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Crime, Inequality, and Unemployment

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Abstract

We construct a dynamic model of the labor market in which agents have the option to engage in crime. Involvement in criminal activities depends on the state of the labor market, the probability of being apprehended and convicted, and the severity of the penal system. We allow both for fines as well as for jail sentences as forms of punishment. We use the model to study the interaction between unemployment, the degree of income inequality and the crime rate, all of which are endogenously determined. We find that criminal activity significantly affects the normal functioning of the labor market. It can induce inequality due to "efficiency-wage" considerations, and it leads to multiple equilibria in a natural way. The model can also be used to compare the effects of alternative policies designed to combat crime.

1. Introduction

Numerous empirical studies have looked at the relationships between inequality, unemployment, and crime.[1] The goal of this paper is to develop a model in which the interactions between these variables, and between policy variables such as law enforcement or the penal system, can be analyzed theoretically. While there exist economic models of criminal activity in the literature[2], the framework presented here is different, in the sense that all three objects of interest – inequality, unemployment, and crime – are determined endogenously. Moreover, even if one is less interested in crime *per se*, another contribution of the paper is the following: we show that allowing for criminal activity in otherwise standard models of the labor market changes their implications in interesting ways. For example, once crime is incorporated, models that without crime predicted a single wage now generate nondegenerate wage distributions, models that predicted continuous wage distributions now generate distributions with gaps due to "efficiency-wage" considerations, models that predicted uniqueness now generate multiple equilibria, and so on.

The basic structure is standard: firms post wages, and unemployed workers search for jobs (in one version we also allow employed workers to search for better jobs). A novel aspect of our model is that workers, whether currently employed or not, randomly encounter opportunities to commit crimes, which they may or may not exploit, and they may also be randomly victimized by others. Criminals face a probability of being caught, which may entail a fine, a prison sentence, or at the very least losing one's job, if one is currently employed. Clearly, agents' decisions to commit crimes will depend on the risk of getting caught, the nature of punishment, their current employment status, their wage, and general labor market conditions. Further, firms' wage setting decisions are influenced by criminal behavior – by paying a higher wage, employers may be able to dissuade workers from engaging in crime, which reduces turnover. Through this mechanism, the model generates inequality (wage dispersion) among homogeneous agents.

A key finding of our analysis is that introducing crime into a standard model can naturally lead to multiple equilibria, for new reasons.[3] For example, in equilibrium the probability of being victimized, and hence the attractiveness of legitimate relative to criminal activity, depends on the number of criminals. This feedback can lead to

[1] Both time-series and cross-sectional studies generally find that unemployment is positively correlated with crime, although the link is not always strong. For a survey of the empirical literature on crime and the labor market see Freeman (1996). That labor-income inequality is positively correlated with the property crime rate across US states is documented in İmrohoroğlu et al. (1999).

[2] Benoît and Osborne (1995), Fender (1999), İmrohoroğlu et al. (1999), Tabarrok (1997), Sah (1991) and of course, the seminal work of Becker (1968) are some examples.

[3] In particular, our multiplicity is not caused by the congestion in law enforcement commonly assumed in the literature (as in Fender 1999, Tabarrok 1997 and Sah 1991, for example). Two other potential sources of multiplicity are described in Murphy et al. (1993). Our interest in the existence of multiple equilibria with different crime rates stems from the empirical work of Glaeser et al. (1996). They argue the most puzzling aspect of crime is that observable local area characteristics seem to account for little of the high variance of crime rates across space (e.g. US cities). This suggests that positive interactions among agents' criminal decisions leading to multiple equilibria may be a key force at work.

multiple equilibria with high or low crime rates. As another example, suppose there are many employers paying wages high enough to dissuade their workers from crime; this also makes legitimate activity attractive relative to crime, which in turn makes it less costly for an individual employer to pay a wage high enough to keep workers honest. Hence, there can be multiple equilibria with either many or few high wage employers, and hence either low or high crime rates. This mechanism could only operate in a model where the wage distribution is endogenous.

The rest of the paper is organized as follows. Section 2 lays out the worker's decision problem. Section 3 first analyzes firm behavior and then characterizes the equilibria of the model for the case when only the unemployed can receive job-offers. Section 4 generalizes Section 3 by allowing workers to search on the job. Section 5 concludes.

2. Workers

Time is discrete and unbounded. There is a $[0, 1]$ continuum of firms and a $[0, L]$ continuum of workers; thus, L is the ratio of workers to firms. All firms, and all workers, are risk neutral and ex ante identical. We begin by analyzing worker behavior, taking firm behavior as given. At any point in time a worker can be in one of three states: employed (at some wage w), unemployed, or in jail. Let the number of workers in each state be e, u, and n, and let the payoff (value function) in each state be $V(w)$, V_0, and J. While employed, each period an agent: receives another job offer with probability λ_1; encounters an opportunity to commit a crime with probability μ_1; falls victim to a crime with probability γ; and suffers an exogenous lay off with probability δ. The probability is zero that more than one of these events occurs in period. Job offers are randomly drawn from a distribution with cumulative distribution function $F(w)$, which for now we take as given. While unemployed, each period an agent: receives an offer from $F(w)$ with probability λ_0; encounters a crime opportunity with probability μ_0; and falls victim to a crime with probability γ.[4]

A crime opportunity is a chance to steal a fraction α of another agent's resources. The victim loses these resources, plus a lump sum d that does not accrue to the criminal, meant to capture emotional distress, medical bills, etc.[5] Potential criminals meet potential victims randomly, and the expected resources of potential victims are denoted $\widetilde{w}$. For now we take $\widetilde{w}$ as given, but later we set it equal to the economy-wide average wage. An agent committing a crime is caught with probability $\overline{\pi}$, while with probability $1 - \overline{\pi}$ he goes free (he cannot be caught at a later date). If caught, the criminal suffers disutility ψ (e.g., a fine) and faces a probability κ of being sent to

[4] Note that while we allow the probability of encountering a crime opportunity or of getting a job offer to differ for employed and unemployed workers, we set the probability of being victimized to be the same. This is possible because we assume that potential criminals cannot observe and therefore cannot condition their actions on potential victims' employment status (or wage).

[5] This way of modeling things may seem to capture property crime better than crimes of pure violence (such as murder, rape or aggravated assault), but it is easy to interpret variables to include any criminal activity that is goal directed and subject to cost benefit calculations by the perpetrator. Only 8.2% of the crimes tabulated by the FBI are crimes of pure violence (Federal Bureau of Investigation 1992).

jail. In jail, each period agents receive payoff z, and with probability ρ are released into the unemployment pool (for simplicity, ρ is time-invariant).

The flow Bellman equation for an agent in jail satisfies

$$rJ = z + \rho\left(V_0 - J\right). \tag{2.1}$$

To derive the other Bellman equations, let $\omega = \alpha\widetilde{w} - \overline{\pi}\psi$ be the expected return net of fine and $\pi = \overline{\pi}\kappa$ the probability of going to jail for a criminal. Then the payoffs employed and unemployed agents get from crime are $K(w) = \omega + \pi J + (1-\pi)V(w)$ and $K_0 = \omega + \pi J + (1-\pi)V_0$. Hence, the Bellman equation for an agent employed at wage w is

$$\begin{aligned} rV(w) &= w - \gamma(\alpha w + d) + \lambda_1 E_x \max\left\{V(x) - V(w), 0\right\} \\ &\quad + \mu_1 \max\left\{K(w) - V(w), 0\right\} + \delta\left[V_0 - V(w)\right]. \end{aligned} \tag{2.2}$$

In words, the flow return is the sum of: instantaneous income; the expected loss from being victimized; the expected value of an outside offer; the expected value of a crime opportunity; and the expected loss associated with a layoff. Similarly, for an unemployed worker,

$$\begin{aligned} rV_0 &= b - \gamma(\alpha b + d) + \lambda_0 E_x \max\left\{V(x) - V_0, 0\right\} \\ &\quad + \mu_0 \max\left\{K_0 - V_0, 0\right\}, \end{aligned} \tag{2.3}$$

where b is the instantaneous income of an unemployed worker.

A worker's utility-maximizing strategy is as follows: while employed, accept any offer paying more than the current wage; while unemployed, accept any job paying more than the *reservation wage* R, where $V(R) = V_0$; while employed, commit crimes when the current wage is less than the *crime wage* w^*, where $K(w^*) = V(w^*)$; and while unemployed, commit crimes iff $K_0 > V_0$. Notice that $K_0 - V_0 = K(R) - V(R)$; this implies that the unemployed engage in crime if and only if a worker employed at the reservation wage engages in crime. In what follows we will use ϕ_0 to denote the probability that an unemployed agent engages in crime; thus $\phi_0 = 1$ if $K_0 - V_0 > 0$, $\phi_0 = 0$ if $K_0 - V_0 < 0$, and $\phi_0 \in (0,1)$ only if $K_0 - V_0 = 0$. Also, $K(w^*) = V(w^*)$ can be rearranged to yield

$$V(w^*) - J = \frac{\omega}{\pi}. \tag{2.4}$$

To derive the reservation wage equation, set $w = R$ in (2.2) and equate it to (2.3) to get

$$(1-\alpha\gamma)(R-b) = (\lambda_0 - \lambda_1)\Delta(R) + (\mu_0 - \mu_1)\phi_0\left[\omega - \pi(V_0 - J)\right], \tag{2.5}$$

where[6]

$$\Delta(R) = \int_R^\infty V'(x)\left[1 - F(x)\right]dx.$$

[6] Using (2.2), we see that the slope of the value function of a worker employed at wage w is

$$V'(w) = \begin{cases} \frac{1-\alpha\gamma}{r+\delta+\mu_1\pi+\lambda_1[1-F(x)]} & \text{if the worker engages in crime} \\ \frac{1-\alpha\gamma}{r+\delta+\lambda_1[1-F(x)]} & \text{if the worker does not engage in crime.} \end{cases}$$

To derive the crime wage equation, manipulate (2.2) and (2.4) to arrive at

$$(1-\alpha\gamma)\,w^{*} = z + \gamma d + (r+\delta)\frac{\omega}{\pi} + (\rho-\delta)\,(V_0 - J) - \lambda_1\Delta(w^{*}). \tag{2.6}$$

To eliminate $(V_0 - J)$ from these equations, subtract (2.1) and (2.3), to obtain

$$(r + \rho + \mu_0\pi\phi_0)\,(V_0 - J) = (1-\alpha\gamma)\,b - \gamma d - z + \lambda_0\Delta(R) + \mu_0\phi_0\omega. \tag{2.7}$$

Inserting (2.7) in (2.5) and (2.6), we have two equations describing the reservation wage and crime wage (R, w^{*}) as a function of parameters (some of which will be endogenized below, including the wage distribution F, as well as γ and $\tilde{w}$).

We now describe the steady state proportions of agents in different states, given the wage distribution F. It is convenient for this purpose to define the conditional distributions of wages above and below the crime wage: $F_H(w) = F(w|w \geq w^{*})$ and $F_L(w) = F(w|w < w^{*})$. Let $\sigma = 1 - F(w^{*})$ denote the fraction of firms paying at least w^{*}, and therefore the fraction of firms where workers do not engage in crime. Also, let e_L and e_H be the numbers of agents employed in jobs paying a wage below and above w^{*}, where $e = e_L + e_H$.

The first thing to do is to derive the distribution of wages across workers, $G(w)$ (to be distinguished form the distribution of wages across firms, F). Given any $w \leq w^{*}$, the number of workers employed at a wage no greater than w at some date t is $G_L(w)\,e_L$, where $G_L(w) = G(w|w \leq w^{*})$ at t. This distribution evolves through time according to[7]

$$\begin{aligned}\frac{d}{dt}G_L(w)\,e_L &= \lambda_0(1-\sigma)\,F_L(w)\,u - \\ &\quad - \{\delta + \mu_1\pi + \lambda_1\sigma + \lambda_1(1-\sigma)\,[1 - F_L(w)]\}\,e_L G_L(w).\end{aligned} \tag{2.8}$$

Similarly, $G_H(w) = G(w|w \geq w^{*})$ evolves according to

$$\begin{aligned}\frac{d}{dt}G_H(w)\,e_H &= (\lambda_0 u + \lambda_1 e_L)\,\sigma F_H(w) - \\ &\quad - \{\delta + \lambda_1\sigma\,[1 - F_H(w)]\}\,e_H.G_H(w).\end{aligned} \tag{2.9}$$

In a steady state where G_H and G_L are constant over time, we have:

$$G_L(w) = \frac{\lambda_0(1-\sigma)\,F_L(w)\,u}{\{\delta + \mu_1\pi + \lambda_1\sigma + \lambda_1(1-\sigma)\,[1-F_L(w)]\}\,e_L} \tag{2.10}$$

and

$$G_H(w) = \frac{(\lambda_0 u + \lambda_1 e_L)\,\sigma F_H(w)}{\{\delta + \lambda_1\sigma\,[1 - F_H(w)]\}\,e_{H.}}. \tag{2.11}$$

The numbers of agents in the various states, (e_H, e_L, u, n), evolve according to:

$$\begin{aligned}\dot{e}_H &= \lambda_1\sigma e_L + \lambda_0\sigma u - \delta e_H \\ \dot{e}_L &= \lambda_0(1-\sigma)\,u - (\delta + \lambda_1\sigma + \mu_1\pi)\,e_L \\ \dot{u} &= \delta\,(e_H + e_L) + \rho n - (\lambda_0 + \mu_0\phi_0\pi)\,u \\ \dot{n} &= \mu_0\phi_0\pi u + \mu_1\pi e_L - \rho n,\end{aligned}$$

[7] We are working under the assumption that $w \geq R$. This will always be true in equilibrium: no firm will offer a wage below the reservation since if it did, it would be unable to attract workers.

where ϕ_0 denotes the fraction of unemployed workers who engage in crime. The unique steady state is $(e_H, e_L, u, n) = (s_1, s_2, s_3, s_4)/\sum_j s_j$, where

$$\begin{aligned} s_1 &\equiv (\delta + \lambda_1 + \mu_1 \pi)\,\sigma\rho\lambda_0 \\ s_2 &\equiv (1-\sigma)\,\rho\delta\lambda_0 \\ s_3 &\equiv (\delta + \lambda_1\sigma + \mu_1\pi)\,\rho\delta \\ s_4 &\equiv [\mu_0\phi_0\,(\delta + \mu_1\pi + \lambda_1\sigma) + \lambda_0\mu_1\,(1-\sigma)]\,\pi\delta. \end{aligned}$$

Finally, one can endogenize the crime rate γ. Since each crime involves one victim and one criminal, we have $(1-n)\,\gamma = e_L\mu_1 + u\mu_0\phi_0$. Hence, in steady state the probability of being victimized is

$$\gamma = \frac{e_L\mu_1 + u\mu_0\phi_0}{1-n}. \tag{2.12}$$

The above model is a natural generalization of the standard decision-theoretic job search model to include crime. It can be used to generate predictions about the effects of parameters, including policy variables such as π, ρ, z and b, on the reservation and crime wages, and hence on the unemployment and crime rates, taking the wage distribution F and other variables as fixed.[8] While this is interesting in its own right, we proceed here to discuss wage determination.

3. Model A: no on-the-job search

In this section, we present a relatively simple version of the model by shutting down on-the-job search; that is we set $\lambda_1 = 0$, and $\lambda_0 = \lambda > 0$. To reduce notation we also set $\mu_0 = \mu_1 = \mu$.

3.1. Firms

Firms have access to a linear production technology with marginal product $p > b$. We assume firms can post and commit to a wage. Without on-the-job search, we can simplify notation significantly by letting w_1 and w_2 denote the reservation and crime wage respectively. A firm paying w_2 hires workers from the unemployment pool at rate λ and loses them at rate δ. If $w_2 > w_1$, then a firm paying w_1 hires at the same rate but sheds workers at rate $\delta + \mu\pi$. This is because workers employed at a wage below the crime wage engage in crime and get convicted to jail with positive probability.[9] Hence in steady-state $l_2 = \lambda u L/\delta$ workers are employed in a firm paying

[8] As a special case, if $\lambda_0 = \lambda_1$ and $\mu_0 = \mu_1$, (2.5) implies that $R = b$. Thus, if job and criminal opportunities arrive at the same rate whether employed or unemployed, then agents accept any offer above b. This case is nice, since we can focus on the effects of changes in the environment on w^*, without worrying about how these changes affect R (something that has been studied extensively in the past).

[9] We assume the tie-breaking rules go in the "right way", namely that a worker accepts a job offer that makes him indifferent between employment and unemployment and rejects a crime opportunity with non-positive expected return. Also, as we show below, $w_2 > w_1$ must be the case in any equilibrium in which more than one wage is offered.

w_2, while the labor force at a firm paying w_1 is $l_1 = \lambda uL / (\delta + \mu\pi)$. Thus with $r \approx 0$, the steady-state profit of a firm paying wage w_i is $\Pi_i = (p - w_i) l_i$. It is easy to show that in this case (i.e. with no on-the-job search) there can never be more than two wages paid in equilibrium: firms either pay the reservation wage or the crime wage.[10] Since all firms are ex-ante identical, $\Pi_1 = \Pi_2$ must hold in an equilibrium with $0 < \sigma < 1$. This equal-profit condition can be explicitly written as[11]

$$(p - w_2) \frac{\delta + \mu\pi}{\delta} = p - w_1.$$

Firms may be indifferent between paying a high and a low wage because paying a high wage commands a larger labor force in the steady state. This is captured by the factor $(\delta + \mu\pi) / \delta$, the size of a firm offering w_2 relative to a firm offering w_1. Intuitively, by paying w_2 a firm keeps its workers away from crime and enjoys lower turnover than a firm paying w_1,whose workers engage in crime and hence are forced to quit their job to go to jail with positive probability. It is convenient to define the map[12]

$$T(\sigma) \equiv \mu\pi p - [(\delta + \mu\pi) w_2(\sigma) - \delta w_1(\sigma)]. \tag{3.1}$$

The first term is essentially the difference between the total revenue of a high and a low-wage firm. It is positive because high-wage firms are larger in the steady state, and the marginal product of labor, p is constant. The second term is basically the difference between the total wage bill of a high and a low-wage firm. In terms of this notation, equal profit obtains when $T(\sigma) = 0$.

Let V_1, and V_2 denote the value of being employed at a (low) wage w_1 and a (high) wage w_2 respectively. With this notation, the worker's flow Bellman equations are

$$rV_0 = (1 - \alpha\gamma) b - \gamma d + \lambda\sigma (V_2 - V_0) + \mu\phi_0 [\omega - \pi (V_0 - J)] \tag{3.2}$$

$$rV_i = (1 - \alpha\gamma) w_i - \gamma d - \delta (V_i - V_0) + \mu\phi_i [\omega - \pi (V_i - J)], \tag{3.3}$$

where as before, $\phi_i \in [0, 1]$ is the criminal decision of an agent in state $i = 0, 1, 2$.[13]

In a wage-posting equilibrium with $\sigma < 1$, a firm offering w_1 will set it so that

$$V_0 = V_1. \tag{3.4}$$

Similarly, if w_2 is offered in equilibrium, then it will be set such that

$$V_2 = \max \{J + \omega/\pi, V_0\}. \tag{3.5}$$

[10] The argument runs as follows. Firms paying less than w_1 would attract no workers, so no firm would offer a wage below the reservation. A firm paying $w' > w_2$ would make smaller profit per worker and have exactly the same steady-state labor force as a firm paying w_2; hence no firm will offer a wage above the crime wage. Similarly, no firm would pay $w'' \in (w_1, w_2)$ since that would imply smaller profit per-worker and the same steady-state labor force as a firm paying w_1.

[11] Another way to obtain this steady-state equal-profit condition is to assume firms have only one vacancy, and to write the value functions of a vacany and of a filled job offering w_i. Then a condition like the one in the text can be obtained by setting the value of a vacancy offering w_1 equal to the value of a vacancy offering w_2. Having $r \approx 0$ just simplifies the algebra.

[12] Here we make explicit the equilibrium relationship between w_i and σ. The precise nature of this relationship is explained below.

[13] We have omitted the term $\lambda (1 - \sigma) (V_1 - V_0)$ from (3.2) in anticipation of the fact that $V_1 = V_0$ must hold in equilibrium.

That is, firms set w_1 so that it makes workers just indifferent between accepting and rejecting the job. Similarly, w_2 is set exactly so that the worker is just indifferent between taking and foregoing a crime opportunity.[14]

3.2. Equilibrium

An equilibrium is a vector of worker criminal decisions $\Phi = (\phi_0, \phi_1, \phi_2)$ and a wage distribution $W = (w_1, w_2, \sigma)$ such that given W, Φ satisfies

$$\phi_i = \begin{cases} 0 & \text{if } V_i - J > \omega/\pi \\ \widetilde{\phi}_i \in [0,1] & \text{if } V_i - J = \omega/\pi \\ 1 & \text{if } V_i - J < \omega/\pi \end{cases}$$

for $i = 0, 1, 2$; and given Φ, W satisfies, (3.4), (3.5), together with $T(0) < 0$ if $\sigma = 0$, $T(1) > 0$ if $\sigma = 1$, or $T(\sigma) = 0$ if $0 \leq \sigma \leq 1$. Notice that (3.4) and (3.5) imply that in equilibrium, a worker employed at w_1 will set $\phi_1 = 1$, while one employed at w_2 will choose $\phi_2 = 0$.[15]

In principle, there could exist four qualitatively different types of equilibria. The first possibility is that neither the employed nor the unemployed agents engage in the criminal activity. The second case has $\sigma = 1$ and $\phi_0 = 1$, which means that all firms pay high enough wages so that no employed workers engage in crime, but every unemployed agent does. The third case $\sigma = 0$, which means that all agents engage in crime regardless of their employment state. The fourth case, has $0 < \sigma < 1$ and $\phi_0 = 1$, which means that there are positive masses of two types of employed workers: those employed at a high wage (w_2) who do not commit crimes, and those employed at a low wage (w_1) who take advantage of every crime opportunity (as do all unemployed agents). Hereafter, we refer to these equilibrium configurations as types I-IV respectively.

To better discriminate the various forces at work in equilibrium, we begin by characterizing "partial" equilibria, namely those obtained while keeping the victimization rate γ and the amount stolen $\alpha\widetilde{w}$ fixed. In turn, we will first incorporate the fact that in a closed system the probability an agent falls victim of a crime is a function of the number of criminals, and finally we will endogenize the instantaneous return to a successful crime for a characterization of the full equilibrium. In any equilibrium, we will require that $V_0 \geq J$. This imposes the common-sense requirement that the value of being in jail be no greater than the value of unemployment and holds if and only if

$$(1 - \alpha\gamma) b \geq z + \gamma d - (\mu\pi + \lambda\sigma)(\omega/\pi). \tag{3.6}$$

[14] The "max" operator in (3.5) is needed to ensure that $V_2 \geq V_0$; namely that employment at w_2 is no worse than unemployment.

[15] And if we restrict ourselves to pure strategies, then $\phi_0 = \phi_1$ must also be the case.

3.3. Equilibrium analysis I (fixed γ and $\widetilde{w}$)

Standard manipulations of (3.2) and (3.3) together with equilibrium conditions (3.4) and (3.5) imply that for a given σ (and $r \approx 0$), the equilibrium wages are

$$w_1(\sigma) = \frac{1}{\rho + \mu\pi + \lambda\sigma}\left[(\rho + \mu\pi)\, b + \lambda\sigma\overline{b}\right] \tag{3.7}$$

$$w_2(\sigma) = \frac{1}{\rho + \mu\pi + \lambda\sigma}\left[(\rho - \delta)\, b + (\delta + \mu\pi + \lambda\sigma)\,\overline{b}\right], \tag{3.8}$$

where

$$\overline{b} \equiv \frac{z + \gamma d + \rho\,(\omega/\pi)}{1 - \alpha\gamma}.$$

For stating our existence results it is convenient to define:

$$p_2(b) \equiv \left\{\left[(\delta + \mu\pi)^2 + \mu\pi\lambda\right]\overline{b} + \mu\pi\,(\rho - \rho^*)\, b\right\}\frac{1}{(\rho + \mu\pi + \lambda)\,\mu\pi} \tag{3.9}$$

$$p_3(b) \equiv \left\{(\delta + \mu\pi)^2\,\overline{b} + \mu\pi\,(\rho - \rho^*)\, b\right\}\frac{1}{(\rho + \mu\pi)\,\mu\pi}, \tag{3.10}$$

where $\rho^* \equiv \delta\,(1 + (\delta + \mu\pi)/\mu\pi)$.

Proposition 1. Let γ and $\widetilde{w}$ be given. An equilibrium always exists. First suppose $b < \overline{b}$. If $\rho \geq \rho^*$ then the equilibrium is unique and:

(a). if $p \geq p_2(b)$, then $\sigma = 1$ and only the unemployed engage in crime (type II).

(b). if $p \leq p_3(b)$, then $\sigma = 0$, and all agents engage in crime (type III).

(c). if $p_2(b) < p < p_3(b)$, then $\sigma \in (0,1)$, with

$$\sigma = \frac{(\rho^* + \mu\pi)\,\overline{b} - (\rho + \mu\pi)\, p + (\rho - \rho^*)\, b}{(p - \overline{b})\,\lambda}. \tag{3.11}$$

and only those agents unemployed and employed at $w_1(\sigma)$ engage in crime (type IV).

Alternatively, let $\rho < \rho^*$:

(d). if $p \geq p_3(b)$, then the equilibrium is unique, and it has $\sigma = 1$ (type II).

(e). if $p \leq p_2(b)$, then the equilibrium is unique and it has $\sigma = 0$ (type III).

(f). if $p_2(b) < p < p_3(b)$, then there exist three equilibria: one with $\sigma = 0$ and all agents engaging in crime (type III), another with $\sigma = 1$ and only the unemployed engaging in crime (type II), and one with $\sigma \in (0,1)$ as given by (3.11) in which the unemployed and those employed at $w_1(\sigma)$ engage in crime (type IV).

Finally, if $b \geq \overline{b}$, then there exists a unique equilibrium: nobody engages in crime and $w_1 = w_2 = b$ (type I).

Proof of Proposition 1.

Plugging (3.7) and (3.8) in (3.1) yields

$$T(\sigma) = \mu\pi p - \left\{ \left[(\delta + \mu\pi)^2 + \mu\pi\lambda\sigma \right] \overline{b} + \mu\pi (\rho - \rho^*) b \right\} (\rho + \mu\pi + \lambda\sigma)^{-1}.$$

First assume a parametrization such that $b < \overline{b}$. Notice that $V_0 - J < \omega/\pi$ if and only if $b < \overline{b}$, hence the unemployed and those employed at w_1 always engage in crime in this case. It is easy to see that

$$\frac{\partial T(\sigma)}{\partial \sigma} = \frac{\lambda\mu\pi \left(\overline{b} - b\right)(\rho^* - \rho)}{(\rho + \mu\pi + \lambda\sigma)^2}$$

is non-positive if and only if $\rho \geq \rho^*$. Hence under this restriction the $T(\cdot)$ map is downward-sloping and the equilibrium is unique. It is straightforward to show that the conditions in parts (a), (b) and (c) are equivalent to $T(1) \geq 0$, $T(0) \leq 0$ and $T(1) < 0 < T(0)$ respectively. The expression for σ in (3.11) satisfies $T(\sigma) = 0$. Conversely, if $\rho < \rho^*$, then the $T(\cdot)$ map is upward-sloping. Conditions (d), (e) and (f) are equivalent to $T(0) \geq 0$, $T(1) \leq 0$, and $T(0) < 0 < T(1)$ respectively. Finally, if parameters are such that $b \geq \overline{b}$ then $V_0 - J \geq \omega/\pi$ and (3.4) and (3.5) imply that $V_0 = V_1 = V_2$ meaning that $b = w_1 = w_2$. ■

The case in which $\rho \geq \rho^*$ is illustrated in Figure 1. From (3.9) and (3.10), it is clear that in this case $p_2(b)$ has a higher intercept (i.e. $p_2(0) > p_3(0)$) and is always flatter than $p_3(b)$. Also, it is easy to verify that $p_2\left(\overline{b}\right) = p_3\left(\overline{b}\right) = \overline{b}$, namely both boundaries intersect when they cross the 45-degree line, at $b = \overline{b}$. The labor market is inactive below the 45-degree line. The case in which $\rho < \rho^*$ is illustrated in Figure 2. In this case, $p_2(0) < p_3(0)$, both boundaries slope down, and it is still the case that $p_2\left(\overline{b}\right) = p_3\left(\overline{b}\right) = \overline{b}$ and that $p_3(b)$ is steeper than $p_2(b)$.

Parts (a)-(c) in Proposition 1 highlight how the type of equilibrium that obtains depends on the productivity parameter p. Recall that the equilibrium map, T, is just the difference between the profit of a high-wage and a low-wage firm. Notice that increases in the productivity parameter p shift the T map up. The reason for this is that due to larger work-force, total revenue rises faster for high than for low-wage firms with increases in p.[16] Hence for p large enough no firm will find it (relatively) profitable to employ workers at the low wage. Conversely, for p low enough no firm will offer the high wage. High and low-wage firms coexist for intermediate productivity levels.

When the release rate ρ is high enough, the equilibrium map, T, is downward-sloping with respect to the fraction of high-wage firms (σ). In other words, the relative profit of a high-wage firm is decreasing in the number of high-wage firms and hence the equilibrium is unique. To see why the equilibrium map slopes down, notice that as

[16] The fact that the total wage bill remains unchanged for both types of firms as productivity increases (since wages do not depend p), is also key here.

σ rises, so does the value of unemployment V_0. (The value of unemployment naturally increases with the fraction of high-wage jobs.) Since (for fixed w_1) an increase in σ increases V_0 relative to V_1 (refer to (3.2) and (3.3)), low wage firms must increase w_1 to keep their jobs acceptable. (That V_0 tends to rise relative to V_1 reflects the fact that σ has a first-order effect on the unemployed since they are searching for offers while those employed are not.) As can be seen in (2.1) and (3.3), a higher value of unemployment means both a higher value of going to jail (J) as well as a higher value of being employed at the high wage (V_2). The increase in V_2 induced by an increase in V_0 is larger than the resulting increase in J when $\delta > \rho$, namely if a worker transits to unemployment from a high-wage job with higher probability than from jail. (This also explains -for instance- why $w_2(\sigma)$ is decreasing in b for $\delta > \rho$.) So $\partial w_1(\sigma)/\partial\sigma > 0$ while $\partial w_2(\sigma)/\partial\sigma < 0$ unless $\rho > \delta$. Hence for the relative profit of a high-wage firm to be decreasing in σ it is necessary that $\partial w_2(\sigma)/\partial\sigma$ be positive and sufficiently large relative to $\partial w_1(\sigma)/\partial\sigma$.[17] In other words, it is necessary that δ be sufficiently smaller than ρ. The increase in the wage bill (resp. the decline in the profit) of a high-wage firm induced by an increase in the fraction of high-wage firms is larger than that of a low-wage firm when $\rho > \rho^*$.

If the converse holds, then it becomes easier to provide incentives for workers to stay honest as the fraction of high-wage firms rises. In this case the relative profit of a high-wage firm is increasing in the number of high-wage firms and multiple equilibria arise for intermediate values of the productivity parameter p. Within this range, for the same parameter values, equilibria with high and low crime rates coexist with a third in which the crime rate is intermediate. No firms are paying high wages in the high-crime (type III) equilibrium. This makes the value of search low, which in turn (because ρ is small relative to δ) tends to make the gap between V_2 and J very narrow, requiring firms to pay a very high w_2 to keep workers honest. This value of w_2 is so high that it makes operating at the high wage unprofitable relative to operating at the low wage. Conversely, all firms are paying high wages in the low crime (type II) equilibrium, which amounts for a large V_0, which in turn (again because ρ is small relative to δ) tends to make the gap between V_2 and J large, requiring a small w_2 to keep workers honest resulting in all firms choosing to operate at w_2.

It is interesting to note that if $b \geq \bar{b}$, then the unique equilibrium has $w_1 = w_2 = b$ and nobody engaging in crime. In this case the flow payoff to being unemployed is so large, that not even a worker who is unemployed forever is willing to risk his state to engage in criminal activities. Aware of this, no firm has an incentive to pay a wage above the minimum acceptable level, b.

3.4. Equilibrium analysis II (endogenous γ, fixed $\widetilde{w}$)

In this section we take another step toward building the full equilibrium by endogenizing the victimization rate γ. By combining (2.12) with the steady-state conditions,

[17] To clarify this, consider an extreme case. Let $\rho \to \infty$. Then $J \to V_0$ and hence high-wage firms must set w_2 so as to keep $V_2 - V_0$ constant. In this case it is easy to verify that $\partial w_1(\sigma)/\partial\sigma = \partial w_2(\sigma)/\partial\sigma > 0$. Clearly, in this case the relative profit of a high-wage firm decreases in σ since w_1 and w_2 rise at the same rate but the high-wage firm must pay the increase to a larger work-force.

we obtain an expression for the crime rate as a function of σ:

$$\gamma(\sigma) = \frac{[\delta + \mu\pi + (1-\sigma)\lambda]\,\delta}{(1-\sigma)\,\delta\lambda + (\delta + \mu\pi)(\delta + \lambda\sigma)}\mu. \tag{3.12}$$

Before stating our existence results it is convenient to define $b_1 = z + \rho\omega/\pi$, as well as

$$P_2(b) \equiv \left\{\left[(\delta + \mu\pi)^2 + \mu\pi\lambda\right]\widehat{b} - \mu\pi(\rho^* - \rho)\,b\right\}\frac{1}{(\rho + \mu\pi + \lambda)\,\mu\pi} \tag{3.13}$$

$$P_3(b) \equiv \left\{(\delta + \mu\pi)^2 b_0 - \mu\pi(\rho^* - \rho)\,b\right\}\frac{1}{(\rho + \mu\pi)\,\mu\pi}, \tag{3.14}$$

where $\widehat{b} = (z + \gamma(1)\,d + \rho\omega/\pi)\,/\,(1 - \alpha\gamma(1))$ and $b_0 = (z + \mu d + \rho\omega/\pi)\,/\,(1 - \alpha\mu)$. Notice that $b_1 < \widehat{b} < b_0$. Even when the rate at which criminals contact victims is endogenous, we require that workers are better off participating of the labor market than in jail. A sufficient condition is that

$$b \geq \frac{z + \mu d - (\mu\pi)(\omega/\pi)}{1 - \alpha\mu} = b_0 - \frac{\rho + \mu\pi}{1 - \alpha\mu}(\omega/\pi),$$

which ensures that agents are better off unemployed than in jail even when all non-institutionalized agents are engaging in crime and all firms offer a wage of b. We know from Proposition 1, that a small enough release rate can be a source of multiplicity. By setting $\rho = \rho^*$, the following proposition shuts down that channel and establishes that an endogenous victimization rate can, on its own, generate multiple equilibria.

Proposition 2. Let $\widetilde{w}$ be given and suppose $\rho \leq \rho^*$. First let $b < b_1$:

(a). if $p \geq p_3(b)$, then the equilibrium is unique, and it has $\sigma = 1$ (type II).

(b). if $p \leq p_2(b)$, then the equilibrium is unique and it has $\sigma = 0$ (type III).

(c). if $p_2(b) < p < p_3(b)$, then there exist three equilibria: one with $\sigma = 0$ and all agents engaging in crime (type III), another with $\sigma = 1$ and only the unemployed engaging in crime (type II), and one with $\sigma \in (0,1)$ as given by (3.11) in which the unemployed and those employed at $w_1(\sigma)$ engage in crime (type IV).

Alternatively, assuming that $b > \widehat{b}$:

(d). if $p \geq p_3(b)$, then the equilibrium is unique and of type I.

(e). if $p \in (p_2(b), p_3(b))$, then an equilibrium of type I, coexists with one of type III and IV.

Finally, suppose $b_1 \leq b \leq \widehat{b}$, then:

(f). if $p \geq p_3(b)$, then an equilibrium of type I coexists with one of type II

(g). if $p \leq p_2(b)$, then an equilibrium of type I coexists with one of type III.

(h). if $p_2(b) < p < p_3(b)$, then an equilibrium of type I coexists with one of type II, III and IV.

Proof of Proposition 2.

Let $\Upsilon(\sigma)$ be defined as $T(\sigma)$ in Proposition 1, but with γ given by (??). It follows that

$$\frac{\partial\Upsilon(\sigma)}{\partial\sigma} = \frac{\partial T(\sigma)}{\partial\sigma} - \frac{(\delta+\mu\pi)^2+\mu\pi\lambda\sigma}{(\rho+\mu\pi+\lambda\sigma)(1-\alpha\gamma)}\left(d+\alpha\bar{b}\right)\frac{\partial\gamma(\sigma)}{\partial\sigma},$$

and hence $\rho < \rho^*$ implies $\partial\Upsilon(\sigma)/\partial\sigma > 0$. Conditions (a), (b) and (c) are equivalent to $\Upsilon(0) \geq 0$, $\Upsilon(1) \leq 0$ and $\Upsilon(0) < 0 < \Upsilon(1)$ respectively. For part (d) notice that $b > \widehat{b}$ implies $V_0 - J > \omega/\pi$ even if all unemployed agents engage in crime, hence a type II equilibrium cannot exist. Also, a type I equilibrium exists whenever $b \geq b_1$ (because this condition implies setting $\phi_0 = 0$ is a best response when everybody else is being honest). Part (e) follows from (c) and (d). Similarly, parts (f)-(g) follow from putting together (a)-(d). ■

A conclusion to be drawn from Proposition 2 is that an endogenous victimization rate is yet another (potential) source of multiplicity of equilibria. Its effect works through two distinct channels. The first one hinges on the effect of crime on the wage distribution while the second works exclusively through the impact of the crime rate on the agents' incentives to engage in crime. Starting with the former, first notice that with an endogenous γ the slope of the equilibrium map is always larger than when γ is kept fixed (i.e. $\partial\Upsilon(\sigma)/\partial\sigma > \partial T(\sigma)/\partial\sigma$). To fix ideas we suppose $\rho = \rho^*$ because the T-map is flat in this case: as σ rises both wages rise, but they rise in a way that makes the relative profit of a high-wage firm remain constant. We have shut off the source of multiplicity discussed in the previous section. Nonetheless, notice that Υ is still strictly increasing. The reason is that the reduction in γ induced by the increase in the fraction of high-wage firms, σ, allows all firms to offer lower wages, and in particular, it allows high-wage firms to reduce w_2 relative to w_1. (To verify this, notice that $\partial w_2/\partial\gamma > \partial w_1/\partial\gamma > 0$.) Intuitively, the crime rate γ acts like a proportional tax on income: increases in σ induce a reduction in the "marginal tax rate" γ. All agents benefit from a reduction in γ but workers employed at high wages profit relatively more than low-wage earners. Specifically, a reduction in γ will tend to increase the gap between V_1 and V_0 (which in equilibrium causes the reservation wage w_1 to fall). The analogous increase in $V_2 - J$ is even larger, and hence high-wage firms are able to reduce w_2 by more than the drop in the reservation wage. This accounts for the relative increase in the high-wage firms' profit. Since γ is a convex function of σ, this effect is likely to be relatively strong for small σ.

The strategic complementarity between the agents' crime decisions is yet another source of multiplicity. The best way to illustrate it is perhaps by focusing on the parameter range where an equilibrium of type III (everybody engages in crime and all firms pay b) coexists with an equilibrium of type I (nobody engages in crime and all firms pay b). The unique equilibrium wage is b in the former because the high crime rate resulting from every worker taking advantage of every crime opportunity tends to make $V_2 - J$ relatively small. This means that the (off the equilibrium path) w_2 required for firms to be able to induce their workforce to stay honest is so high that offering it is not profitable relative to offering the low wage. No firm has incentive to pay more than b in the type I equilibrium because all workers are foregoing every crime opportunity already. Given that the same wage b is offered in both equilibria (albeit for different reasons), why do workers all choose to stay honest in one and be criminals in the other? The answer resides in the fact that criminals impose negative externalities on others: when there are many criminals, each agent in the labor force gets victimized more often, and this tends to narrow the gap between V_0 and J. In words, the value of being in the labor force is small relative to the value of being convicted to jail when the crime rate is high, and this in turn rationalizes engaging criminal activities (by reducing the expected cost relative to the –for now– constant return).

4. Model B: on-the-job search

In this section, we let $\lambda_1 > 0$, and hence we allow for on-the-job search. The model generalizes Burdett and Mortensen (1998).

4.1. Firms

In this section we begin by describing the problem of an individual firm taking as given behavior of workers (as described by R and w^*) and the behavior of other firms (as described by F). For simplicity, we focus on the case where $r \to 0$. Let p denote the per-period revenue generated from employing any worker. Hence a firm that pays wage w earns steady-state profit

$$\Pi\left(w|w^*, R, F\right) = (p - w)\, l\left(w|w^*, R, F\right)$$

where $l\left(w|w^*, R, F\right)$ denotes the steady state measure of workers employed at the firm given it pays w. Again, it will be convenient to distinguish between the number of workers employed by firms paying wages above and below w^* by $l_L\left(w|w^*, R, F\right)$ and $l_H\left(w|w^*, R, F\right)$. Standard arguments yield[18]

$$\begin{aligned} l_L\left(w|w^*, R, F\right) &= \frac{G_L'(w)}{(1-\sigma)\, F_L'(w)} e_L \\ l_H\left(w|w^*, R, F\right) &= \frac{G_H'(w)}{\sigma F_H'(w)} e_H. \end{aligned}$$

Using (2.10) and (2.11), we have

$$l_L\left(w|w^*, R, F\right) = \frac{\lambda_0\left(\delta + \lambda_1 + \mu_1 \pi\right) u}{\left\{\delta + \mu_1\pi + \lambda_1\sigma + \lambda_1 (1-\sigma)\left[1 - F_L(w)\right]\right\}^2} \tag{4.1}$$

$$l_H\left(w|w^*, R, F\right) = \frac{\left(\delta + \lambda_1\sigma\right)\left(\lambda_0 u + \lambda_1 e_L\right)}{\left\{\delta + \lambda_1\sigma\left[1 - F_H(w)\right]\right\}^2} \tag{4.2}$$

[18] This assumes F_i and G_i, $i = H, L$ are differentiable. We will show below that this is indeed the case. Intuitively, the measure of workers employed at a firm paying w equals the number of workers earning w divided by the number of firms paying w. See Burdett and Mortensen (1998) for a more rigorous argument.

Taking as given w^*, R, and F, each employer posts a wage that maximizes its steady state profit. Given worker behavior, an equilibrium in the wage-posting game is a wage distribution F such that

$$\begin{aligned} \Pi(w|w^*, R, F) &= \Pi^* \quad \text{if } w \in \text{supp}F \\ \Pi(w|w^*, R, F) &\leq \Pi^* \quad \text{if } w \notin \text{supp}F \end{aligned}$$

where $\Pi^* = \max_w \Pi(w|w^*, R, F)$. In words, all wages on the support of F yield equal profit Π^*, while wages off the support yield profit no greater than Π^*.

The first thing to observe is that F cannot have a mass points (i.e., there cannot be a strictly positive measure of firms paying any wage w). To see this, suppose there was a mass point at $w' < p$. Then any firm paying w' could earn strictly greater profit by paying $w' + \varepsilon$ for some $\varepsilon > 0$, since this would imply a discrete increase in the number of workers it employs. It implies a discrete increase because now the firm can hire workers currently earning w', and it meets workers earning w' with positive probability, given the mass point. Hence, there can be no mass point at $w' < p$. It is not hard to see that no firm pays $w \geq p$.[19] Hence, there can be no mass point at any w.

Consider any equilibrium in the wage posting game with $0 < \sigma < 1$ (i.e., with some firms paying above and some firms paying below w^*). It is immediate that the lowest wage paid by any firm above w^* is exactly w^*.[20] Let $\overline{w}$ be the upper bound of the support of F_H, and let $\underline{w}$ and $\widehat{w}$ denote the upper and lower bounds of F_L. Since all firms paying wages below w^* must earn the same profit, we have

$$(p - \underline{w})\, l_L(\underline{w}) = (p - w)\, l_L(w) \ \text{ for all } w \leq w^*. \tag{4.3}$$

Similarly, firms must be indifferent to offering any wage at least as high as w^*, namely

$$(p - w^*)\, l_H(w^*) = (p - w)\, l_H(w) \ \text{ for all } w^* \leq w. \tag{4.4}$$

Substituting (4.1) in (4.3) we see that the conditional wage offer distribution below w^* consistent with equal profit is

$$F_L(w) = \frac{\delta + \lambda_1 + \mu_1 \pi}{\lambda_1 (1 - \sigma)} \left(1 - \sqrt{\frac{p - w}{p - \underline{w}}}\right). \tag{4.5}$$

Similarly, for firms paying above w^* (4.2) and (4.4) imply

$$F_H(w) = \frac{\delta + \lambda_1 \sigma}{\lambda_1 \sigma} \left(1 - \sqrt{\frac{p - w}{p - w^*}}\right). \tag{4.6}$$

The next result is that there can be no gaps on the support of F, except possibly at w^*. To see this, suppose there is an non-empty interval $[w', w'']$, with $w^* \notin [w', w'']$,

[19] A firm paying $w' \geq p$ makes nonpositive profit; but under the maintained assumption $R < p$, it could pay $w \in (R, p)$ and make strictly positive profit, since it would still attract some workers.

[20] Suppose $w' > w^*$ is the lowest wage above w^*. Given w' cannot be a mass point, the firm paying w' can strictly increase its profit by paying w^*, because in doing so it does not lose workers any faster.

with some firm paying w'' and no firm paying $w \in [w', w'']$. Then the firm paying w'' can make strictly greater profit by paying $w'' - \varepsilon$ for some $\varepsilon > 0$. This is because such a firm loses no more workers to firms offering higher wages than it did before. However, there must be a gap between $\widehat{w}$ (the upper bound of F_L) and w^*. To see this, note that a firm paying $w^* - \varepsilon$ loses workers at discretely greater rate than a firm paying exactly w^* because workers paid less than w^* engage in crime and go to jail with positive probability.

To summarize, an equilibrium in the wage-posting game with $0 < \sigma < 1$ necessarily entails a connected support between $\underline{w}$ and $\widehat{w}$, a gap between $\widehat{w}$ and w^*, and a connected support between w^* and $\overline{w}$. To conclude the characterization of the wage distributions taking worker behavior as given, it only remains to characterize the endpoints. First, $\underline{w} \geq R$ since a firm paying $w < R$ would attract no workers. Moreover, in the case $0 < \sigma < 1$, we have $\underline{w} = R$ since if $\underline{w} > R$ the firm paying $\underline{w}$ can increase its profit by paying R. To find the upper bounds, we use (4.5) and (4.6) to solve $F_L(\widehat{w}) = F_H(\overline{w}) = 1$:

$$\widehat{w} = p - (p - R)\left(\frac{\delta + \mu_1\pi + \lambda_1\sigma}{\delta + \mu_1\pi + \lambda_1}\right)^2 \quad (4.7)$$

$$\overline{w} = p - (p - w^*)\left(\frac{\delta}{\delta + \lambda_1\sigma}\right)^2. \quad (4.8)$$

At this point we know everything about the two conditional distributions F_L and F_H as functions of (R, w^*) and σ. To determine σ, we use the condition that profits must be equal for firms paying wages above and below w^*:

$$\Pi(R|w^*, R, F) = \Pi(w^*|w^*, R, F).$$

Using (4.1) and (4.2) we can rewrite this condition as

$$\frac{p - R}{\delta + \mu_1\pi + \lambda_1} = \frac{(\delta + \mu_1\pi + \lambda_1)(p - w^*)}{(\delta + \sigma\lambda_1)(\delta + \mu_1\pi + \sigma\lambda_1)}. \quad (4.9)$$

This is a quadratic equation in σ with at most one positive root.[21]

This completes the description of the wage distribution taking as given worker behavior, (R, w^*), for the case $0 < \sigma < 1$. It is also possible to have $\sigma = 0$, which means that all workers are criminals, or $\sigma = 1$, which means no employed workers engage in crime. The wage distributions for these cases are found by setting $\sigma = 0$ in (4.5) and (4.7) or $\sigma = 1$ in (4.6) and (4.8).

[21] This unique (potentially) positive root is

$$\frac{-(2\delta + \mu_1\pi) + \left\{(2\delta + \mu_1\pi)^2 - 4\left[\delta(\delta + \mu_1\pi) - (\delta + \mu_1\pi + \lambda_1)^2 \frac{p - w^*}{p - R}\right]\right\}^{1/2}}{2\lambda_1}.$$

4.2. Equilibrium

An equilibrium is a vector (R, w^*, F) such that: (R, w^*) solve (2.5) and (2.6) given F; F satisfies (4.5), (4.6), (4.7), (4.8) and (4.9) given (R, w^*).[22] In principle, there could exist four qualitatively different types of equilibria. The first case has $\sigma = 1$ and $w^* \leq R = \underline{w} < \overline{w}$, which means that all firms pay high enough wages so that no employed workers engage in crime, and moreover no unemployed agents do either; this is essentially Burdett and Mortensen (1998). The second case has $\sigma = 1$ and $R < w^* = \underline{w} < \overline{w}$, which means that all firms pay high enough wages so that no employed workers engage in crime, but unemployed agents do engage in crime. The third case has $\sigma = 0$ and $R = \underline{w} < \overline{w} < w^*$, which means that all employed and unemployed agents engage in criminal activity. The final case, and the one in which we are most interested, has $0 < \sigma < 1$ and $R = \underline{w} < w^* < \overline{w}$, which means that there are positive masses of two types of employed workers: those with $w \geq w^*$ who do not commit crimes, and those with $w < w^*$ who do (as do all unemployed agents).

We now proceed to characterize the different types of equilibria. In what follows, we specialize the analysis to the case in which employed and unemployed workers face the same criminal and job opportunities (i.e. $\mu_0 = \mu_1$ and $\lambda_0 = \lambda_1$). As noted earlier, this implies unemployed workers accept any job paying at least b (see (2.5)) and allows us to focus on the criminal involvement decision exclusively.

Let $\eta_1 = (\pi/\rho)\,\Omega$, where

$$\Omega = \left(\frac{\lambda}{\delta+\lambda}\right)^2 p + \left[1 - \left(\frac{\lambda}{\delta+\lambda}\right)^2\right] b.$$

An equilibrium of type I –namely one in which nobody engages in crime– exists if $v < \eta_1$. The corresponding allocations are

$$\begin{aligned} \sigma &= 1,\ \phi_0 = \gamma = 0 \\ w^* &< \underline{w} = b < \overline{w} \\ \overline{w} &= p - (p-b)\left(\frac{\delta}{\delta+\lambda}\right)^2 \text{ and} \\ F(w) &= \frac{\delta+\lambda}{\lambda}\left[1 - \left(\frac{p-w}{p-b}\right)\right]^{1/2}. \end{aligned}$$

Hence if the flow return to crime is "very low", then there is an equilibrium in which nobody engages in crime and the model collapses to the basic framework analyzed in Burdett and Mortensen (1998).

[22] In principle there is one more variable required to complete the definition of equilibrium – the probability that unemployed workers engage in crime, φ_0. The equilibrium condition is simply

$$\phi_0 = \begin{cases} 0 & \text{if } K_0 - V_0 < 0 \\ \widetilde{\phi}_0 \in [0,1] & \text{if } K_0 - V_0 = 0 \\ 1 & \text{if } K_0 - V_0 > 0. \end{cases}$$

As stated earlier, we will focus mainly on the case where $\sigma < 1$, which guarantees $\varphi_0 = 1$.

If the flow return to crime is "large enough but not too large", then a type-II equilibrium exists: every unemployed worker engages in crime while no employed worker does. Formally, let

$$\begin{aligned}\eta_{2L} &= (\pi/\rho)\left[\left(1-\alpha\frac{\delta\mu}{\delta+\lambda}\right)\Omega-\frac{\delta\mu}{\delta+\lambda}d\right] \text{ and}\\ \eta_{2H} &= (\pi/\rho)\left[\left(1-\alpha\frac{\delta\mu}{\delta+\lambda}\right)\widehat{\Omega}-\frac{\delta\mu}{\delta+\lambda}d\right],\end{aligned}$$

with $\widehat{\Omega}=\widehat{\varepsilon}p+(1-\widehat{\varepsilon})\,b$ and

$$\widehat{\varepsilon}=\frac{\mu\pi\,(\delta+\lambda)\,(\rho+\mu\pi)+(\delta+\mu\pi)\,\lambda^2}{(\delta+\lambda)\,(\delta+\mu\pi)\,(\delta+\mu\pi+\lambda)}.$$

An equilibrium of type II exists if $\upsilon\in(\eta_{2L},\eta_{2H})$,[23] and the equilibrium allocation is given by:

$$\begin{aligned}\sigma &= \phi_0=1,\ \gamma=\frac{\delta\mu}{\delta+\lambda}\\ b &< \underline{w}=w^*<\overline{w}\\ \overline{w} &= p-(p-w^*)\left(\frac{\delta}{\delta+\lambda}\right)^2\\ F(w) &= \frac{\delta+\lambda}{\lambda}\left[1-\left(\frac{p-w}{p-w^*}\right)\right]^{1/2} \text{ and}\\ w^* &= \left\{\frac{(\delta+\mu\pi)}{\left(1-\alpha\frac{\delta\mu}{\delta+\lambda}\right)}\left[(\rho/\pi)\,\upsilon+\frac{\delta\mu}{\delta+\lambda}d\right]-(\delta-\rho)\,\Omega_o\right\}\frac{1}{\xi},\end{aligned}$$

where

$$\begin{aligned}\Omega_o &= \left[b+\frac{\delta+\mu\pi}{\delta-\rho}\left(\frac{\lambda}{\delta+\lambda}\right)^2p\right] \text{ and}\\ \xi &= (\rho+\mu\pi)\left[1-\frac{\delta+\mu\pi}{\rho+\mu\pi}\left(\frac{\lambda}{\delta+\lambda}\right)^2\right].\end{aligned}$$

For completeness, note that if $\upsilon\in[\eta_{2L},\eta_1]$ then a mixed-strategy equilibrium in which no employed worker engages in crime –as in types I and II– but a fraction

$$\phi_0=\left(\frac{\delta+\lambda}{\delta\mu}\right)\frac{\Omega-(\rho/\pi)\,\upsilon}{d+\alpha\Omega}\in[0,1]$$

[23] It can be shown that $\eta_{2L}<\eta_{2H}$ if and only if

$$\rho>\frac{\delta\left[\lambda^2-\mu\pi\,(\delta+2\lambda)\right]}{(\delta+\lambda)^2}.$$

of the unemployed do, exists.[24] The rest of the equilibrium allocations look as in a type-I equilibrium, except for the fact that $\underline{w} = b = w^*$ and

$$\gamma = \frac{\Omega - (\rho/\pi)\, v}{d + \alpha\Omega}.$$

An equilibrium of type III (i.e. one in which all agents behave like criminals) exists if the flow return to crime is "very large". More precisely, letting $\eta_3 = (\pi/\rho)\left[(1-\alpha\mu)\overline{\Omega} - \mu d\right]$ and $\overline{\Omega} = \overline{\varepsilon} p + (1-\overline{\varepsilon})\, b$, with

$$\overline{\varepsilon} = \frac{(\lambda + \mu\pi)^2 + \rho\,(2\lambda + \mu\pi)}{(\delta + \mu\pi + \lambda)^2},$$

a type-III equilibrium exists if $v > \eta_3$. The corresponding allocations are given by

$$\begin{aligned}
\sigma &= 0,\ \phi_0 = 1,\ \gamma = \mu \\
\underline{w} &= b < \overline{w} < w^* \\
\overline{w} &= p - (p-b)\left(\frac{\delta + \mu\pi}{\delta + \mu\pi + \lambda}\right)^2 \\
F(w) &= \frac{\delta + \mu\pi + \lambda}{\lambda}\left[1 - \left(\frac{p-w}{p-b}\right)\right]^{1/2} \text{ and} \\
w^* &= \left\{\frac{\delta + \mu\pi}{1-\alpha\mu}\left[(\rho/\pi)\, v + \mu d\right] - (\delta - \rho)\,\underline{\Omega}\right\}\frac{1}{(\rho + \mu\pi)},
\end{aligned}$$

where

$$\underline{\Omega} = \left[\left(\frac{\lambda}{\delta + \mu\pi + \lambda}\right)^2 p + \left[1 - \left(\frac{\lambda}{\delta + \mu\pi + \lambda}\right)^2\right] b\right].$$

Notice that and equilibrium of type II (resp. III) can be seen as a special case of a type-IV equilibrium in which the lower (resp. higher) "branch" of the wage distribution is "inactive".

4.3. Policy Implications

In this section we explore the effects that changes in the apprehension probability π, the parole probability ρ, and the severity of punishment z have on the crime rate and other labor market variables. We also analyze the consequences of changes in the productivity parameter p and the level of unemployment income b.

In an equilibrium of type II, the crime rate (defined as the number of criminals per worker in the labor force) is $\delta/(\delta + \lambda)$ and is insensitive to policy. In this type of equilibrium $K_0 - V_0 > 0$, so all unemployed workers take advantage of every crime opportunity they get. Employed workers are always paid $w \geq w^*$ and hence choose never to engage in crime. Firms only offer wages above the crime wage w^*

[24] It is easy to show that $\eta_{2L} < \eta_1$ for all parametrizations in accordance to our maintained assumptions.

because $\Pi(b|w^*, R, F) < \Pi(w^*|w^*, R, F)$. A firm paying $w < w^*$ employs workers who regularly engage in crime. Each period these firms loose some of its employees to jail, hence their steady state labor forces are much smaller than those of firms paying at least the crime wage. In this type of equilibrium the difference in steady state labor forces is relatively large while the distance between w^* and b is relatively small, so no firm finds it profitable to offer wages that will induce workers to engage in crime.

Although anti-crime policies have no effect on the crime rate in this type of equilibrium, they have an impact on labor market outcomes. For instance, notice that a decrease in the apprehension probability π, an increase in the parole probability ρ or an increase in z (i.e. a decrease in the severity of punishment) raise the workers' crime wage w^*. A higher crime wage induces a wage distribution that stochastically dominates the former one[25]. In addition, a higher crime wage also reduces wage inequality as measured by the 90-10 wage differential.[26] Virtually all previous work on the relationship between crime and the labor market has focused on how the state of the latter affects the former. This analysis shows there may be other potentially important mechanisms working in the opposite direction.

Since all (and only the) unemployed engage in crime in a type-II equilibrium, the unemployment rate equals the crime rate. This equilibrium predicts the strongest possible relationship between unemployment and crime: each additional worker joining the unemployment pool is a potential criminal.[27] The steady state fraction of the labor force in jail is $\mu\pi\delta/(\delta+\lambda)$.

In an equilibrium of type III the crime rate is 1: all workers regardless of employment status take advantage of every crime opportunity they get. In this case, w^* is so high relative to the worker's reservation wage, that firms cannot afford to hire workers and pay them wages that will keep them from engaging in crime. Since $w^* > \overline{w}$, (small) changes in policy parameters cannot affect the crime rate through their effect on the crime wage. But, as in an equilibrium of type II, reductions in the apprehension probability induce an equilibrium wage distribution which is more favorable to workers.[28] The unemployment rate is $(\delta+\mu\pi)/(\delta+\mu\pi+\lambda)$ and the fraction of the labor force in jail is $\mu\pi/\rho$.

An interesting feature of the outcome in a type-IV equilibrium is that while some workers are employed in high-paying jobs and remain honest, others work for firms that pay less than the crime wage and hence choose to become criminals. Firms on the lower "branch" of the wage distribution are able to earn the same profit as those paying $w \geq w^*$ because the per-worker wage savings associated with paying wages discretely below the crime wage allow them to compensate for the higher turnover

[25] In the first-order sense.

[26] In an equilibrium where the employed don't engage in crime, the 90-10 wage differential is given by

$$\frac{p-(p-\underline{w})\left(1-\frac{.9\lambda}{\delta+\lambda}\right)^2}{p-(p-\underline{w})\left(1-\frac{.1\lambda}{\delta+\lambda}\right)^2},$$

which is strictly decreasing in $\underline{w}$ and strictly increasing in p (for a fixed $\underline{w}$).

[27] In the sense that he will take every crime opportunity he gets.

[28] To see this set $\sigma=0$ in (4.5) and notice that $\partial F_L(w)/\partial\pi > 0$ for all w.

they suffer. The unemployment rate is given by

$$\frac{(\delta+\mu\pi+\lambda\sigma)\,\delta}{(\delta+\mu\pi+\lambda)\,(\delta+\lambda\sigma)},$$

and is decreasing in σ and increasing in π. The crime rate, $\delta/(\delta+\lambda\sigma)$, is decreasing in the fraction of firms paying wages above the crime wage; while the fraction of the labor force in jail, $(\mu\pi/\rho)\,(\delta/(\delta+\lambda\sigma))$, is proportional to the crime rate, increasing in the apprehension probability and decreasing in the release probability.

5. Concluding remarks

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