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Banks with Peso – Denominated Deposits in Small Open Economies with Aggregate Liquidity Shocks

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Abstract

I extend the traditional Diamond Dybvig framework with aggregate liquidity shocks to small open economies. Currency board may imply perfect risk sharing (with perfect credit markets), contrary to Chang and Velasco 's findings (2000). With interim-date borrowing constraints and fixed exchange rates, Wallace' s (1990) partial suspension of convertibility of deposits is obtained. A banking system with an international lender may implement both allocations without runs. Flexible exchange rates with local-currency denominated deposits improves risk sharing relative to fixed exchange rates when borrowing constraints are present. It also avoids equilibrium bank runs.

1. Introduction (change)

This paper presents an extension of the traditional Bryant (1981) and Diamond and Dybvig (1983) framework, embedded in a small open economy with aggregate preference shocks. In a three period economy, each consumer may become impatient (early consumer) or patient (late consumer). In the model, the number of impatient consumers in the interim period may be either high or low. As a benchmark, I first check that the socially efficient allocation with an (unlimited) international credit line from an *international lender* implies perfect risk sharing, meaning that a non-random and constant per-capita consumption for all agents. It is also obvious that the availability of international credit always allows the implementation of the optimal contract by a competitive banking system. This implementation is free of bank runs given that the international institution also acts as a lender of last resort with unlimited funds.

Since credit lines in reality are far from being unlimited as in the first case, I introduce borrowing constraints in the social planner's problem. I assume that total available international funds are bounded above. In this context it is shown that the constrained optimal contract implies partial suspension of convertibility of deposits. An inefficient bank run equilibrium reappears, provided that no extra credit is available for banks with a sufficiently illiquid long term asset when facing a panic. However, if an international lender of last resort is able to provide extra funds at an interest rate which is bounded above by a ratio of long term to short term deposits, then the run equilibrium disappears. This last result shows the importance of institutions such as the IMF as a liquidity-based-bank-run preventing device in borrowing-constrained small open economies. This issue has been one of the central topics of the discussion about the international financial architecture since the occurrence of the Asian Crisis in 1997.

This paper also adds an interesting analysis on the role of the exchange rate policy in the stability of the banking system. This work extends the discussion presented in Chang and Velasco (2000) about how flexible exchange rates make the financial system less fragile to the case of aggregate liquidity shocks. This paper shows that with flexible exchange rates it is possible to implement better allocations than with fixed exchange rates when the banking system is borrowing-constrained. Hence, fixed exchange rates or even currency boards could only implement a better allocation when no borrowing constraints are added. The partial - suspension - of - convertibility result is also replaced by a depreciation of the nominal exchange rate. Thus, the exchange rate policy is used to manage aggregate liquidity shocks concerning international currency.

This model allows an alternative interpretation of the rescue packages sent by international institutions to Argentina during the Mexican Crisis in 1995. The

theoretical model have interesting implications in terms of policy recommendations. In the context of the recent crisis in Argentina, one of the most important points of discussion was the de-dollarization of the financial system and a subsequent change in the exchange rate regime. This paper shows as a corollary that a banking system with liabilities in local currency combined with flexible exchange rates is able to offer better ex-ante contracts to its depositors than with fixed exchange rates. This model predicts then that a depreciation of the local currency may be the result of an increase in the liquidity needs of the population. In other words, transitory illiquidity may imply the need of a depreciation or devaluation of local currency.

Section 2 discusses the literature on bank runs in closed and small open economies. Section 3 presents the economy. Section 4 analyzed the benchmark case in which the social planner has unrestricted credit in period 1. Section 5 adds borrowing constraints to the planner's problem of the economy in section 3. Section 6 analyzes the case of flexible exchange rates. Section 7 discusses some policy implications. Finally section 8 presents concluding remarks and points of future research.

2. Related Literature

As mentioned before, the model is built on the Bryant (1980) and Diamond and Dybvig (1983) tradition [1]. The main feature in the current paper is the presence of two currencies instead of the typical unique type of money in the literature. However the two papers share with the standard literature the potential existence of two equilibria, one involving runs.

The two main antecessors are the papers by Chang and Velasco (1998a and 2000). They construct a Bryant - Diamond - Dybvig model in a small open economy. In the first paper (1998b) the long term investment is financed partially by international borrowing (to be paid in the last period). Unlike Chang and Velasco (2000), in my model *both* types of consumers (impatient and patient) derive utility from local (real) currency holdings. The consumption for impatient agents is financed by short term international funds to be paid at the end of the economy.

Aggregate uncertainty is modelled as in Wallace (1988 and 1990). The amount of short run withdrawals is stochastic. In the first paper Wallace (1988) shows that the two run-preventing regulatory regimes studied by Diamond and Dybvig cannot be implemented, due to the non-observability of proportion of impatient consumers. In the second paper Wallace (1990) presents a special case in which the

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[1]For a survey on the bank runs literature see Freixas and Rochet (1997, chapter 7).

banking system 's manager can learn the proportion through the order in which consumers withdraw in the interim period. He shows that partial suspension of convertibility in deposit contracts characterizes the planner's optimal allocation. I use this special device for the case of two currencies. More recently, Green and Lin (2000) have shown that with a finite number of depositors (and then, with *aggregate uncertainty* in terms of the proportion of each type of agents) there is a unique equilibrium involving no runs. However an important assumption in this work is the fact that depositors know their position in line, which is absent in Wallace (1990). Actually, Peck and Shell (2001) have recently shown that without this assumption, runs could be part of the optimal contract.

In terms of the empirical literature that motivates this work, the papers by Chang and Velasco's (1998b) and Corsetti, Pesenti and Roubini 's (1998) constitute competing interpretations on the causes of the recent Asian Crisis. The point of view adopted in this paper is close to what Chang and Velasco (1998b) call an *international - illiquidity - based* crisis. In a sense, what constitutes a crisis in my model can be viewed as a liquidity shock. However, as we see below, the interpretation of a *crisis* from the model is twofold. It can be viewed either as part of the fundamentals or as a self-fulfilling crisis. This discussion is better developed in section 6.

3. The Economy.

The economy lasts for three periods: $t = 0, 1, 2$. There are two currencies, called *home* currency, or *pesos*, and *foreign* currency or *dollars*. I also use the term *money* interchangeably with the term currency. There is only one consumption good, which is the numeraire. The economy is small and open. Hence the price in dollars of this good is assumed to be one, identifying then the consumption good directly with the foreign currency. The two usual technologies in the literature are assumed here. There is a storage technology that returns one dollar in period $t + 1$ for each dollar invested in period t (with $t = 0, 1$). On the other hand there is a long-term investment opportunity. For each unit of the good invested in this technology at date 0 it returns $R > 1$ units of the good in period 2, but only $r \in (0, 1)$ in period 1.

There is a Lebesgue measure one of ex-ante identical consumers. At the beginning of period 1 each person receives an idiosyncratic preference shock. This determines whether the consumer survives until period 2 or dies at period 1. The ex-ante probability of dying in period 1 is π. The person who survives until date 1 is called *impatient*, otherwise she is *patient*. This probability is stochastic and unknown ex-ante. In period 0 the proportion of impatient is a random variable. For simplicity I adopt the device presented by Wallace (1990). Assume that π

4

can be either $p\alpha + (1-p)$ with probability q_1 and $p\alpha$ with probability q_2. The complement is the set of patient agents. This is common knowledge. Let $s = 1$ be the aggregate state in which $\pi = p\alpha + (1-p)$ and $s = 2$ be such that $\pi = p\alpha$.

In this section I assume that in period 1, state $s = 1$, all impatient consumers form a queue in order to get their corresponding consumption allocation. In particular, assume that, if state $s = 1$ occurs, then the whole queue containing the impatient agents can be divided into two groups. Let us call the *early* group those who are first and the *late* group to that with agents coming afterwards. Any person is within the early group with probability p and within the late group with probability $(1-p)$. In state 2, the group of patient consumers are divided in two groups. The early group corresponds to the first $p(1-\alpha)$ patient consumers who withdraw a certain amount of pesos at date 1. The late group corresponds to the second $1 - p$ patient consumers. Assume that any impatient agent 's utility function is $u(c_1)$, while if she is patient it is $\left[u(c_2) + v\left(\frac{m_2}{e_2}\right) \right]$. Here c_t denotes consumption of dollars at date t, m_2 is the amount of pesos that a patient consumer holds between periods 1 and 2, and e_2 is the price of the consumption good in pesos (i.e., the nominal exchange rate). The function u is C^2, strictly increasing and strictly concave. The function v is also C^2, strictly concave and it possesses a unique global maximum, called μ.

The ex-ante utility function is:

$$
p\alpha u\left(c_1^1\right) + q_1 \left[(1-p) u\left(c_1^2(1)\right) + p(1-\alpha) \left[u\left(c_2^1(1)\right) + v\left(\frac{m_2^1(1)}{e(1)}\right) \right] \right] \tag{3.1}
$$
$$
+ q_2 \left[p(1-\alpha) + 1 - p \right] \left[u\left(c_2(2)\right) + v\left(\frac{m_2(2)}{e(2)}\right) \right]
$$

where c_1^1 is the amount of dollars that each impatient consumer gets were she first in line (that is, if she is among the first $p\alpha$ consumers), $c_1^2(1)$ is the consumption of each impatient agent in state 1 if she is among those in the second group (whose proportion is $1 - p$ in this state). Also, $c_2(s)$ denotes the dollar consumption by each patient agent in state s (period 2) and $m_2(s)$ is the nominal amount of pesos that each patient consumer holds between periods 1 and 2 at that state. Note that given the proportion of consumers withdrawing at date 1 the planner learns the proportion of patient agents before date 2 starts.

Assume that the planner does *not* observe the aggregate state s at the beginning of date 1. Instead, she must infer that by observing the number of impatient consumers withdrawing at date 1. If after αp impatient consumers nobody else shows up, the planner infers (correctly) that the proportion of impatient people is αp and then the state is $s = 1$. If after those first αp consumers more agents withdraw at date 1, the proportion of impatient agents must clearly be equal to

5

$(\alpha p + (1 - p))$ (provided that nobody lies). Hence whenever the planner observes that there are more impatient consumers than αp, then she infers that the state is $s = 2$. However, in any case, people who were lucky showing up first (among the first αp) should receive the same consumption in period 1 since the planner cannot know the state at that stage, due to the sequential service constraint. This explains the fact that c_1^1 is independent of s.

4. Benchmark: The planner's problem with unlimited funds at date 1.

Suppose that the planner must borrow from abroad an amount of $\bar{d}$ units of the consumption good at date 0. Then she allocates this amount between the short run technology and the long run asset. Let y be the amount of the good invested in the short run technology and x the amount invested in the long run asset. At the beginning of period 1 the preference shock is privately known to each consumer. As explained before, the planner does not know directly whether the proportion of impatient agents is high or low. She must learn this through the actual amount of agents withdrawing at date 1. For now, suppose that the planner is able to differentiate patient consumers from impatient ones. Impatient consumers are ordered through a queue. In state 2 all impatient consumers receive the same amount of consumption c_1^1. In state 1, the first $p\alpha$ impatient consumers also get c_1^1, but the rest of impatient agents gets a (potentially different) amount $c_1^2(1)$, since the planner must learn through the queue whether the true state is 1 or 2. In each case the planner potentially borrows from abroad an additional amount of $z(s)$ dollars to be repaid at date 2. In this last period, the planner pays off an amount of $c_2^j(s)$ dollars to those patient consumers in state s. This is done after repaying the total debt, at a gross rate of 1. Assume also that the exchange rate at all dates is equal to one (this is better explained below).

The problem for the planner is then to maximize (3.1) subject to the constraints

$$x + y \leqq \bar{d} \tag{4.1}$$

$$p\alpha c_1^1 + (1 - p) c_1^2(1) \leqq y + z(1) \tag{4.2}$$

$$p\alpha c_1^1 \leqq y + z(2) \tag{4.3}$$

$$p(1 - \alpha) c_2^1 + (\bar{d} + z(1)) \leqq Rx \tag{4.4}$$

$$[p(1 - \alpha) + (1 - p)] c_2(2) + (\bar{d} + z(2)) \leqq Rx \tag{4.5}$$

6

for every t and s. The (sufficient) first order conditions give the following result.

Proposition 4.1. *The planner's solution implies that:*

$$c_1^1 = c_1^2 (1) = c_2^j (2) = c_2 (1)$$

for $j = 1, 2$. That is, there is perfect risk sharing at the solution of the planner's problem. Moreover the optimal consumption of dollars is equal to

$$\bar{c} = (R - 1) \bar{d}$$

Each patient consumer gets exactly the optimal amount μ.

Proof. See Appendix. ∎

In this economy clearly perfect capital markets in period 1 are enough to ensure perfect risk sharing for all agents. The presence of perfect credit markets at date 1 implies market completeness. This is the underlying argument of the last proposition, which contrasts clearly with the optimality-of-partial-suspension result by Wallace (1990). In his article he showed that partial suspension is optimal in a one - currency economy with fixed endowments. Proposition 4.1 confirms the fact that Wallace's result is a consequence of some type of market incompleteness.

4.1. Implementation of the planner's solution: the case of a currency board.

This subsection shows how to implement the optimal allocation considered above through financial institutions. Consider a mutual fund bank owned by all consumers that pools resources and offers peso - denominated contracts to consumers. There is also a Central Bank that may exchange pesos for dollars. In this subsection it is assumed that this institution acts as a *currency board*. This means that the Central Bank commits itself to buy and sell dollars for pesos at the fixed rate of 1. At date 0, the private bank borrows $\bar{d}$ dollars from abroad and invests this amount in the long run and the short run asset. The commercial bank issues demand deposit contracts to consumers, specifying the amount of pesos to be withdrawn in the corresponding period.

At the beginning of date 1, the state s is realized. Neither the commercial bank nor the Central Bank observe this realization. All consumers show up at the private bank to withdraw pesos. Those who claim to be impatients want to get c_1 pesos, while those who claim to be patient want μ pesos (to be stored until date 2). The commercial bank would learn s through the amount of consumers who claim their type. Starting date 1, the bank knows for sue that at least a proportion

7

$p\alpha$ of the population is impatient and a proportion of at least $p(1-\alpha)$ agents are patient. However, bankers do not know at that time whether the remaining proportion of $1-p$ consumers is of an impatient or of a patient type. Hence, banks pay a certain amount of c_1^1 pesos to the first $p\alpha$ consumers claiming impatience and a certain amount of m_2^1 pesos to the first $p(1-\alpha)$ agents claiming patience. These amounts are independent of the state s, since they pay this before banks learn whether the remaining $1-p$ agents are patient or impatient. Let $c_1^2(1)$ be the amount of pesos that impatient consumers within this second group receive in period 1, state 1. Let $m_2^2(2)$ be the amount of pesos that *late* patient consumers receive in period 1, state 2.

At the beginning of date 1 the commercial bank may borrow from abroad (from an international lender) an amount to satisfy exactly the pesos to be withdrawn by impatient agents. Although there are several ways to assume sequential service constraints here, for simplicity I adopt the following sequence of actions. The commercial bank first may borrow dollars from abroad, which are completely sold to the Central Bank in exchange for an equal amount of pesos (the exchange rate is assumed to be equal to one). Each consumer withdraws pesos from banks. Then impatient agents sell those pesos for dollars at the Central Bank (at an exchange rate equal to one). Impatient consumers consume dollars and disappear. Patient consumers store the pesos until date 2. After this, the Central Bank returns the unused amount of dollars to the commercial banks in exchange for unused pesos, and these banks return the unused amount of dollars to the foreign lender.

At the beginning of date 2 commercial banks must repay their outstanding foreign debt using a portion of the receipts from the long run investments. After repaying foreign debt, the remaining dollars are sold to the Central Bank in exchange for pesos (also at an exchange rate of one). Then the financial intermediary pay pesos to the patient consumers. These pesos are then sold to the Central Bank. Finally, agents consume the dollars sold by the Central Bank and the economy disappears.

The commercial bank then maximizes

$$p\alpha u\left(c_1^1\right) + q_1\left[(1-p)\,u\left(c_1^2(1)\right) + p(1-\alpha)\left(u\left(c_2^1(1)\right) + v\left(m_2^1\right)\right)\right] \quad (4.6)$$
$$+ q_2\left[p(1-\alpha)\left(u\left(c_2^1(2)\right) + v\left(m_2^1\right)\right) + (1-p)\left(u\left(c_2^2(2)\right) + v\left(m_2^2(2)\right)\right)\right]$$

subject to the constraints (4.1) and

$$p\alpha c_1^1 + p(1-\alpha)\,m_2^1 + (1-p)\,c_1^2(1) \;\leqq\; z(1) + y$$
$$p\alpha c_1^1 + p(1-\alpha)\,m_2^1 + (1-p)\,m_2^2(2) \;\leqq\; z(2) + y$$
$$p(1-\alpha)\,c_2^1(1) \;\leqq\; p(1-\alpha)\,m_2^1 + Rx - \bar{d} - z(1)$$
$$p(1-\alpha)\,c_2^1(2) + (1-p)\,c_2^2(2) \;\leqq\; p(1-\alpha)\,m_2^1 + (1-p)\,m_2^2(2)$$
$$+ Rx - \bar{d} - z(2)$$

8

and the incentive compatibility constraint:

$$\max \left\{ u\left(c_1^1\right); u\left(c_1^2\left(1\right)\right) \right\} \leqq \max_s \left[u\left(c_2\left(s\right)\right) + v\left(m_2\left(s\right)\right) \right]$$

It is not difficult to show the conditions under which the planner's allocation presented above can be implemented in this currency board regime. The following proposition shows when this is possible.

Proposition 4.2. *With unlimited credit in period 1 the planner's solution can be implemented in a decentralized banking system equilibrium with a currency board.*

The proof of this result is in the appendix. The intuition is immediate. Under these assumptions it is straightforward to show that banks will not invest in the liquid asset, since they have an unlimited credit line at date 1. Since at the beginning of this period commercial banks do not know the state s, they borrow a total of $\bar{c} + \mu$ dollars coming from abroad. Banks sell these dollars to the Central Bank in exchange for pesos. All impatient consumers withdraw exactly $\bar{c}$ pesos at commercial banks, which are immediately sold to the Central Bank to obtain and consume $\bar{c}$ dollars. All patient consumers withdraw μ pesos to be held until period 2. At the end of period 1, banks resell $(1 - \pi\left(s\right))\bar{c} + \pi\left(s\right)\mu$ pesos to the Central Bank at an exchange rate of one to get the corresponding amount of dollars to be repaid to the foreign lenders before ending $t = 1$. At the beginning of period 2, commercial banks obtain the output from the long term investment to repay the remaining external debt. To do this, patient consumers return their μ pesos withdrawn at date 1 to the commercial banks, which are sold to the Central Bank at an exchange rate of one. Commercial banks are able to pay $\bar{c}$ pesos to the each patient consumer, who finally sell them to the monetary authority, so that each patient consumer can also get $\bar{c}$ dollars.

4.2. No-runs with infinitely available credit with unit gross rate.

This subsection briefly addresses the question of bank runs within the framework above. Assume that an international institution commits to lend (at zero net interest rates) any amount of dollars above $z\left(1\right)$. The next result shows that this commitment is enough to eliminate the run equilibrium in both exchange rate systems.

Proposition 4.3. *Suppose that the international lender lends any amount of dollars to the banking system at zero net interest rates. Hence the run equilibrium is eliminated under a currency board.*

9

The proof is immediate and omitted. The intuition is standard. The fact that $R > 1$ implies that the long term investment is always able to honor all debt, including deposits withdrawn by patient consumers who withdraw early. Hence no bank can fail (in the fixed exchange rate regime, the Central Bank always have enough dollars to be sold in exchange for pesos). Since the equilibrium consumption allocation of dollars satisfies the incentive compatibility constraint then all patient consumers prefer to wait (in the absence of bank failure). Thus the international lender of last resort acts as a coordinating device that ensures that the unique equilibrium is the one without runs.

5. The planner's problem with borrowing constraints.

The last section presents an economy with perfect international credit markets. This clearly contradicts evidence. This is seriously a problem since most of the recent crises were somehow caused by borrowing constraints. This section adds restrictions in the borrowing of the social planner (or the commercial banks in the banking system) in period 1. These constraints are somehow justified by the way institutions such as the IMF lend in practice to developing countries. In particular, official documents from the IMF (2000) confirm that members face *quotas* of credit funding, based on the member 's relative size. These quotas make plausible that even a potential planner may face credit constraints that impedes perfect risk sharing. I then extend the analysis above considering a constrained optimum problem.

More formally, assume that the social planner maximizes (3.1) subject to the constraints (4.2), (4.4), (4.5) and the following equations.

$$x + y \leqq d \tag{5.1}$$

$$d + z\left(s\right) \leqq \bar{d}, \quad s = 1, 2 \tag{5.2}$$

where now d is the decision variable and $\bar{d}$ denotes now the (exogenous) total credit availability at date 1. Here it is assumed that the planner does not observe s at the beginning of period 1. Instead the impatient consumers form a line in front of the pay-off window of the planner, and this learns whether s is 1 or 2 through the amount of impatient agents who show up at date 1. In particular, if the planner only ends up receiving αp agents in period 1, then she interprets that $s = 1$. If however after paying to αp agents there is at least an extra consumer showing up for payment, then she learns that $s = 2$. The probability that, in state 1, each consumer is included in the first αp agents is equal to p. In this case, for every impatient agent who shows up in period 1 for payment the planner may borrow a certain amount of dollars from the international lender to pay off the

10

consumer. Hence the international lender also learns the state s exactly as the social planner does. Relaxations of this assumption seem important but it involves more complicated issues on contract design. This is left for future research.

5.1. Characterization of the constrained optimum.

Due to the borrowing restrictions at date 1, the optimal consumption allocation under these constraints may not be deterministic as in the unrestricted credit case. In particular, it is possible that the consumption allocation of dollars may not be the same for the impatient consumers who are paid first and those who are paid secondly. The next proposition shows that first - in - line depositors get a higher level of consumption than the those in the second group.

Proposition 5.1. *The second best allocation implies that $c_1^1 > c_1^2(1)$. In this allocation all patient agents consume the satiation level of pesos, μ.*

The proof is presented in the appendix. Hence, partial suspension of convertibility (in the sense of Wallace, 1990) reappears here, even though *limited* credit markets are available for the planner in the first two periods. This proposition implies that it is enough to impose an exogenous constraint to the planner (borrower) involving periods 0 and 1 loans to generate this result. In a sense, a binding borrowing constraint is associated with high liquidity needs in the economy. In this situation, the proposition would predict that banks will partially suspend the convertibility of certificates of deposit in an event of aggregate *illiquidity*, which is signaled by facing binding borrowing constraints, although this does not imply any intrinsic problem in the financial system. In other words, partial suspension is also part of the constrained optimal allocation.

In the proof of the last proposition it is shown that impatient agents's consumption can be financed in different ways. However, it is characterized by strictly positive borrowing in period 1, state 1, and also strictly positive investment in the short run asset and/or strictly positive borrowing at date 1, state 2. Hence it can be assumed without loss of generality that the consumption of the first αp consumers is entirely financed by the liquid investment and, in state 1, the rest of payments is financed through borrowing.

5.2. Implementation of the constrained efficient allocation

This subsection explores the relationship between the implementation of second-best allocation and the exchange rate regime within a similar banking system to the one presented in section 4. Consider first the banking system in subsection 4.1. As above, banks borrow from a (private) international lender an amount d

11

of dollars and they invest it in both the liquid and the illiquid asset. The idea is that commercial banks (as defined before) face the borrowing constraint (5.2) at date 1. Therefore financial intermediaries can only borrow up to $\bar{d}$ dollars in total. The rest of actions are as in subsection 4.1.

The question is whether a currency board implements this. The shows that this cannot happen:

Proposition 5.2. *Under a currency board with limited credit as assumed above, the second best allocation cannot be implemented as an equilibrium of a banking system similar to that of section 4.2.*

The formal proof is in the appendix. The main problem is that within this exchange rate system, the amount of pesos withdrawn by the early patient consumers in period 1 (the first $p(1-\alpha)$ agents) is strictly less than μ.

Therefore, a local lender of last resort, providing transitory liquidity in pesos for at least a group of impatient agents, may be needed to implement the second best allocation. Consider the following timing of actions. Actions at date 0 are identical to the system described above in this section. In period 1, each commercial bank sells dollars coming from either liquid investments or from foreign debt borrowed at that date to the Central Bank at a one-to-one rate. In addition, the Central Bank lend μ pesos (per-capita) to the commercial banks. These institutions pay the corresponding amount of pesos to each consumer (the amount is the one specified in the second best allocation). Patient consumers store the μ pesos until date 2. Impatient consumers sell their pesos to the Central Bank at a one - to - one rate. The commercial bank return the unused borrowed pesos to the Central Bank at the end of this period.

In period 2 the commercial banks liquidate their long term investment. They first pay their foreign debt. The remaining is sold to the Central Bank. At the same time, patient consumers return their μ pesos to commercial banks, which at the same time return them to the monetary authority. The commercial banks give $c_2(s)$ pesos to each patient consumer (these pesos are those obtained from selling dollars to the Central Bank). The patient agent sells these pesos to the Central Bank, always at a one-to-one exchange rate.

This system is able to implement the second best allocation, as expressed in the following proposition:

Proposition 5.3. *Implementation of the constrained optimum can be done within a fixed exchange rate regime with a local lender of last resort.*

The proof is in the appendix. The logic of the argument is that all the *consumption* of pesos by patient consumers are completely financed by pesos issued by the Central Bank and lent to commercial banks.

12

5.3. Bank Runs and Lenders of Last Resort

With credit constraints, it is natural to re-think how inefficient bank runs arise in equilibrium. Clearly, as it is the case in the literature the conditions for which bank runs constitute another equilibrium is linked to the return of early liquidation of the long run asset, r, as well as to the consumption allocation at the constrained optimum. (The proof is found in the appendix).

Proposition 5.4. *Assume that*

$$r < R \left(\frac{c_1^2 (1)}{c_2 (1)} \right)$$

Then the contract that implements the constrained optimum is subject to runs. Hence, there is an equilibrium in which the Central Bank fail in period 1. Otherwise no failure takes place.

This is an extension of the result by Chang and Velasco (2000). Suppose that the inequality above holds. In a run, the commercial banks need to liquidate early all of the long term investment. Recall that if more than a proportion of αp consumers withdraw at date 1 then financial intermediaries and the Central Bank infer that the true state is $s = 1$. But then each consumer should get $c_1^2 (1)$. However the inequality implies that total date-1 assets are less than total dollars demanded in period 1, implying the failure of either the intermediary (in the currency board regime) or the monetary authority (in the fixed exchange rate regime, due to its role as local lender in pesos).

Thus, it is clear that the existence of an international lender of last resort constitutes a run-preventing device. Suppose that this international institution provides liquidity in dollars in period 1 above the debt constraint imposed by the private international institution, $\bar{d}$ dollars. Then, it is easy to show the following result (the proof is in the appendix).

Proposition 5.5. *Assume that an international lender of last resort is able to lend any amount whenever a threat of panic arises in the banking system. Then, as long as the gross interest rate is less than or equal to $c_2 (1) \, / \, c_1^1 (2)$ the run equilibrium is eliminated.*

The main conclusion is that this international lending institution eliminates the run equilibrium allowing for implementation of the second best allocation without panics, as long as the interest rate charged by the international lender is *not too high*. Therefore the international lender of last resort is always sufficient to prevent liquidity-based bank runs in a two-currency economy, where the local currency is used only intra-period.

13

5.4. An example: linear quadratic case

To illustrate these conditions I present a numerical example based on linear quadratic utility functions. Assume that

$$u\left(c\right) = -\frac{\gamma}{2}c^2 + \beta c + \delta$$

where all coefficients are strictly positive and $\beta > R\bar{d}$. This is so to assume that, on the relevant domain, $u'\left(c\right) > 0$. Assume also that

$$v\left(m\right) = -\frac{1}{2}m^2 + \pi m + \nu$$

Then:

$$\mu = \pi$$

Given the utility function u above we have that the first order conditions can be reduced to the following linear system.

$$
\begin{aligned}
c_1^1 - q_1 c_1^2\left(1\right) - q_2 c_2\left(2\right) &= 0 \\
-\gamma c_1^1 + \gamma R q_1 c_2\left(1\right) + \gamma R q_2 c_2\left(2\right) &= \beta\left(R-1\right) \\
R\alpha p c_1^1 + R\left(1-p\right) c_1^2\left(1\right) + p\left(1-\alpha\right) c_2\left(1\right) &= \left(R-1\right)\bar{d} \\
R\alpha p c_1^1 + \left(R-1\right)\left(1-p\right) c_1^2\left(1\right) + \left(p\left(1-\alpha\right)+1-p\right) c_2\left(2\right) &= \left(R-1\right)\bar{d}
\end{aligned}
$$

which can be written in the following way

$$
\begin{bmatrix}
1 & -q_1 & 0 & -q_2 \\
-\gamma & 0 & \gamma R q_1 & \gamma R q_1 \\
R\alpha p & R\left(1-p\right) & p\left(1-\alpha\right) & 0 \\
R\alpha p & \left(R-1\right)\left(1-p\right) & 0 & \left(p\left(1-\alpha\right)+1-p\right)
\end{bmatrix}
\begin{bmatrix}
c_1^1 \\
c_1^2\left(1\right) \\
c_2\left(1\right) \\
c_2\left(2\right)
\end{bmatrix}
=
\begin{bmatrix}
0 \\
\beta\left(R-1\right) \\
\bar{d}\left(R-1\right) \\
\bar{d}\left(R-1\right)
\end{bmatrix}
$$

The proof of this is directly derived from the first order conditions and left to the reader. As the explicit solution does not give a specially intuitive condition, I prefer to report the solutions to these problems for numerical examples. Assume the following values for the parameters.

p	α	q_1	R	β	γ
0.25	0.20	0.5	1.5	10	1

The following table shows the second best allocations for three values of $\bar{d}$.

allocation\value of $\bar{d}$	4	5	6
c_1^1	1.1742	1.5419	1.9097
$c_1^2\left(1\right)$	0.5548	0.9484	1.3419
$c_2\left(1\right)$	6.4387	6.5871	6.7355
$c_2\left(2\right)$	1.7935	2.1355	2.4774

14

As it is clear, a softening in the foreign borrowing constraint (i.e., an increase in $\bar{d}$) decreases the percentage of the partial suspension for impatient agents. For example, when $\bar{d} = 4$, the variable $c_1^2(1)$ represents 47.2% of c_1^1, while when $\bar{d} = 4$ it represents 70.26%.

The next table shows gives the upper bound for r so that illiquidity is verified. It also has upper bounds for the interest rate charged by an international lender of last resort in order to prevent runs.

variables\value of d	4	5	6
r	0.1292	0.2159	0.2988
int. rate	11.605	6.9455	5.0194

Note that the maximum interest rate that can be charged by the international lender of last resort seems to decrease in $\bar{d}$. A possible interpretation states that, as the availability of credit in the second best problem increases, the disparity between consumption assigned to (second group) impatient agents and the patient consumers at date 2 decreases. But the upper bound of the interest rate on the credit line provided by the international liquidity provider is lower the closer are these two consumption values. Basically, the greater $\bar{d}$, the more generous is the payment to the impatient consumers in period 1 and the greater is the need of credit in this same period to prevent panics. Hence the interest rate must be less tight to implement this procedure.

6. Constrained optimum and Flexible Exchange Rates

In the last section, implementation of the constrained optimal allocation implies that, when $\mu > c_1^2(1)$, impatient agents who withdrew pesos must return them to the Chang and Velasco (2000) demonstrated that, in the absence of aggregate uncertainty, a *flexible* exchange rate regime is able to implement the optimal allocation without runs. This policy can be interpreted in fact as a threat of devaluation, since in their equilibrium the exchange rate remains fixed at 1. Hence, their model does not generate any equilibrium exchange rate depreciation. The model presented in last section can be adapted so that the implementation of a constrained optimum implies exchange rate devaluations.

Suppose an economy as in section 5. Consider the planner problem first. Assume now that the planner can observe the aggregate state s at the beginning of period 1. In this case, there is clearly only one consumption variable for that state. That is, $c_1(s)$ denotes consumption of dollars for the agent given that she is impatient in period 1 and given the state s. Hence, ex-ante preferences are now defined as follows.

$$q_1 \left[(p\alpha + 1 - p) u\left(c_1\left(1\right)\right) + p\left(1 - \alpha\right) \left(u\left(c_2\left(1\right)\right) + v\left(\frac{m_2\left(1\right)}{e\left(1\right)}\right) \right) \right] \quad (6.1)$$

$$+q_2 \left[p\alpha u\left(c_1\left(2\right)\right) + \left(p\left(1 - \alpha\right) + 1 - p\right) \left(u\left(c_2\left(2\right)\right) + v\left(\frac{m_2\left(2\right)}{e\left(2\right)}\right) \right) \right]$$

where $c_t\left(s\right)$ denotes the consumption of dollars by the agent at date $t = 1, 2$, aggregate state s, and $m_t\left(s\right)$ denotes the amount of pesos consumed at date t, state s. The problem for the planner now is to maximize this utility function subject to the same constraints as the planner in section 5 (I maintain the borrowing constraint). It is clear that the following proposition must be true (The proof is in the appendix).

Proposition 6.1. *The solution to the planner's problem in the case of a publicly observed aggregate state s is incentive - compatible for each consumer and implies a higher ex-ante expected utility relative to that with the informational restrictions.*

Therefore, this allocation is Pareto - improving relative to that studied in last section. It is still less preferable than the one in section 4, given that $c_1\left(1\right) < c_1\left(2\right)$ at the solution. That is, optimal consumption at date 1 is stochastic.

6.1. Implementation: Flexible (Contingent) Exchange Rate Policy

This subsection embeds the same banking system described in section 5 into a flexible exchange rate regime. This exchange rate policy is similar to that used in Chang and Velasco (2000). Consider the banking system analyzed in section 5. This means that commercial banks cannot directly learn the realization of s at the beginning of date 1. Note that with *fixed* exchange rates, commercial banks cannot implement the allocation in the last subsection, since the optimal allocation implies no difference in consumption for all impatient agents in state s. However, a similar banking system can work here if coped with a *flexible* exchange rate.

Consider the following financial system. Each commercial bank borrows d dollars from abroad at date 0 and invests in both the illiquid long term and the liquid short term assets. Each consumer has the right to withdraw μ pesos in period 1, in any state and $c_2\left(s\right)$ at date 2, state s. At the beginning of date 1, the realization of s is not observed by anyone in the economy. The realization of types are observed only by each individual consumer. Then the Central Bank issues $(p\alpha + 1 - p)\,\mu$ pesos to the commercial banks. Each commercial bank pays

16

μ pesos to the impatient consumers withdrawing in the interim period. If the aggregate state is $s = 2$, then the commercial bank pays returns to the Central Bank a total of $(1 - p)\mu$ pesos after honoring the deposits of the $p\alpha$ impatient consumers. They immediately try to sell μ pesos at the Central Bank in exchange for dollars. Following Chang and Velasco (2000), I assume that there is no sequential service constraint at the Central Bank when selling dollars for pesos in period 1. Instead, there is an auction to sell those dollars. But then the Central Bank learns immediately the realization of s since it observes the size of consumers selling pesos for dollars. Therefore the banking authority fixes the exchange rate between pesos and dollars in the following way

$$e_1(s) \equiv \frac{\mu}{c_1(s)} \tag{6.2}$$

This implies that each impatient consumer gets exactly $c_1(s)$ dollars to consume at date 1. Finally, at the beginning of period 2, the Central Bank issues $(1 - \pi(s))c_2(s)$ pesos, where $\pi(s)$ is the proportion of impatient consumers at state s. These pesos are delivered to the commercial banks. Each private bank receives $(1 - \pi(s))\mu$ pesos from the patient consumers, to be returned to the Central Bank. The private bank also pays off $c_2(s)$ pesos to each patient consumer. After this, the private bank gets the revenue from the long term asset, returns foreign debt dollars and deliver the remaining dollars and the returned pesos to the Central Bank. Then, each consumer sells the $c_2(s)$ pesos to the Central Bank at an exchange rate equal to one. It can be easily seen that each patient agent gets $c_2(s)$ dollars to be consumed at the end of period 2. Therefore, this financial system implements the optimal allocation. The next proposition summarizes this result.

Proposition 6.2. *A banking system as described in section 5 can implement the optimal allocation if the Central Bank fixes the period 1 exchange rate as in equation 6.2 with a unit exchange rate in period 2.*

Thus, exchange rates do depend on the state and time. In other words, the exchange rate policy that implements the optimal allocation is contingent. The important point is that, even with foreign credit restrictions and lack of information about the realization of the aggregate shock, this banking system is able to implement a better allocation than that of section 5. However, it seems that a flexible exchange rate is important for this implementation to occur.

6.2. Exchange Rate Policy as a Bank-Run Preventing Device

Chang and Velasco (2000) showed that a similar exchange rate policy prevents bank runs. In this paper, the same conclusion can be easily obtained after some

17

adaptations of that same result.

Proposition 6.3. *There exists a contingent (perfectly credible) exchange rate policy such that the unique equilibrium corresponds to the optimal allocation. Hence bank runs cannot occur in equilibrium.*

Proof. To show this, I extend the depreciation policy to the case of runs. The Central Bank issues at the beginning of date 1 first an amount of $(p\alpha + 1 - p)\mu$ pesos to be delivered to commercial banks. Suppose that $s = 1$. If, more than $p\alpha + 1 - p$ people (say, a proportion $[(p\alpha + 1 - p) + p(1 - \alpha) - \pi]$) try to sell μ pesos in the auction, the Central Bank sets simply the exchange rate equal to

$$\widehat{e}_1(1) \equiv \frac{[(p\alpha + 1 - p) + p(1 - \alpha) - \pi]\mu}{(p\alpha + 1 - p) c_1(1)}$$

Each consumer gets in period 1 a total of $\frac{(p\alpha + 1 - p)}{[(p\alpha + 1 - p) + p(1 - \alpha) - \pi]} c_1(1)$ dollars, which is clearly less than $c_1(1)$. Note that the Central Bank does not suffer any problem since the total amount of dollars to pay here is equal to $(p\alpha + 1 - p) c_1(1)$. Since the optimal allocation implies that $c_2(s) \geq c_1(1)$, then by waiting until date 2 every patient person gets strictly more than by running. Therefore no patient consumer will find optimal to withdraw at date 1.

Suppose that the true state is $s = 2$. Suppose that the proportion of consumers selling pesos in period 1 to the Central Bank is strictly greater than $p\alpha$. Let $p\alpha + \eta$ denote the actual proportion of agents withdrawing μ pesos in period 1. Banks wrongly perceive that the state is $s = 1$, so commercial banks borrow according to what the allocation dictates to borrow in state 1. In this case, then the Central Bank sets the exchange rate equal to

$$\begin{aligned}
\widehat{e}_1(2) &\equiv \frac{[p\alpha + \eta]\mu}{(p\alpha + \eta) c_1(1)} \\
&= \frac{\mu}{c_1(1)}
\end{aligned}$$

Thus, each consumer selling pesos at date 1 gets $c_1(1)$ dollars, which is (weakly) dominated by $c_2(s)$. Hence, no patient consumer chooses to withdraw at date 1 in state 2. This shows that the optimal allocation constitutes the unique equilibrium. ∎

Note that the argument uses the fact that the exchange rate policy *will* be applied in period 1 with certainty. If the Central Bank cannot commit to this policy ex-ante, then the same argument may fail, depending upon the incentives to deviate by the monetary authority. This is specially true if the depreciation of the peso is not interpreted as the outcome of an auction but as a direct devaluation

18

policy. Kawamura (2001) shows that, in absence of aggregate uncertainty, the devaluation threat introduced by Chang and Velasco (2000) may not be credible, in the sense that a benevolent Central Bank may want to deviate from that policy. However the same result has not been extended to the aggregate uncertainty case.

7. Empirical and Policy Implications

The results in this paper sheds some light on the discussion about contingent credit lines for financial systems. Propositions 4.1 and 5.1 state that international institutions may provide funds with high liquidity needs in the short run, so that withdrawals do not have to be suspended. This allows for an alternative interpretation of the banking distress observed in Argentina, in 1995, after the Mexican crisis. The model suggests that the help from the International Monetary Fund, the Inter American Development Bank and the World Bank was mainly directed to provide funds due to a fundamental liquidity shock faced by the banking system of this country (besides the banking system restructuring process, see Camdessus, 1995). It is interesting to compare this view to the traditional self-fulfilling run interpretation of such a crisis. The banking problem in Argentina in 1995 may be viewed as a high realization of the liquidity shock. This interpretation seems to be more consistent with the model presented here than with the usual (inefficient) equilibrium interpretation. As Chang and Velasco (1998a) and others have shown, in a traditional Diamond - Dybvig model without aggregate uncertainty a lender of last resort always prevents runs. Hence runs cannot occur in these equilibria. This is inconsistent with the evidence. Proposition 5.1 specially implies that, in state 1 (when the proportion of impatient consumers is high) banks must reduce payments at some point. It also implies that the borrowing constraint is binding. This can be interpreted as a situation in which the government negotiates an increase in loans (which is not necessary with lower liquidity needs). This negotiation actually happened in 1995 (see the document IMF News, 1995). The actual increase in Disbursements from 1994 to 1995 to Argentina was larger than 2.5 times (from 611.95 millions of ADR's to 1,558.966 millions).

On the other hand, proposition 5.3 states that a suitable local lender of last resort is needed to implement the optimal allocation. In a sense this result suggests that both the international and the local lender could complement each other. However such a local lender cannot have a loose behavior. The main risk of such a situation is to decrease the foreign reserves stock when facing *too many* customers with local currency holdings selling them to the Central Bank. Thus the purpose of the local lender must be limited only to cover local currency liquidity needs. In a different model, Antinolfi, Huybens and Keister (2001) emphasize the need of restrictions imposed on the local lender. In particular, they show that a

19

costless local lender (as it is in my model) implies a continuum of hyperinflationary equilibria. This clearly calls for either high interest rates or for upper bounds on local currency supply to rule this situation out.

It is worth emphasizing that this paper does not try to answer general questions on social optimality of a currency board compared to other exchange rate regimes. The scope of this work is more modest. However this paper predicts that, for economies with *low* level of demand for local currencies, a currency board is consistent with *full* insurance against liquidity shocks. As long as either the return on long term investments is high enough (and certain) or the availability of credit in dollars is high enough, then a currency board is able to provide perfect risk sharing to the customers of a banking system with simple peso-denominated deposit contracts. More in general this implies that countries with a very low relative demand for local currency (relative to foreign currency) may be more propense to use currency boards. Unfortunately I have found no evidence on this issue yet. However this prediction can be contrasted using indirect indicators such as foreign - currency - denominated deposit contracts. A more direct way to contrast the prediction is to look at the units of account used in several main transactions. Casual evidence in Argentina suggests that most important types of transactions (such as real estate and durables) are made using the American dollar as the unit of account. I leave this problem for future research.

The analysis in section 6 implies that, when foreign credit restrictions are binding, then the optimal allocation may call for flexible exchange rates. This statement *must* be qualified. Results in last section only refer to the virtues of flexibility of exchange rates in terms of absorbing liquidity shocks. However, there is a well-known large body of literature about the credibility issues that flexible exchange rates with loose fiscal behavior imply in emerging economies. In a sense, section 6 is not intended to defend flexible exchange rates *per-se*. On the contrary, it confirms the necessity of rules (although these are contingent) and *commitment* to make the policy credible (recall that the exchange rate policy in section 6 is credible due to perfect commitment by the Central Bank).

This analysis also seems helpful to reflect on recent views about the recent Argentine crisis. Several policy suggestions were given as a way out of the currency board regime. Among others, perhaps the Haussman's proposal (2001) was the best known not only in the academic area, but also was certainly partially applied in the real situation. In any case, Haussman proposed a de-dollarization of liabilities in Argentina as an essential pre-requisite to floating exchange rates. In terms of the model in section 6, this proposal seems to make sense for banks with dollar - denominated assets (under the caveats discussed in the paragraph above). The model predicts that flexible rates with peso - denominated bank liabilities implies a better insurance against aggregate liquidity shocks than either

a fixed exchange rate, a currency board or even a dollarized[2] banking system. However, the model is silent about the efficacy of such policies with banks with peso-denominated assets. Since in the current situation most of these assets are also denominated in local currency, the model needs to be modified to address the influence of exchange rate floating on the banking system performance.

On the other hand, as long as the funds from a foreign credit line given by an international lender of last resort are used to cover *transitory illiquidity*, then proposition 5.4 specifies that these credit lines, *regardless of the exchange rate regime*, always prevents runs. Also, proposition 5.4 suggests upper bounds for the interest rate that the international lender of last resort must charge in order to make repayment feasible. Once more the main problem here is to measure deposits with different horizons so that interest rates on these credit lines still allow its preventing role. I do not suggest to take these ratios literally, but they constitute a major guide for interest rate negotiations.

From the paragraphs above it is clear that implementing such institutions is not easy. Monitoring costs (in the sense of keeping track of deposits) and the problem of measuring the liquidity needs in each currency are difficult. This does not mean that they are infeasible in practice, but the implicit informational assumptions give a warning in terms of how to implement them. In any case all these regulatory regimes implied by the results deal with *liquidity* problems. It does not say anything in terms of solvency issues. The main challenge in practice is to discover whether certain financial distress phenomena were caused by liquidity or solvency problems. This still remains an open question for the policy makers.

8. Concluding remarks and possible extensions

This paper has presented an extension of the Diamond - Dyvbig framework to a banking system with two currencies. It was also assumed that the proportion of impatient consumers is unknown (ex-ante). The first point is that a banking system within a currency board regime implements a first best allocation as an equilibrium only when perfect credit markets are available. Otherwise, a local lender of last resort is needed to implement the optimal allocation.

This model is also able to predict a reduction in deposit payments when the liquidity shock is high. The model also predicts a binding borrowing constraint. These two features resembles events in which local banking systems received funds from international institutions when facing some situation of *distress*. The model also emphasizes the importance of the interest rate level on those funds. These cannot be too high so that repayment is ensured. The upper bound is related

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[2]The dollarization case was not discussed since it is a trivial extension of Wallace 's (1990) work.

to the ratio of long term over short term deposits. This has implications on the design process of contingent credit lines.

A possible direction for future work is the construction of a version of this model in a world integrated economy with two tradable currencies, following also similar ideas as in Allen and Gale (2000). There are several issues that can be addressed with this framework. Perhaps one of the most discussed issues is the incentive to constitute the reserves for the international lender of last resort. In the paper I have presented such problem could not be studied since the economy was of the small open type. A world integrated economy with well-defined participants could help to see when each country is willing to deposit funds in an international institution.

Another important point to be addressed is the role of non-tradeables. In this model, only the patient consumers derive utility from pesos. However this is clearly artificial, as it is discussed in Chang and Velasco (2000). The intuition that pesos are used for local transactions (specially, for transactions in non - tradeables) must be transformed into a model that incorporates explicitly non tradeables. The model by Chang and Velasco (1999) can be extended to address this problem.

More fundamental shocks can be introduced, making either the short term rate (as in Chang and Velasco, 2001) or the long investment return (as in Allen and Gale, 1998) stochastic. This would allow to study solvency - based runs and the role of the lenders of last resort to prevent such runs, if these are not optimal. Nevertheless, problems of asymmetric information could make things worse. The reason is that, when returns are risky, adverse selection may not allow for availability of an international lender of last resort. This issue should be studied in combination with a world-integrated environment.

A related topic to the solvency problem is the explicit separation between managers and depositors. By study a version of this banking model in which managers do not have the same objective as the depositors the moral hazard considerations mentioned above could be seriously addressed. That is, moral hazard considerations are to be studied in settings where those objectives are discordant, since it is obvious that when they are the same hidden action problems cannot arise. There are several alternatives for modelling this. There is a vast literature on incomplete contracts in banking (see Dewatripont and Tirole, 1993 and 1994). Chang and Velasco (1998a) also present a model in which the banking sector is monopolistic. Any of these frameworks could be helpful to study moral hazard and lenders of last resort. Finally, issues on insurance schemes (in the spirit of Druck, 2000, for example) can also be considered in the international setting.

A. Proofs

A.1. Proof of Proposition 4.1

Given that the utility function is strictly concave and the constraints are linear, the first order conditions are necessary and sufficient to characterize the optimal allocation. Those conditions are the following:

$$R\left[\phi_2\left(1\right) + \phi_2\left(2\right)\right] = \phi_0 \tag{A.1}$$
$$\left[\phi_1\left(1\right) + \phi_1\left(2\right)\right] \leqq \phi_0 \tag{A.2}$$
$$\phi_1\left(s\right) \leqq \phi_2\left(s\right) \tag{A.3}$$

$$u'\left(c_1^1\right) = \phi_1\left(1\right) + \phi_1\left(2\right) \tag{A.4}$$
$$q_1 u'\left(c_1^2\left(1\right)\right) = \phi_1\left(1\right) \tag{A.5}$$
$$q_s u'\left(c_2\left(s\right)\right) = \phi_2\left(s\right) \tag{A.6}$$
$$v'\left(m_2\left(s\right)\right) = 0 \tag{A.7}$$

The first four expressions correspond to the first derivative of the Lagrangian with respect to x, y, $z\left(1\right)$ and $z\left(2\right)$ respectively. The last four equalities are the first derivatives with respect to c_1^1, $c_1^2\left(1\right)$, $c_2^1\left(1\right)$, $c_2\left(s\right)$ and $m_2\left(s\right)$. This last equality implies that $m_2\left(s\right) = \mu$ for all s.

I first show that $y^* = 0$. Using contradiction, assume that $y^* > 0$. Then from A.1 and A.2 we have

$$\left[\phi_1\left(1\right) + \phi_1\left(2\right)\right] = R\left[\phi_2\left(1\right) + \phi_2\left(2\right)\right]$$
$$> \left[\phi_2\left(1\right) + \phi_2\left(2\right)\right]$$

On the other hand, we have
$$\phi_1\left(s\right) \leqq \phi_2\left(s\right)$$

for every s, and therefore

$$\left[\phi_1\left(1\right) + \phi_1\left(2\right)\right] \leq \left[\phi_2\left(1\right) + \phi_2\left(2\right)\right]$$

a contradiction. Hence $y^* = 0$.

Then this implies that $\phi_1\left(s\right) = \phi_2\left(s\right)$ for $s = 1, 2$. From the expressions ??, A.7 and A.4 we get

$$u'\left(c_1^2\left(1\right)\right) = u'\left(c_2\left(1\right)\right)$$
$$u'\left(c_1^1\right) = q_1 u'\left(c_2^1\left(1\right)\right) + q_2 u'\left(c_2\left(2\right)\right)$$

23

Obviously, from the first equation we get $c_1^2(1) = c_2^1(1)$. On thee other hand, from the constraints (holding with strict equality)

$$p\alpha c_1^1 + (1-p)\, c_1^2(1) = z(1)$$

$$p\alpha c_1^1 = z(2)$$

$$p(1-\alpha)\, c_2^1(1) + \left(\bar{d} + z(1)\right) = R\bar{d}$$

$$p(1-\alpha)\, c_2^1(2) + (1-p)\, c_2^2(2) + \left(\bar{d} + z(2)\right) = R\bar{d}$$

implies that

$$
\begin{aligned}
&p(1-\alpha)\, c_2^1(1) + p\alpha c_1^1 + (1-p)\, c_1^2(1) \\
={}& (R-1)\,\bar{d} \\
={}& [p(1-\alpha) + (1-p)]\, c_2(2) + p\alpha c_1^1
\end{aligned}
$$

and therefore

$$p(1-\alpha)\, c_2^1(1) + (1-p)\, c_1^2(1) = [p(1-\alpha) + (1-p)]\, c_2^2(2)$$

But $c_1^2(1) = c_2^1(1)$. This implies that $c_1^2(1) = c_2^1(1) = c_2^2(2) \equiv \bar{c}$. But then, from

$$u'\left(c_1^1\right) = q_1 u'\left(c_2^1(1)\right) + q_2 u'\left(c_2(2)\right)$$

we get that $c_1^1 = \bar{c}$, showing that perfect risk sharing is the only solution to the planner's problem.

Then, from the constraint

$$p(1-\alpha)\, c_2^1(2) + (1-p)\, c_2^2(2) + \left(\bar{d} + z(2)\right) = R\bar{d}$$

we have that

$$[p(1-\alpha) + (1-p)]\,\bar{c} + z(2) = (R-1)\,\bar{d}$$

But

$$z(2) = p\alpha\bar{c}$$

so that

$$[p(1-\alpha) + (1-p)]\,\bar{c} + p\alpha\bar{c} = (R-1)\,\bar{d}$$

and then

$$\bar{c} = (R-1)\,\bar{d}$$

This ends the proof. ∎

24

A.2. Proof of Proposition 4.2

Ignoring the incentive compatibility constraints for a moment, the first order conditions of the commercial bank problem are as before.

$$R\left[\phi_2\left(1\right)+\phi_2\left(2\right)\right] = \phi_0 \tag{A.8}$$

$$\left[\phi_1\left(1\right)+\phi_1\left(2\right)\right] \leqq \phi_0 \tag{A.9}$$

$$\phi_1\left(s\right) \leqq \phi_2\left(s\right) \tag{A.10}$$

$$u'\left(c_1^1\right) = \phi_1\left(1\right)+\phi_1\left(2\right) \tag{A.11}$$

$$q_1 u'\left(c_1^2\left(1\right)\right) = \phi_1\left(1\right) \tag{A.12}$$

$$q_2 u'\left(c_2^j\left(s\right)\right) = \phi_2\left(s\right),\, j=1,2 \tag{A.13}$$

$$v'\left(m_2^1\right) = \sum_{s=1}^{2}\left(\phi_1\left(s\right)-\phi_2\left(s\right)\right) \tag{A.14}$$

$$q_2 v'\left(m_2^2\left(2\right)\right) = \phi_1\left(2\right)-\phi_2\left(2\right) \tag{A.15}$$

These are the same first order conditions as above with the addition of the last equation. Now, by the same arguments as before, $y^* = 0$ and so $\phi_1\left(s\right) = \phi_2\left(s\right)$, since $z\left(s\right) > 0$ for $s = 1, 2$. But then $v'\left(m_2^1\right) = v'\left(m_2^2\left(2\right)\right) = 0$, so $m_2^1 = m_2\left(2\right) = \mu$. On the other hand, it is clear that $c_2^1\left(2\right) = c_2^2\left(2\right) \equiv c_2\left(2\right)$. Therefore, the first order conditions of the optimal allocation arise. Therefore the currency board allocation is the optimal allocation, since this allocation satisfies with equality the incentive compatibility constraint. This ends the proof. ∎

A.3. Proof of Proposition 5.1

First, it is clear that the optimal amount of local currency is again μ, since the cost of printing pesos is always zero. Next, the necessary and sufficient first-order conditions with respect of x, y and $z\left(s\right)$ of the second best problem are the following.

$$R\left[\phi_2\left(1\right)+\phi_2\left(2\right)\right] = \phi_0 \tag{A.16}$$

$$\left[\phi_1\left(1\right)+\phi_1\left(2\right)\right] \leqq \phi_0 \tag{A.17}$$

$$\phi_1\left(s\right) \leqq \phi_2\left(s\right)+\tau\left(s\right) \tag{A.18}$$

where $\tau\left(s\right)$ is the multiplier of the constraint $d+z\left(s\right) \leqq \bar{d}$. The FOC corresponding to the consumption allocations are as follows.

$$u'\left(c_1^1\right) = \phi_1\left(1\right)+\phi_1\left(2\right) \tag{A.19}$$

$$q_1 u'\left(c_1^2\left(1\right)\right) = \phi_1\left(1\right) \tag{A.20}$$

$$q_s u'\left(c_2\left(s\right)\right) = \phi_2\left(s\right) \tag{A.21}$$

25

Finally, the FOC with respect to d is

$$\phi_0 = \sum_{s=1}^{2} \tau(s) + \sum_{s=1}^{2} \phi_2(s) \qquad (A.22)$$

This condition must hold since it must be the case that $d > 0$. Otherwise $x = 0$ but then consumption of patient agents is always zero. Since the objective function is strictly concave, the solution to be characterized must be unique. I show now that this equilibrium is characterized by $z(1) > 0$, $z(2) \geq 0$ and $y \geq 0$. Under these conditions it must be the case that

$$R[\phi_2(1) + \phi_2(2)] = \phi_0 \qquad (A.23)$$
$$[\phi_1(1) + \phi_1(2)] \lesseqgtr \phi_0 \qquad (A.24)$$
$$\phi_1(s) \lesseqgtr \phi_2(s) + \tau(s) \qquad (A.25)$$
$$\phi_0 = \sum_{s=1}^{2} \tau(s) + \sum_{s=1}^{2} \phi_2(s) \qquad (A.26)$$

and so

$$R[\phi_2(1) + \phi_2(2)] = \sum_{s=1}^{2} \tau(s) + \sum_{s=1}^{2} \phi_2(s) \qquad (A.27)$$

This implies

$$\sum_{s=1}^{2} \tau(s) = (R-1)[\phi_2(1) + \phi_2(2)] > 0 \qquad (A.28)$$

which means that for at least one s, $\tau(s) > 0$. On the other hand it must be the case that

$$R[\phi_2(1) + \phi_2(2)] = [\phi_1(1) + \phi_1(2)] \qquad (A.29)$$

This is because either $y > 0$, or, if this is zero, then it must be the case that $z(s) > 0$ for $s = 1, 2$. In both cases we arrive to this last equality. Hence we have:

$$u'(c_1^1) = R[q_1 u'(c_2(1)) + q_2 u'(c_2(2))] \qquad (A.30)$$

On the other hand we have that if $z(1) > 0$, then

$$q_1 u'(c_1^2(1)) = q_1 u'(c_2^2(1)) + \tau(1) \qquad (A.31)$$
$$\geq q_1 u'(c_2^2(1))$$

which implies $c_1^2(1) \leq c_2^2(1)$. Also, from the date 1 and 2 budget constraints:

$$p(1-\alpha)c_2(1) + (1-p)c_1^2(1) = [p(1-\alpha) + (1-p)]c_2(2) \qquad (A.32)$$

26

since they hold with equality. Therefore it must be the case that

$$c_1^2(1) \leqq c_2(2) \leqq c_2^2(1) \tag{A.33}$$

However, the fact that the date 1 budget constraints hold with equality implies

$$z(1) = (1-p)c_1^2(1) + z(2) \tag{A.34}$$

but so $z(1) > z(2)$. On the other hand, it is clear that

$$R[\phi_2(1) + \phi_2(2)] = \sum_{s=1}^{2} \tau(s) + \sum_{s=1}^{2} \phi_2(s) = [\phi_1(1) + \phi_1(2)]$$
$$\phi_1(s) \leqq \phi_2(s) + \tau(s), \quad s = 1, 2$$

But the first two equalities imply that for each s, $\phi_1(s) = \phi_2(s) + \tau(s)$. Suppose, instead, that for some s, $\phi_1(s) < \phi_2(s) + \tau(s)$. Then, summing over s we have $[\phi_1(1) + \phi_1(2)] < \sum_{s=1}^{2} \tau(s) + \sum_{s=1}^{2} \phi_2(s)$, which contradicts the second equality on the first line. Hence it must happen that $\phi_1(s) = \phi_2(s) + \tau(s)$ for every s. On the other hand, since $z(1) > z(2)$, clearly it is true that $z(1) > 0$. Therefore it must be the case that $\tau(1) > 0$. If this were not the case, then $\tau(2)$ should be positive, since for at least one s, $\tau(s) > 0$. But then $z(2) + d = \bar{d}$, but then $z(1) + d > \bar{d}$, violating the constraint. Therefore $\tau(1) > 0$ and so $z(1) + d = \bar{d}$. But then $z(2) + d < \bar{d}$ and so $\tau(2) = 0$. This implies that:

$$\begin{aligned} u'(c_1^1) &= [\phi_1(1) + \phi_1(2)] \\ &= q_1 u'(c_1^2(1)) + \phi_1(2) \\ &= q_1 u'(c_1^2(1)) + \phi_2(2) \\ &= q_1 u'(c_1^2(1)) + q_2 u'(c_2(2)) \end{aligned} \tag{A.35}$$

and so

$$q_1 [u'(c_1^1) - u'(c_1^2(1))] = q_2 [u'(c_2(2)) - u'(c_1^1)] \tag{A.36}$$

meaning that

$$sgn[c_1^1 - c_1^2(1)] = sgn[c_2(2) - c_1^1] \tag{A.37}$$

I show now that $sgn[c_1^1 - c_1^2(1)] > 0$, which proves partial suspension of convertibility. Suppose that this is not the case, that is, $sgn[c_1^1 - c_1^2(1)] < 0$. Then $c_2(2) < c_1^1$. But then $c_2(2) < c_1^2(1)$, contradicting the statement above. Therefore it must be that $c_1^1 \geq c_1^2(1)$. However, if $c_1^1 = c_1^2(1)$ then it must be true that

$$c_1^1 = c_1^2(1) = c_2(1) = c_2(2) \tag{A.38}$$

27
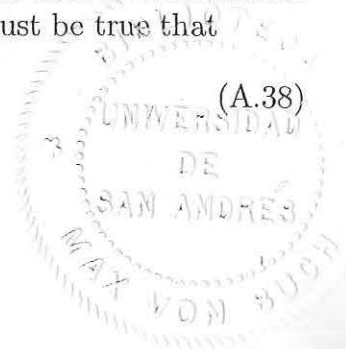
But this implies that $q_1 u'\left(c_1^2\left(1\right)\right) = q_1 u'\left(c_2^2\left(1\right)\right)$. But then, from

$$q_1 u'\left(c_1^2\left(1\right)\right) = q_1 u'\left(c_2^2\left(1\right)\right) + \tau\left(1\right) \tag{A.39}$$

then $\tau\left(1\right) = 0$, contradicting the result above. This implies that $c_1^1 > c_1^2\left(1\right)$ showing that partial suspension of convertibility of deposits must hold. This also implies that $c_2\left(2\right) > c_1^1$, showing that the optimal allocation is incentive compatible. This ends the proof of this proposition. ∎

A.4. Proof of proposition 5.2

The problem of a currency board system with limited credit is to maximize the expected utility function 4.6 subject to the constraints

$$
\begin{aligned}
d + z\left(s\right) &\leqq \bar{d} \\
x + y &\leqq d \\
p\alpha c_1^1 + p\left(1 - \alpha\right) m_2^1 + \left(1 - p\right) c_1^2\left(1\right) &\leqq z\left(1\right) + y \\
p\alpha c_1^1 + p\left(1 - \alpha\right) m_2^1 + \left(1 - p\right) m_2^2\left(2\right) &\leqq z\left(2\right) + y \\
p\left(1 - \alpha\right) c_2^1\left(1\right) &\leqq p\left(1 - \alpha\right) m_2^1 + Rx - z\left(1\right) - d \\
p\left(1 - \alpha\right) c_2^1\left(2\right) + \left(1 - p\right) c_2^2\left(2\right) &\leqq p\left(1 - \alpha\right) m_2^1 + \left(1 - p\right) m_2^2\left(2\right) \\
&\quad + Rx - z\left(2\right) - d
\end{aligned}
$$

The first order conditions with respect to d, x, y and $z\left(s\right)$ are as in the second best program. The same is true with respect to the first derivatives of the Lagrangian with respect to c_1^1, $c_1^2\left(1\right)$, m_2^1 and $c_2^2\left(2\right)$. The first order conditions with respect to pesos are the following

$$
\begin{aligned}
v'\left(m_2^1\right) &= \sum_{s=1}^{2}\left(\phi_1\left(s\right) - \phi_2\left(s\right)\right) \\
q_2 v'\left(m_2^2\left(2\right)\right) &= \phi_1\left(2\right) - \phi_2\left(2\right)
\end{aligned}
$$

The arguments in the proof of the proposition (5.1) presented above shows that $\tau\left(1\right) > \tau\left(2\right) = 0$. On the other hand, the same proof shows that $\phi_1\left(s\right) = \phi_2\left(s\right) + \tau\left(s\right)$ for every s. So we obtain that $v'\left(m_2^1\right) = \sum_{s=1}^{2}\tau\left(s\right) = \tau\left(1\right) > 0$ and $q_2 v'\left(m_2^2\left(2\right)\right) = \tau\left(2\right) = 0$. Therefore, even though this last equality implies $m_2^2\left(2\right) = \mu$, however the first implies that $m_2^1 < \mu$. Hence, a banking system within currency board regime cannot implement the second best allocation.

A.5. Proof of Proposition 5.3

Ignoring the incentive compatibility constraints, the problem can be written as the maximization of

$$p\alpha u\left(c_1^1\right) + q_1\left[(1-p)\,u\left(c_1^2\left(1\right)\right) + p\left(1-\alpha\right)\left(u\left(c_2^1\left(1\right)\right) + v\left(m_2^1\right)\right)\right] \quad \text{(A.40)}$$
$$+q_2\left[p\left(1-\alpha\right)\left(u\left(c_2^1\left(2\right)\right) + v\left(m_2^1\right)\right) + (1-p)\left(u\left(c_2^2\left(2\right)\right) + v\left(m_2^2\left(2\right)\right)\right)\right]$$

subject to

$$x + y \leqq d \tag{A.41}$$

$$p\alpha c_1^1 + p\left(1-\alpha\right)m_2^1 + (1-p)\,c_1^2\left(1\right) \leqq z\left(1\right) + y + h_1 \tag{A.42}$$

$$p\alpha c_1^1 + p\left(1-\alpha\right)m_2^1 + (1-p)\,m_2^2\left(2\right) \leqq z\left(2\right) + y + h_2 \tag{A.43}$$

$$p\alpha c_1^1 + (1-p)\,c_1^2\left(1\right) \leqq z\left(1\right) + y \tag{A.44}$$

$$p\alpha c_1^1 \leqq z\left(2\right) + y \tag{A.45}$$

$$p\left(1-\alpha\right)c_2\left(1\right) \leqq p\left(1-\alpha\right)m_2^1 - h_1 + Rx - d - z\left(1\right) \tag{A.46}$$

$$p\left(1-\alpha\right)c_2^1\left(2\right) + (1-p)\,c_2^2\left(2\right) \leqq p\left(1-\alpha\right)m_2^1 + (1-p)\,m_2^2\left(2\right) \tag{A.47}$$
$$+Rx - d - z\left(2\right) - h_2 \tag{A.48}$$

$$d + z\left(s\right) \leqq \bar{d} \tag{A.49}$$

where the inequalities $(A.44)$ and $(A.45)$ are imposed so that banks cannot finance date 1 consumption of dollars entirely with pesos issued by the Central Bank (this is similar to Chang and Velasco, 2000). Let $\eta\left(s\right)$ be the multiplier of each constraint. The first order conditions with respect to x, y, $z\left(s\right)$, h_s and d are as follows.

$$R\left[\phi_2\left(1\right) + \phi_2\left(2\right)\right] = \phi_0 \tag{A.50}$$

$$\sum_{s=1}^{2}\left[\phi_1\left(s\right) + \eta\left(s\right)\right] \leqq \phi_0 \tag{A.51}$$

$$\phi_1\left(s\right) + \eta\left(s\right) \leqq \phi_2\left(s\right) + \tau\left(s\right) \tag{A.52}$$

$$\phi_1\left(s\right) \leqq \phi_2\left(s\right) \tag{A.53}$$

$$\phi_0 = \sum_{s}\left[\phi_2\left(s\right) + \tau\left(s\right)\right]$$

First, I consider an equilibrium where $h_s > 0$ for both s. Hence, we must have $\phi_1\left(s\right) = \phi_2\left(s\right)$. On the other hand, this implies then that $\eta\left(s\right) \leqq \tau\left(s\right)$ for every s. If $y = 0$ then $z\left(s\right) > 0$ for both s and therefore $\eta\left(s\right) = \tau\left(s\right)$). If $y > 0$ then $\phi_0 = \sum_s\left[\phi_1\left(s\right) + \eta\left(s\right)\right] = R\left[\sum_s \phi_2\left(s\right)\right] = \sum_s\left[\phi_2\left(s\right) + \tau\left(s\right)\right]$. Hence this implies that $\sum_s \tau\left(s\right) = \sum_s \eta\left(s\right)$. But then since $\eta\left(s\right) \leqq \tau\left(s\right)$ for every s, it must be true in fact

29

that $\eta(s) = \tau(s)$ for every s. But then, since $R\left[\sum_s \phi_2(s)\right] = \sum_s \left[\phi_2(s) + \tau(s)\right]$ and $R > 1$, again we have that at least for some s, $\tau(s) = \eta(s) > 0$. It must be shown for which s this is true.

The rest of the first order conditions are as follows.

$$u'\left(c_1^1\right) = \sum_s \left[\phi_1(s) + \eta(s)\right] \tag{A.54}$$

$$q_1 u'\left(c_1^2(1)\right) = \phi_1(1) + \eta(1) \tag{A.55}$$

$$q_s u'\left(c_2^j(s)\right) = \phi_2(s), \quad j = 1,2, \ s = 1,2 \tag{A.56}$$

$$v'\left(m_2^1\right) = \sum_{s=1}^{2} \left(\phi_1(s) - \phi_2(s)\right) = 0 \tag{A.57}$$

$$q_2 v'\left(m_2^2(2)\right) = \phi_1(2) - \phi_2(2) = 0 \tag{A.58}$$

Therefore it must happen that $m_2^1 = m_2^2(2) = \mu$. On the other hand, we again have

$$u'\left(c_1^1\right) = q_1 u'\left(c_1^2(1)\right) + \left[\phi_1(2) + \eta(2)\right]$$

On the other hand, it is easy show that the constraints $(A.44)$ and $(A.45)$ must hold with equality. If this were not true, then it is possible to increase the consumption of dollars for all consumer types (the details are left to the reader). Therefore, the solution to this problem satisfies exactly the same constraints of the planner's second best problem. Also this shows that $h_1 = p(1-\alpha)\mu$ and $h_2 = (p(1-\alpha) + 1 - p)\mu$. Then, this again implies that $z(1) = z(2) + (1-p)c_1^2(1)$ and so $z(1) > z(2)$. If this holds and if at least for some s, $\tau(s) > 0$, then it must be true that $\tau(1) > \tau(2) = 0$, following the same arguments as in the proof of proposition 5.1. Therefore $\eta(2) = \tau(2) = 0$. But then we have that $u'\left(c_1^1\right) = q_1 u'\left(c_1^2(1)\right) + \phi_1(2) = q_1 u'\left(c_1^2(1)\right) + \phi_2(2) = q_1 u'\left(c_1^2(1)\right) + q_2 u'\left(c_2(2)\right)$. Also it must happen that $\sum_s \left[\phi_1(s) + \eta(s)\right] = R\left[\sum_s \phi_2(s)\right]$, which is the same as $u'\left(c_1^1\right) = R\left[\sum_s q_s u'\left(c_2(s)\right)\right]$. Thus, the solution to this problem also satisfies the same first order conditions as in proposition 5.1. Hence, this solution must coincide with that of proposition 5.1. ∎

A.6. Proof of proposition 5.4

The condition

$$r < R\left(\frac{c_1^2(1)}{c_2(1)}\right) \tag{A.59}$$

30

is equivalent to $r < \frac{p(1-\alpha)c_1^2(1)}{\left(\frac{p(1-\alpha)c_2(1)}{R}\right)}$, which is true if and only if

$$r < \frac{p\left(1-\alpha\right)c_1^2\left(1\right)}{\left(\frac{p(1-\alpha)c_2(1)+\bar{d}-\bar{d}}{R}\right)} \tag{A.60}$$

However, the second best implies that $\bar{d} = z\left(1\right) + d$, so

$$r < \frac{p\left(1-\alpha\right)c_1^2\left(1\right)}{\left(\frac{p(1-\alpha)c_2(1)+(z(1)+d)-(z(1)+d)}{R}\right)} \tag{A.61}$$

and also at the solution of the second best problem we had that $Rx - d - z\left(1\right) = p\left(1-\alpha\right)c_2\left(1\right)$, so that the last inequality is equivalent to

$$r\left(x - \frac{z\left(1\right)+d}{R}\right) < p\left(1-\alpha\right)c_1^2\left(1\right) \tag{A.62}$$

Suppose all patient consumers believe that the others withdraw from the commercial banks at date 1. The intermediaries pay c_1^1 to the first αp and $c_1^2\left(1\right)$ consumers, financing both by borrowing $z\left(1\right)$ dollars from abroad and or with the total amount of short run investment y (if this is positive). Note again that although the true state may be $s = 2$, if $\alpha p + 1 - p$ agents show up then the commercial bank thinks that the true state is $s = 1$. If more consumers show up the intermediaries must still pay $c_1^2\left(1\right)$ pesos to each one, which will be exchanged for dollars at the Central Bank. But the amount of resources left is equal to $r\left(\bar{x} - \frac{d+z(1)}{R}\right)$. This happens because the intermediary liquidates $\left(\bar{x} - \frac{d+z(1)}{R}\right)$ units of the long term investment at date 1 in order to satisfy withdrawals. Since this is strictly less than payments needed to be done if all the rest of agents withdraw then the bank fails (in the sense that not all consumers are satisfied, although debt at date 2 is perfectly honored).

All agents know that this happens if everybody runs against the bank. Then if every individual consumer thinks that the rest of the population runs, then it is optimal to withdraw early. This is because the expected utility of withdrawing early is strictly greater than $u\left(0\right) + v\left(0\right)$. But then in this case the commercial bank fails.

Suppose that the opposite inequality holds. I show now that there is no run. Suppose then that

$$r \geq R\left(\frac{c_1^2\left(1\right)}{c_2\left(1\right)}\right) \tag{A.63}$$

31

Therefore, by the same argument as before,

$$r\left(x - \frac{z(1) + d}{R}\right) \geq p(1 - \alpha) c_1^2(1) \tag{A.64}$$

Consider now the following situation. Suppose that a total measure of $p\alpha + 1 - p + \widehat{\pi}$ consumers try to withdraw from commercial banks at date 1, with $\widehat{\pi} < p(1 - \alpha)$. Again the first $p\alpha$ consumers get c_1^1 while the rest of agents get $c_1^2(1)$. In order to satisfy withdrawals by $\widehat{\pi}$ consumers commercial banks must liquidate a portion of the long term investment. Let $\widehat{l}$ be the total liquidation of this investment. This must satisfy

$$\widehat{l} = \frac{\widehat{\pi} c_1^2(1)}{r} \tag{A.65}$$

In period 2 total per-capita resources are given by:

$$Rx - \bar{d} - R\widehat{l} \tag{A.66}$$
$$= p(1 - \alpha) c_2(1) - R\frac{\widehat{\pi} c_1^2(1)}{r}$$

But since $r \geq R\left(\frac{c_1^2(1)}{c_2(1)}\right)$, then $\frac{R}{r} c_1^2(1) \leqq c_2(1)$ and therefore

$$p(1 - \alpha) c_2(1) - R\frac{\widehat{\pi} c_1^2(1)}{r} \geq p(1 - \alpha) c_2(1) - \widehat{\pi} c_2(1) \tag{A.67}$$
$$= [p(1 - \alpha) - \widehat{\pi}] c_2(1)$$

This means that commercial banks have enough resources so that patient consumers waiting until period 2 consume $c_2(1)$ dollars and μ pesos. Hence banks do not fail and patient consumers do not find optimal to run. This is because, by running they get at most c_1^1 dollars (if included first in line) that can be stored until til period 2. Since they are patient consumers, they consume a null amount of local currency. Hence each patient consumer is strictly better off by waiting until date 2. This concludes the proof. ∎

A.7. Proof of Proposition 5.5

If the interest rate is equal to ρ then the amount of dollars available at date 2 is

$$Rx - (\bar{d} + z(1)) - \rho\widehat{\pi} c_1^2(1) \tag{A.68}$$

Replacing in x and $z(1)$ using the constraints of the second best problem, it can be shown that this last expression is equal to

$$p(1 - \alpha) c_2(1) - \rho\widehat{\pi} c_1^2(1) \tag{A.69}$$

32

Since $\rho \leqq c_2(1) / c_1^2(1)$ then the last expression is at least equal to $c_2(1) \geq c_1^2(1)$. This implies again that for any patient consumer it is best to wait until period 2. This ends the proof. ∎

A.8. Proof of proposition 6.1

Clearly, it must be the case that $m_2(s) / e_2(s) = \mu$ since printing pesos is free for the planner. The first order conditions that characterizes the optimal dollar consumption in this case are the following.

$$q(s) u'(c_t(s)) = \phi_t(s)$$

with $t = 1, 2$, and $s = 1, 2$,

$$\phi_0 = R[\phi_2(1) + \phi_2(2)]$$
$$\phi_0 = \sum_{s=1}^{2}(\phi_2(s) + \tau(s))$$
$$\phi_1(1) + \phi_1(2) \leqq \phi_0$$
$$\phi_1(s) \leqq \phi_2(s) + \tau(s), \quad s = 1, 2$$

where the first expression is the first derivative of the Lagrangian with respect to $c_t(s)$, the second two equalities are the derivatives of the Lagrangian with respect to x and d (both should be positive to ensure that $c_2(s) > 0$) and the two weak inequalities are the first derivatives of the Lagrangian with respect to y and $z(s)$ respectively. It is clear again that either $y > 0$ or, if $y = 0$, then $z(s) > 0$ for both s. Therefore

$$R[\phi_2(1) + \phi_2(2)] = \sum_{s=1}^{2}(\phi_2(s) + \tau(s)) = \phi_1(1) + \phi_1(2)$$

This implies that $\phi_1(s) = \phi_2(s) + \tau(s)$ for each s. Otherwise the second equality would be violated. The first equality implies that at least for some s, $\tau(s) > 0$, since $R > 1$. Therefore, for both s, $u'(c_2(s)) \leq u'(c_1(s))$, with at least for some s, the inequality is strict. Hence it is true that $c_2(s) \geq c_1(s)$ for both s, and with strict inequality for at least one s. Hence the optimal solution is incentive compatible for patient consumers.

To show that this allocation is better from the ex-ante point of view for agents, consider the following. Take the optimal consumption allocation from the planner's problem in section 5. Consider the following alternative consumption allocation

$$\widehat{c}_1(1) \equiv \left(\frac{p\alpha}{p\alpha + 1 - p}\right) c_1^1 + \left(\frac{1 - p}{p\alpha + 1 - p}\right) c_1^2(1)$$

33

$$\widehat{c}_1(2) \equiv c_1^1$$
$$\widehat{c}_2(s) \equiv c_2(s)$$

It is clear that the *hat* allocation, together with the investment and borrowing plan $(x, y, d, z(1), z(2))$ from the solution satisfy all constraints. This is because

$$
\begin{aligned}
& [p\alpha + 1 - p]\,\widehat{c}_1(1) \\
&= [p\alpha + 1 - p]\left[\left(\frac{p\alpha}{p\alpha + 1 - p}\right)c_1^1 + \left(\frac{1-p}{p\alpha + 1 - p}\right)c_1^2(1)\right] \\
&= p\alpha c_1^1 + (1-p)\,c_2^1(1) \\
&= y + z(1)
\end{aligned}
$$

and clearly, $\widehat{c}_1(2)$ and $\widehat{c}_2(s)$ also satisfy the other constraints by construction. However, this *hat* allocation gives a strictly higher ex-ante utility. This is because of the following. Original preferences are represented by equation (3.1). Given that the solution implies that $\frac{m_2(s)}{e_2(s)} = \mu$ for all s, then the relevant part of the utility function (the part that only includes utility for dollars) can be written as

$$
\begin{aligned}
& q_1\left[p\alpha u\left(c_1^1\right) + (1-p)\,u\left(c_1^2(1)\right) + p(1-\alpha)\,u\left(c_2(1)\right)\right] \\
& + q_2\left[p\alpha u\left(c_1^1\right) + (p(1-\alpha) + 1 - p)\,u\left(c_2(2)\right)\right]
\end{aligned}
$$

But since u is strictly concave

$$
u\left(\widehat{c}_1(1)\right) > \left(\frac{p\alpha}{p\alpha + 1 - p}\right)u\left(c_1^1\right) + \left(\frac{1-p}{p\alpha + 1 - p}\right)u\left(c_1^2(1)\right)
$$

and hence

$$
p\alpha u\left(c_1^1\right) + (1-p)\,u\left(c_1^2(1)\right) < [p\alpha + 1 - p]\,u\left(\widehat{c}_1(1)\right)
$$

Therefore ex-ante utility under the *hat* allocation is strictly higher than under the optimal solution with non public revelation of s (solution of the planner's problem of section 5). Therefore the solution to the planner's problem in section 6 must yield a strictly higher utility than the solution to the planner's problem in section 5. This ends the proof.

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