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An introduction to Allocation Mechanisms"

Alberto Porto

(Universidad Nacional de La Plata)

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Admission Process to Higher Education.
An Introduction to Allocation Mechanisms

Huberto M. Ennis
Cornell University and
Universidad Nacional de La Plata
E-mail: hme1@cornell.edu

Alberto Porto*
Universidad Nacional de La Plata
E-mail: aporto@netverk.com.ar

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Abstract

The paper studies the problem of students admission to higher education institutions. We introduce a simple model where some of the fundamental issues can be analyzed. After providing a justification for the use of quantity restrictions in a system of public provision of higher education, we proceed to review the literature on Allocation Mechanisms pioneer by the work of Gale and Shapley (1962). This literature deals with the different ways to allocate a number of limited vacancies to a set of candidates. The main existing theorems are introduced and their relevance for the college admission problem is carefully discuss. Finally, we present some preliminary ideas that relate the work in this paper with the current situation of the higher education system in Argentina.

*Address: *Departamento de Economía, U.N.L.P., Calle 6 y 47, 5to piso, La Plata 1900, Buenos Aires, Argentina.*

1 Introduction

Discussions about the best way to organize the admission to public universities are of everyday occurrence in current Argentinian political and economic forums. The subject has been under discussion though, since the early days of the creation and diffusion of universal public higher education. Lately, the debate has been concentrated on two opposing positions. On one side there is those that advocate for the status quo, with public provision of the service (higher education) and unrestricted access to it. On the other side, those that propose the implementation of tuition fees that not only would improve the financing of the system but would also act as a way of rationing the service.

In the present paper, we attempt to motivate a third alternative approach to the problem. We recognize that the two views mentioned above have valid points to make in the finding of a solution to the problem and that each fails to address the opposing side concerns. With this in mind, we present the problem mixing elements of both positions and find that a natural answer to the query could be the implementation of a system with public provision of the service and restricted access through the use of an appropriate allocation mechanism. Interesting enough, systems similar to this one are functioning already in several other countries in the world. This fact has motivated an important body of theoretical literature on allocation mechanisms. We proceed to give an overview of the current knowledge on the subject. This, we think, is essential for our argument because proposing quantity restrictions immediately rise the question of how to better assign the limited number of vacancies among the candidates. There is some work that have been done on the subject and there is an organic way to follow that research. That is, we know some things about the problem and we can hope to learn more. This is clearly of great importance for the implementability of the system that our study of the issue seems to suggest.

The paper is organized as follows. In the next section, we present a very simple general equilibrium model where some of the essential features of the problem can easily be handle and studied. Using this model, we show that quantity restrictions may be an interesting alternative in the search for a solution to the questions on the provision of higher education. Section 3 then, gives an overview of the literature on allocation mechanisms that can be use to assign a number of limited vacancies to a set of given candidates in the best possible way. Section 4 includes a discussion of some of the most important aspects of the Argentinian situation and how these relate to the analysis in the previous sections. Some preliminary conclusions are also included at the end of that section.

2 Towards Quantity Restrictions

We chose to start our inquires by introducing a very simple general equilibrium model that will show appropriate to precisely describe and analyze some of the issues that we think are essential to the problem. We certainly recognize that the present exposition will appear extremely narrow in the treatment of a subject that have multiple aspects and where details sometimes come to be extremely important.[1] However, we also think that the model will illustrate with sheer clarity, partly due to its simplicity, some of the points that are always in play in any discussion of these issues. Three elements provide the substance to our model and, of course, drive our results: the students tastes, the cost of education and its returns.

2.1 The Model

Assume we have an economy with two group of agents. Agents live two periods, and chose to obtain education in the first period of their life to enjoy its return in the second period. The groups contain each a continuum of heterogeneous agents with names in the unit interval. There is two possible activities (careers) that agents can choose, 1 and 2. Agents are heterogeneous in their taste towards learning one or the other activity. Group 1 agents have no endowment in the first period and agents in group 2 have an endowment ϖ. Learning activity 2 in the first period implies a cost $\alpha > 0$ in goods that needs to be payed at that very moment. This should be interpreted as a differential cost in the learning of activity 2 (and not 1), where we choose to simplify notation and normalize the cost of learning activity 1 to zero. We assume that this economy have access to funds at a fixed gross interest rate

[1] There are several other dimensions to the problem of how to finance higher education. They all constitute a permanent concern in the design of a general policy on the matter. One example of these is how to assign the total budget for higher education among the different institutions. For a recent treatment of this subject, with especial relevance for Argentina, see Delfino and Gertel (1996).

R.[2] Agent $i \in [0,1]$ in either of the two groups have preferences that can be represented by the following lifetime utility function,

$$\sigma \theta_i \nu + u(c_1) + \beta u(c_2) \tag{1}$$

where $\theta_i \in [0,1]$, $\nu = 1$ if the agent chose to learn activity 2 and $\nu = 0$ otherwise, and β is the discount factor. In this formulation, θ_i indicates the taste of agent i for learning activity 2 and σ is a parameter of the utility function that indicates whether the mere act of education induce a utility benefit ($\sigma > 0$) or a utility cost ($\sigma < 0$) upon private agents. Accordingly, when $\sigma > 0$, agents with a high value of θ (close to one) are agents that greatly enjoy learning activity 2 (if $\sigma < 0$ then agents dislike learning activity 2 but they may still chose to do it if the second period returns are high enough). We assume that the tastes (i.e. θ) of the population in each group is uniformly distributed over the interval $[0,1]$. Assume that the utility function is linear to avoid issues of intertemporal consumption smoothing. Also assume that $R > 1/\beta$ and $\varpi > \alpha$. Let b be first period debt. If agents have direct access to the credit market the budget constraints are as follows. For agents of group 1 we have,

$$c_1 = -\alpha\nu + b \geq 0, \tag{2}$$

and

$$c_2 = w - Rb, \tag{3}$$

where w is the second period wage rate and depend on whether the agent learn to do activity 1 or 2. Similarly, for agents of group 2 we have

$$c_1 = \varpi - \alpha\nu + b \geq 0, \tag{4}$$

and

$$c_2 = w - Rb. \tag{5}$$

Since we have assumed that $R > 1/\beta$, only agents of group 1 that want to learn activity 2 will choose to borrow in equilibrium (they will chose $b = \alpha$ to be able to pay the costs of education).

[2] This is clearly a separation from the general equilibrium premise but it comes to be not essential for our analysis which is greatly simplified by this assumption. The assumption will allow us to make our points in a much more clear and transparent way.

For the second period, we assume that there exist a number of profit maximizing competitive firms that produce consumption goods using a technology represented by the production function $f(L_1, L_2)$, where L_i is the number of agents knowing activity i ($i = 1, 2$) employed in the production process. Let f have the standard properties: strictly increasing, strictly concave and Inada conditions. The usual result follows: $w_i = f_i$, i.e. factors are remunerated their marginal productivity.

Assume an interior solution for the pair $\{\theta^1, \theta^2\}$ defining an equilibrium in this economy. Let θ^1 be the value of θ that solves the following equation

$$\sigma\theta^1 + u(0) + \beta u(w_2 - R\alpha) = u(0) + \beta u(w_1), \tag{6}$$

and θ^2 the one that solves

$$\sigma\theta^2 + u(\varpi - \alpha) + \beta u(w_2) = u(\varpi) + \beta u(w_1). \tag{7}$$

Let $\sigma = 1$. It is not hard to see that the agents of group 1 with $\theta > \theta^1$ (and only those) will chose to learn activity 2 and that the agents of group 2 with $\theta > \theta^2$ (and only those) will also chose to learn activity 2. By the market clearing conditions for the labor markets in the second period we have

$$L_1 = \theta^1 + \theta^2 \tag{8}$$

and

$$L_2 = 2 - L_1 \tag{9}$$

because agents supply labor inelastically. An equilibrium for this economy can be found by solving the following system of equations in $\{\theta^1, \theta^2\}$,

$$\theta^1 + \beta u[f_2(\theta^1 + \theta^2, 2 - \theta^1 - \theta^2) - R\alpha] = \beta u[f_1(\theta^1 + \theta^2, 2 - \theta^1 - \theta^2)], \tag{10}$$

$$\theta^2 + u(\varpi - \alpha) + \beta u[f_2(\theta^1 + \theta^2, 2 - \theta^1 - \theta^2)] =$$

$$= u(\varpi) + \beta u[f_1(\theta^1 + \theta^2, 2 - \theta^1 - \theta^2)]. \tag{11}$$

Call the solution to this system (θ^1_C, θ^2_C). It is not hard to see that $\theta^1_C > \theta^2_C$ because $R > 1/\beta$, i.e. agents in group 1 with no endowment in their first period of life find more costly to undertake the project of learning activity 2 (they need to borrow at the gross interest rate R); hence only those with relatively higher preference towards activity 2 will choose that path.

2.2 Credit Constraints

Suppose now that young individuals find credit constraints as they wish to borrow resources to finance the education costs.[3] In this case, $b \equiv 0$ in equilibrium and no agent of group 1 would be able to afford learning activity 2, i.e. $\theta^1 = 1$. Agent in group 2 have available some endowment and the credit constraint turns out to be non-bidding for them. As a consequence, in equilibrium $L_1 = 1 + \theta^2$ and $L_2 = 1 - \theta^2$. The equilibrium is then given by the solution of the following equation in θ^2 (to be compared with (11)),

$$\theta^2 + u(\varpi - \alpha) + \beta u[f_2(1 + \theta^2, 1 - \theta^2)] = u(\varpi) + \beta u[f_1(1 + \theta^2, 1 - \theta^2)]. \tag{12}$$

Call the solution to this equation θ^2_{NC}.

Claim 1 *The following two inequalities hold: a)* $\theta^2_C > \theta^2_{NC}$ *and b)* $2 - \theta^1_C - \theta^2_C > 1 - \theta^2_{NC}$.

Proof. a) Suppose not, suppose $\theta^2_C \leq \theta^2_{NC}$. Since $\theta^1_C < 1$, we have that $2 - \theta^1_C - \theta^2_C > 1 - \theta^2_{NC}$. By the assumptions on the production function we have that in the credit constrained economy the wage rate for activity 2 would be higher than with no credit constraints and the wage for activity 1 would be lower. But then

$$\theta^2_C + u(\varpi - \alpha) + \beta u(w_2^{NC}) > u(\varpi) + \beta u(w_1^{NC}) \tag{13}$$

which implies that $\theta^2_C > \theta^2_{NC}$ and we have reach a contradiction.

b) Suppose not, suppose $1 - \theta^2_{NC} \geq 2 - \theta^1_C - \theta^2_C$. For this last inequality to hold, and given that $\theta^1_C < 1$, we need to have that $\theta^2_{NC} < \theta^2_C$. But since this also implies that $w_2^C \geq w_2^{NC}$ and $w_1^C \leq w_1^{NC}$ and therefore

$$\theta^2_C + u(\varpi - \alpha) + \beta u(w_2^{NC}) < u(\varpi) + \beta u(w_1^{NC}), \tag{14}$$

we have that for θ^2_{NC} to be an equilibrium it has to be greater than θ^2_C, which stands in contradiction. ∎

[3] There is an extensive body of literature that discuss this possibility. Information asymetrics that derive in moral hazard and adverse selection problems are the standar arguments to justify this type of assumption. Additionally, the data tends to suggest that the phenomenon is quite relevant for Argentina (see Table 1 in Section 4 below).

The claim just proved is useful to evaluate the effects of credit constraints in the welfare of the individuals within the different groups. On one side, we see that more agents in group 2 tend to choose activity 2 whenever agents in group 1 are credit constrained. However, the increase in the number of agents in group 2 performing activity 2 cannot compensate the reduction in the number of agents in group 1 that are not able to incur the cost of education and work on that activity. Hence, under standard conditions for the production function, the wage rate for activity 2 would tend to increase. In summary, more workers in group 2 (that were already favored by the fact that they own some endowment when young) are able to perform activity 2 that now is relatively better remunerated. We understand that one may consider this situation not only inefficient but also politically unfair for agents in group 1 and may want to implement some kind of financing system for education to help counterbalance this situation.

2.3 Public Unrestricted Provision

One possibility is to introduce a system of public provision of the education service (i.e., let the government pay the cost α per student of learning activity 2). Consider for example the following policy scheme. Suppose the government have access to the credit market and faces the fixed gross interest rate R. Also assume that there is no feasible way for the government to a priori determine whether an specific agents belong to either group 1 or 2.[4] Then, the government could implement the following system: 1) allow free and unrestricted access to education in activity 2, financing the cost through borrowing in the credit market, 2) charge a uniform tax in the second period to every agent and repay the public debt. Even though this is only one possible alternative for the government policy, we think that it captures some of the main aspects of the system at work in Argentina and specially those that were our intention to discuss.[5] In this context, the balanced budget restriction for the government is given by

$$\left(2-\theta^1-\theta^2\right)\alpha R = 2T, \tag{15}$$

[4] This fact is essential for our argument. Being the government able to separate agents in group 1 from the agents in group 2, a simple tax-subsidy policy could solve all our problems. See Section 4 for further discussions on this matter.

[5] For a comment on other government policies see the end of this subsection.

where T' is the second period tax. Following the same ideas as before, define θ^1 as the solution to

$$\theta^1 + u(0) + \beta u(w_2 - T) = u(0) + \beta u(w_1 - T), \tag{16}$$

and θ^2 as the solution to

$$\theta^2 + u(\varpi) + \beta u(w_2 - T) = u(\varpi) + \beta u(w_1 - T). \tag{17}$$

Again as before, agents in group i with $\theta < \theta^i$ will chose to learn activity 1 and those with $\theta \geq \theta^i$ will chose activity 2. Plugging the second period labor market conditions and the government budget constraint (15) in the system formed by equations (16) and (17) we can obtain the new equilibrium values for θ^1 and θ^2. Note that $\theta^1 = \theta^2$. Call this common value θ_{PU} (where PU stands for "public unrestricted").

Claim 2 *The following two inequalities hold: a) $\theta_{PU} < \theta^1_C < \theta^1_{NC} = 1$ and b) $\theta_{PU} < \theta^2_{NC} < \theta^2_C$.*

Proof. Remembering the linear utility assumption and by comparison with (10) and (12), the claim follows directly (as part of it was proved in the previous claim). ■

The claim shows that the proportion of the population choosing activity 2 under the public unrestricted provision of education is not only higher than in the case of credit constraints but also higher than when individuals have to afford the private costs. We may say that in an effort to solve one problem, the "under-supply" of agents performing activity 2, the government have gone all the way to create an "excess" of this supply. In our section of discussions (Section 4 below), we present a table (Table 1) with total university enrolment in Argentina during several different periods. The systems that were in place at those times may partially resemble the credit constraints and the public unrestricted schemes. The differences between number-of-students in those periods are substantial and we think they illustrate the relevance of our analysis.

2.4 Quantity Restrictions

Suppose now that a benevolent government is able to set quantity restrictions over θ^i, $i = 1, 2$.[6] Suppose as before that the government can not identify

[6] Several issues not considered here can also be suggestive of the convenience of implementing quantity restrictions. In the short run capacity tends to be fixed and big changes

agents from different groups but somehow can assign the vacancies for learning activity 2 to the agent that most prefer it. Call the quantity restriction set by the government θ_R, which implies that the government would only let the $2(1-\theta_R)$ agents that most like activity 2 to access to the education program for activity 2. It is not hard to show that the benevolent government would seek to maximize the following objective choosing $\theta_R \in [0,1]$,

$$(1-(\theta)^2)+2\beta\left(\theta u\left[w_1(\theta)-(1-\theta)\alpha R\right]+(1-\theta)u\left[w_2(\theta)-(1-\theta)\alpha R\right]\right)$$

where the notation $w_i(\theta)$ shows that the government realizes that wages will depend on the number of people choosing each activity.[7] From the first order condition to this problem we have that θ_R must satisfy the following equation

$$\theta_R + u(w_2 - T) = \beta u(w_1 - T) + \Lambda, \tag{18}$$

where $\Lambda = [\theta_R w_1' + (1-\theta_R) w_2' + \alpha R]\, u'$ and w_i' is the derivative of the wage rate with respect to θ. Note that $w_1' < 0$ and $w_2' > 0$ and in general Λ will tend to be positive. In that case, it is easy to see by comparison with (16) that $\theta_R > \theta_{PU}$ and the government restriction to education in activity 2 would be binding.[8] For the importance of this result in our discussions we wish to state it with a formal claim.

Claim 3 *For most cases the following inequality holds, $\theta_R > \theta_{PU}$.*

Proof. See the argument above. ∎

This claim states the fundamental fact that if a benevolent government were to implement a policy of public provision of education, it would best achieve its objectives when being able to set a quantity restriction in the number of vacancies for learning the costly activities. Essential for the argument above was the assumption that the government can assign the limited vacancies to the best candidates (in this case, those that most like to learn

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in demand can cause important inefficiencies under the public unrestricted system. For example, we think that the adjustment-through-quality mechanism, with all its associated inefficiencies, have been of widespread operation in Argentina in the last couple of decades.

[7] This objective can be obtain by summing the utility of all the agents in the economy (which of course involves integration over types) and replacing the market clearing conditions and the government budget constraint. The expression in the text is the same up to a constant as the one obtain following the steps indicated in this note.

[8] See again Table 1 in Section 4 for its empirical counterpart.

activity 2). How can this be done and what are the properties of the different possible procedures is the very subject of the literature on allocation mechanisms that we intend to review in the following section.

Before going to the next section, we shall briefly discuss another possible policy scheme that the government may want to implement in a situation like the one presented in the previous subsection. In particular, the government may be able to charge different second period tax rates to agent according to the activity they choose to perform. For example, suppose the government decides to charge a zero tax rate to agents that perform activity 1 in the second period. In this case, the tax to agents working in activity 2 would have to be $T = \alpha R$ (using the government budget constraint) and the equilibrium 0^1 and 0^2 would be given by the following two equations,

$$0^1 + \beta u(w_2 - \alpha R) = \beta u(w_1), \tag{19}$$

and

$$0^2 + \beta u(w_2 - \alpha R) = \beta u(w_1). \tag{20}$$

Clearly, by comparing equation (19) with equation (6) we can see that using this method the government can set $0^1 = 0^1_C$. However, 0^2 will in general be different (lower) than 0^2_C because βR was assumed greater than 1. This fact uncovers an inefficiency already present with the previous policy. The government by financing through credit the education of agents in group 2 is in fact creating an inefficiency since credit is relatively more costly and this agents actually have available some own funds that they could use to pay the education costs. We think though that this sort of inefficiencies may possibly show empirically relevant and therefore it seems worth to have them identified in the analysis. Also, it is interesting to note the trade-off in efficiency inherent to this policy. The government, trying to solve the problem of credit restrictions for group 1 agents, incurs another inefficiency in the equilibrium allocation for agents in group 2.[9]

3 Allocation Mechanisms

Consider the following problem. Suppose we have a number of students and a number of colleges-faculties with a limited number of vacancies each. Every

[9] See the comment on the "tax-to-graduates" in Section 4.

student have a complete order of preference over the colleges-faculties and every college-faculty can somehow come up with an also complete order of preference over the set of students. The question is how should one allocate the students to the available vacancies. This problem was first studied by Gale and Shapley in their 1962 seminal paper. A number of other papers have appeared since then. They carefully characterize the different possible allocation mechanisms and introduce some interesting extensions. The purpose of this section is to provide an overview of the main definitions, concepts and results that constitute the core of this theory.

In general, all this type of social choice problems have two constituent parts. First, one needs to precisely define the specific problem under consideration. Second, one needs to design an allocation mechanism and study the properties of the matching allocations that such a procedure would finally select for each particular problems.

We will start then with the formal definition of the College Admission Problem that constitutes the basic first step in our investigation (see Balinski and Sonmez, 1999).

Definition 4 *A College Admission Problem is a set of students* $S = \{s_1, ..., s_n\}$*, a set of colleges* $C = \{c_1, ..., c_m\}$*, a capacity vector* $q = (q_{c_1}, ..., q_{c_m}\}$ *where* q_{c_i} *is the capacity of college* c_i*, a list of student preferences* $P_S = (P_{s_1}, ..., P_{s_n})$ *where* P_{s_i} *is the preference relation of student* s_i *over colleges including the no-college option* c_0*, and a list of college preferences* $P_C = (P_{c_1}, ..., P_{c_m})$ *where* P_{c_i} *is the preferences of college* c_i *over students that includes the no-student option* s_0*.*

In general, S and C are fixed and the triple (P_S, P_C, q) will define a College Admission Problem. We will be studying allocation mechanisms that systematically select a student-college matching arrangement with certain desirable properties whichever the triple (P_S, P_C, q) is. In practice usually (P_S, P_C, q) are the components that show harder to find out or actually know. Therefore, it is clearly a very important general requirement to be able to determine whether the proposed mechanism is able to select a matching with certain characteristics no matter what (P_S, P_C, q) are.

Next we define a matching as an allocation of college slots to students such that no student occupies more than one college slot.

Definition 5 *A matching is a function* $\mu : S \rightarrow C \cup \{c_0\}$ *such that* $|\mu^{-1}(c)| \leq q_c$ *for all* $c \in C$*, where* $|x|$ *denotes cardinality of the set* x *and* $q_{c_0} = |S|$*.*

Note that $\mu(s) = c_0$ means that student s is not assigned any college slot. The condition $|\mu^{-1}(c)| \leq q_c$ says that the matching function assigns to each college c a number of students that does not exceeds the corresponding capacity restriction.

The preference relation P_s of student s (initially defined over $C \cup \{c_0\}$) can be extended to the set of matchings with the following rule: student s prefers matching μ to matching μ' if and only if he/she prefers $\mu(s)$ to $\mu'(s)$. The concept of a matching is central in the theory of allocation mechanisms.

Definition 6 *An Admissions Mechanism is a systematic procedure that selects a matching for each admission problem* (P_S, P_C, q).

Properties of the matchings that are selected by each particular allocation mechanism are the main elements used in its characterization.

Definition 7 *A matching* μ *is individually rational if no student is assigned to a college that is worse than the no-college option.*

The first theorem that was proof by the literature is refer to the desirable property that the matching that result from a certain mechanism be actually stable in the sense that the agents can not change the proposed allocation by ex-post mutual arrangements. The concept of stability is closely linked to the concept of the Core from the Theory of General Equilibrium. To define stability we first need to introduce the following idea.

Definition 8 *A student-college pair* $(s, c) \in S \times C$ *blocks a matching* μ *if*

$$cP_s\mu(s) \text{ and } \left|\mu^{-1}(c)\right| < q_c, \tag{21}$$

or

$$cP_s\mu(s) \text{ and } sP_c\widetilde{s} \text{ for some } \widetilde{s} \in \mu^{-1}(c). \tag{22}$$

Condition (21) says that if student s prefers college c to college $\mu(s)$ (the one that was assign to her under the current matching μ) and if the college c have an excess of vacancies under matching μ, them this two participants (student s and college c) would block the matching μ by closing a mutual deal. Similarly, condition (22) says that if student s prefers college c to $\mu(s)$ and college c prefers student s rather than student $\widetilde{s}$ (currently being assign to it by the matching μ), then again this pair of participant will block the matching μ. With this concepts in hand we are ready to introduce the definition of stability.

Definition 9 *A matching μ is stable if it is individually rational and is blocked by no student-college pair.*

Let $\mathcal{S}(P_S, P_C, q)$ denote the set of stable matchings for the college admission problem (P_S, P_C, q).

Theorem 10 [10] *(Gale and Shapley, 1962) The set $\mathcal{S}(P_S, P_C, q)$ is non-empty.*

The proof of the theorem is constructive. Gale and Shapley propose an algorithm that always finds a stable matching for any college admission problem. The algorithm is call the **Student Proposing Deferred Acceptance Algorithm** and works as follows. In the first step each student proposes to his/her top choice among those colleges for which he/she is acceptable (i.e., that the student is better than the no-student option for the respective college). Each college c accepts the best q_c students among those who proposed to it and place them in a waiting list for admission. In the next step, each student who has been rejected in the previous step proposes to his/her top choice among those colleges that has not as yet rejected him/her and for which he/she is acceptable (if there is no such college the student stops proposing). Each college c accepts the best q_c students among those students who have just proposed and those that the college have in its waiting list. A new waiting list is form. Identical steps follows. The algorithm terminates when every student is either on a waiting list or has been rejected by all the colleges to which he/she is willing and able to apply. At that point, each college admits everyone in the waiting list. It is not hard to see that the resulting matching is stable.

Fact 1: In general, $\mathcal{S}(P_S, P_C, q)$ is not a singleton.

Fact 2: If all colleges have the same preferences in a college admissions problem, then there is a unique stable matching, i.e.$\mathcal{S}(P_S, P_C, q)$ is a singleton.

Fact 3: For each college admissions problem there is a stable matching that is preferred to any other stable matching by all the students. It is called the **Student Optimal Stable Matching** and denoted by $\mu^S(P_S, P_C, q)$. This stable matching is also the worst stable matching for all the colleges.

[10] The theorem may look fairly simple but it certainly have some content. Gale and Shapley provide an example of a closely related problem, the "problem of the roommates," and show that those problems not always admit a stable matching (see Gale and Shapley (1962), Example 3).

Fact 4: Similarly, for each college admission problem there is a stable matching that is preferred to any other stable matching by all the colleges. It is called the College Optimal Stable Matching and is denoted by $\mu^C(P_S, P_C, q)$. This is also the worst stable matching for all the students.

Lemma 11 *a)* $\mu^S(P_S, P_C, q) = \mu^C(P_S, P_C, q)$ *if and only if* $\Im(P_S, P_C, q)$ *is a singleton.*

b) (Gale and Shapley, 1962) The ***Student Proposing Deferred Acceptance Algorithm*** *selects the* ***Student Optimal Stable Matching*** *from the set of stable matchings for any college admission problem,* $\Im(P_S, P_C, q)$.

Definition 12 *An admission mechanism is student-strategy-proof if no student can ever benefit by unilaterally misrepresenting his/her preferences.*

Definition 13 *The* ***Gale-Shapley Student Optimal Mechanism*** *selects the student optimal stable matching for each college admission problem by use of the Student Proposing Deferred Acceptance Algorithm.*

Theorem 14 *(Dubins and Freedman, 1981) The Gale-Shapley Student Optimal Mechanism is student-strategy-proof.*

The theorem says that truth-telling is a dominant strategy for all students under the Gale-Shapley mechanism. In other words, no student can improve their fate by lying about their preferences. Indeed, Dubins and Freedman show that no coalition of students can simultaneously improve the lot of all its members if those outside the coalition state their true preferences.

Two other important properties of matching outcomes worth mentioning are the following.

Definition 15 *The matching* μ *is non-wasteful if* $cP_s\mu(s)$ *implies* $|\mu^{-1}(c)| = q_c$ *for every* $s \in S$ *and every* $c \in C$.

In other words, a matching μ is non-wasteful if whenever a student prefers a college c to his/her assignment, the college c has all its slots filled.

Definition 16 *A matching* η *Pareto dominates a matching* μ *if* $\eta(s_i) R_{s_i} \mu(s_i)$ *for every* $s_i \in S$ *and* $\eta(s_j) P_{s_j} \mu(s_j)$ *for some* $s_j \in S$ *where* R_s *denotes the at-least-as-good-as relation associated with the preference relation* P_s *for every* $s \in S$.

Definition 17 [11] *A matching μ is Pareto efficient if it is not Pareto dominated by any other matching.*

Note that Pareto efficiency implies individual rationality and non-wastefulness.

One of the most interesting and controversial issue associated with the college admission problem is how do colleges come up with a complete order of preferences over students. This is in our view a very important and non-trivial problem that will deserve extensive consideration in the section for discussion and conclusions (Section 4).

One actual possibility is the implementation of placements tests previous to the admission procedure.[12] This bring us to the definition of a Student Placement Problem.

Definition 18 *A (Student) Placement Problem consists of a set of students S, a set of colleges-faculties C, a capacity vector q, a list of students preferences P_S, a set of skill categories $T = \{t_1, ..., t_k\}$, a list of student test scores $f = (f^{s_1}, ..., f^{s_n})$ where $f^{s_i} = (f^{s_i}_{t_1}, ..., f^{s_i}_{t_k})$ are the test score of student s_i in each category, and a function $t : C \to T$ where $t(c)$ is the skill category required by college-faculty c.*

In general, S, C, T, and t are considered fixed and hence the triple list (P_S, f, q) defines a placement problem. Each placement problem (P_S, f, q) have associated a college admission problem (P_S, P_C, q) where the preference relation P_c for each college c is constructed as follows. For every $c \in C$, the preference P_c is such that

$$sP_c\widetilde{s} \text{ if and only if } f^{s}_{t(c)} > f^{\widetilde{s}}_{t(c)} \text{ for every } s, \widetilde{s} \in S \tag{23}$$

and

$$sP_cs_0 \text{ for every } s \in S \tag{24}$$

where s_0 denotes the no-student option.

[11] Note that this concept of Pareto efficiency only considers the preferences of students (but not that of colleges) as the basic elements driving the definition. This will show more appropriate for the Student Placement Problems (to be studied next) where colleges are just "mechanic" entities (see Balinski and Sonmez (1999) for a careful comparison between these two type of problems).

[12] This is how it works in the Turkish system (for a complete description see Balinski and Sonmez, 1999).

Definition 19 *A matching μ for a placement problem (P_S, f, q) is fair if for all students $s, \widetilde{s} \in S$ with $\mu(\widetilde{s}) = \widetilde{c}$, we have that*

$$\widetilde{c} P_s \mu(s) \text{ implies } f^{\widetilde{s}}_{t(\widetilde{c})} > f^{s}_{t(\widetilde{c})} \tag{25}$$

In other words, this definition says that a matching is fair if a student prefers college $\widetilde{c}$ to his/her assignment then all the students that have been assign to college $\widetilde{c}$ have better scores in the relevant category. It simply requires that students with better test scores are assigned to their better choices.

Fact 5: (Balinski and Sonmez, 1999) A matching is individually rational, fair, and non-wasteful for a placement problem if an only if it is stable for its associated college admissions problem.

Definition 20 *A placement mechanism is a systematic procedure that selects a matching for each placement problem.*

Obviously, there exist a placement mechanism that can be obtain by using the Student Proposing Deferred Acceptance Algorithm of Gale and Shapley (to select a matching from the college admission problem associated to each corresponding placement problem). This placement mechanism is also called the **Gale-Shapley Student Optimal Mechanism.**

If there is only one category (and hence only one test score for each student) then there is only one placement mechanism that is fair and Pareto efficient, i.e. that it always selects a fair and efficient matching for any placement problem.[13] This mechanism is call **Serial Dictatorship** (see Balinski and Sonmez, 1999) and it is such that the student with the highest test score is assigned his/her top choice, the student with the next highest score is assigned his/her top choice among the remaining slots, and so on. This is the system used for example to match medical interns and hospitals in Argentina.

For a thorough analysis of the multi-category placement problem see Balinski and Sonmez (1999).[14] One important result that we think worth mentioning before closing this section is the following.

[13] Note that Pareto efficiency implies individual rationality and non-wastefulness. By **Fact 5**, if the matching is fair and Pareto efficient then it is stable. But having only one category derive in the same ordering for all colleges and by **Fact 2**, there exist a unique stable matching. Therefore the fair and Pareto efficient matching ought to be unique.

[14] The Turkish system works as a Student Placement Problem with a Multi-Category Serial Dictatorship Mechanism that is analyzed in detail by Balinski and Sonmerz (1999).

Theorem 21 *(Balinski and Sonmez, 1999) The Gale-Shapley Student Optimal Mechanism Pareto dominates any other fair placement mechanism.*

This theorem, together with **Fact 3**, shows that the Student Optimal Stable Matching Pareto dominates any other attainable stable and fair matching.

The present overview of the theory of allocation mechanisms does not intend to be exhaustive or conclusive.[15] The main purpose of the section is to show that there exist an organic and fairly developed way to handle the problem of how to best assign a limited number of vacancies to a given set of candidates. This question was shown to be relevant when we study in Section 2 of the paper some of the essential issues concerning the financing and admission to the higher education system. We think that this section works to strengthen our suggestion that the public provision of education services combined with a certain arrangement of quantity restrictions may be a valid alternative worth further consideration.

4 Discussion and Conclusions

We have started the paper with a mention to the lively debate on the financing of higher education that dominates the political and economic arena in Argentina. Two main positions about the possible ways to deal with the problem were identified. On one side those that advocate for a public unrestricted system and, on the other side those that think that the implementation of tuition fees can solve most of the problem. The spirit of these two opposing views have been reflected in diverse manners throughout the Argentinian legislation all along the years. The Decreet-Law 7631/57 and the Law 17245/66 were in the line of the second position. The Law 20654/74 changes towards the first position and banned tuition fees. With Law 22207/76, legislation shows a come-back towards the second position and with Laws 23151/84 and 23659/88, it returns, once again, to the first position. This history of the evolution of the legislation has been mixed with ideological issues and miss-conceived generalizations. We think our paper can represent an item in the set of long awaited systematic tools to be used in the study of this essential question.

[15] See Roth and Sotomayor (1985), and Roth (1985) and Hylland and Zeckhauser (1979) for interesting applications.

Interesting enough, Argentinian data on total enrolment in public universities also corresponds to the fluctuation in the Legislation just described. The following Table documents this.

Year	System	Total Enrolment
1965	Unrestricted	208,284
1970	Restricted	214,253
1979/80 to 1983	Restricted	319,794 (average)
1984	Unrestricted	425,122
1985	Unrestricted	587,657
1990	Unrestricted	873,883

It is worth mentioning that during the second half of the sixties the system did combine restricted admission and partial charge of tuition fees. The explosion on the total enrolment in the last fifteen years of public unrestricted provision of the service of higher education is striking.

The model presented in Section 2 have broad applicability to understand this history in Argentina. It, in fact, admits two different interpretation. One possibility could be to think that what we have called activity 1 is the path taken by agents that choose not to attend a higher education institution. Probably, for that interpretation, the value of σ should be taken negative. Proceeding with the "education path" tends to generate high opportunity cost in the early periods of active life.[16] However, we have chosen to direct our presentation to the interpretation of the model as one where agents choose between either of two possible "education path". Activity 1 is relatively less costly than activity 2 but at the same time agents obtain relatively more utility from studying activity 2. The later is in fact what causes that throughout the presentation the second period payoff for activity 2, w_2, is generally lower than w_1.[17] The interpretation we have favored have some advantages. We think that it allows us to discuss certain questions that have received less attention in previous work (see however Olivera (1964) and

[16] Obviously, education have higher *direct* cost than no education too. This again would be represented by the cost α in the model. For Argentina, the total budget for the higher education system is currently around 1600 million dollars.

[17] This result is reversed when $\sigma < 0$, i.e. w_2 would be higher than w_1 for two reasons: first, there is a direct cost α of learning activity 2 and, second there is also a utility cost $\sigma\theta_i$ associated with such a decision. The second period return of activity 2 needs to be higher to compensate the early investment (See Becker (1964) and Becker (1971), Chapter IX).

(1967) for an early treatment of some of these issues, especially on the recognition of a consumption and an investment component of higher education). It also motivates the idea of the implementation of an scheme of quantity restriction over the admission process. This help to stress the importance for the subject of the literature on Allocation Mechanisms. Additionally, the acknowledgment of the existence of differential direct costs between alternative "education path" is a common feature recognized among the people in charge of the administration of the system in Argentina. For example, in the National University of Cuyo (Argentina), some preliminary calculations show that the most expensive career is 5.7 times more costly than the less expensive one (see Ginestar, 1992). The C.I.N. (Inter-Universities National Council) constructs an index of academic complexity usually used for the distribution of funds among colleges-faculties. This coefficient reflects a difference in relative costs within careers of about 3 to 1. Petrei and Cartas (1989) suggests that those differences also hold across universities despite the disparate characteristics (total size, number of college-faculties and departments, etc.).

The model also highlights the perverse consequences of ignoring the existence of binding credit constraints. Note that when credit constraint are in operation, more agents of the group with own endowments would chose to perform the pleasant activity (2) and, not only that, but also the wage rate for this activity actually increase due to the credit constraints. Agent in group 1 (with no endowment) are not able to access to this (activity 2-) "market". In a simplified way, the model suggest why a system of tuition fees may not be able to guarantee the "equal opportunity" principle established by the Argentinian National Constitution and in this particular case, by the Federal Law of Education (article 3 and 5, clause F).[18]

Another important issue that may be partially analyzed using the model in Section 2 is the proposal for implementation of a system of vouchers (see

[18]Article 5 starts saying that "the National State should set lines of policy on education that reflect the following rights, principles and criteria:..." and next, clause F enumerates, "the accomplishment of authentic equality of opportunities and possibilities for everyone and the repudiation of any kind of discrimination." Note however, that in the consecutive clause G, the article states that equity should be pursued "through the fair distribution of education services, looking for the accomplishment of the best possible quality and results taking into account the heterogeneity of the population." In our model of Section 2, agents' heterogeneity is one of the driving forces of the results and we think our discussions are in close correspondence with the principles suggested by the Federal Law.

Piffano, 1993). In general, the idea is that the universities would charge a tuition fee but the government would provide students with a number of vouchers that can be used to pay those costs. Usually, the stock of vouchers per capita would be enough to pay the *average* cost of education. It is clear in the model though, that the essential element for most of the arguments we have made is the existence of differential costs. In this respect, the voucher system does not improve the situation. However, several other advantages and disadvantages of the voucher can not be study within our dry set up.[19]

All sort of information problems complicate the basic question in this paper, which is the efficient allocation of resources to higher education. Some of them were determinant for the results obtain in the model presented in Section 2. All along we have assume that the government can not differentiate an agent that have own resources to invest in education from one that does not have them. This is essential since otherwise there exist simple direct tax-subsidy policies that would totally solve the allocation problems. This information constraint is also relevant at the moment of studying systems of fellowships and grants (or special credits for education).[20] Note however, that we do consider cases where the government may be able to ex-post differentiate among agents according to the carrier they have chosen in previous years. The "tax-to-graduates" is not a foreign concept on the discussions about the subject in Argentina.[21] We show however that such a tax is not a panacea.

Another important aspect that was totally ignored in the previous sections is the fact that agents need to somehow predict the future return of learning certain activities. We think that the collection of information by agents at the moment of deciding their future occupation is quite difficult and costly and that there may be a role for the government in such a process. One possibility is that the government centrally process the information and set a structure of quantity restrictions in the admission process that would

[19] In fact, the voucher system may for example improve the production efficiency of the education service taking the universities as "production units" (see Piffano, 1993 and Olivera, 1964 and 1967).

[20] One proposal that has been made is to charge tuition to high income agents and partly use it to finance fellowships for low income ones. This system not only have the obvious information problem but also probably have high administration cost, danger of social fragmentation and limitations on the capacity to "capture" the contributors.

[21] There has been several proposals to implement a "tax-to-graduates". It was even suggested in the Decreet-Law 7361/57, without though a clear exposition of its actual scheme.

help to avoid future over-supply of certain type of labor (consequence of the potential miscalculation of individual agents). In this environment, again, allocation mechanisms would play a central role.

As we said in Section 3, one of the most controversial component that needs to be carefully considered in the configuration of an allocation problem is the genesis of the final colleges' rankings over students. Note that in the Gale-Shapley Algorithm one is able to avoid the unpleasant comparison of absolute levels of student preferences (when for example two students like a college that only have one available position) through the use of the colleges' ordering of candidates.[22] This of course makes clear how determinant this component of the College Admission Problem can be. In the Student Placement Problem, the exclusive criteria used by colleges is the students scores in a complete arrangement of capacity tests. We do discuss this alternative in the paper but we wish to make clear that we do not advocate the use of tests as the final solution to the issue. We tend to think that the actual implementation of capacity test can derive in several induced behaviors that may maintain high levels of "unfairness" in the system. One well known "artificial" behavior resulting from this kind of system is the appearance of specialists that prepare students for taking the tests in exchange for considerable economic remuneration. This again, tends to marginate the agents with credit constraints (in the spirit of what was illustrated by the model of Section 2). However, we also realize that the global problem is extremely complex and that to reach any conclusion in the matter a much more detail study (beyond the scope of this paper) would need to be undertaken. Possible alternatives though, are the improvement of official pre-university introductory courses and the clear determination for maintainig the "rules-of-the-game" so that students can start preparing for the admission tests with enough time and resources.

The elements that we have presented in the review of the literature on Allocation Mechanisms can be applied to several current, and previous, Argentinian social and economic situations. For example, there were periods with restricted admission to universities where the colleges' rankings over students were based on a test score. Students though, first had to choose the university and the college-faculty within the university to only after that be

[22]This is one of the main differences between the College Admission Problem and the Problem of Assigning Individuals to Positions. For an excellent study of the later see Hylland and Zeckhauser (1979).

able to take a test that ranked them among the ones enrolled in that particular college-faculty division. Some ad-hoc versions of this method are at work today in several college-faculties of public universities (Medical School in the National University of La Plata, for example). It is not hard to show (with an explicit example, say) that this methods do not select *fair* matchings as defined in Section 3. A further sophistication of the previous method is the one that was used by secondary schools in Argentina two decades ago. In that case, students were allowed to take the test in more than one school. Under certain conditions this mechanism can improve the fairness of the resulting matching. Other examples of problems that could be understood by using the theory on Allocation Mechanisms include the filling of vacancies for professorships at universities, the selection of fellowship assignments for researchers in the CONICET (National Scientific and Technical Research Commission in Argentina), etc.

We have touch several important and controversial issues along the exposition in the paper. The main intention was always to try to provide new elements and ideas that can be use in the search for some, at least partial, solutions to the problems and questions that so much dominate the subject. We realize though that there is still a long way to go to be able to reach any solid conclusions.

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