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"Business Cycles and Firm Dynamics in Small Emerging Economies"

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Business Cycles and Firm Dynamics in Small Emerging Economies*

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Abstract

I present a small open economy model to analyze the role of firms in the macroeconomic dynamics of the business cycles. The only shock is through the interest rate, and the main transmission mechanism is an asymmetric information problem between small firms and banks. Banks can infer the average quality of firms by observing their age and net worth. This introduces heterogeneity among different generations of firms that live at the same period of time. I present three results. First, unexpected increases in the interest rate produce endogenous long-lasting recessions because both the average "net worth" of the firms and their "reputation" are important in generating business cycles. Second, by adding externalities in production the model is able to mimic fairly well macroeconomic and microeconomic dynamics observed along some business cycle episodes in small emerging economies. Third, government's stabilizing policies can be welfare improving.

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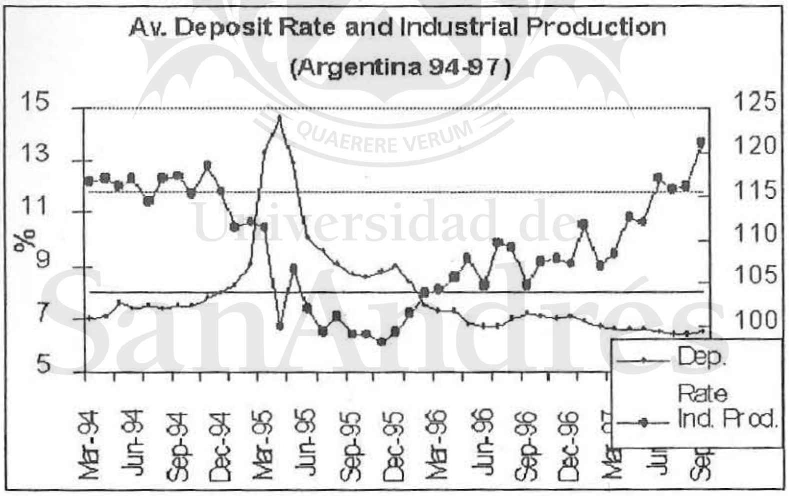
1 *Introduction*

In the last decade small open developing economies have suffered from speculative attacks, contagious effects, and in some cases financial crises, uncovering puzzles that still have not been resolved. In this paper I address one of these puzzles: unexpected increases in the interest rate faced by these countries are responsible for long recessions, despite the fact that this source of external disturbances exhibits very weak serial correlation. The impact of the Mexican crisis that took place in December 1994 on the Argentinean economy, is an example of the link between weakly correlated interest rate shocks and poor macroeconomic performance in the years that follow. The average deposit interest rate in Argentina increased in the first quarter of 1995, and returned to its original levels right away. Yet, this short-period shock had long-lasting and profound effects on this economy, which entered in a recession that lasted almost three years as the series on the deviations from trend of the Industrial Production Index below shows.[1]

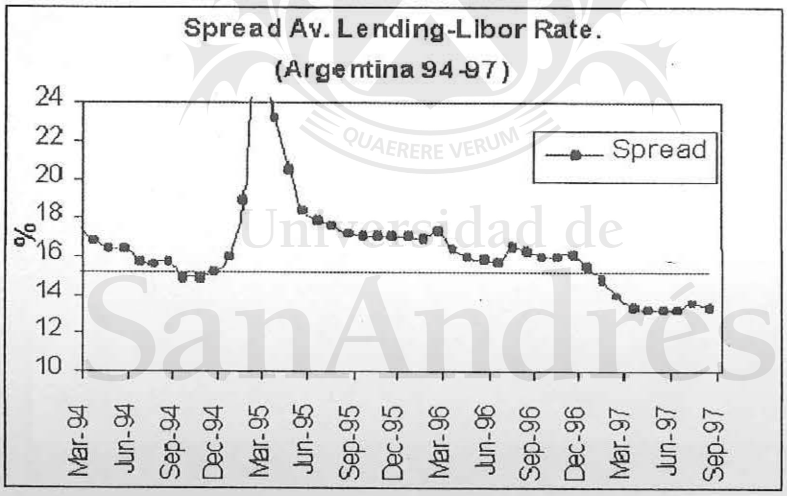


This fact seems to suggest that there are strong aggregate endogenous transmission mechanisms at work, something that the standard real business cycle literature is not able to explain. While this branch of influential literature -led by Lucas, Prescott and Kynland- and its application to small open economies -by Mendoza (1995) and Correia, Neves and Rebelo (1995)-

[1] Appendix A describe the series utilized in the Introduction.

helps us to understand the nonlinear comovement between the main macroeconomic aggregates when exogenous perturbations occur, it is incapable of generating the autocorrelation observed in these aggregates without highly correlated shocks.

Aside from these macroeconomic facts, policy makers in Argentina have repeatedly shown concern regarding the inability of small firms to recover from an external shock because of the difficulty encountered by these firms in accessing credit in the periods that followed the shock on interest rates. This concern is backed up by the evolution of the spread between average bank lending rates in Argentina and Libor (180 days) along the episode. Since the beginning of the Convertibility Plan implemented in 1991 and especially after a series of reforms to the financial system implemented in 1993, this spread continuously decreased until December 1994. The shock that occurred in the first quarter of 1995 not only sharply increased the lending rates in the first quarter of 1995, as expected, but also had a persistent effect on the spread. While these interest rates decreased in subsequent periods, the spread did not return to December 1994 levels until February 1997, more than two years later.[2]



In this work I attempt to explain the empirical observation that an unexpected and uncorrelated shock on interest rates is capable of generating a

[2]The spread between lending rates to small firms and libor rate over the downturn of the business cycles is likely to be underestimated in this graph. In Appendix A I present evidence suggesting that small firms suffered from more severe credit constraints during this period.

long lasting recession through an endogenous transmission mechanism, rationalizing policy makers' concern about the credit market imperfections faced by small firms in the economy.

I study a small open economy that produces tradable and nontradable goods where the nontradable good is only used as an input of production in the tradable sector. Firms in the tradable sector produce with a constant returns technology and have perfect access to financial markets. Firms in the nontradable sector are owned by entrepreneurs who have access to a decreasing returns to scale technology where management is a fixed and indivisible factor of production. Entrepreneurs can only borrow from banks. The most important feature of this economy is the existence of an asymmetric information problem between entrepreneurs and banks about each entrepreneur's productivity. While entrepreneurs know their own productivity, banks are unable to observe them.

At every period a constant mass of entrepreneurs is born with access to technology to produce nontradable goods. They start up a firm and continue operating it as long as they are successful producers. At every period of the firms' life the project undertaken by the firm can come up "successful" or "unsuccessful", where the success probability is each entrepreneur's private characteristic. The entrepreneurs keep the same success probability over time. Whenever the entrepreneurs get an "unsuccessful" outcome they retire.

Because entrepreneurs know more about the quality of the investment project to be undertaken than banks do, the amount borrowed depends on the firms' net worth, as casual observation suggests. The higher the net worth, the greater the ability of banks to infer that the entrepreneur has a high success probability. For this reason each firm's net worth determines its credit conditions and financial contracts.

Also firms with lower "success" probability are more likely to default and exit, implying that the average productivity of surviving firms belonging to the same cohort improves over time.[3] Thus, the firm's age is useful observable information and financial contracts also depend on it.

It is assumed that all entrepreneurs have the same wealth at the moment of starting up their firms and that this wealth is not even close to what an entrepreneur with the highest possible productivity would need to fully

[3] Jovanovic (1982) introduces a similar screening process of firms' quality, although in his model there is no asymmetry of information since the quality is not even known by the entrepreneur who learn it over time.

finance the project by himself. For that reason, at the beginning of each cohort's life entrepreneurs need to finance a higher proportion of the firms' costs by borrowing from banks. In equilibrium entrepreneurs with different productivity end up sharing the same financial contract which turns out to be inefficient since highly productive entrepreneurs pay the same cost of external finance as entrepreneurs with lower productivity. This result follows because highly productive entrepreneurs are unable signal their type to banks since they don't have enough net worth.

As time goes on successful firms build up net worth that help highly productive entrepreneurs to separate from lower productive types. As firms are getting old, the total amount of output produced by high quality firms increases. This occurs not due to technological reasons -because firms have the same technology since birth-, but due to financial ones. The banks' perception about the firms' productivity is updated each period based on age and net worth. Older and wealthier firms are perceived as better firms by banks, implying a lower cost of external finance. As older firms pay lower rates, they also produce more.

Eventually, when the highest quality firms have accumulated enough net worth the asymmetric information problem for all members of the cohort is solved, since banks are able to perfectly infer each firm's success probability. Nonetheless, it takes a long time for this to happen, and in the meantime high quality firms contract credit at a higher lending rate than the one they should be charge were information perfect. Because banks make zero profits in equilibrium, some lower quality firms contract credit at lower interest rates than they would under perfect information. This inefficiency is only fully resolved once the highest quality firms have accumulated enough wealth to truthfully signal their type.

The model is capable of producing a long-lasting endogenous transmission mechanism after a one period shock. This happens due to two reasons. First, the speed at which information is revealed is slowed down when firms are surprised by a bad shock that reduces their net worth. Slow recovery of the firms' net worth leads to a slow information revelation process because good firms in each cohort pay higher interest rate for more periods than under steady state (while bad firms pay lower rates), implying that aggregate economic performance deteriorates due to this inefficiency.

Second, since macroeconomic conditions deteriorate, more firms exit the industry on impact than in normal times, destroying not only present but also future output since the production levels of exiting firms can only be resumed once younger generations pass through the costly screening process of producing over time. Again this process is costly because younger firms with high productivity are unable to convince banks to finance large investment projects since firms similar in age and equity but with low productivity have private incentives to free ride on those contracts. Hence, there is an informational loss at the aggregate level that weakens economic activity.

While the model with both "net worth" and "reputation" effects is able to generate strong serial output correlation after a one period shock to the interest rate, it fails to replicate the sizable economic downturns experienced in these economies. I show that by introducing an externality in production the model economy can resemble important recessions. I also show that externalities alone cannot explain long recessions.

The firms dynamics are also studied in this work, not only under macroeconomic steady state conditions but also along the business cycle after a bad shock. Time series and cross sectional information for firms drawn by simulations shows that the information revelation process is slowed down in the business cycle. This is reflected in temporally higher lending rates, lower net worth and hence lower input-output scales of firms along the business cycles compared to steady state levels.

Finally the source of business fluctuations comes from a market failure, leaving room for policy analysis.

1.1 Related Literature on the Credit Channel

The differences between this work and the literature on the credit channel deserve a special comment. In the last fifteen years there has been an increasing mass of literature emphasizing the importance of asymmetric information problems in financial relationships to the credit cycle. Most of the literature focuses on the idea that it is costly for lenders to verify the output produce by ex-ant identical borrowers. Stephen Williamson (1987), Bernanke and Gertler (1989), Gertler (1992), Fuerst (1995) and its comment by Gertler (1995) and Cooley and Nam (1998) are part of this literature. The ex-ante similarity among agents and other assumptions in these models guarantees a simplifying result: there is only one optimal financial contract to solve for in the economy at each period, making models easily tractable. Yet, this simplification comes at a cost of neglecting the role of firms' dynamics over the business cycles. Since all firms are equal to each other at every point in time, there can be no differential access to credit markets among them.

To the best of my knowledge the only work that uses heterogeneity to study the credit channel in the business cycle is Bernanke, Gertler and Gilchrist (1998). They present a model where heterogeneity is due to ex-post realizations of output. In this model firms face borrowing constraints that depend on the firms' net worth. Because of the assumption that the cost of monitoring is linear in the amount of capital used by firms, the demand for capital for each firm depends linearly on the firms' net worth. This allows computation of the aggregate level of capital in the economy. In equilibrium, the level of leverage defined as the ratio of debt to net worth, and the lending rate are the same for all firms, leading to very simple firm dynamics.[4] Thus, there is no role for reputation in financial contracts as there is in my work, since credit access depends on productive history only through net worth and each firm's productivity follows an i.i.d. process only known after contracts have been signed.

In all the credit channel models mentioned before the agency problem arises because it is costly for lenders to monitor firms' output. Bernanke and Gertler (1990) argue that this simplifying assumption has one important drawback: "...agency costs in the model are identified with monitoring costs, which empirically are too small to rationalize first-order effects for financial fragility". With this idea in mind, they introduced a different type of asymmetric information problem into the literature similar in spirit to the one introduced in this work: firms differ ex-ante in their probability of having a high output performance. In their model, there is no feasible contract able to align borrowers' incentives to the lender's objective function. In equilibrium then, some firms free ride on others seeking private profits even though the social value of such actions is negative. The free riding problem adds a cost to the financial contract which becomes the agency cost. This agency cost depends on the initial financial state of the firms or "net worth". Since all projects require a fixed investment, the bigger the firms' endowment the lower the agency costs in the economy simply because higher equity reduces borrowers' incentive to free ride. While this idea brings more realism to the model, it also brings heterogeneity, making it more difficult to handle: whenever firms are ex-ante different, production reveals information about

[4]This feature is similar to Kiyotaki and Moore (1997), except for the fact that in Bernanke, Gertler and Gilchrist (1998) the leverage is endogenously determined given prices.

the firms' quality, and that information should be incorporated into future financial contracts since age becomes an observable variable upon which contracts can be based. In their model, they avoid dealing with the problem by constructing a two period model, where entrepreneurs get to play this game only once. In my work the main asymmetric information problem is somewhat similar to Bernanke and Gertler (1990), but I extend the model by letting all firms live for many periods and allowing banks to update their beliefs about the firms' productivity by taking into account all relevant past information available to them.

Cooley and Quadrini (1998) develop a model to explain some stylized facts for US firms. Some of these stylized facts are also explained by the model economy I present. In their model they introduce moral hazard to let firms borrowing depend (proportionally) on the amount self- financed. In the present work I also have adverse selection which is eventually resolved once firms build up enough net worth. Also, I obtain a level of leverage (defined as the debt-equity ratio) that is endogenously determined and dependent on the firms' age. Although I loose some of the realism they get in their model, I am able to study the life cycle of firms not only in the steady state (as they do) but also along the business downturn. This is important because I believe that the fundamentals behind the firms' life cycle, say the information revelation process, contribute to a persistent poor macroeconomic performance when small firms are surprised by a bad shock.

2 *The model*

There are two types of agents in the economy, workers and entrepreneurs, and three sectors, the tradable and nontradable goods sectors and the financial sector. Workers and entrepreneurs consume tradables, which are produced using capital and the non-tradable good. The non-tradable good is produced using capital and labor.

There is a mass μ of infinitely lived homogeneous workers. They are infinitely endowed with labor at every period of life and they consume only tradable goods. Their intertemporal utility function is given by:

$$U_t^W = E_t \sum_{j=t}^{\infty} (\frac{1}{r})^{j-t} \frac{\left(c_j^W - a_1 l_j^{a_2}\right)^{1-\sigma}}{1-\sigma} \qquad a_i, \sigma > 0 \tag{1}$$

where c_t and l_t represent consumption of tradable goods and labor supplied respectively at time t. Superscript W stands for worker. Preferences are convex and satisfy usual assumptions. Labor can be supplied at the market wage rate w_t. The discount parameter is set equal to $1/r$, where r is one plus the long run international interest rate faced by this economy. This assumption guarantees existence of a steady state equilibrium consumption path.

At each period of life, workers decide how much of their wealth to allocate to consumption and to savings. Savings are carried via three riskless assets: bonds, capital in the tradable sector and capital in the nontradable sector. For simplicity, I assume that all assets holdings between period t and $t+1$ are represented by portfolio Γ_t expressed in consumption goods. Hence, the workers intratemporal budget constraint at every period t is given by,

$$c_t^w + \Gamma_t \leq w_t l_t + r_{t-1}\Gamma_{t-1} \qquad \forall t \geq 0 \tag{2}$$

where r_t is the international interest rate between period t and $t+1$.[5]

Entrepreneurs are also infinitely lived agents and consume only tradable goods. A unit mass of them is born every period and they are risk neutral agents with preferences given by

$$U_t^E = E_t \sum_{j=t}^{\infty} \gamma^{j-t} c_j^E$$

where superscript E stands for entrepreneur. Entrepreneurs have a discount factor $\gamma < \frac{1}{r}$. Although entrepreneurs are assumed to be more impatient than workers, they will end up saving more because they have access to very profitable investment opportunities. These infinitesimal agents are endowed with labor in their first period of life and with a project to produce nontradable goods in all remaining periods, contingent on having been successfully productive in the past. Entrepreneurs are assumed to be the only type of agents capable of managing inputs to produce nontradable goods in this economy. These good cannot be stored. Although all entrepreneurs have the same preferences, their productivity might differ. That productivity constitutes their individual characteristic, and the second source of heterogeneity in the model, relevant to generate some important results. To understand how this

[5] Although the return on assets should be derived in equilibrium, I simplify notation by letting it be equal to the international interest rate from the beginning.

characteristic is modeled, I introduce the production technology embodied in these agents. Production of nontradable goods at time $t+1$ requires capital (k^N) -which is a tradable good- and labor (l) to be input at t, and it is only possible through the following technology belonging to each entrepreneur.

$$y_{t+1}^N = \theta_{t+1}(k_t^N)^\alpha(l_t^N)^\beta \qquad \alpha, \beta > 0, \quad i.i.d. \ \theta_{t+1} = \begin{cases} \overline{\theta} & with \ prob \ p \\ 0 & o.w. \end{cases}$$

where y_t^N stands for nontradable output. The random variable θ_{t+1} can take two values high, $\overline{\theta}$, or 0, and it is realized once inputs have been chosen. If the outcome of the project is "unsuccessful" $(\theta_{t+1} = 0)$ then the entrepreneur looses the licence to produce non-tradable goods and the firms disappears..

All the parameters in this production function except for the probability p are the same across entrepreneurs. This probability constitutes each entrepreneur's characteristic and it is only observed by herself. While the parameter p is non-verifiable private information, it is drawn from a publicly know density function $f(p)$ where $p\epsilon[0,1)$. I assume that the density function is well behaved and the production function exhibits decreasing returns to scale.

Assumption 1: $\alpha + \beta$ <1.

Hence, management can be interpreted as a fixed indivisible factor of production in a constant returns to scale technology. Assumption 1 imposes an upper bound on the size of the firms given equilibrium input and output prices.

I assume capital in this sector can be rented at r_k per unit of time and depreciates at a rate δ^N.

Firms exit the industry for two reasons. The first one, mentioned above, is due "unsuccessful" outcomes and it is more has to do with financial reasons. When entrepreneurs are unsuccessful they are unable to pay back debt. This triggers a bankruptcy process that I assume end up destroying the firm. The second one is due to reasons such as market conditions. I assumed that the exiting rate due the latter argument is exogenous in this model.

Assumption 2: Entrepreneurs' become unproductive with probability $q_t = \xi^{1+\chi_t}$, where $\chi_t > 0$ represents an adverse shock to the demand of nontradable goods and $\chi_t = 0$ implies no shock.

Thus, the probability that an entrepreneur becomes unproductive for reasons other than financial ones depends on macroeconomic conditions. In good times this probability is just ξ, while in bad times it is assumed to be increasing on the magnitude of the shock.

The tradable sector is composed of a mass of firms producing tradable goods.[6] I assume that this sector can produce tradables at time $t+1$ by inputting a tradable capital good (k^T) and nontradable goods (y^N) at time t.[7] The technology used by this sector is given by the following generic production function,

$$y_{t+1}^T = A\ F(k_t^T, y_t^N) \tag{3}$$

where y_{t+1}^T is the firm's total output of tradables at time $t+1$, and $F(\cdot)$ is a constant returns to scale production function, with the usual assumptions on marginal products and concavity.[8] Capital utilized in this sector is assumed to depreciate at the rate δ^T.

Finally, the model is completed with the financial sector. There is a mass of infinitesimal banks, and technology in this sector is trivial. They transform one unit of tradable goods borrowed into one unit of tradable good lent at no cost (fixed or marginal). They raise funds by issuing debt (deposits) to workers and other international investors, and they lend those funds to small entrepreneurs.[9] This sector is introduced to keep the economy decentralized and to make clear assumptions on debt contracts.

Assumption 3: Banks observe only the firms' age and net worth.

Banks do not observe the entrepreneurs' characteristic. They only observe the type of contract that their clients are taking. Since in equilibrium there are separating contracts, the banks can infer what is the exact productivity of their client when they take these separating contracts. I assume that banks don't observe contracts that firms sign with other banks.

Assumption 4: Only one period debt contracts are enforceable.

Assumption 4 is introduced for different reasons. From the theoretical point of view, this assumption rules out the possibility that banks offer contracts where they get to keep all the firms revenues for a certain number of periods before finally letting the surviving firms recover control of their revenues. In the environment of this model such contracts would come up in equilibrium since they dominate simple debt contracts because there is no

[6] For simplicity, no specific agent operates this sector. One might assume that the sector is operated by managers that get zero payoff in equilibrium.

[7] Labor can be easily introduce as an input of production but it doesn't add any insight to the model.

[8] For simulation purposes, I assume that $F(\cdot)$ is a CES production function.

[9] Banks are owned by foreign agents.

need for high quality firms to pay out dividends in all these periods. In a more realistic set up where firms have outside options for their funds, this dominance might be reverse. Assumption 4 has the purpose of ruling these alternative contracts out without having to further complicate the environment. From the empirical point of view, there is evidence on small open economies suggesting that firms' working capital is financed in short term basis and only a small fraction of total liability correspond to longer maturity debt.

Regardless of Assumption 5, banks are allowed to commit to offer any one period debt contract they want in the future. I come back to this point later when I solve for the equilibrium contracts. Also, I assume that firms are unable to commit to future production plans.

In the rest of the paper I analyze the limiting case where the probability of having an adverse shock to the interest rate goes to zero. Then I hit the economy with a one period shock. This case study allows for tractability while still giving insights regarding the transmission mechanisms that work along business cycles downturns in these economies, which is the principal focus of the paper.

In the next subsections I present the tradable, financial and nontradable sectors' problems.

2.1 *The tradable sector's problem*

In this sector, the objective is to maximize intertemporal profits. Thus, the problem at each period of time is

$$\max_{\{y_t^N,k_t^T,x_t\}_{t=0}^{\infty}} \pi_t^T = \sum_{t=0}^{\infty}\left(\prod_{j=0}^{t}\frac{1}{r_j}\right)\left[AF(k_{t-1}^T,y_{t-1}^N)-P_t^N y_t^N - x_t\right] \quad (4)$$

$$s.t. \quad k_t^T = (1-\delta^T)k_{t-1}^T + x_t \quad ; \quad x_t > 0 \quad (5)$$

$$y_{t-1}^N, k_{t-1}^T, \{P_t^N, r_t\}_{t=0}^{\infty} \quad given.$$

where x_t denotes the firm's investment level. At any period t, and given the timing of production, total tradable output has already been chosen. There is no uncertainty for this sector since the interest rate at t is known when inputs are decided. Also note that investment becomes capital –or productive– right away. Finally, capital in the sector is irreversible.

The first order conditions of this problem are

$$AF_{y^N}(k_t^T, y_t^N) = P_t^N r_t \tag{6}$$

$$AF_k(k_t^T, y_t^N) = (r_t - 1 + \delta^T) \tag{7}$$

Both conditions implies that the value of the marginal product of both inputs should equal their marginal cost at the optimum.

Now we turn to the nontradable firm's problem.

2.2 *The entrepreneurs' problem*

The entrepreneur's problem is more complex due the asymmetry of information between them and the rest of the agents in this economy. As was mentioned before, only small firm owners have the technology to produce nontradables and each of them is embodied with a privately known probability p of having a high output performance. Because of this heterogeneity and the fact that entrepreneurs keep their characteristic through time if they have successfully produced in the past, not all problems for different owners will be the same. The setup of the problem will differ across entrepreneurs' characteristics and ages, since useful information is revealed over time.[10] For this reason I denote with subscripts nt an entrepreneur of age n at time t.

An entrepreneur's first-period problem is trivial: he supplies all his labor endowment and save all their income. The problem becomes less trivial for subsequent periods. In all these periods, a small firm owner decides how to allocate his wealth NW_{nt} between consumption c_{nt}^E and savings. He can save by investing some of the savings in his small firm (e_{nt}) and/or by investing in safe assets at the international interest rate. Nonetheless, an entrepreneur never saves in safe assets given assumptions on preferences and the subjective discount rate.

Investment within the firm is allocated between capital (k_{nt}^N) and labor (l_{nt}^N) to produce nontradables, given input prices, expected output prices P_{t+1}^N, entrepreneurs' wealth and available financial contracts.

The assumptions made in the model restrain the financial agreements to simple debt contracts. These contracts will depend on the firm's "net worth" and its age but not on its owner' characteristic, since that is non-verifiable

[10] Note the non-recursive structure of each entrepreneur's problem.

private information. The contract is a tuple $\{M_{nt}(e_{nt}), i_{nt}(e_{nt})\}$, where M_{nt} stands for the size of the loan and i_{nt} for one plus the lending interest rate charged to an entrepreneur of age n at date t.[11] The contract is a function of the firm's "net worth" because this variable is one of the bank screening devices to imperfectly infer the entrepreneur's characteristic. This point will become clearer once I set up the financial sector's problem and show how to solve for the equilibrium of the model.

Before specifying the entrepreneur's problem I present the maximization problem that allows computation of the return to investing in the firm.

Because the entrepreneur's discount rate is higher than the interest rate by assumption, she will always borrow from the bank –as long as she is productively successful– keeping a positive firm leverage level. This allows us to compute the return to the investment project. The gross expected return on investment e_{nt} under external finance per period is denoted as $TR_{nt}(e_{nt}, p)$. Taking contracts as given, this return function is computed through the following problem

$$\max_{\{k^N_{nt}, l_{nt}\}} TR_{nt}(e_{nt}, p) = p[P^N_{t+1}\ \bar{\theta}\ (k^N_{nt})^{\alpha}(l^N_{nt})^{\beta} - i_{nt}(e_{nt})M_{nt}(e_{nt})] \tag{8}$$

subject to

$$r_{k_t} k^N_{nt} + w_t l^N_{nt} \leq e_{nt} + M_{nt}(e_{nt}) \tag{9}$$

Thus, the entrepreneur's expected return of investing e_{nt} in her small firm having characteristic p, is given by total output in case of good productive performance minus the amount due next period, the loan's principal plus interest. Equation (9) is a budget constraint: total cost of investment has to be financed with internal or external funds, where the external funding comes only from the bank. It is worth noting that maximizing this one period return for the firm will result in a maximization of the entrepreneurs' utility as long as the sequence of net worth chosen is optimal.

Having described how returns are computed, and letting τ be the number of period that have passed since the entrepreneur was born, I next set up the entrepreneur's problem assuming it starts once labor has been supplied. It is

[11] To simplify notation I assume that a sufficient contract only specifies "net worth" and age, but the reader should keep in mind that contracts are also over production plans.

at this stage of the problem that total wealth is optimally divided between present consumption and savings via the firm's net worth. Thus,

$$\max_{\{c^E_{nt}, e_{nt}\}} U^E_{nt} = E_t \sum_{j=t}^{\infty} \gamma^{j-t} c^E_{(j-\tau)j} \tag{10}$$

subject to

$$\begin{aligned} c^E_{nt} + e_{nt} &\leq NW_{nt} \quad \forall nt. \\ NW_{n't'} &= \begin{cases} w_t & for \quad n = 1 \\ TR_{nt}(e_{nt}, p) & \forall n > 1 \end{cases} \end{aligned} \tag{11}$$

where subscript $n't'$ denotes the entrepreneur's decision variables at $t+1$.

Given the assumptions in the model, the firm's total revenue function at all times is differentiable with respect to e_{nt}. Thus, the first order condition with respect to e_{nt} (and consumption) can be computed for all periods and ages.

$$1 \leq \gamma \frac{\partial E_t \left[TR_{nt}(e_{nt,}p)\right]}{\partial e_{nt}} \qquad if < 0, \; then \; e_{nt} = NW_{nt} \tag{12}$$

which means that all of an entrepreneur's wealth should be allocated to the firm if marginal returns there are higher than $1/\gamma$. If this condition holds with equality, an interior consumption solution arises. For this we need to solve for the return function $TR_{nt}(e_{nt,}p)$, which I do after defining equilibrium.

Next, I complete the productive structure of this economy with the financial sector.

2.3 The financial sector's problem

As mentioned above, this sector is composed of infinitesimal financial institutions offering standard debt contracts to entrepreneurs and raising funds at the international interest rate from workers or foreign investors through bank deposits. As is usual, banks participate if they make non negative expected profits.

Banks' choice variables are the size of the loan and the lending interest rate under all types of contracts. Thus the bank's objective function, assuming there is some demand for loans, will be.

$$\max_{\{M_{nt}, i_{nt}\}} E\pi^F_{t+1} = \overline{p}_{nt}(e_{nt}) i_{nt} M_{nt} - r M_{nt} \tag{13}$$

subject to

$$\overline{p}_{nt}(e_{nt}) = E_t \left[p \mid p > \in PC(e_{nt}, i_{nt}, M_{nt}),\ f_{nt}(p) \right] \tag{14}$$

where $\overline{p}_{nt}$ is the average quality of a firm of age n at t engaging in this credit contract. Note that this average is computed as the average entrepreneur's quality of those who are willing to participate in the contract $\{i_{nt}, M_{nt}\}$ and who have the same net worth e_{nt}, given all other alternative financial contracts. The mass of entrepreneurs of age n at time t with characteristic p comes from a known density function $f_{nt}(p)$.

It is useful to see for future reference that: a) if only one type of entrepreneur is willing to participate in a contract, then the average quality is given by that type, and b) if all types $p > p^*_{nt}$ are willing to participate, the average type can also be computed. While the bank is unable to observe individuals' characteristics, it knows $f_{nt}(p)$ and it is able to compute the lowest quality type that will participate in the contract (from the participation constraint). This density function can be computed using the density function of firms of age n alive at every period t. Assuming that there was no bad aggregate shock in the history of these firms, this function is $f(p)(\xi p)^{n-1}$. In other words, it is density function of firms that were born together conditional on being alive n periods later. Thus, the density function of those alive in their first period of life ($n = 1$) is just the density function of the newborns. Moreover, because p^*_{nt} can be inferred by the bank in every period, the bank also knows the state variable $p^*_{(n-1)(t-1)}$ or the previous period's lower bound on types participating in the financial contract. Now we have everything we need to compute the average quality type participating in the pool today.

$$\overline{p}_{nt} = E_t(p/p > p^*_{nt}, f^n(p)) = \frac{\int_{p^*_{nt}}^{1} f(p)\ p^n dp}{\int_{p^*_{nt}}^{1} f(p)\ p^{n-1} dp} \qquad with\ \ p^*_{nt} \in [p^*_{(n-1)(t-1)}, 1] \tag{15}$$

where p^*_{nt} is the lowest quality type for whom the participation constraint binds.

2.4 The worker's problem

Workers are passive players in this model. As mentioned before they supply labor and buy and sell assets to maximize intertemporal utility. Since the mass of firms in the financial and tradable sectors is the same as the mass of workers, I let all firms to be owned by workers. This reduces accountability problems and simplifies notation without changing any results. Then, the consumers solve the following problem.

$$\max_{\{c_t^W, l_t, \Gamma_t\}_0^\infty} U_t^W = E_t \sum_{j=t}^{\infty} (\frac{1}{r})^{j-t} \frac{\left(c_j^W - a_1 l_j^{a_2}\right)^{1-\sigma}}{1-\sigma} \qquad a_i, \sigma > 0 \tag{16}$$

subject to

$$c_t^w + \Gamma_t \leq w_t l_t + r_{t-1}\Gamma_{t-1} \qquad \forall t \geq 0 \tag{17}$$

$$\Gamma_0 \; and \; \{w_t, r_{t-1}\}_{t=0}^{\infty} \; given.$$

$$\lim_{t \to \infty} \frac{\Gamma_t}{\Pi_{\tau=0}^t r_\tau} \geq 0 \tag{18}$$

Equation (18) rules out *Ponzi* schemes. The first order conditions for this problem in the limiting case where the probability of the shock goes to zero give us:

$$c_t^W - a_1 l_t^{a_2} = \left(\frac{r}{r_t}\right)^{1/\sigma} \left(c_{t+1}^W - a_1 l_{t+1}^{a_2}\right) \quad \forall t > 0 \tag{19}$$

$$l_t = \left(\frac{w_t}{a_1 a_2}\right)^{\frac{1}{a_2 - 1}} \quad \forall t > 0 \tag{20}$$

$$\sum_{t=0}^{\infty} [c_t^w + \Gamma_{t+1} - w_t l_t - r_{t-1}\Gamma_t] \leq 0 \tag{21}$$

and the transversality conditions for assets.

Equation (19) is the law of motion for consumption and Equation (20) is the labor supply in the nontradable sector. From the assumptions on preferences we are able to derive a labor supply that is independent of present or future consumption –and therefore independent of income. This is important to compute the equilibrium transition from one steady state to the other, after the economy is perturbed by an exogenous shock.

Note that with these preferences, workers try to smooth $\left(c_t^W - a_1 l_t^{a_2}\right)$ but not consumption. Having completed the description of the workers' problem, I closed the model with aggregation details to finally define equilibrium for this economy.

3 Definition of Equilibrium

To avoid postponing the definition of equilibria, I present a heuristic description over the types of equilibrium contracts that arise in this economy.[12] There are two types of equilibrium contracts, and they are

Definition 1 *A pooling financial contract* $\{i_{nt}^{Pool}(e), M_{nt}^{Pool}(e)\}$ *is a simple debt contract in which more than one type of entrepreneurs participate.*[13]

Definition 2 *A separating financial contract* $\{i_{nt}^{Sep}(e(\hat{p})), M_{nt}^{Sep}(e(\hat{p}))\}$ *is a simple debt contract in which only those entrepreneurs that truthfully reveal the same type participate, where* $\hat{p}$ *is the announcement of each entrepreneur's type.*

In fact, those entrepreneurs that belong to the same cohort and with characteristic $p > p_{nt}^*$ will participate in the same pooling contract, sharing the same production plan and the same ex-post output (although the probability of getting a high output will differ across those with different p). Entrepreneur that belong to this cohort with characteristic $p < p^*$ will be engaged in truth telling ($\hat{p} = p$) –separating– financial contracts from then on. As mentioned before this contracts are a function of the firms age and net worth only.

The model is closed by specifying the mass of agents of each type. As mentioned before, at each moment in time there is a mass μ of firms producing tradable goods, banks and workers. Computing the mass of entrepreneurs is not a trivial task due heterogeneity. To define equilibria we need to know –for each cohort– the mass of firms of the same age taking a truth telling contract (those that have characteristic parameter lower than p_{nt}^*) and the mass corresponding to those from the same cohort taking a pooling contract

[12] A formal proof is presented in the next section.

[13] Note that the pooling contract does not depend on the characteristic parameter.

(with characteristic parameter higher than p^*_{nt}), since the amounts of labor and capital inputs allocated to each entrepreneur's firm (and hence aggregate output and equilibrium prices) will depend on the amount financed to each type.

Before computing this, note that the mass of entrepreneurs productively active at each moment in time is the sum of those that are one, two and so periods old. In the absence of an aggregate bad shock history $q_t = q \forall t$, this total mass can be computed in the following way.

$$\begin{aligned} M_t^E &= \int\limits_{p\epsilon[0,1)} [f(p) + qpf(p) + (qp)^2 f(p) + ...]dp \\ &= \int\limits_{p\epsilon[0,1)} [\frac{f(p)}{1-qp}]dp \end{aligned} \tag{22}$$

where M_t^E is the mass of firms at each point in time and is finite and independent of time if the macroeconomic variables stay at steady state levels (if there is no history of aggregate shocks).

Also we are able to distinguish the total mass of firms under a pooling contract and the total mass under a truth telling –or separating– contract.

Variables p^*_{nt} define the threshold for each cohort n at date t that separates those firms taking truth telling contracts from those still in a pooling contract. Thus, for a cohort n at time t, a fraction

$$\eta_{nt} = \int\limits_{P^*_{nt}}^{1} (q_t p)^n \ f(p)dp$$

will take the pooling contract and a fraction

$$\int\limits_{0}^{P^*_{nt}} (q_t p)^n \ f(p)dp$$

will take a truth telling contract. This is true for all cohorts. Note that if p^*_{nt} reaches a value one for some cohort at some time, everybody in this group will take truth telling contracts and all asymmetry of information is solved among them from then on. I come back to this point later.

Because this is a small open economy model, equilibrium is determined by emptying the labor and the nontradable good markets and requiring intertemporal resource constraints for workers to be satisfied.

Let $\Omega(\mu, \{(\eta_{nt})_{n=1}^{\infty}\}_{t=1}^{\infty}, f(p))$ be the economy described above, where $\{(\eta_{nt})_{n=1}^{\infty}\}_{t=1}^{\infty}$ determines the mass of all firms alive at t that were born at $t-n$ and come from a density function $f(p)$ which by assumption is constant over time.

Definition 3 *A competitive equilibrium for economy* $\Omega(\mu, \{(\eta_{nt})_{n=1}^{\infty}\}_{t=1}^{\infty}, f(p))$ *is a collection of state variables* $\{[\eta_{nt}, p^*_{(n-1)(t-1)}, NW_{nt}(p)_{p=0}^{p^*_{nt}}]_{n=1}^{\infty}\}_{t=0}^{\infty}$*, a collection of inputs, financial contracts and output for the entrepreneurs taking a pooling contract,* $\{(k_{nt}^N, l_{nt}, i_{nt}^{Pool}(e), M_{nt}^{Pool}(e), y_{n+1t+1}^N)_{n=1}^{\infty}\}_{t=0}^{\infty}$*, a collection of inputs, financial contracts and output for all entrepreneurs taking separating contracts,* $\{(k_{nt}^N(p), l_{nt}(p), i_{nt}^{Sep}(e(\hat{p})), M_{nt}^{Sep}(e(\hat{p})), y_{n+1}^N(p))_{n=1}^{\infty}\}_{t=0}^{\infty}$*, inputs and output for the tradable sector,* $\{Y_t^N, K_t^T, Y_{t+1}^T\}_{t=0}^{\infty}$[14]*, all entrepreneurs' consumption allocations* $\{(c_{nt}^{Ep})_{n=1}^{\infty}\}_{t=0}^{\infty}$*, workers' consumption allocation, labor supplied and portfolios* $\{c_t^w, l_t, \Gamma_t\}_{t=0}^{\infty}$ *and prices* $\{r_t, w_t, P_t^N\}_{t=0}^{\infty}$ *such that,*

- $\{(k_{nt}^N, l_{nt}, i_{nt}^{Pool}(NW_{nt}), M_{nt}^{Pool}(NW_{nt}), y_{n+1}^N)_{n=1}^{\infty}\}_{t=0}^{\infty}$ *is the solution to all entrepreneurs' problems of age n at time t with parameter* $p \geq p^*_{nt}$ *and net worth* NW_{nt}.
- $\{(k_{nt}^N(p), l_{nt}(p), i_{nt}^{Sep}(e(p)), M_{nt}^{Sep}(e(p)), y_{n+1}^N(p))_{n=1}^{\infty}\}_{t=0}^{\infty}$ *is the solution to all entrepreneurs' problems for all owners of firms of age n at time t with parameter* $p < p^*_{nt}$ *and wealth* $NW_{nt}(p)$.
- *Pooling and separating contracts solve the Banks' problem and they participate.*
- $\{Y_t^N, K_t^T, Y_t^T\}_{t=0}^{\infty}$ *is the solution to the tradable sector's problem,*
- $\{(c_{nt}^{Ep})_{n=1}^{\infty}\}_{t=0}^{\infty}$ *are the consumption allocations of entrepreneurs of type p and age n at every period t.*
- $\{c_t^w, l_t, \Gamma_t\}_{t=0}^{\infty}$ *is the solution to the workers' problem. Finally,*
- *Markets clear:*

[14] A competitive equilibrium can be solved assuming there is only one firm producing tradable goods.

Equilibrium in the labor markets

$$\sum_{n=1}^{\infty}\left[\eta_{nt}\ l_{nt}+\int_{0}^{P_{nt}^{*}} l_{nt}(p)\ (q_t p)\ f(p)dp\right]=b+\mu l_t \qquad \forall t\geq 0.$$

or aggregate labor demand (demand across firms by type and mass) equal labor supply.

Equilibrium in the nontradable market or

$$\sum_{n=1}^{\infty}\eta_{nt}\left[\int_{P_{nt}^{*}}^{1} p\ y_{n+1t+1}^{N}\ (q_t p)^{n}\ f(p)dp+\int_{0}^{P_{nt}^{*}} p\ y_{t+1}^{N}(p)\ (q_t p)^{n}\ f(p)dp\right]=Y_t^N \quad \forall t\geq 0.$$

Note that labor demand is the sum of labor demanded by firms under pooling contracts (all of them having the same production plan), plus labor demanded by firms under separating contracts (each having different production plans). Also note that total nontradable output is computed following the same reasoning, although the total output produced under a pooling plan will be the expected output knowing that each entrepreneur with characteristic p in the same pooling contract will produce an average nontradable output of $p\ y_{n+1}^N$. Because this happens for all types in the pool, aggregation is given by expression $y_{n+1}^N \int_{P_{nt}^*}^{1} p\ (q_t p)^n\ f(p)dp$.

4 Analysis of the model

To prove existence of equilibrium I present some analytic results that are also useful to get some insights of the model's predictions. Due to the huge source of heterogeneity the reader might think that the problem cannot be solved. Nonetheless, the model is solvable not only for the steady state but also out of it.

In the next subsection I work under the assumption that all prices in the economy are constant over time, and I show that the types of contracts described above are actually equilibrium contracts. I explain how to solve for separating and pooling contracts for members of the same cohort. This implies determining which entrepreneurs' types of the same cohort end up with a pooling contract and which with separating ones.

Then I show how contracts are allocated between members of the same cohort in successive periods. This is useful to observe how inefficiencies vanish over time in the same cohort. In other words, I show that over time, the set of entrepreneurs' types taking a pooling contract shrinks, meaning that more and more types will take a contract that only fits themselves and that the asymmetry of information is eventually resolved in the cohort. In this process we observe how banks learn the firms' productivity as these entrepreneurs build up "net worth".

Later I show how the shock to the interest rate affects the price of non-tradable goods, surprising firms in this sector.

Finally I explain that both types of financial contracts mentioned before are also equilibrium contracts (with some minor changes) after the economy is perturbed by the shock to the interest rate. It is at this stage where the assumption that banks learn their clients' type by observing the type of contracts they took in the past comes into play. Since some information about an entrepreneurs' types has been revealed (since they have some reputation) banks will make use of this information after the shock, even if the "net worth" of the firms (that helps to signal entrepreneurs' types to banks) is drastically reduced. Although the economy doesn't loose information already acquired, the information revelation process is slowed down after a bad shock since firms loose "net worth". Because of the fact that information is never destroyed I refer to this information revelation process as reputation acquisition. Once firms get to build up some reputation, they will keep it as long as they are productively successful.

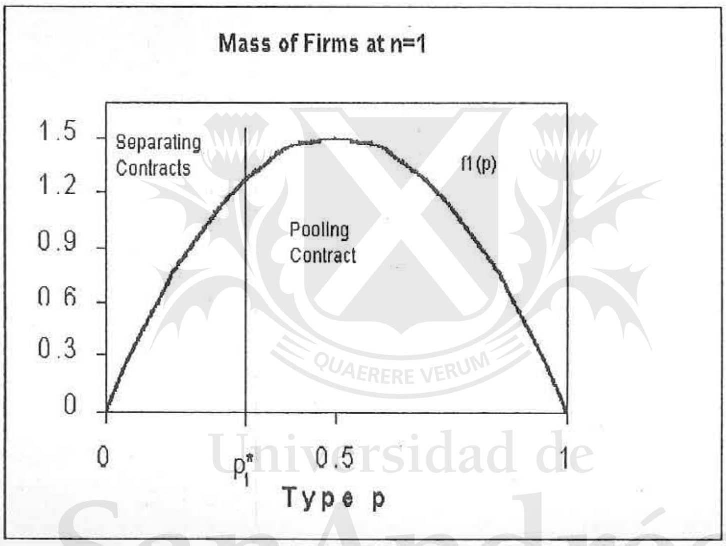
This efficient use of the information explains the value of the lender-borrower relationships analyzed by Petersen and Rajan (1995) and Petersen and Rajan (1994). Because it is important to have a relationship with a bank, there might be incentives for firms to keep borrowing from the same intermediary throughout time.

The reputation acquisition feature of the model challenges previous work on the credit channel where reputation is absent. Because the loss of information in such a world are overstated, the role of the credit channel as a propagation mechanism might be overrated. In the present work, I show that although reputation reduces the damage in the economy when shocks arise, weakening the "net worth" effect stressed in the literature, it also introduces the feature that it takes a long time for firms to build up reputation. Thus, if some firms die along the downturn of the business cycle, it takes a long time for the economy to replace them.

4.1 Financial Contracts in steady state

Assume we are looking at a newborn cohort of entrepreneurs that are just starting up their firms after having supply their labor endowment, which by assumption happens only once. For the time being assume that prices are at steady state levels.

Figure 1 shows the mass of these newborn entrepreneurs that belong to the same cohort under the assumption, as in the simulation exercises followed later, that $f(p) = 6p(1-p)$.[15]



The approach followed is that I have a candidate for the type of equilibrium contracts in this economy. To prove this I show that all agents are maximizing expected utility given market prices. Entrepreneurs are assumed to take contracts as given. Banks can come up with new contracts if the ones proposed by other banks in the market are not equilibrium ones.

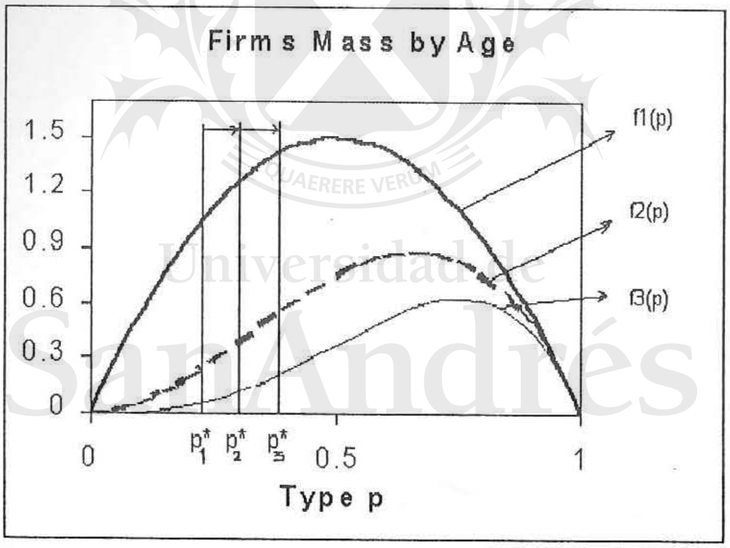
As I mentioned before, there are two types of equilibrium contracts, separating and pooling contracts. The former ones have the characteristic that

[15]More general density functions do not change the result as long as $f(p) > 0$ for all $p \in [0, 1]$.

each type will get a different contract while in the latter ones more than one type participates.

The equilibrium contracts for a 1 year old cohort are such that all types in this cohort with $p < p_1^*$ –those with success probability below some threshold p_1^*– will take separating contracts (see Figure 1). In other words, they will take a financial contract that no other type will be willing to take. Also all types with age 1 and with $p \geq p_1^*$ will share the same pooling financial contract. In this section I explain why these are equilibrium contracts and where the p^* threshold is coming from. I Also explain that this threshold is increasing over time until it reaches the upper bound of the distribution of types. It is then when the asymmetry of information in the cohort is resolved.

Figure 2 shows how the mass of entrepreneurs changes over time due to the fact that unsuccessful entrepreneurs disappear. If the types of newborn firms is given by the density function assumed above, when the cohort is n periods old the density function is $f_n(p) = 6p^n(1-p)$.



The thresholds p_n^* show the cut off points between types taking separating and pooling contracts over time. In steady state p_n^* is a non-decreasing function of n, the cohort's age. In the picture p_n^* is a strictly increasing function of n because of assumptions on $f_1(p)$. This help to facilitate computations although it doesn't alter the results. I'll discuss this point in detail later.

Letting the threshold be also indexed by time, the first result obtained is given by the following proposition.

Proposition 4 *The average quality firm in a pooling contract,* $\overline{p}_{nt}^{pool}$*, is an increasing function of both* p_{nt}^{*} *and* n.

Proof. See Appendix.

The average quality increases with the age of the cohort (holding the lowest type participating in the pooling contract constant) because as time passes lower types die with higher probability as the reader can see from Figure 2. For the same cohort this average also increases with p_{nt}^{*} as lower types exit pooling contracts.

To understand how contracts work, let's focus on the equilibrium along a steady state path. For this reason I drop time subscript on prices for the purpose of this subsection.

By Assumption 4 banks cannot commit to multiperiod financial contracts, although they can commit to offer any kind of one period financial contracts in the future. An equilibrium contract is a pair composed by a lending rate and a loan, $\{i_{nt}(e), M_{nt}(e)\}$, specifying age and net worth e (which is observable information), such that firms maximize profits subject to: 1) technological constraints, 2) available financial contracts and 3) banks getting at least zero profits.

These contracts are solved using a principal agent approach. As was explained in Section 2, the entrepreneur's problem can be divided in two steps. First, we solve for the returns of the firm in a period by period basis as a function of the firm's "net worth" and then we solve for the optimal allocation of the entrepreneurs' wealth between consumption and investment in the firm. After having done the last step, we can go back to check whether the financial contract that comes out of the first step is actually consistent with the equilibrium conditions defined in Section 3.

An entrepreneur with characteristic p (assumed to be high enough) and internal funds e_{nt} can compute his own return function $TR_{nt}(e_{nt}, p)$ by solving the following problem[16]

$$\max_{\{k_{nt}^{N}, l_{nt}^{N}, i_{nt}, M_{nt}, \bar{p}, p^{*}\}} E_t[TR_{nt}(e_{nt}, p)] = p[P^{N}\, \bar{\theta}\, (k_{nt}^{N})^{\alpha}(l_{nt}^{N})^{\beta} - i_{nt}M_{nt}] \qquad (23)$$

subject to

$$\bar{p}_{nt}\; i_{nt}M_{nt} - rM_{nt} \geq 0 \qquad (24)$$

[16]The problem looks the same whether the economy is in steady state or not, though it simplifies notation to assume it is, since time subscripts for all prices can be dropped.

$$r_k k_{nt}^N + w l_{nt}^N = e_{nt} + M_{nt} \tag{25}$$

$$\overline{p}_{nt} = E_t[p \,|p \in PC(e_{nt}, i_{nt}, M_{nt}),\ f_{nt}(p)] \tag{26}$$

Note that all the microeconomic variables –especially the financial contract– depends on the age of the firm because it is observable. The choice variables are inputs (i.e. capital and labor), financial contracts (i.e. principal plus interest) and the average quality firm participating in the contract. Implicitly we need to find the lower quality type in the contract. If it is a pooling contract the lower quality type is p_{nt}^* and if it is a separating contract it is just p, since there is only one type taking it. Finally the objective function is the expected return for the firm with success probability p. It is interesting to notice that the problem for firms with different characteristics that take the same (pooling) contract looks the same except for the fact that the objective function of one type is a positive transformation of the others. This feature will facilitate aggregation across types taking the same contract.

Equation (24) is the bank's participation constraint. Total expected return on loans should be at least equal to the cost of funds (given by the international interest rate). Equation (25) is a budget constraint: total cost of production must be financed with either internal funds or loans. Equation (26) defines the average quality, which is computed by averaging across the types $p \in PC$ in the same financial contract, and knowing $f_{nt}(p)$, the density function of the firms of age n at time t. The types $p \in PC$ are determined in equilibrium.

Before solving the problem under asymmetric information, it is worth noting that in a fully informed environment there is no free riding since financial agreements would internalize the default probability by raising the lending rate of the contract as in Modigliani and Miller (1967). Thus,

Proposition 5 : *(Modigliani and Miller's Neutrality Theorem). Under complete information, the optimal amount of labor and capital hired to produce nontradables is independent of the firms' wealth .*
Proof. *See Appendix A.*

The basic intuition behind this theorem is that if the entrepreneur and banks have the same information regarding the success probability of the firm, then there is no conflict of interests among them and they will work out a financial contract such that the efficient scale of production is implemented. In this world of full information, shocks to the entrepreneurs' "net worth" do not change the aggregate production level. Moreover, firms do not grow over time since they start up right away at the efficient level of production.

In a world with asymmetric information matters are different. I study this world since it opens interesting dynamics at the firm level that impact on the macroeconomy both at the steady state and along the downturn of the business cycles.

In this case, an analytic solution for financial contracts and inputs is not possible. Nonetheless, the optimal level of capital and labor can be solved as a function of $\bar{p_{nt}}$, the average quality type in the same financial contract (which is an endogenous variable of the problem).

Proposition 6 *Solutions for inputs under one-period debt contracts are given by*

$$k_{nt}^{N} = \left[\frac{\bar{p_{nt}}\, P^{N}\, \bar{\theta}\, \alpha^{1-\beta}\beta^{\beta}}{w^{\beta}\, r\, r_{k}^{1-\beta}}\right]^{\frac{1}{1-\alpha-\beta}} \tag{27}$$

$$l_{nt}^{N} = \left[\frac{\bar{p_{nt}}\, P^{N}\, \bar{\theta}\, \alpha^{\alpha}\beta^{1-\alpha}}{w^{1-\alpha}\, r\, r_{k}^{\alpha}}\right]^{\frac{1}{1-\alpha-\beta}} \tag{28}$$

Proof. *See Appendix.*

Variable $\bar{p_{nt}}$ can be interpreted as the banks' perception about the average quality firm taking the contract. Note that Proposition 6 also holds for a truth telling separating contract (by letting $\bar{p_{nt}} = p$).

Although we are not able to solve analytically for the average quality of firms taking the contract, some interesting insights arise. Inputs depend negatively on their prices and positively on the price of the final good and the productivity parameter $\bar{\theta}$. More meaningfully, both inputs depend positively on the average quality of the pool since the loan interest rate depends on it. A better average reduces the interest rate on loans and increases the demand for both inputs. It is interesting to notice that the actual productivity doesn't appear in Equation (27) and (28). Thus, the total output is determined only by the bank's perception about the firms average productivity ($\bar{p}$). This occurs because banks are the marginal suppliers of funds when entrepreneurs do not have access to other financial sources.

Using Equation (27) and (28) we can collapse the entrepreneur's problem even further. Now total return for firms becomes

$$\max_{\{k_{nt}^N, l_{nt}^N, \bar{p}, p^*\}} E_t[TR_{nt}(e_{nt}, p)] = p\left[(1-\alpha-\beta)\left(\frac{\bar{p}^{\alpha+\beta}\ P^N\ \bar{\theta}\ \alpha^\alpha \beta^\beta}{w^\beta\ r^{\alpha+\beta}\ r_k^\alpha}\right)^{\frac{1}{1-\alpha-\beta}} + \frac{r}{\bar{p}}e_{nt}\right] \tag{29}$$

subject to

$$\overline{p}_{nt} = E_t[p \mid p \in PC(e_{nt}, i_{nt}, M_{nt})\ ,\ f_{nt}(p)] \tag{30}$$

The expected return on e_{nt} is increasing in the average quality of the firm for low "net worth" levels. It will be shown later that in equilibrium the return is always increasing in the average quality.

Next, I address the question of whether it is possible for banks to offer (non-linear) financial contracts such that every entrepreneur taking a contract would be willing to truthfully reveal his own type. These contracts exist under two conditions. First, the level of net worth invested in the firm within the period has to be big enough to make the entrepreneur's type announcement credible. Note that in the extreme case where entrepreneurs finance all the cost of production, they have no incentives to lie. In equilibrium, banks will lend to firms since entrepreneurs have a subjective discount factor that is bigger than the interest rate. By making financial contracts where the amount self finance ("net worth") is increasing on the announcement banks can make sure that all types reveal truthfully. Thus any intermediate type faces a trade off: announce a higher type, invest more and pay lower borrowing rates if successful, or announce his own type and invest a lower amount which lead him consume the difference sooner for sure.

Second, all future contracts have to be as demanding as the first truth telling contract in terms of the amount financed internally. Otherwise, some entrepreneurs may imitate others for a number of periods knowing that they can free ride on these others' future contracts. This condition is satisfied since banks can commit to offer the same type of contracts in the future. Then, if there is no gain from free riding in the present, there is no gain from doing it in the future because contracts are expected to be the same over time.

Let p be the firm's true characteristic and $\hat{p}$ its announcement. A truth telling contract is $\{i_{nt}(e(\hat{p})), M_{nt}(e(\hat{p}))\}$, where the entrepreneur has incentives to announce $\hat{p} = p$.

Proposition 7 *A truth telling contract is given by*

$$\hat{p} = p$$

$$e(p) = \frac{(1-\alpha-\beta)\gamma r(\alpha+\beta)}{[1-\gamma r(\alpha+\beta)]}\left(\frac{P^N\,\bar{\theta}\,\alpha^\alpha\beta^\beta}{w^\beta\,r\,r_k^\alpha}\right)^{\frac{1}{1-\alpha-\beta}} p^{\frac{1}{1-\alpha-\beta}} \tag{31}$$

$$\begin{aligned} i_{nt}(e_{nt}(p)) &= \frac{r}{p} \\ M_{nt}(e_{nt}(p)) &= r_k k_{nt}^N(p) + w l_{nt}(p) - e(p) \end{aligned}$$

Proof. *See Appendix.*

Interestingly, the amount financed internally under a truth telling contract increases with p, parameter that also represents the size of the project, and with γ, indicating that banks will lend proportionally more when entrepreneurs are more impatient. Note that by letting $\gamma r = 1$, the net worth required becomes

$$e(p) = (\alpha+\beta)\left(\frac{P^N\,\bar{\theta}\,\alpha^\alpha\beta^\beta}{w^\beta\,r\,r_k^\alpha}\right)^{\frac{1}{1-\alpha-\beta}} p^{\frac{1}{1-\alpha-\beta}} \tag{32}$$

which is the total cost of production for a firm with characteristic p.[17] This implies that $M_{nt}(e_{nt}(p)) = 0$: the owner will only have incentives to reveal his characteristic when there is no borrowing! When the subjective discount rate is higher than the interest rate, the bank will be able to make a truth telling loan contract since only those firms with a high enough probability of surviving are willing to postpone consumption to invest in the firm.

Again, these contracts are only truth telling if the firm take the same contract in the future, which happens in equilibrium. Otherwise, the asymmetry of information would persist because there would be incentives for the lower types to mimic good types knowing that they would get better contracts in

[17] This can be seen by computing the total cost as

$$TC_{nt}(p) = \left(r_k k_{nt}^N(p) + w l_{nt}^N(p)\right)$$

the future (contracts that allow them to invest less and get the same lending rate).

Entrepreneurs will qualify for this last type of contract only if they have enough wealth. Since all entrepreneurs in each cohort start with the same net worth, high quality types –the ones with more productive potential– spend more periods without being able to engage in truth telling contracts. What do they do then?

Without the appropriate level of wealth, firms end up engaging in financial contracts that are not truth telling. Their problem is to maximize (29) subject to Equation (30). As it was mentioned before, all those $p > p^*_{nt}$ will participate, and the problem reduces just to pin down p^*_{nt}.

Every firm that has not taken a truth telling contract in the past, chooses between participating in a pooling contract and participating in a truth telling contract (contingent on having enough net worth). Note that in principle the bank can set up different pooling contracts (for different best quality types in different pools). Nonetheless, the following statement holds.

Proposition 8 *In equilibrium, every entrepreneur that belongs to the some cohort with characteristic $p \geq p^*_{nt}$ and with the same net worth will participate in the same pooling contract.*
Proof. *See Appendix.*

Corollary 9 *All entrepreneurs in the some cohort that belong to a pooling contract will have the same net worth.*
Proof. *See Appendix.*

This Corollary follows from Proposition 8. Given that everybody participates in the same pooling contract as long as they don't take a truth telling one, and that all entrepreneurs in the same pool started with the same net worth (coming from labor endowment), we get the result that everybody that succeeded in the past will have the same net worth independently of their type.

Thus entrepreneurs with quality $p > p^*_{nt}$ will take a pooling contract if and only if total return under the pooling contract is at least equal to total return under the truth telling one. The lowest type can choose to take the latter type of contract only when her net worth is big enough. Thus,

$$TR_{nt}(e_{nt}, \bar{p}_{nt}(p^*_{nt}), p^*_{nt}) \geq TR_{nt}(e(p^*_{nt})) + \frac{1}{\gamma}[e_{nt} - e(p^*_{nt})] \qquad (33)$$

The return under a pooling contract depends on the average quality $\bar{p}_{nt}$, which is obviously a function of the worse type p^*_{nt} and is based on total wealth of the best entrepreneurs in the cohort (since they are willing to invest as much as they have, $NW_{nt} = e_{nt}$). Total return under truth telling is the sum of the return from the firm, based on "net worth" $e(p^*_{nt})$, and the return coming from utility (or consumption), based on $NW_{nt} - e(p^*_{nt})$, which is consumed right away.[18] It is worth highlighting that the participation constraint in Equation (33) only takes into account the present trade off between free riding and taking a truth telling contract. This occurs because under reasonable assumptions regarding the density function $f(p)$, an entrepreneur that is indifferent between free riding or taking a separating contracts (one with characteristic p^*_{nt}) will strictly prefer to reveal himself tomorrow, since the wealth of the best entrepreneurs in the cohort that survive one more period will be even greater, and they will be willing to re-invest all their revenues[19]. This implies that if there are no gains from free riding on today's pooling financial contract, there won't be any gains from free riding on tomorrow's pooling contract. A simple proof of consistency to see whether Equation (33) is the right participation constraint is to check $p^*_{nt} < p^*_{n+1t+1}$.

By using $e(p^*_{nt})$ from Equation (31), plugging it into the last expression and simplifying we are able to get the participation constraint.

$$\left(\frac{P^N\,\bar{\theta}\,\alpha^\alpha\beta^\beta}{w^\beta\,r\,r_k^\alpha}\right)^{\frac{1}{1-\alpha-\beta}}\left[\bar{p}_{nt}^{\frac{\alpha+\beta}{1-\alpha-\beta}}-\frac{(1-\alpha-\beta)}{(1-r\gamma(\alpha+\beta))}p_{nt}^{*\frac{\alpha+\beta}{1-\alpha-\beta}}\right]\geq e_{nt}\frac{(\bar{p}_{nt}-r\gamma p^*_{nt})}{(1-\alpha-\beta)r\gamma p^*_{nt}\,\bar{p}_{nt}} \tag{34}$$

The participation constraint will be always binding in the steady state. Those types participating in the pool today are only the ones that were in the pool in previous periods (unless this is a newborn cohort). While in the steady state this constraint always holds with equality (regardless of the age

[18] It can be trivially proved that no entrepreneur has incentives to undertake a truth telling contract for a type worse than her own. By staying in the pool she will get a subsidize until it is optimal for her to truthfully reveal her own characteristic. And all these contract are cheaper than the one she could get by mimicing a lower type.

[19] If under the present specification we get that for some t, $p^*_{nt} > p^*_{n+1t+1}$ it means that at time t there was a type below p^*_{nt} that would have prefered to choose to free ride on the pooling contract. These cases, although they can be handled, only happens under extreme assumptions on $f(p)$ since it has to be the case that the average quality of firms in the pooling contracts sharply increases between t and $t+1$ even for the same p^* (see Proposition 4). Thus, we need a lot of mass on low values of p since $f_{n+1t+1}(p) = pf_{nt}(p)$.

of the cohort), when the economy is out of the steady state –after a shock for example– the entrepreneur's net worth can be so low that every member of the cohort that was in the pool in the previous period will be willing to participate in it today. I'll come back to this point later.

Also it is worth noting that if e_{nt} becomes high enough then this equation will only hold for $p^*_{nt} = \bar{p}_{nt} = 1$. The "net worth" level that makes the participation constraint binding for a lowest type $p^*_{nt} = 1$ is given by:

$$\left(\frac{P^N \, \bar{\theta} \, \alpha^\alpha \beta^\beta}{w^\beta \, r \, r_k^\alpha} \right)^{\frac{1}{1-\alpha-\beta}} \left[\frac{(1-\alpha-\beta) r\gamma(\alpha+\beta)}{(1-r\gamma(\alpha+\beta))} \right] = e_{nt} \qquad (35)$$

which is the net worth required by a truth telling contract to an entrepreneur announcing $\hat{p} = 1$!

More generally, the following result holds.

Proposition 10 *The lowest and average type participating in a pooling financial contract, p^*_{nt} and $\bar{p}_{nt}$ are nondecreasing functions of the entrepreneurs net worth e_{nt}.*

Proof. See appendix.

Proposition 10 means that as the amount financed internally increases, the average quality of the pool improves. This happens because incentive problems between low quality firms and banks decreases when firms put more "at stake" in the investment project.

In the next subsection I describe how the interest rate shocks impact on the price of nontradable goods, and hence on the firms revenues.

4.2 Macroeconomic effects of the shock

As it was mentioned before, technology in the nontradable sector is given by a constant returns to scale production function. Moreover, assumptions on technology in this sector allow us to state first order conditions as follows.

$$AF_{Y^N} = r_t P^N_t \qquad (36)$$

$$AF_k = (r_t - 1 + \delta^T) \qquad (37)$$

Let r_l and r_h be the interest rates in normal and crisis times respectively, the next result follows.

Proposition 11 *If labor supply is infinitely elastic, there is only one possible equilibrium nontradable good price corresponding to each in interest rate,* $P^N(r_l)$ *and* $P^N(r_h)$ *with* $P^N(r_l) \geq P^N(r_h)$.

Proof. See Appendix A.

The fall in the intermediate nontradable output pricehas two effects. On one hand, it surprises firms that were expecting good macroeconomic conditions and high prices. The fall in the nontradable price triggers a net worth effect in the nontradable –or bank dependent– sector. On the other hand, it increases the exit probability for small firms. Both together put the economy in a recession because it takes time for surviving firms to recover their net worth and for the economy to replace the firms that exit with good financial reputation. The severity and duration of this effect depends on parameter values.

4.3 Financial Contracts out of the steady state

It is worth noting that Propositions 4 to 10 also hold out of the steady state. In particular, even though for some type of entrepreneur her future net worth might not be big enough to satisfy the financial contract given in Proposition 7, after a shock for example, banks will finance the firm as long as the entrepreneur invests all her wealth. This situation continues until net worth is reestablished to normal levels. Would this be violating the commitment undertaken by banks in previous period? The answer is no. The purpose of the commitment is to avoid having some types be free ridden by worse ones. After the shock, the banks can renegotiate the truth telling contracts because the expected probability of such shocks is negligible, implying that no agent was expecting it. Thus, even when the banks renegotiate with firms after a shock, that fact that this shocks are unexpected make Equation (33) the correct participation constraint before the shock.

Note that if the entrepreneurs' net worth collapses to zero, everybody will want to participate since Equation (34) hold with strict inequality because the left hand side of this expression is always positive,[20] even for a type $p^*_{nt} = 0$. Nonetheless even if the "net worth" of all entrepreneurs in the same cohort collapses to zero, not all members of the same cohort will be taking the

[20] See that $\frac{(1-\alpha-\beta)}{(1-r\gamma(\alpha+\beta))} \leq 1$ by assumption and that $\bar{p}_{nt} \geq p^*_{nt}$ for all cohorts.

same pooling contract because banks learn the productivity of their clients by observing the type of contracts they took in the previous period. Then, banks can distinguish those that took a pooling contract in the previous period and will offer them a financial contract using this information. This implies that once the bank knows that a certain type has characteristic p bigger than p^*_{n-1t-1} , they will never offer them a contract where a type lower than p^*_{n-1t-1} is willing or able to take.

4.4 Equilibrium

I have shown that the financial contracts proposed are equilibrium contracts both in and out of the steady state. Now existence of equilibrium follows by showing that the allocations derived from this contracts describe well behaved aggregate excess demand functions for all goods in this economy.

Proposition 12 *Equilibrium exists for an economy* $\Omega(\mu, \{(\eta_{nt})_{n=1}^{\infty}\}_{t=1}^{\infty}, f(p))$ *both in and out of the steady state.*
Proof. *See Appendix A.*

5 Simulations

In this section I first set up the parameters of this model to then carry a comparison between three simulation exercises. The first exercise has the property of switching the reputation mechanism off so we can focus on the implications of extending the Bernanke and Gertler's "net worth" approach to a dynamic setting where firms live for many periods. I call this simulation the literature's benchmark. This is done by letting all firms have the same survival probability after the shock on interest rates as in normal times.

The second simulation exercise differs from the benchmark case in that the survival probability changes on impact as shown in Section 2. The third simulation exercise is similar to the second, but it also includes an externality in the tradable sector.

Lastly, the model allows me to analyze the microeconomic performance of all types of entrepreneurs not only in the steady state, but also after a shock. This information is a by- product of the model, which requires computing for financial contracts at each period.

5.1 Parameters

The tradable goods production function adopted for the simulation is a standard CES

$$Y_{t+1}^{T} = A\left[\phi\left(K_t^T\right)^{-\rho} + (1-\phi)\left(Y_t^N\right)^{-\rho}\right]^{-\frac{1}{\rho}} \tag{38}$$

where $A > 0$, $\rho > -1$ and $\phi \, \epsilon \, (0,1)$, where $\frac{1}{1+\rho}$ is the elasticity of substitution between capital and nontradable inputs.

The parameter values were chosen to roughly match shares of labor and capital in total output and to produce a fall in the intermediate good's prices of 10% as a response to a strong shock on the interest rate. These parameters are listed in the following table

$A \cong 1.349$
$\rho_1 = 7.0678$
$\phi = 0.2039$

For simulation purposes the interest rate levels are $\{r_l, r_h\} = \{1.0147, 1.035\}$.

The elasticity between capital and the nontradable good is required to be low enough to generate a fall in prices of approximate 10% and a fall in the capital stock of only, say 4%. This high complementarity can be relax at the cost of increasing the volatility of investment in the tradable sector.

The sum of the distributional parameters on the nontradable production function was set as large as possible given Assumption 1. This matches microeconomic evidence for the US[21] about technology at the plant level, firms growth and evolution of financial sources.[22] Thus, $\alpha = .35$ and $\beta = .61$, capturing the idea that small firms are labor intensive. The Solow parameter in this sector ($\bar{\theta} = 3.12$) was chosen to obtain the result that the labor demanded by the biggest firm be 150 times the labor demanded by the smallest firm in the nontradable sector, where this ratio was arbitrarily chosen.

Parameters for the worker's utility function are given in the following table.

$a_1 \cong 2.06$
$b_1 = .33$
$\sigma = 3$

[21] Since I wasn't able to obtain microdata from developing economies, I took evidence for the US as a gross substitute to it. Future research should address this question.

[22] See Cooley and Quadrini (1998) and Davis, Haltinwanger and Schuh (1996) for a discussion on these issues.

The parameters corresponding to the labor supplied in the nontradable sector were calibrated to normalize steady state wages in this sector to one and to match evidence that labor elasticity is equal to $\frac{1}{2}$ in developing economies.[23] Finally, the intertemporal elasticity of substitution, represented by the parameter σ is assumed to have a value of three, to mimic some evidence in emerging economies.[24]

The mass of workers μ is set in the following way. For fixed wages and nontradable prices, total labor demand in this sector is given. To normalize labor supplied by each worker to one, I let the mass of workers be equal to labor demand minus labor supplied by entrepreneurs. For simulation purposes I assume that $\Gamma_0 = 0$, no initial wealth is held by workers, meaning that all workers' wealth comes from wages.

The entrepreneur discount rate was chosen to match a "reasonable" leverage level for a firm that has solve all agency problems (the biggest firm for example) and letting it be bigger than the interest rate at all times, good or bad. Thus, $\gamma = \frac{1}{r_h+.01}$.

The density function utilized in this numeric example is $f^1(p) = 6p(1-p)$, where the numbers were set to let the function integrate to one and to match reasonable average spreads between deposit and lending rates. From this density function, it can be seen that there is no mass of firms with characteristic parameter one or zero, implying that everybody produces something and that no firm leaves for ever.

Finally, I assume that the probability that firms exit the industry for non-financial reasons is 2% in steady state and 4% on impact. I believe these are conservative rates since Cooley and Quadrini (1998) argue that for the US the steady state rate of exit due to non-financial reasons is above 3%.

Finally, depreciation rates for capital in the tradable and nontradable sectors were arbitrarily fixed at 6%. Results in the model have shown to be robust to different depreciation rates, although lower depreciation rates require higher complementarity between capital and nontradable inputs in the tradable production function to be able to produce a 10% drop in nontradable prices on impact.

[23] See Rebelo and Vegh (1995).

[24] See ....

5.2 Three simulation exercises

Before entering into the actual comparison of the three simulations, I present the nature of the externality assumed in the third. This externality is introduced by letting the total factor productivity in the tradable sector depend on aggregate nontradable output. For concreteness, I assume

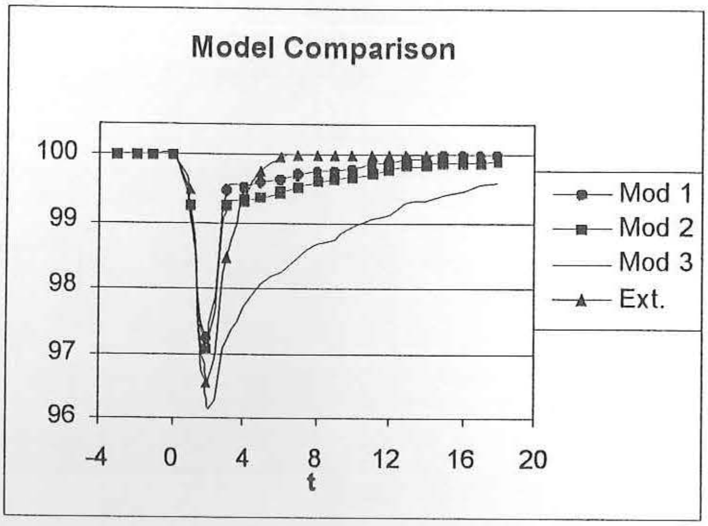
$$A_t(Y_{t-1}^N, \bar{Y^N}) = A\left[1 - \nu\left(\frac{\left|\bar{Y^N} - Y_{t-1}^N\right|}{\bar{Y^N}}\right)\right] \tag{39}$$

where $\nu > 0$. For any scale bigger or lower than the long run aggregate nontradable output scale, $\bar{Y^N}$, tomorrow's total productivity decreases. The idea behind this assumption, is that the nontradable output is a composite of many different goods that are needed for production. When the economy enters into a recession, and the amount produced decreases, the marginal productivity of tomorrows' tradable sector decreases due to coordination problems between sectors, adjustment costs, etc.

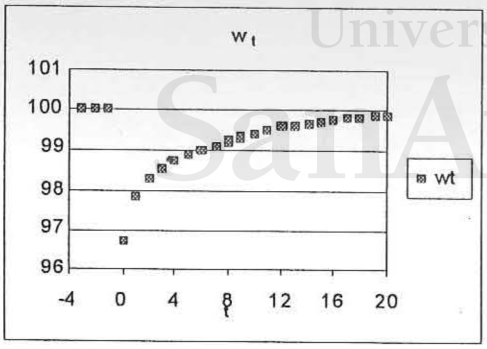
The parameter ν determines the relative importance of the externality. Because obtaining a measure for this parameter is difficult I approach the problem in the following way: I pick a parameter value that do well in matching the evolution of aggregate output in this small open economy. In this simulation I have adopted a parameter $\nu = .25$, implying that a one percent drop in total nontradable output at t decreases total factor productivity by 0.25% in the period that follows.

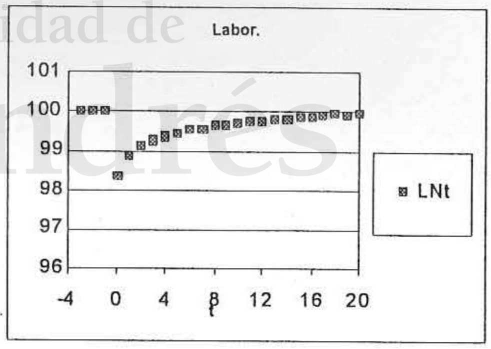
I compare the macroeconomic performance of these three models in one dimension, aggregate tradable output. It is worth noting that across all three cases, all variables are the same in the steady state, since there is no bankruptcy of firms with high output performance and there are no externalities because total nontradable output is being produced at its long run scale.

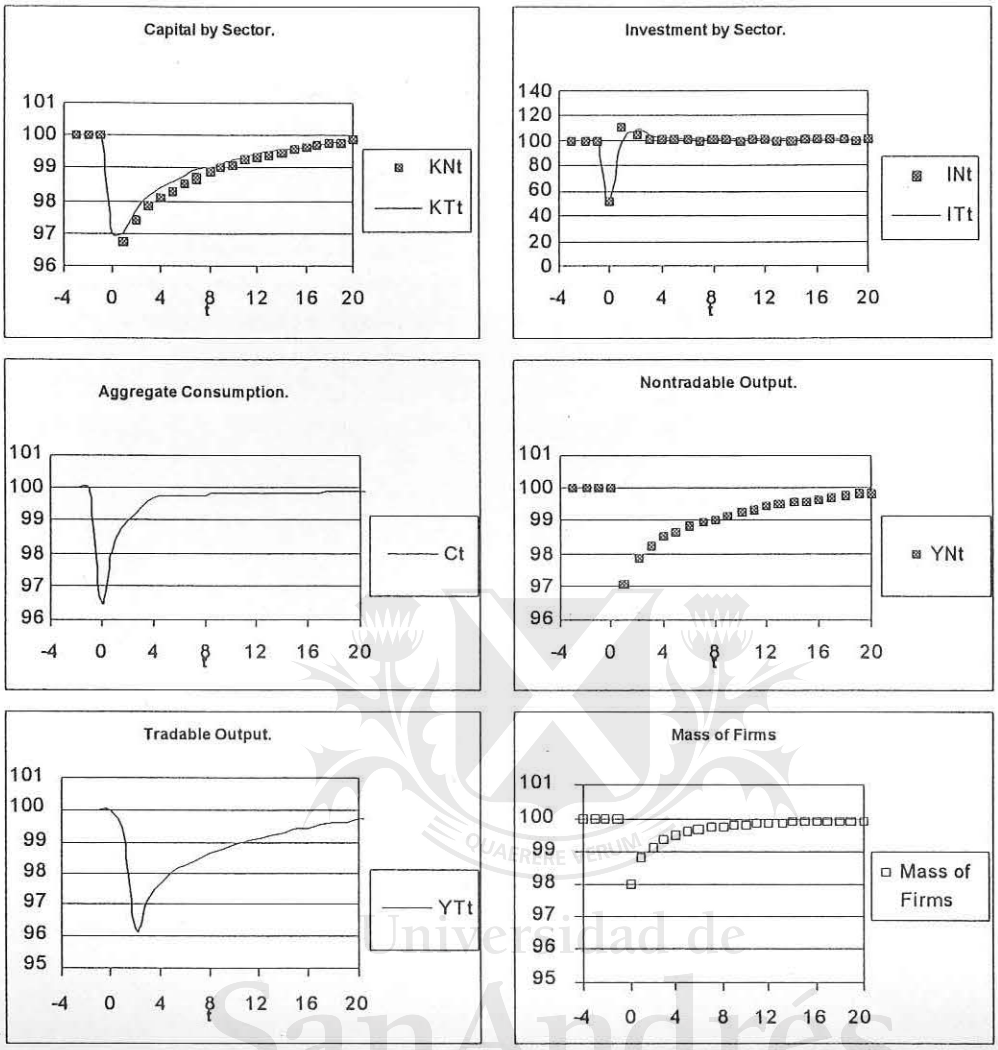
The comparison can be observed in the following chart, where Model 1 refers to case where only the net worth channel is at work, Model 2 refers to the model where the exit rate increases, and finally Model 3 is equivalent to the second case adding externalities to the economy. Also, just as a theoretical exercise, I show the evolution of total output when only the externalities are present (Ext.). This is done by letting all firms have the same net worth on impact instead of in the steady state and the same survival rates.



The model with externalities and bankruptcy is capable of producing more severe business cycles downturns for the same interest rate shock even though externalities alone have very weak serial correlation. Because models without externalities underestimate the business cycles experienced by these economies, I continue by presenting all the macroeconomic variable simulated under this last case. All the main macroeconomic variables are presented in the graphs below.







The simulation, as in the previous cases, was done by assuming that the interest rate increases at period 0, and it returns to normal levels right away. Wages in the nontradable sector are procyclical. Employment decreases as a response to lower wages. Capital in both sector decreases on impact due higher interest rate. After the shock, capital remains low because nontradable output is lower than under steady state, and the two are highly complementary by assumption. Investment in both sector drops sharply on impact and then increases so that capital steadily recovers its steady state level. Aggregate consumption is mostly workers' consumption,[25] and it is highly

[25]Although it also includes entrepreneurs' consumption, this is around 2.5 percent of

correlated with output. This is due the assumption on workers preferences since the sum of both leisure and consumption are smoothed out over time. Nontradable output decreases on impact and it remains depressed through many periods. This is due to externalities: exiting and agency problems between banks and firms. I come back to this last point below. Tradable output is temporary reduced after the shock since it takes time for the economy to recovery due to problems in the nontradable sector. Finally, the mass of firms drops 2% by assumption and although it recovers quickly, tradable output doesn't recover because it takes time for new good firms to build up their net worth and thus to get lower interest rates in financial contracts.[26]

As is shown in the simulations there is transmission of the shock through time despite the fact that this shock happens only at $t = 0$. Wages, tradable and nontradable output, investment and consumption experience depression an it takes a while for the economy to return to it's full potential output and consumption levels. This model shows how externalities, exiting and agency costs drive the cycle after the shock.[27]

Higher agency costs are incurred through two informational channels. The first channel –which I call "net worth" mechanism– takes place when all firms experience losses after a bad shock; the result is that wealth is drastically reduced, and that the proportion of free riders within the same pool becomes higher than it would otherwise be. The main reason for this is that incentive problems between firms and banks are positively correlated with leverage, which is much bigger after the bad shock since firms are financially devastated.

The second channel –which I call the reputational mechanism– is due the loss of information when exit occurs. The firms that exit due to the macroeconomic shock destroy not only present but also future output since the production levels of exiting firms can only be regained once younger generations pass through the costly screening process of producing over time. Again, this process is costly because younger firms with a high productivity parameter are unable to convince banks to finance large investment projects since firms similar in age and equity but with a low productivity parameter have private incentives to free ride on those contracts.

Due to these agency problems in the nontradable sector, the shock puts

aggregate consumption.

[26]The trade balance is sharply improved on impact mostly due to the drop in investment.

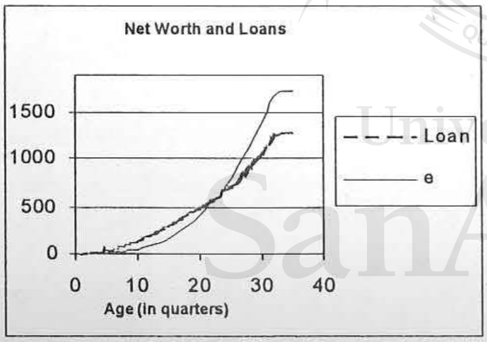
[27]The interest rate paid by firms increases during the recession in this model economy.

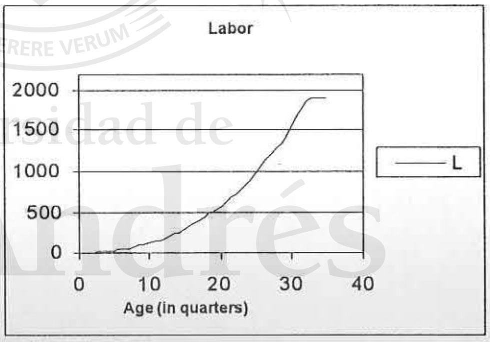
the economy in a long-lasting and recessional path, a situation that is aggravated by the presence of externalities. While this externalities were chosen to contribute to the economic downturn by only 30%, equilibrium effects are stronger, because pecuniary externalities are also important in the model. When total factor productivity decreases, nontradable prices are also reduced, driving nontradable output down with it. This is the reason why externalities add so much to the business cycles.

To complete the analysis of the model, in the next Subsection I present some microeconomic information drawn from the simulation.

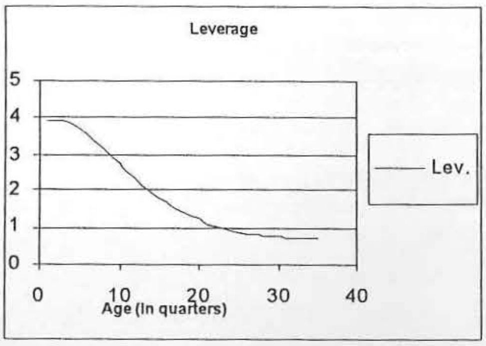
5.3 Microeconomic information

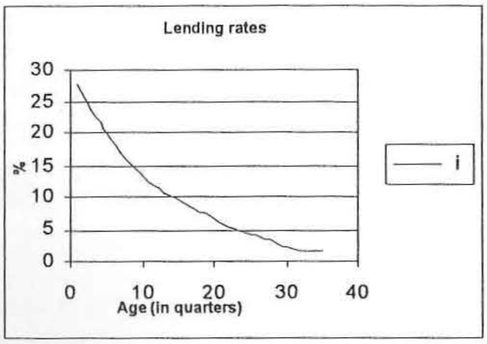
In this subsection I present firm data simulated for the model with externalities, both in and out of the steady state.[28] To analyze this information in the steady state, it is better to concentrate on the data generated by a firm owned by an entrepreneur with the highest characteristic parameter p. Remember that this entrepreneur keeps his productivity over time, as long as he is productively successful. The graph below shows the main firm variables as a function of the age of the firm (per quarter), assuming the economy is at its steady state (or prices of inputs and output are constant).





[28] Again, microeconomic data corresponding to steady state levels are the same for the three models.





Net worth and amounts loaned are positively correlated, evidence that the banks utilize the firms' wealth as a revealing informational screening device. It is worth mentioning that in a symmetric informational environment, these variables would not be correlated. Also they increase with age, since by assumption the (highest quality) firm is productively successful in all these periods. The net worth has an upper bound because of the assumption that technology in this sector exhibits decreasing returns to scale. The simulation shows that only after 33 quarters, these firms are able to take truth telling contracts that fully solve the asymmetric information problem with banks.[29]

Inputs and output also increase with age, as can be observed in the graph for labor demanded by firms.

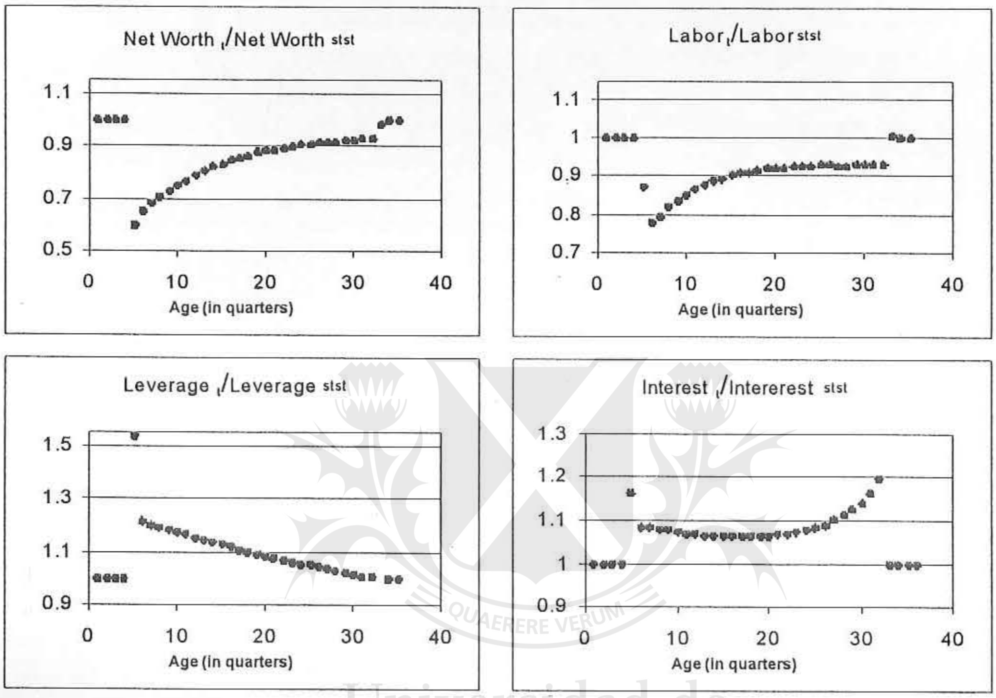
Leverage, expressed as the ratio of loans to net worth, is monotonically decreasing with the firm's age. As firms get older, the fraction of spending that is self-finance converges to the fraction in truth telling contracts, meaning that net worth grows proportionally faster than bank loans in the firm's first periods of life. This fraction stabilizes once the firms take truth telling contracts.

Finally, the interest rates paid on loans by these firms decreases with age as the bank's perception of the firms' quality improves. Younger firms pay higher rates because their reputation -and their access to credit markets- has not been developed.

In the next graphs I present micro-data for the simulation with externalities after the economy was hit by the external shock. When the economy is at its steady state, as in the previous graphs, time series micro-data coincides with cross sectional data. In contrast, after the economy is impacted with a

[29] Note that the performance of a lower type entrepreneur gives a similar graph except that the pooling financial contracts would be dropped at an earlier stage. This statement holds by Corollary 8.

high interest rate, time series and cross sectional data differ because a firm's performance will depend on the age of the firm at the moment of the shock. In the next set of graphs, I show time series data for the highest quality firms that belong to a five period old cohort at impact. For comparison, I present the information on this cohort as a ratio of actual data to the time series data that would have been produced by these firms if no shock had -occurred.



The ratio of actual net worth to that in steady state conditions is lower than one, showing that firms that are hit by the shock will only recover after 35 periods. During this time, labor demand will also be lower since agency costs are higher. Actual leverage is temporary higher than the steady state level of leverage in all these periods, because banks do no require firms to finance in the same proportions as at steady state, since some information about this cohort average quality has been already revealed. Clearly, the information revelation process takes longer in recessions due to the net worth effect. Finally, the ratio of actual interest rates paid by firms during recessions to those rate paid in the steady state are higher throughout the recession.

6 Policy analysis

The size of the economic recession, given assumptions on technology and preferences, is in direct relationship with the size of the external interest rate rise. Higher rates imply lower unexpected nontradable prices in the economy and this will increase the exit probability and reduce even further the net worth of those firms still producing. The deeper the interest rate crisis the deeper and longer lasting the recessions due to higher agency costs, exiting rates and externalities, leaving room for policy analysis.

The shock reduces welfare in two different ways. On the one hand, workers have a cost in terms of expected welfare because their utility function is concave in the sum of consumption and leisure, and the shock reduces expected utility by Jensen's inequality. On the other hand, agency problems add welfare costs to both entrepreneurs and workers since profits and wages are reduced throughout the economic downturn.

Any stabilizing policy that neutralizes sudden changes in entrepreneurs' wealth might improve the overall performance of this economy. Thus there are different policies that might be implemented. A subsidy to the interest rate in bad states or any policy that inflates the demand in the nontradable sector will help to reduce the recession. A sterilization policy, used directly or indirectly in emerging economies, implies that the interest rate is subsidized when the bad shock occurs. Then, the government should collect taxes in good times and subsidize interest rates in bad times, where this can be done even if taxes are collected after the subsidy takes place.

Under such a policy total welfare would be greatly increased. A first order measure of welfare gains can be approximated as the area delimited by the full capacity level and the actual performance of the economy's tradable output along the cycle in net present terms.[30]

A more realistic policy would be one where the government collects liquid international resources in good times to subsidize interest rates in bad times. This policy can be implemented at the cost of keeping productive resources underutilized. The cost of keeping these reserves will determine the optimal degree of intervention in each economy.

Regardless of the intervention levels, such a policy might always be welfare improving in economies (or episodes) that face severe and unexpected

[30] Note that this measure is a lower bound on the total welfare gains of this stabilization policy since this policy also increases expected worker's utility given the concavity assumption on their preferences.

increases in interest rates –without including the cost of the policy– since the aggregate agency cost is a monotonic function of the change of these rates.

7 Concluding remarks

This paper shows that financial frictions might be a strong transmission mechanism for the propagation of shocks in small open economies, both from the qualitative and quantitative points of view. Of all the different ways to model financial frictions developed in the literature, I took the one proposed by Bernanke and Gertler (1990) since the asymmetric information problem emphasized there seems the most appropriate and representative one in financial relationships. When this friction is incorporated into a dynamic macroeconomic model, we obtain two effects that impact on the incentive side of financial contracts -and hence on the macroeconomic performance of the economy-: "'net worth" and "reputation". The first one was analyzed by Bernanke and Gertler (1990) in a static environment, concluding that the firms' financial health might have an important role explaining aggregate agency costs and output performance. In this work, I show that although the "net worth" effect is present and important in a dynamic setting, the "reputation" effect might be also important when there is information to be learned about firms' quality from their performance over time: if firms with good reputations die in the presence of unexpected bad news it takes a long time to replace them.

Under the present setup, most of the macroeconomic variables in the simulation are well behaved. Interest rates and investment in the tradable sector are the leading indicators of the cycle. Low investment levels depress the small firms' output price putting firms in a fragile financial situation since their revenues are less than expected. This declines aggregate performance because firms are less able to convince the banks to finance large investment projects. If the shock implies a greater exit probability, then the economy will perform even more poorly for some periods following the shock because firms that have developed a good financial reputation disappear and it takes time before new firms develop their own. Aggregate performance declines even further in the presence of externalities. This dynamic leads to countercyclical agency costs and procyclical employment and consumption. These features of the model match empirical evidence.

In this environment, sterilization policies might be welfare improving depending on the cost of implementation. Neutralizing capital volatility –when it can be done at a relatively low social cost– will help the economy to perform more closely to its full productive potential. From the theoretical point of view, any policy that reduces the nontradable price uncertainty would improve total welfare.

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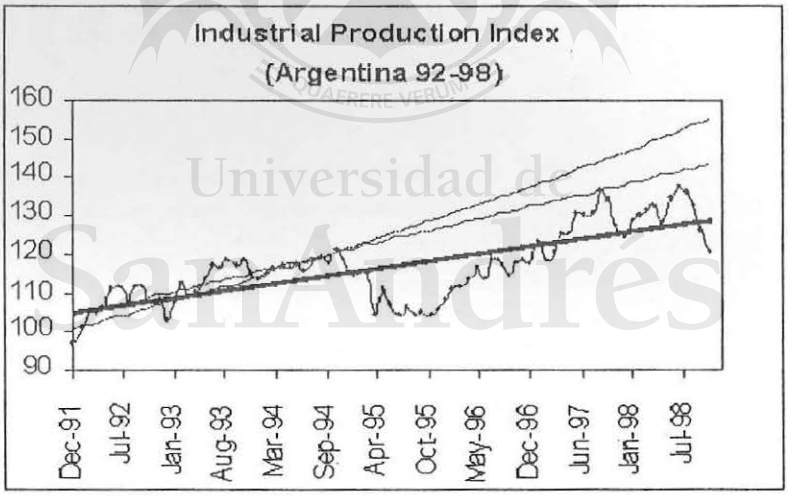
Appendix A

The Industrial Production Index (with 1986=100) includes the following industries: Food, Beverages, and Tobacco, Aparel, Paper, Chemical, Contruction, Metalic and Machines and Equipment.

The series presented in the Introduction is the Industrial Production Index for Argentine and modified as follows. First, I replaced all February's observations by the average of January's and March's observations, since the Index exhibits a sharp decline on each Febraury due to vacations.[31] Second, I computed a linear trend for two periods: February 1992 to December 1994 and March 1994 to December 1994. I utilized the second linear trend since it is the most conservative one (not show in the Graph below). The Graph shows an exponential and a linear trend based on the period 1992-1994, as well as a linear trend based on the period 1992-1998. The graph shows that by computing deviations from the linear trend based on the period 1992-1994, the Industrial Production Index would not recover trend until September 1997. This implies an even longer recession than the one presented in the Introduction.

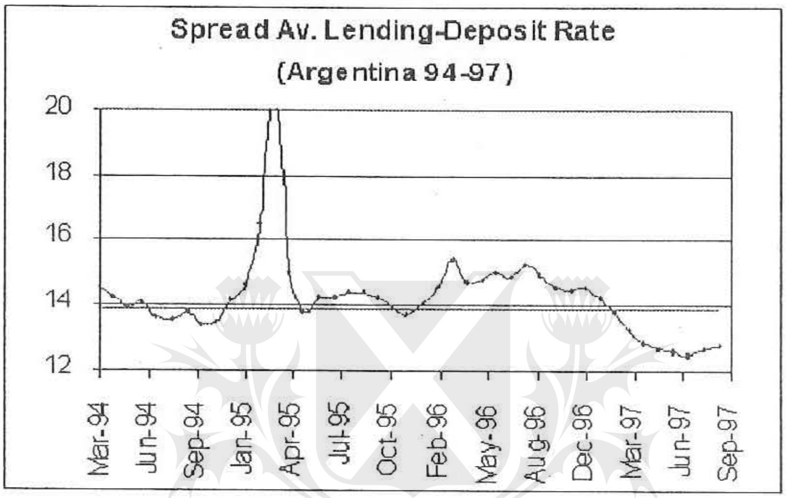


The series for the average deposit interest rates of commercial banks in the Argentinean financial system are build as a weighted average of the average interest rate paid to deposits denominated in pesos and in dollars.

[31] Leaving the Index intact would increase the trend rate (fictitiuosly), reinforcing the argument that the economy entered in a long recession.

The average lending rate is the weighted average interest rate charged to loans denominated in pesos and in dollars to local firms, big and small. There is no information on the interest rates paid on bank loans by small firms neither on loans to small or AAA firms. This is a problem since in this article I focus on the dynamics of small firms along the business cycles.

As a reference, I present the spread between the average deposit rate and the average lending rate of the financial system.



As the graph shows, the spread returned to normal levels right away after the sharp spike on impact. It seem that the persistence in the lending premium is nonexistent. Nonetheless, this result is driven by changes in the composition of lending to small and AAA firms during the downturn.

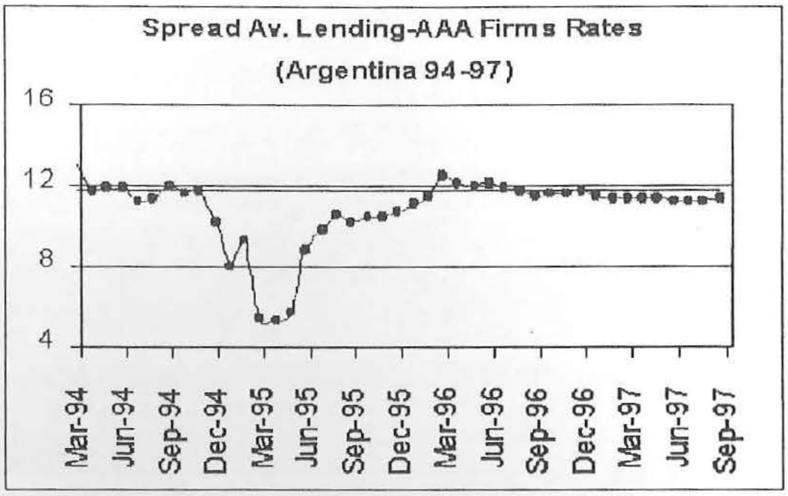
By definition the average lending rate, regardless of denomination issues, is

$$r_t^L \equiv \alpha_t r_t^S + (1-\alpha_t) r_t^b$$

where the interest rates are the average lending rate, the lending rates for loans to small firm and big firms (or AAA firms) respectively, and α_t is the fraction of lending to small firms. Rearranging this expression we get the spread in the previous graph.

$$r_t^L - r_t^b \equiv \alpha_t (r_t^S - r_t^b) \tag{40}$$

I present this information in the following chart.



This spread shows that the average lending rate and the lending rate to AAA firms get significantly close in the first months after the interest rate shocks. As Equation (40) shows, this result is consistent with changes in α_t or in $(r_t^S - r_t^b)$ or in both. Although we have an identification problem, anecdotal evidence points that small firms face relatively higher interest rates. This implies that the drop in the spread shown in the previous graph must be driven by a sharp fall in α_t. Since overall lending of the financial system fall over this period, credit to small firms must have fallen even further.

To see that note that in June 1995, the spread between the Average lending rate and the rate for loans to AAA firms is almost 4% below the same spread before and after the crisis. Also, the AAA rate was 5 percentage points above steady state values. If α_t did not fall, then this observations imply that while AAA rate was 5 percentage points above trend, the lending rate for small firms was only 1% above trend. This scenario is refuted by anecdotal evidence.

Appendix B

Proof. of Proposition 4. Dropping subscripts and taking partial derivatives to expression (17) give us

$$\frac{\partial\,\bar{p}}{\partial p^*}=\frac{f(p^*)p^{*n}}{\int_{p^*}^{1} f(x)x^{n-1}dx}\left(\frac{\bar{p}}{p^*}-1\right)>0\;\forall\,p^*\in[0,1)$$

where it is easy to further show that this derivative goes to one from below as $p^* \rightarrow \bar{p}$. Also,

$$\frac{\partial\,\bar{p}^{Pool}}{\partial n}=\frac{n}{(n-1)}\frac{\int_{p^*_{nt}}^{1} f(p)\;p^{n-1}dp}{\int_{p^*_{nt}}^{1} f(p)\;p^{n-2}dp}>0\;\forall p^*\in[0,1)$$

■.

Proof. of Proposition 5. Under full information, the firms problem becomes[32]

$$\max_{\{k^N,l^N,i,M\}} E_t[TR(e,p)]=p[P^N\,\bar{\theta}\,(k^N)^{\alpha}(l^N)^{\beta}-iM] \tag{41}$$

subject to

$$p\,iM-rM\geq 0 \tag{42}$$

$$r_k k^N+wl^N=e+M \tag{43}$$

where it can be seen that there is no adverse selection since the bank lends at a rate that take into account the true entrepreneur characteristic p. The solution to this problem is just given by

$$k^N = \left[\frac{pP^N\,\bar{\theta}\,\alpha^{1-\beta}\beta^{\beta}}{w^{\beta}\,r\,r_k^{1-\beta}}\right]^{\frac{1}{1-\alpha-\beta}} \tag{44}$$

$$l^N = \left[\frac{pP^N\,\bar{\theta}\,\alpha^{\alpha}\beta^{1-\alpha}}{w^{1-\alpha}\,r\,r_k^{\alpha}}\right]^{\frac{1}{1-\alpha-\beta}} \tag{45}$$

[32] I loose unnecessary notation.

where productions plans only depend on each entrepreneurs characteristic and not on the initial net worth e ■.

Proof. of Proposition 6.Let the bank participation constrained and the budget constraint collapse into one equation to solve for the total amount due next period, $i_{nt}M_{nt}$.

$$i_{ny}M_{nt} = \frac{r}{\overline{p}}(r_k k_{nt} + w l_{nt}^N - e_{nt}) \tag{46}$$

Plugging this expression into the objective function, simplify the problem to

$$\max_{\{k_{nt}^N, l_{nt}^N, \overline{p}, p^*\}} E_t[TR_{nt}(e_{nt}, p)] = p[P^N \bar{\theta} (k_{nt}^N)^\alpha (l_{nt}^N)^\beta - \frac{r}{\overline{p}}(r_k k_{nt}^N + w l_{nt}^N - e_{nt})] \tag{47}$$

subject to

$$\overline{p} = E_t[p \mid p \in PC(e_{nt}, i_{nt}, M_{nt}),\ f_{nt}(p)] \tag{48}$$

and solution follows from solving this problem ■.

Proof. of Proposition 7. Given a truth telling contract offered by the bank, the entrepreneur solves the following problem today and in every subsequent period:

$$\max_{\{\hat{p}\}} E_t[\pi_t^N(p, \hat{p})] = p\left[(1-\alpha-\beta)\left(\frac{\hat{p}^{\alpha+\beta} P^N \bar{\theta}\, \alpha^\alpha \beta^\beta}{w^\beta\, r^{\alpha+\beta}\, r_k^\alpha}\right)^{\frac{1}{1-\alpha-\beta}} + \frac{r}{\hat{p}} e(\hat{p})\right] - \frac{1}{\gamma} e(\hat{p})$$

By taking first order conditions, imposing the truth telling incentive condition ($\hat{p} = p$) and rearranging terms we can obtain the following differential equation on $e(\hat{p})$.

$$(\alpha+\beta)\gamma\left(\frac{P^N \bar{\theta}\, \alpha^\alpha \beta^\beta}{w^\beta\, r^{\alpha+\beta}\, r_k^\alpha}\right)^{\frac{1}{1-\alpha-\beta}} \hat{p}^{\frac{\alpha+\beta}{1-\alpha-\beta}} = \frac{r\gamma}{\hat{p}}\, e(\hat{p}) + (1 - r\gamma)\, e'(\hat{p}) \tag{49}$$

Fortunate enough, a closed form solution to this differential equation exist[33]. Finally, by noting that an entrepreneur with characteristic $p = 0$ never invest ($e(0) = 0$), the proof is completed ■.

Proof. of Proposition 8. This can be easily proved by contradiction. Suppose that are two different equilibrium pooling contracts for types in the same cohort and with the same net worth. Then, one of these type will have and average parameter $\bar{p}$ bigger than the other, implying a lower interest rate on loans. Since types cannot be screened but through age and net worth, being they the same in the two pooling contracts, all entrepreneurs would try to participate in the debt contract that charges lower interest rate. ■

Proof. of Proposition 9. All entrepreneurs in the same cohort start with the same net worth given by labor endowment. This means that firms in a new born cohort participate in the same pooling contract and have the same production plan. Those surviving a period ahead, will have the same net worth regardless of their type. The subset of these taking a new pooling contract will, again end up with the same wealth. This process continues until no pooling contract exist for member of the cohort (until the best quality firms have accumulated enough wealth to take truth telling contract) ■.

Proof. of Proposition 10. Making use of Proposition 1 the proof consist on showing that p^*_{nt} is a nondecreasing function of the entrepreneurs net worth e_{nt}. There are two cases. On one hand, if the participation constraint is not binding, the local changes in the entrepreneur net worth does not change p^*_{nt}. On the other hand, when the participation constraint is binding, then p^*_{nt} will change with e_{nt}. Dropping subscripts and rearranging the participation constraint, we obtain

$$e = C\,\frac{p^*\ \bar{p}\left[\bar{p}^{\frac{\alpha+\beta}{(1-\alpha-\beta)}} - \frac{(1-\alpha-\beta)}{(1-r\gamma(\alpha+\beta))}p^{*\frac{\alpha+\beta}{(1-\alpha-\beta)}}\right]}{(\bar{p}-r\gamma p^*)}$$

where C is a constant that depends on parameter values. Call [1] the expression between brackets. Differentiating the participation constraint with

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[33]This differential equation fits into the following general type of linear differential equations

$$w(\hat{p}) = u(\hat{p})e(\hat{p}) + e'(\hat{p})$$

and its closed form solution is given by

$$e(\hat{p}) = \exp(-\int u\,d\hat{p})\ \left(A + \int w\exp(\int u\,d\hat{p})d\hat{p}\right)$$

respect to p^*, and simplifying gives as

$$\begin{aligned}\frac{\partial e}{\partial p^*} &= \frac{\bar{p}}{(\bar{p} - r\gamma p^*)}\left[\bar{p}\,[1] - (\bar{p} - r\gamma p^*)\frac{\alpha+\beta}{(1-r\gamma(\alpha+\beta))}p^{*\frac{\alpha+\beta}{(1-\alpha-\beta)}}\right] + \\ &\frac{\partial\,\bar{p}}{\partial p^*}\frac{p^*}{(\bar{p} - r\gamma p^*)^2}\left[-r\gamma p^*[1] + (\bar{p} - r\gamma p^*)\frac{\alpha+\beta}{(1-\alpha-\beta)}\;\bar{p}^{\frac{\alpha+\beta}{(1-\alpha-\beta)}}\right]\end{aligned}$$

Where $r\gamma \leq 1$ by assumption. Now, let [2] and [3] be the first and second expressions between brackets in this derivative. The proof follows by showing that this two expression are positive for all possible values of p^*. Since $\frac{\partial \bar{p}}{\partial p^*}$ is always positive, then $\frac{\partial e}{\partial p^*} > 0$ for all values of p^*.

Rearranging terms, [2] becomes

$$\begin{aligned}[2] &= \frac{\bar{p}^{\frac{1}{(1-\alpha-\beta)}}}{(1-r\gamma(\alpha+\beta))}\left[(1-r\gamma(\alpha+\beta)) - x^{\frac{\alpha+\beta}{(1-\alpha-\beta)}} + r\gamma(\alpha+\beta)x^{\frac{1}{(1-\alpha-\beta)}}\right] \text{ or} \\ &\frac{\bar{p}^{\frac{1}{(1-\alpha-\beta)}}}{(1-r\gamma(\alpha+\beta))}F(x)\end{aligned}$$

where $x = \frac{p^*}{\bar{p}} \in [0,1)$. It is easy to show that $F(0) > 0$, $F(1) = 0$, and $F'(x) < 0\ \forall x$. This implies that $[2] > 0$. Similarly,

$$\begin{aligned}[3] &= \frac{\bar{p}^{\frac{1}{(1-\alpha-\beta)}}}{(1-\alpha-\beta))}\left[\frac{r\gamma(1-\alpha-\beta)^2}{(1-r\gamma(\alpha+\beta))}x^{\frac{1}{(1-\alpha-\beta)}} - r\gamma x + (\alpha+\beta)\right] \text{ or} \\ &\frac{\bar{p}^{\frac{1}{(1-\alpha-\beta)}}}{(1-\alpha-\beta))}G(x)\end{aligned}$$

where x is defined as before. Now, $G(0) > 0$, $G(1) \geq 0$, and $G'(x) < 0\ \forall x$. ■

Proof. of Proposition 11. See that the indirect profit function for this firms is only a function of P_t^N and r_t. Zero profit condition $\pi_t^T(P_t^N, r_t) = 0$ implies that there is one possible price of nontradable goods corresponding to each interest rate level. If $r_l < r_h$ are the interest rates in normal time and crisis time, then $P^N(r_l) \geq P^N(r_h)$.

Proof. of Proposition 12.

In a small open economy there is no need to for excess demand for tradable goods to be zero. Then we only worry about aggregate excess demand for nontradables (Y^N) and labor. The aggregate demand for nontradable goods is well behaved with respect to P_t^N and w_t, and so is the aggregate supply of labor. Thus, we just need to show that aggregate supply of Y^N and aggregate demand of labor are well behaved functions of P_t^N and w_t.

First, note that each firms' supply of nontradable output and demand of nontradable skilled labor are not continues functions of prices. An entrepreneur with characteristic p^* in cohort n at t is indifferent between participating in the pooling contract or taking a truth telling one. Equation (28) in Proposition 6 shows the firms' labor demand for all cohorts. Under a pooling contract $\bar{p}_{nt}=\bar{p}_{nt}^{Pool}$ while under a truth telling contract $\bar{p}_{nt}= p$. In equilibrium, an entrepreneur taking a truth telling has a success probability of $p \leq p_{nt}^* < \bar{p}_{nt}^{Pool}$. Thus, each firm demand for labor is not continues in prices since for a type p_{nt}^* that is indifferent between one type of contract or the other, a small change in prices will make it switch to the other type of contract. Also note that this is the only source of discontinuity, since the entrepreneurs only participate in either of this two types of contracts by Proposition 8 and since labor demand is well behaved when the entrepreneurs' type is different from p_{nt}^*.

From the individual demand (and supply) functions we construct the aggregate demand by computing the mass of firms taking truth telling contract and the mass taking a pooling contract for each cohort. These individual demands are locally continues functions of prices for every type but type p^* in each of this cohort. Nonetheless these types have zero mass in the cohort, implying that demand for the whole cohort is globally continues in the prices space since for all prices there is at most a type with mass zero whose demand is discontinuous being the everybody else's demand continuous in the same cohort. Aggregate demand accounted as the sum of each cohort demand is continuous and finite for every positive price by assumptions on $f(p)$.

Finally since aggregate excess labor demand and nontradable output supply are well behaved we conclude that equilibrium exist. ■