \frac{1}{J}\right) \geq \rho_{P'}(w(s', v_s)) - \frac{\rho_P(w(s, v_s))}{J}$$

Considering that $1 - \frac{1}{J} = \frac{J-1}{J}$, $\rho_P(w(s, v_s)) = \frac{1}{\Psi(s)}$ and $\rho_{P'}(w(s', v_s)) = \frac{1}{\Psi(s)+1}$, I get to:

$$\frac{F}{\Theta} \geq \left(\frac{1}{\Psi(s)+1} - \frac{1}{\Psi(s)J}\right)\frac{J}{J-1}$$

Which is the result presented in Proposition 3. Also, note that:

$$\frac{d}{d\Psi(s)}\left[\left(\frac{1}{\Psi(s)+1} - \frac{1}{\Psi(s)J}\right)\frac{J}{J-1}\right] = \frac{J}{J-1}\left(\frac{1}{\Psi(s)^2 J} - \frac{1}{(\Psi(s)+1)^2}\right)$$

Which is always negative for $J \geq 3$ and $\Psi(s) \geq 2$, which are imposed conditions, meaning that it is lower for a higher number of political parties $\Psi(s)$. Alternatively,

$$\frac{d}{dJ}\left[\left(\frac{1}{\Psi(s)+1} - \frac{1}{\Psi(s)J}\right)\frac{J}{J-1}\right] = \frac{\frac{J-1}{\Psi(s)+1} - \left(\frac{1}{\Psi(s)+1} - \frac{1}{\Psi(s)J}\right)J}{(J-1)^2}$$

Which can be rewritten as:

$$\frac{d}{dJ}\left[\left(\frac{1}{\Psi(s)+1} - \frac{1}{\Psi(s)J}\right)\frac{J}{J-1}\right] = \frac{1}{\Psi(s)\cdot(\Psi(s)+1)\,(J-1)^2}$$

Which is always positive for any number of politicians J, meaning that as the more crowded parties are, the lower the relation between the fixed cost F and the information factor Θ has to be in order for the equilibrium condition for a given number of political parties not to be met. This is intuitive, when understanding that as parties are more crowded, power is divided among more politicians and the utility for staying in a given political party is reduced. Therefore, a lower utility for deviation can be enough for politicians to have an incentive to deviate and choose to start a new party.

Proof: Proposition 4

In the simple scenario, there are between two or three politicians for each political party, and there are always greater incentives to deviation for those where there are three politicians. Based on this, I can study the incentives for a politician who is a part of a political party conformed by three politicians.

When the politician is indifferent between the two possibilities, I assume that he chooses to keep his current strategy instead of changing it. A politician keeps its strategy if:

$$u_j(s, v_s, F) = \frac{\left(\rho_P(w(s, v_s)) - \dfrac{F}{\Theta}\right)}{3} \geq u'_j(s', v_s, F) = \left(\rho_{P'}(w(s', v_s)) - \frac{F}{\Theta}\right)$$

Hence, the five possible cases are:

i $\Psi = 2$ if $4F \geq \Theta$

$$\begin{aligned} \frac{\frac{1}{2} - \frac{F}{\Theta}}{3} &\geq \frac{1}{3} - \frac{F}{\Theta} \\ \frac{1}{6} - \frac{F}{3\Theta} &\geq \frac{1}{3} - \frac{F}{\Theta} \\ \frac{F}{\Theta} - \frac{F}{3\Theta} &\geq \frac{1}{3} - \frac{1}{6} \\ \frac{2F}{3\Theta} &\geq \frac{1}{6} \\ 4F &\geq \Theta \end{aligned}$$

ii $\Psi = 3$ if $\frac{24}{5}F \geq \Theta > 4F$

$$\begin{aligned} \frac{\frac{1}{3} - \frac{F}{\Theta}}{3} &\geq \frac{1}{4} - \frac{F}{\Theta} \\ \frac{1}{9} - \frac{F}{3\Theta} &\geq \frac{1}{4} - \frac{F}{\Theta} \\ \frac{F}{\Theta} - \frac{F}{3\Theta} &\geq \frac{1}{4} - \frac{1}{9} \\ \frac{2F}{3\Theta} &\geq \frac{5}{36} \\ \frac{24}{5}F &\geq \Theta \end{aligned}$$

iii $\Psi = 4$ if $\frac{40}{7}F \geq \Theta > \frac{24}{5}F$

$$\begin{aligned} \frac{\frac{1}{4} - \frac{F}{\Theta}}{3} &\geq \frac{1}{5} - \frac{F}{\Theta} \\ \frac{1}{12} - \frac{F}{3\Theta} &\geq \frac{1}{5} - \frac{F}{\Theta} \\ \frac{F}{\Theta} - \frac{F}{3\Theta} &\geq \frac{1}{5} - \frac{1}{12} \\ \frac{2F}{3\Theta} &\geq \frac{7}{60} \\ \frac{40}{7}F &\geq \Theta \end{aligned}$$

iv $\Psi = 5$ if $\frac{20}{3}F \geq \Theta > \frac{40}{7}F$

$$\begin{aligned} \frac{\frac{1}{5} - \frac{F}{\Theta}}{3} &\geq \frac{1}{6} - \frac{F}{\Theta} \\ \frac{1}{15} - \frac{F}{3\Theta} &\geq \frac{1}{6} - \frac{F}{\Theta} \end{aligned}$$

$$\frac{F}{\Theta} - \frac{F}{3\Theta} \geq \frac{1}{6} - \frac{1}{15}$$
$$\frac{2F}{3\Theta} \geq \frac{3}{30}$$
$$\frac{20}{3}F \geq \Theta$$

v $\Psi = 6$ if $\Theta > \frac{20}{3}F$, which will hold true given that six was the maximum amount of political parties allowed in order to limit the example.

Proof: Proposition 5

Under the preference for consensus setup, politicians have less incentives to deviate and create a new political party. Now, a politician keeps its strategy if:

$$u_j(s, v_s, F, \Theta, \Psi(s)) = \left(\rho_P(w(s, v_s)) - \frac{F}{\Theta}\right) \frac{y^{\overline{(1+\Theta)}}}{(J\Psi(s))} \geq \left(\rho_{P'}(w(s', v_s)) - \frac{F}{\Theta}\right) \frac{y^{\overline{(1+\Theta)}}}{(\Psi(s)+1)} = u'_j(s', v_s, F, \Theta, \Psi(s))$$

Where $\rho_P(w(s, v_s)) = \frac{1}{\Psi(s)}$ and $\rho_{P'}(w(s', v_s)) = \frac{1}{\Psi(s)+1}$.

$$\frac{F}{\Theta}\left(\frac{1}{\Psi(s)+1} - \frac{1}{J\Psi(s)}\right) \geq \frac{1}{(\Psi(s)+1)^2} - \frac{1}{J\Psi(s)^2}$$
$$\frac{F}{\Theta}\frac{(J\Psi(s) - (\Psi(s)+1))}{(\Psi(s)+1)J\Psi(s)} \geq \frac{J\Psi(s)^2 - (\Psi(s)+1)^2}{(\Psi(s)+1)^2 J\Psi(s)}$$
$$\frac{F}{\Theta} \geq \frac{\Psi(s)^2 J - (\Psi(s)+1)^2}{(\Psi(s)+1)\Psi(s)(J\Psi(s) - (\Psi(s)+1))}$$

Which is the result presented in Proposition 5. Also, note that:

$$\frac{d}{d\Psi(s)}\left(\frac{\Psi(s)^2 J - (\Psi(s)+1)^2}{(\Psi(s)+1)\Psi(s)(J\Psi(s) - (\Psi(s)+1))}\right) =$$
$$= -\frac{\left(J^2 - 2J + 1\right)\Psi(s)^4 + (4-4J)\Psi(s)^3 + (6-4J)\Psi(s)^2 + (4-2J)\Psi(s) + 1}{\Psi(s)^2 \cdot (\Psi(s)+1)^2 \left((J-1)\Psi(s) - 1\right)^2}$$

Which is always negative for $J \geq 4$ and $\Psi(s) \geq 2$, which are imposed conditions, meaning that it is lower for a higher number of political parties $\Psi(s)$. Alternatively,

$$\frac{d}{dJ}\left(\frac{\Psi(s)^2 J - (\Psi(s)+1)^2}{(\Psi(s)+1)\Psi(s)(J\Psi(s) - (\Psi(s)+1))}\right) = \frac{t}{(t+1)(tx - t - 1)} - \frac{t^2 x - (t+1)^2}{(t+1)(tx - t - 1)^2}$$

Which can be rewritten as:

$$\frac{d}{dJ}\left(\frac{\Psi(s)^2 J - (\Psi(s)+1)^2}{(\Psi(s)+1)\Psi(s)(J\Psi(s) - (\Psi(s)+1))}\right) = \frac{1}{(tx - t - 1)^2}$$

Once again, this is always positive for any number of politicians J, meaning that the more crowded parties are, the lower the relation between the fixed cost F and the information factor Θ has to be in order for the equilibrium condition for a given number of political parties not to be met.

I also stated that the general condition for a certain number of political parties to be an equilibrium under the preference for consensus setup is always less strong than the condition found on Section 2.2. This can be proved by noting that the difference between them is always negative.

$$\frac{\Psi(s)^2 J - (\Psi(s)+1)^2}{(\Psi(s)+1)\Psi(s)(J\Psi(s) - (\Psi(s)+1))} - \left(\frac{1}{\Psi(s)+1} - \frac{1}{\Psi(s)J}\right)\frac{J}{J-1}$$

Operating on the second term, I get to:

$$\frac{\Psi(s)^2 J-(\Psi(s)+1)^2}{(\Psi(s)+1)\Psi(s)(J\Psi(s)-(\Psi(s)+1))}-\frac{(J-1)\Psi(s)-1}{(\Psi(s)+1)\Psi(s)(J-1)}$$

And now I expand:

$$\frac{\left(\Psi(s)^2 J-(\Psi(s)+1)^2\right)(J-1)}{(\Psi(s)+1)\Psi(s)(J\Psi(s)-(\Psi(s)+1))(J-1)}-\frac{((J-1)\Psi(s)-1)\,(\Psi(s)J-(\Psi(s)+1)}{(\Psi(s)+1)\Psi(s)(J-1)(J\Psi(s)-(\Psi(s)+1))}$$

Operating:

$$\frac{\Psi(s)^2 J(J-1)-(\Psi(s)+1)^2(J-1)-\Psi(s)^2 J(J-1)+\Psi(s)(\Psi(s)+1)(J-1)+\Psi(s)J-(\Psi(s)+1)}{(\Psi(s)+1)\Psi(s)(J\Psi(s)-(\Psi(s)+1))(J-1)}$$

Cancelling, expanding again and rearranging, I get to:

$$\frac{\Psi(s)^2+2\Psi(s)+1-\Psi(s)^2 J-2\Psi(s)J-J+\Psi(s)^2 J+\Psi(s)^2 J-\Psi(s)^2+\Psi(s)J-\Psi(s)+\Psi(s)J-\Psi(s)-1}{(\Psi(s)+1)\Psi(s)(J\Psi(s)-(\Psi(s)+1))(J-1)}$$

Which cancels out to:

$$\frac{-J}{(\Psi(s)+1)\Psi(s)(J\Psi(s)-(\Psi(s)+1))(J-1)}$$

Which is always negative for $J \geq 4$ and $\Psi(s) \geq 2$. This means that, for a given number of political parties $\Psi(s)$ and politicians J, the threshold required for it to be an equilibrium is always higher under the Section 2.2 setup.

Proof: Proposition 6

On section 2.4, the threshold for voter participation is studied. Maximal differentiation implies that given a number of political parties $\Psi(s)$, a voter i located at distance $d \in \left(0, \frac{1}{\Psi(s)}\right)$ will be indifferent between voting for political party P if:

$$\Theta d = \Theta\left(\frac{1}{\Psi(s)} - d\right)$$

And the distance that marks the breaking point is:

$$d = \frac{1}{2\Psi(s)}$$

The threshold for participation is set when:

$$Y(\Theta, \Psi(s)) - \Theta d \geq Z$$

$$Y(\Theta, \Psi(s)) - \Theta d \geq Y(\Theta, \Psi(s)) - C$$

$$-\Theta d \geq -C$$

$$d \leq \frac{C}{\Theta}$$

As the circle is of single unit, the number of voters who vote is also the proportion of voter participation or turnout. When the distance threshold for participation is high enough, the restriction is not active and everyone votes, so $t = 1$. When it is low enough to become an active restriction for voters, each party's power is determined by their weight given by the amount of votes they receive equal to:

$$\rho_P(w(s, v_s)) = w(s, v_s) = \int_{i=0}^{d^*} 2i\, di = d^{*2} = \left(\frac{C}{\Theta}\right)^2$$

And the total amount of voters equals the total amount of votes, which is:

$$t = \Psi(s)\left(\frac{C}{\Theta}\right)^2$$

Proof: Proposition 7

As before, politicians do not have incentives to deviate when:

$$u_j(s, v_s, F) = \frac{\left(\rho_P(w(s, v_s)) - \dfrac{F}{\Theta}\right)}{J} \geq u_j'(s', v_s, F) = \left(\rho_{P'}(w(s', v_s)) - \frac{F}{\Theta}\right)$$

When the model includes voter participation, politicians may face three different scenarios with different incentives. The first case is when the distance threshold for participation is inactive (i.e., when $\dfrac{C}{\Theta} > \dfrac{1}{2\Psi(s)}$.), and politicians' problem remains the same as in Proposition 3. The second one is when the restriction is active, but would not be active anymore if an additional political party joined the competition. Formally, this happens when $\dfrac{1}{2\left(\Psi(s)+1\right)} \leq \dfrac{C}{\Theta} \leq \dfrac{1}{2\Psi(s)}$. In this case, $\rho_P(w(s, v_s)) = \left(\dfrac{C}{\Theta}\right)^2$ and $\rho_{P'}(w(s', v_s)) = \dfrac{1}{\Psi(s)+1}$. The problem is:

$$u_j(s, v_s, F) = \frac{\left(\left(\dfrac{C}{\Theta}\right)^2 - \dfrac{F}{\Theta}\right)}{J} \geq u_j'(s', v_s, F) = \left(\frac{1}{\Psi(s)+1} - \frac{F}{\Theta}\right)$$

Note that, in this scenario, the initial number of votes that each political party receives must be lower than the case where the distance threshold for participation was not active (due to some voters abstaining from voting), and higher than the case where a new political party joins the competition. Formally, it must be true that: $\dfrac{1}{\Psi(s)+1} \leq \left(\dfrac{C}{\Theta}\right)^2 \leq \dfrac{1}{\Psi(s)}$. Rearranging I get to:

$$\frac{F}{\Theta}\left(1 - \frac{1}{J}\right) \geq \frac{1}{\Psi(s)+1} - \frac{\left(\dfrac{C}{\Theta}\right)^2}{J}$$

Considering that $1 - \dfrac{1}{J} = \dfrac{J-1}{J}$ I get to:

$$\frac{F}{\Theta} \geq \left(\frac{1}{\Psi(s)+1} - \frac{\left(\dfrac{C}{\Theta}\right)^2}{J}\right)\frac{J}{J-1}$$

Since $\left(\dfrac{C}{\Theta}\right)^2 \leq \dfrac{1}{\Psi(s)}$ in this scenario, the difference between politicians' initial individual power and their potential one if they deviate and create a new political party is greater than on Proposition 3, and the right-hand side of the equation is greater as well. This means that politicians have greater incentives to deviate. Also, note that the direction of the effect of a rise in the number of political parties or politicians is the same as in Proposition 3. Formally:

$$\frac{d}{d\Psi(s)}\left[\left(\frac{1}{\Psi(s)+1} - \frac{\left(\dfrac{C}{\Theta}\right)^2}{J}\right)\frac{J}{J-1}\right] = \frac{J}{J-1}\left(-\frac{1}{(\Psi(s)+1)^2}\right)$$

Which is always negative, and:

$$\frac{d}{dJ}\left[\left(\frac{1}{\Psi(s)+1}-\frac{\left(\frac{C}{\Theta}\right)^{2}}{J}\right)\frac{J}{J-1}\right]=-\frac{\left(\frac{1}{\Psi(s)+1}-\frac{C^{2}}{\Theta^{2}J}\right)J}{(J-1)^{2}}+\frac{C^{2}}{\Theta^{2}\cdot(J-1)\,J}+\frac{\frac{1}{\Psi(s)+1}-\frac{C^{2}}{\Theta^{2}J}}{J-1}$$

Which can be rewritten as:

$$\frac{d}{dJ}\left[\left(\frac{1}{\Psi(s)+1}-\frac{\left(\frac{C}{\Theta}\right)^{2}}{J}\right)\frac{J}{J-1}\right]=\left(\frac{C}{\Theta}\right)^{2}\left(\frac{\Psi(s)+1-\left(\frac{\Theta}{C}\right)^{2}}{(\Psi(s)+1)\,(J-1)^{2}}\right)$$

Since $\frac{1}{\Psi(s)+1}\leq\left(\frac{C}{\Theta}\right)^{2}\leq\frac{1}{\Psi(s)}$, it must be true that $\Psi(s)+1\geq\left(\frac{\Theta}{C}\right)^{2}\geq\Psi(s)$, so the derivative is always positive for any number of politicians J.

The third scenario politicians may face is when the distance threshold for participation is active, and would still be active even if an additional political party joined the competition. Formally, this happens when $\frac{C}{\Theta}\leq\frac{1}{2\,(\Psi(s)+1)}$. In this case, $\rho_{P}(w(s,v_s))=\rho_{P'}(w(s',v_s))=\left(\frac{C}{\Theta}\right)^{2}$. The problem is:

$$u_j(s,v_s,F)=\frac{\left(\left(\frac{C}{\Theta}\right)^{2}-\frac{F}{\Theta}\right)}{J}\geq u'_j(s',v_s,F)=\left(\left(\frac{C}{\Theta}\right)^{2}-\frac{F}{\Theta}\right)$$

Since J is always positive, politicians do not have incentives to deviate if $\left(\frac{C}{\Theta}\right)^{2}\leq\frac{F}{\Theta}$, and they always have incentives to do so otherwise. Thus, if $\left(\frac{C}{\Theta}\right)^{2}>\frac{F}{\Theta}$, politicians always have incentives to create a new political party, until $\frac{1}{2\,(\Psi(s)+1)}\leq\frac{C}{\Theta}\leq\frac{1}{2\Psi(s)}$ and we are back to the second scenario, or until $\frac{C}{\Theta}>\frac{1}{2\Psi(s)}$ and we are back to the first scenario where the distance threshold for participation is inactive.