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## "A Sequential-Trade Model with

Heterogenous Information Structures. Two
Examples"

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# A Sequential - Trade Model with Heterogeneous Information Structures. Two Examples. 

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#### Abstract

In this short paper two examples of "heterogeneous" information partition are presented in the context of sequential trade models. In the first one the insider does not infer the true value of the asset exactly. Instead she has a partition of the state space which is different from the one possesed by the dealer. The insider is able to infer the true value only under certain realizations but not able to do so under other realizations. This implies that there will be speculative trade only when insider can differentiate the true realization in an exact way.

In the second example I use the concept of "bounded rationality" due to Geanakoplos. In this case the insider only could see a signal that says if the value of the asset is low or not. However, whenever the insider does not observe this signal, she does not infer if the value of the asset is low or high. This is equivalent to say that the information sets do not form a partition, in a similar way that Geanakoplos applied to games.


## 1. Introduction

In this paper I present two examples of a two-period version of the sequential - trade models of Glosten - Milgrom [6] and Easley - O'Hara [1] type where the insider does not have full information about the realization of the payoffs of the asset and where the market maker possesses a different information sets from the insider ${ }^{1}$. Some related work is presented by Foster and Viswanathan [4] and by Madrigal and Scheinkman [8]. In the first case there is a set of heterogeneously informed insiders. The market maker in that case has still to infer the information through trading. This is an example of an "information aggregation" problem, where the specialist is the one "in charge" of aggregating different types of information. In the second mentioned paper, the main result is the presence of a discontinuity of the equilibrium price function which is interpreted as "price crashes", although there is no dynamics involved. This is also

[^0]the outcome of an information aggregation process by the market maker. However none of these two work with the case where the specialist and the insider have different (though not necessarily better) information. ${ }^{2}$

In the first example the insider does not have informational advantage all the time. In the first case I assume that the insider only knows the true value when this is the lowest possible. For any other realization the insider is unable to make any distinction among all those values. On the other hand the market maker posts bid and ask quotes that are "measurable" with respect to the information partition that she has. This implies that the bid and the ask must be the same as long as the dealer cannot differentiate between two or more realizations of the value of the asset. Therefore the insider could infer the "true" value of the asset as long as the intersection of the elements of her partition and the elements of the dealer's partition gives a singleton. This is because the insider could observe her own initial partition as well as the partition induced by the bid and ask quotes. In the equilibrium of the first case of the first example the asks and bids are such that the insider is able to differentiate two out of four possible realizations, but not the other two. That is, in equilibrium the insider infers correctly the true value of the asset whenever the realization of that value is either the lowest one or the second lowest. This of course depends crucially on the partitions of each agent. I show this by changing the information structure of both traders types. In the second case I assume, for the same partition of the insider, that the dealer knows perfectly the value of the asset if the realization is the highest possible. In this case there will be no trade if the value of the asset is the highest since both types of agents know the true value for that realization. The reason is that the insider infers perfectly the true value when this is the highest one from the bid-ask distribution. In this second case the insider will buy the security in some state where before, in the first case, she was not buying it. If the information sets are interchanged then the result is that in the lowest possible value there will be no trade while the insider will sell under states where in the first case of the first example the insider was not trading. All this demonstrates that the equilibrium that arises in these types of models is very dependent of the assumption about the availability of information for each agent. In a sense this would say that the usual sequential trade models (at least in its simple one-period version) are not very robust to the information assumptions. However this also confirms that "speculation" can arise also in situations where the insider does not have "perfect" signals about the true realization of the value of the security. As the cases 2 and 3 of the first example show, the insider may buy or sell in states where she does not fully identify the true value. This is true provided that the insider still observes an information set which is smaller than the one observed by the dealer. I provide a certain generalization of the cases at the end of section 2 .

In section 3 I present a second example. This is different from the first one in the sense that the insider possesses information sets that are not partitional. I follow closely Geanakoplos [5]. To see the idea, assume a standard sequential trade model

[^1]of the Easley - O'Hara type, where there are only two possible realizations. In the second example I assume that the insider infers correctly the true realization whenever this is the lowest value. However if the realization is the high value the signal does not convey any information. In other words, if the "high" value is realized, then the insider thinks that either the low or the high are still possible. This may be interpreted in different ways. Nevertheless it is natural to have this as an approximation of some type of "pessimistic" behavior. Think about an investor who has a person who gets secret information about the firm. If the investor is "pessimistic" enough then he will tend to believe the informer only when he gives to the insider "bad" news. But when the person has good news the insider will still tend to think that the bad situation is still possible. I also give alternative interpretations when I describe the model. In any case the result is that, with this assumption, there is a bid-ask spread even though there is no formal "adverse selection" in one of the states. This is because when the true realization is high the information set faced by the insider is exactly the same as the one observed by the market maker. Therefore the intersection of those two gives the same set. On the other hand if the true realization is the low value therefore there is adverse selection. The quotes must still be measurable with respect to the market maker's information. Since in this second example the dealer has no information about the true value, then the quotes must be the same regardless of the realization. Since in one of the states there is adverse selection, then a bid-ask spread is generated in the usual way. The combination of this and the measurability of the quotes gives the result. The important point is that non-partitional information implies that it may suffice to have adverse selection under only one possible state of the world to generate a bid-ask spread. Another implication of these assumptions is that the speculative trading will be lower than in the standard models. In the second example the insider sells only when the asset has a low value, but she never buys the security. This is again "consistent" with a type of "pessimistic" behavior.

Section 2 describes the first example. Section 3 presents the second example. Section 4 gives an interpretation for each case as well as the implications of them. Finally section 5 gives some concluding remarks and directions for future research.

## 2. The First Example.

### 2.1. The Model

There are two periods, $t=0$ and $t=1$. It is important to know the development of revelation of information. There is a unique asset whose "true" value is only revealed completely at the beginning of period 1. The asset trading session, though, takes place previous to that moment. The true value $V$ is considered random at the beginning of period 0 and it could take four values. Without loss of generality I assume that $V \in\{1,2,3,4\}$. The common priors are given by $\operatorname{Pr}[V=i]=q_{i}$, for $i=1,2,3,4$ with $q_{i} \in(0,1]$ and $\sum_{i} q_{i}=1$. The reason for this will be explained in the next paragraphs.

There are three types of market participants. There is a measure $\rho$ of liquidity traders. This is a usual assumption in these information - based models. These agents have no information about the environment. Instead they buy one unit of the asset with probability $\varepsilon$ and sell one unit with probability ( $1-\varepsilon$ ). Assume that $0<\varepsilon<1$. There is a measure ( $1-\rho$ ) of risk neutral "insiders". Note that I do not call informed traders since they may not have perfect information about the realization of the true value of the asset (see the description of events below). They have some knowledge about the value of the asset through some partition of the state space. The same is true for the dealers. I assume a measure one of risk neutral market makers. They also have some information partition once the true value is chosen by Nature. I describe this in the following paragraph. The important point so far is that only the dealer and the insider will have some decisions to make. The liquidity traders will trade in a total random way. The reader is referred to the classic paper by Milgrom and Stokey [9] for a justification of this assumption.

In the next subsection I present the first case where the insider only knows the true value when this is the lowest possible. I will consider other possible information sets in subsequent subsections.

### 2.2. Case 1

The sequence of events is as follows. At the beginning of period 0 Nature chooses a determined value for $V$. This value is not known to neither the dealer nor the insider. However each of them have information partitions of $\{1,2,3,4\}$ that reveal some information to each of the informed participants (the dealer and the insider) although not necessarily in a perfect way. In this case I assume the following partitions for the dealer and for the insider.

for the dealer and

$$
\mathcal{F}=\{\{1\},\{2,3,4\}\}
$$

for the insider. This information partition is common knowledge for everybody.
The interpretation of these refers to the situation where the insider only has "perfect information" whenever the true value of the asset is the lowest possible. Let us identify the state 1 as a "bankruptcy" state for the firm. This can be interpreted as the case where the insider knows only if the firm could go "bankrupt" or not. However this same insider does not know otherwise how well the firm will do. In other words,
if the firm is "lucky enough" not to go bankrupt, then the insider does not know how well the firm will do in the next period. On the other hand the dealer does not know if the firm will go bankrupt or not. However the dealer has some information whether the firm will perform "well" or "not so well". Here the "well" state is given by the element $\{3,4\}$ while the "not so well" is implied by the element $\{1,2\}$. In this last situation the bankruptcy state is included by not differentiated with the state 2 element.

After knowing this the dealer posts bids and asks and trade takes place under these quotes as in the usual sequential trade models. The ask is the expected value of the asset conditional to a buy observed by the dealer and the element of $\mathcal{B}$ revealed to the dealer. The bid is the expected value of the asset conditional to a sell observed by the market maker and the element of $\mathcal{B}$ revealed to the dealer.

In the next subsection I describe the equilibrium conditions and present the results for this paper.

### 2.3. Equilibrium behavior in case 1.

Denote $A$ the ask and $B$ the bud posted by the dealer. Denote $\omega$ a typical element of $\{1,2,3,4\}$. Following Glosten and Milgrom the insider buys if

$$
Z(\omega)>A(\omega)
$$

where

$$
Z(\omega)=E[V \mid H, A, B](\omega)
$$

and where

$$
H \in \mathcal{F}
$$

Similarly the insider sells if

$$
Z(\omega)<B(\omega)
$$

where $Z$ is defined as above.

I make the following assumption about the insiders.
Assumption 1. The insiders will not buy if $Z(\omega)=A(\omega)$. Similarly the insiders will not sell if $Z(\omega)=B(\omega)$.

This will become important in the proof of the results.
On the other hand the bids and asks are given by

$$
\begin{aligned}
& A(\omega)=E[V \mid J, \text { buy }](\omega) \\
& B(\omega)=E[V \mid J, \text { sell }](\omega)
\end{aligned}
$$

where $J \in \mathcal{B}$.
The computation of equilibria will require to consider the actual realization of $\omega$.

### 2.3.1. Case $\omega=1$.

Suppose that Nature chooses 1, that is, the firm going bankrupt. Therefore the insider knows this perfectly. This implies that $Z(1)=1$.

Note that in this case the set containing 1 for the dealer is the set $\{1,2\}$. Therefore the ask and the bid are given by

$$
\begin{aligned}
A(1) & =E[V \mid\{1,2\}, \text { buy }](1) \\
& =P[V=1 \mid\{1,2\}, \text { buy }] 1+P[V=2 \mid\{1,2\}, \text { buy }] 2 \\
B(1) & =E[V \mid\{1,2\}, \text { sell }](1) \\
& =P[V=1 \mid\{1,2\}, \text { sell }] 1+P[V=2 \mid\{1,2\}, \text { sell }] 2
\end{aligned}
$$

This is because $P[V=i \mid\{1,2\}$, buy $]=P[V=i \mid\{1,2\}$, sell $]=0$, for $i=3,4$. The expressions for the bid and the ask imply that $A(1) \in(1,2)$ and $B(1) \in(1,2)$. This is because $q_{i}>0$ for all $i$.

To get the values for $A(1), B(1)$ we must compute the conditional probabilities using Bayes rule. First, we have

$$
=\begin{aligned}
& P[V=1 \mid\{1,2\}, \text { buy }] \\
& = \\
& P[\{1,2\}, \text { buy } \mid V=1] P[V=1]+P[\{1,2\}, \text { buy } \mid V=2] P[V=2]
\end{aligned}
$$

Now, note that if $V=1$ or $V=2$, then it is clear that $V$ is in $\{1,2\}$. Therefore we have

$$
\begin{aligned}
& P[\{1,2\}, \text { buy } \mid V=1] \\
= & P[b u y \mid V=1]
\end{aligned}
$$

and

$$
\begin{aligned}
& P[\{1,2\}, b u y \mid V=2] \\
= & P[b u y \mid V=2]
\end{aligned}
$$

I compute the ask first. If $V=1$ is true, clearly the dealer would infer that $Z=1$. This due to the common knowledge assumption. However, since $A \in(1,2)$ then $A>V$. This implies that if $V=1$ there is no insider buying the asset. Therefore we have

$$
P[b u y \mid V=1]=\rho \varepsilon
$$

On the other hand, if $V=2$ were true then the dealer infers that $Z$ is not 1 but 2. This is because the dealer knows that if $V=2$, the insider would observe $\{2,3,4\}$. However, since $A(2) \neq A(3)=A(4)$ (shown below) then the insider would correctly infer that $V=2$. As long as $Z^{(2)}>A(1)$ the insider would buy the asset and if the contrary is true the insider will not buy the asset (given that $V=2$ ). However, note that

$$
\begin{aligned}
Z^{2} & =2 \\
A(1) & \in(1,2)
\end{aligned}
$$

This implies that $Z^{2}>A(1)$. Then the insider will buy if $V=2$.

As a consequence:

$$
P[b u y \mid V=2]=\rho \varepsilon+(1-\rho)
$$

This implies that

$$
=\frac{P[V=1 \mid\{1,2\}, b u y]}{\rho \varepsilon q_{1}} \begin{aligned}
& \frac{P \varepsilon q_{1}+[\rho \varepsilon+(1-\rho)] q_{2}}{}
\end{aligned}
$$

In a similar fashion

$$
\begin{gathered}
P[V=2 \mid\{1,2\}, b u y] \\
= \\
\frac{[\rho \varepsilon+(1-\rho)] q_{2}}{\rho \varepsilon q_{1}+[\rho \varepsilon+(1-\rho)] q_{2}}
\end{gathered}
$$

and then

$$
A^{*}(1)=\frac{\rho \varepsilon q_{1}+2[\rho \varepsilon+(1-\rho)] q_{2}}{\rho \varepsilon q_{1}+[\rho \varepsilon+(1-\rho)] q_{2}}
$$

This is the first equilibrium ask for the realization $\omega=1$.
What about the bid? We must compute the probabilities

$$
\begin{aligned}
& P[\text { sell } \mid V=1] \\
& P[\text { sell } \mid V=2]
\end{aligned}
$$

Note that if $V=1$ then the dealer knows that $Z=1$. This implies that $Z<B(1)$. This is because $B(1)$ is strictly greater than 1 according to our assumptions. Therefore the insider will sell the security if $V=1$. This implies

$$
P[\text { sell } \mid V=1]=\rho(1-\varepsilon)+(1-\rho)
$$

On the other hand, if $V=2$, the dealer knows that $Z=Z^{(2)}$ given above. Since $Z^{2}>2$ and $B(1)<2$, then $B(1)<Z^{(2)}$. This implies that if $V=2$ the dealer knows that only the liquidity traders would sell the security. This implies that

$$
P[\text { sell } \mid V=2]=\rho(1-\varepsilon)
$$

Therefore

$$
B^{*}(1)=\frac{[\rho(1-\varepsilon)+(1-\rho)] q_{1}+2 \rho(1-\varepsilon) q_{2}}{[\rho(1-\varepsilon)+(1-\rho)] q_{1}+\rho(1-\varepsilon) q_{2}}
$$

In order to assure that these are the actual equilibrium quotes for $\omega=1$, we must show that $A^{*}(1)=A^{*}(2)$, and that $B^{*}(1)=B^{*}(2)$. This is because the quotes must be $\mathcal{B}$ - measurable functions of $\{1,2,3,4\}$. This measurability condition is implied by the fact that the dealer should not be able to differentiate between $\omega=1$ and $\omega=2$. Otherwise she could infer the true state of the world. Therefore we must compute the equilibrium quotes $A^{*}(2)$ and $B^{*}(2)$ and compare these with the obtained prices.

### 2.3.2. Case $\omega=2$.

By the same general argument as before, it is true that

$$
\begin{aligned}
A(2) & =E[V \mid\{1,2\}, \text { buy }](2) \\
& =P[V=1 \mid\{1,2\}, \text { buy }] 1+P[V=2 \mid\{1,2\}, \text { buy }] 2
\end{aligned}
$$

But this is the same formula as $A(1)$. Therefore $A^{*}(2)=A^{*}(1)$. In a similar way

$$
\begin{aligned}
B(2) & =E[V \mid\{1,2\}, \text { sell }](2) \\
& =P[V=1 \mid\{1,2\}, \text { sell }] 1+P[V=2 \mid\{1,2\}, \text { sell }] 2
\end{aligned}
$$

Therefore $B^{*}(2)=B^{*}(1)$. Then this model satisfies the measurability condition for $\omega=1$ and $\omega=2$.

Note that when the true value is 2 the insider is not perfectly informed anymore. However, due to the heterogeneous partitions, the insider will still buy the asset under this event. This implies that the "speculative" motive for transactions is still present even though the information possessed by the insider may not be better than the information that the market maker has.
2.3.3. Case $3: \omega=3$ and $\omega=4$.

In this case the quotes are given by the following:

$$
\begin{aligned}
A(3) & =E[V \mid\{3,4\}, \text { buy }](3) \\
& =P[V=3 \mid\{3,4\}, \text { buy }] 3+P[V=4 \mid\{3,4\}, \text { buy }] 4 \\
& =E[V \mid\{3,4\}, \text { buy }](4) \\
& =A(4)
\end{aligned}
$$

Again we have that $A(3)=A(4)$ and then measurability is automatically satisfied.
We must compute now

$$
=\frac{P[V=3 \mid\{3,4\}, \text { buy }]}{} \quad \frac{P[\{3,4\}, \text { buy } \mid V=3] P[V=3]}{P[\{3,4\}, \text { buy } \mid V=3] P[V=3]+P[\{3,4\}, \text { buy } \mid V=4] P[V=4]}
$$

By the same reason as before we must just care of

$$
\begin{aligned}
& P[b u y \mid V=3] \\
& P[b u y \mid V=4]
\end{aligned}
$$

Now, if $\omega=3$, or $\omega=4$ were true then the dealer knows that $Z$ is given by

$$
Z^{(3)}=Z^{(4)}=\frac{3 q_{3}+4 q_{4}}{q_{3}+q_{4}}
$$

This must be true since the function $Z$ is $\mathcal{F}$ - measurable. On the other hand we do not include the value $V=2$. This is because the insider can differentiate between 2 and $\{3,4\}$ since the quotes will be different. The only problem is that the insider cannot differentiate between 3 and 4. This the justification of this formula. Note then that $Z^{(3)}$ and $Z^{(4)}$ are both in $(2,4)$. On the other hand, with all our assumptions $A(3) \in(3,4)$. Now we do not have a straightforward relationship between $A(3)$ and $Z^{(3)}$ and between $A(4)$ and $Z^{(4)}$. This would complicate the computation of the conditional probabilities. However I will show that we could not have $A(j)<Z^{(j)}$
for $j=3,4$ later on. In fact I show that $A(j)=Z^{(j)}$. This will imply inaction from the part of the insider.

To start with, we must get the values of $P[b u y \mid V=3]$. Note that there will be $\rho$ liquidity traders buying with probability $\varepsilon$. Assume first that $A(3)<Z^{(3)}$. If this is the case then the insider will buy. In this case

$$
P[b u y \mid V=3]=\rho \varepsilon+(1-\rho)
$$

Since $A(3)=A(4)$ and $Z^{(3)}=Z^{(4)}$ then

$$
P[b u y \mid V=4]=\rho \varepsilon+(1-\rho)
$$

Therefore $P[$ buy $\mid V=3]=P[b u y \mid V=4]$. This implies that

$$
\begin{aligned}
& P[V=3 \mid\{3,4\}, \text { buy }]=\frac{q_{3}}{q_{3}+q_{4}} \\
& P[V=4 \mid\{3,4\}, \text { buy }]=\frac{q_{4}}{q_{3}+q_{4}}
\end{aligned}
$$

Therefore

$$
\begin{aligned}
\bigcup \cap(3) & =A(4) \\
& =\frac{3 q_{3}+4 q_{4}}{q_{3}+q_{4}}
\end{aligned}
$$

However this is exactly $Z^{(3)}=Z^{(4)}$. Therefore the initial conjecture that $A(3)<$ $Z^{(3)}$ is not true under our assumptions. This is also true for obvious reasons for $A(4)<Z^{(4)}$. We have proven the following lemma.

Lemma 2.1. It is impossible to have $A(j)<Z^{(j)}$ for each $j=3$, 4. In fact $Z(j)=$ $A(j)$ for each $j=3,4$.

Therefore we should have that $A(j)=Z^{(j)}, j=3,4$. This implies that

$$
\begin{aligned}
& P[b u y \mid V=3]=\rho \varepsilon \\
& P[b u y \mid V=4]=\rho \varepsilon
\end{aligned}
$$

Note that this still implies that

$$
\begin{aligned}
A^{*}(3) & =\frac{3 q_{3}+4 q_{4}}{q_{3}+q_{4}} \\
A^{*}(4) & =\frac{3 q_{3}+4 q_{4}}{q_{3}+q_{4}}
\end{aligned}
$$

These are the equilibrium ask quotes whenever $\omega=3$ and $\omega=4$. Note that under these two states the insiders will not buy the security, although in a more standard sequential trade model the insider would buy at least for the realization $\omega=4$.

The bid quotes can be calculated in a similar fashion. The bids for $\omega=3$ and $\omega=4$ are given by the following expressions.

$$
\begin{aligned}
B(3) & =B(4) \\
& =P[V=3 \mid\{3,4\}, \text { sell }] 3+P[V=4 \mid\{3,4\}, \text { sell }] 4
\end{aligned}
$$

This again implies to calculate

$$
\begin{aligned}
& P[\text { sell } \mid V=3] \\
& P[\text { sell } \mid V=4]
\end{aligned}
$$

In each of the cases we have that with probability $\rho(1-\varepsilon)$ a liquidity trader will sell the security. On the other hand, if $V=3$ were true then the value $Z^{(3)}=Z^{(4)}$ is given by the former formula. That is

$$
\begin{aligned}
Z^{(3)} & =Z^{(4)} \\
& =\frac{3 q_{3}+4 q_{4}}{q_{3}+q_{4}}
\end{aligned}
$$

On the other hand, we again must assume the value of $B(3)$ to get the probabilities. Suppose that $Z^{(j)}<B(j)$ for $j=3,4$. This would imply that

$$
\begin{aligned}
& P[\text { sell } \mid V=3] \\
= & P[\text { sell } \mid V=4] \\
= & \rho(1-\varepsilon)+(1-\rho)
\end{aligned}
$$

But then

$$
\begin{aligned}
& P[V=3 \mid\{3,4\}, \text { sell }] \\
= & \frac{q_{3}}{q_{3}+q_{4}}
\end{aligned}
$$

and similarly

$$
\begin{aligned}
& P[V=4 \mid\{3,4\}, \text { sell }] \\
= & \frac{q_{4}}{q_{3}+q_{4}}
\end{aligned}
$$

Therefore

$$
\begin{aligned}
B(3) & =B(4) \\
& =\frac{3 q_{3}+4 q_{4}}{q_{3}+q_{4}}
\end{aligned}
$$

But then $B(j)=Z^{(j)}$ for $j=3,4$. This implies that our guess that $Z^{(j)}<B(j)$ was not correct. Then we will get that $Z^{(j)}=B(j)$ for $j=3,4$. This implies that

$$
\begin{aligned}
& P[\text { sell } \mid V=3] \\
= & P[\text { sell } \mid V=4] \\
= & \rho(1-\varepsilon)
\end{aligned}
$$

This gives also

$$
\begin{aligned}
& P[V=3 \mid\{3,4\}, \text { sell }] \\
= & \frac{q_{3}}{q_{3}+q_{4}}
\end{aligned}
$$

and similarly

$$
\begin{aligned}
& P[V=4 \mid\{3,4\}, \text { sell }] \\
= & \frac{q_{4}}{q_{3}+q_{4}}
\end{aligned}
$$

Therefore

$$
\begin{aligned}
B^{*}(3) & =B^{*}(4) \\
& =\frac{3 q_{3}+4 q_{4}}{q_{3}+q_{4}}
\end{aligned}
$$

Therefore $B^{*}(j)=A^{*}(j), j=3,4$. There is no trade in these two states. Therefore we can summarize this result in the following proposition.

Proposition 2.2. There is an equilibrium in the first case presented in section 2. The equilibrium bid and ask quotes are given by the following


Moreover the insider will sell the security only when $V=1$ and buy when $V=2$. There is no trade in either state $\omega=3$ or $\omega=4$.
2.4. How robust are the examples to the information specification? Cases 2 and 3.

In the last section we got the result that says that, for the information specification given in the first sub-section, the insider would not trade for speculative reasons even if the true value is high enough. However I will show that this is not a robust result in the sense that, for a different information specification we will get very different results.

Suppose now the following partitions.

$$
\mathcal{B}=\{\{1,2,3\},\{4\}\}
$$

for the dealer and

$$
\mathcal{F}=\{\{1\},\{2,3,4\}\}
$$

for the insider.
This is the case where the dealer has better a-priori information than the insider whenever the realization is the highest value.

It can be shown that the following is an equilibrium.

Proposition 2.3. There is an equilibrium in the economy presented in section 2 under the last information specification. The equilibrium bid and ask quotes are given by the following

| $j \backslash$ price | A(j) | $B(J)$ |
| :---: | :---: | :---: |
| 1 | $q_{1} \rho \varepsilon+[\rho \varepsilon+(1-\rho)]\left[2 q_{2}+3 q_{3}\right]$ | $q_{1}[\rho(1-\varepsilon)+1-\rho]+\rho(1-\varepsilon)\left[2 q_{2}+3 q_{3}\right]$ |
| , |  | $q_{1}[\rho(1-\varepsilon)+1-\rho]+\rho(1-\varepsilon)\left[q_{2}+q_{3}\right]$ |
|  | $q_{1} \rho \varepsilon+\left[\rho \varepsilon+(1-\rho)\left[q_{2}+q_{3}\right]\right.$ | $q_{1}[\rho(1-\varepsilon)+1-\rho]+\rho(1-\varepsilon)\left[q_{2}+q_{3}\right]$ |
| 3 | $\frac{q_{1} \rho \varepsilon+\rho \varepsilon+(1-\rho)\left[2 q_{2}+3 q_{3}\right]}{q_{1} \rho \varepsilon+\left[\rho \varepsilon+(1-\rho)\left[q_{2}+q_{3}\right.\right.}$ | $q_{1}[\rho(1-\varepsilon)+1-\rho)+\rho(1-\varepsilon)\left[2 q_{2}+3 q_{3}\right]$ |
| 4 | $q_{1} \rho \varepsilon+[\rho \varepsilon+(1-\rho)]\left[q_{2}+q_{3}\right]$ | $q_{1}[\rho(1-\varepsilon)+1-\rho]+\rho(1-\varepsilon)\left[q_{2}+q_{3}\right]$ |

Moreover the insider will sell the security only when $V=1$. The insider buys when $V=2$ and $V=3$. There is no trade in state $V=4$.

## Proof. See appendix $A$.

This demonstrates that these types of sequential trade models are not very robust to the assumptions about the information availability. In fact we can get exactly the opposite result if we change the information sets as follows.

Suppose now the following partitions.

$$
\mathcal{B}=\{\{1\},\{2,3,4\}\}
$$

for the dealer and

$$
\mathcal{F}=\{\{1,2,3\},\{4\}\}
$$

for the insider.
We state the following proposition.

Proposition 2.4. There is an equilibrium in the economy presented in section 2 under the last information specification. The equilibrium bid and ask quotes are given by the following


Moreover the insider will sell the security when $V=2$ or $V=3$ The insider buys only when $V=4$. There is no trade in state $V=1$.

Proof. See appendix $B$.
This clearly shows that the just switching the information sets give totally opposite results. I refer to section 4 for further comments and interpretations.

### 2.5. A generalization

Some of the results stated above could be generalized under certain conditions. Suppose now that $V \in \Omega=\left\{s_{1}, s_{2}, \ldots, s_{p}\right\}$. Assume without loss of generality that $s_{1}>0$ and $s_{i+1}>s_{i}$. Let $q_{i} \equiv \operatorname{Pr}\left[V=s_{i}\right]$. Let also $\mathcal{F}$ be the partition of the market maker. For notational purposes, assume that $\mathcal{F}=\left\{D_{1}, \ldots, D_{n}\right\}$, where $n<p$. Here $D_{i}$ is a subset of $\Omega$, such that $D_{i} \cap D_{j}=\emptyset$ for $i \neq j$ and $\cup_{i} D_{i}=\Omega$. On the other hand define $\mathcal{B}=\left\{I_{1}, \ldots, I_{m}\right\}$, with $m \leq p$ the partition for the insider.

Take any $s$ in $\Omega$. Define $M(s) \equiv D_{i}(s) \cap I_{j}(s)$, some $i$ and $j$. This is the set that is the intersection of the elements of the partitions $\mathcal{F}$ and $\mathcal{B}$ containing the state $s$. Therefore we can state the following result.

Proposition 2.5. Suppose that $M(s)=\{s\}$. Then the insider knows this state with probability 1 whenever it is realized. Suppose instead that $M(s)$ is not a singleton. Suppose furthermore that $D_{i}(s) \subset I_{j}(s)$ strictly, where both sets are such that $M(s)=D_{i}(s) \cap I_{j}(s)$. Then, for every element $x$ in $M(s)$ the bid-ask spread is 0 and there is no speculative trade whenever $V=x$.

Proof. See appendix D.
To prove this, we use the following lemma shown in appendix $C$.

Lemma 2.6. The insider forms her own expected value $Z^{(s)}$ using an partition induced by her own initial partition and the dealer's partition.

The last proposition just constitutes a generalization of proposition (2.2). Note that in the first case of the first example the set $\{3,4\}$ is included in $\{2,3,4\}$, the first one being an element of the dealer's partition and the second one being an element of the insider's partition.

One may wonder what would be the generalizations of the other two cases in the first example. Those are possible only under some special assumptions about the priors $q_{s}$.

Proposition 2.7. Let $M(s)$ defined as before. Suppose it is not a singleton. Moreover, if $D_{i}(s)$ and $I_{j}(s)$ are such that

$$
\begin{equation*}
\frac{\left[\sum_{j: s_{j} \in M} q_{j} s_{j}\right]}{\left[\sum_{j: s_{j} \in M} q_{j}\right]}>\frac{\left[\sum_{\substack{i: s_{i} \in D_{i} \\ s_{i} \notin I_{j}}} q_{i} s_{i}\right]}{\left[\sum_{\substack{i: s_{i} \in D_{i} \\ s_{i} \in I_{j}}} q_{i}\right]} \tag{}
\end{equation*}
$$

If the insider does not buy under any state $s^{\prime}$ such that $s^{\prime} \in D_{i} \backslash M$, then the insider will buy the asset under any $x$ in $M$. Moreover the insider does not sell the security in $M$.

## Proof. See appendix $E$.

The condition (**) says that the expected value conditional to the set $M$ is greater than the expected value conditional to the set $D_{i} \backslash M$. This condition is satisfied in case 2 of the first example. This then says that it suffices to have a higher conditional expected value under these realizations to "induce" the insider to buy the asset (if the insider does not buy under the realizations in the set $\left.D_{i} \backslash M\right)$.

There is also a "generalization" for the case 3 of the first example.

Proposition 2.8. Let $M(s)$ defined as before. Suppose it is not a singleton. Moreover, if $D_{i}(s)$ and $I_{j}(s)$ are such that

$$
\begin{equation*}
\frac{\left[\sum_{j: s_{j} \in M} q_{j} s_{j}\right]}{\left[\sum_{j: s_{j} \in M} q_{j}\right]}<\frac{\left[\sum_{\substack{i: s_{i} \in D_{i} \\ s_{i} \notin I_{j}}} q_{i} s_{i}\right]}{\left[\sum_{\substack{i: s_{i} \in D_{i} \\ s_{i} \in I_{j}}} q_{i}\right]} \tag{***}
\end{equation*}
$$

If the insider does not sell under any state $s^{\prime}$ such that $s^{\prime} \in D_{i}, s^{\prime} \notin M$ then the insider will sell the asset under any $x$ in M. Moreover the insider does not buy the security in $M$.

Proof. The argument is symmetric from proposition 2.7 and it is left to the reader.

The interpretation of this proposition is also symmetric from the one of proposition 2.7.

## 3. The Second Example: Non-Partitional Information Structures.

### 3.1. The Model

This second example is more similar to the usual sequential trade models. There are two periods and the asset has a future value that could only take two values, $\{1,2\}$. We still interpret $V=1$ as the case where the firm goes bankrupt. The timing is as in section 2. The notation is similar as in last section. There is a measure $\rho$ of liquidity traders who buy with probability $\varepsilon$ and sell with probability $(1-\varepsilon)$. Assume that $\rho \in(0,1)$ and $\varepsilon \in(0,1)$. There is a measure $(1-\rho)$ of insiders. The information structure is as follows. The market maker does not have any way of differentiating between the two values. Therefore the information structure for the dealer is just given by:

$$
\mathcal{B}=\{1,2\}
$$

The structure for the insider's information is somehow non-standard. I follow closely Geanakoplos [5] for this. If the true realization is $V=1$ then I assume that the insider can know this entirely. That is, if $V=1$, the set observed by the insider is $\{1\}$. However, if the true realization is $V=2$ the insider does not have any way of figuring this out. This implies that if $V=2$ the insider observes $\{1,2\}$, the entire state space.

What is the sense of this? We can interpret this as an investor who could only know correctly if the firm will go bankrupt. That is, she receives some news about the firm going bankrupt. However, if the insider does not receive any bad news she cannot infer that the firm will not go bankrupt. The ignorance of news does not preclude the bankruptcy possibility, at least from the insider's point of view. Notice the difference with the section 2 model. In that case the signal received by the insider was clear enough to know if the firm will or will not go bankrupt, i.e., whether $V=1$ or $V>1$. In this second case considered here the signal observed by the insider is not that clear. If the true state is $V=1$ then the signal conveys true information to the insider. But if the true state were $V=2$ then the signal is so "confusing" that the insider still thinks it is possible to have the firm going bankrupt. Another interpretation is that the insider is "bounded rational" in the following sense. The information processing of the signal by the insider is such that she could only infer correct information from one of the values of the signal, the "bad" state value. However the insider is not able to get any information whenever she receives a "good" signal. (Maybe we could say that the insider is "pessimistic" in the sense that even though she receives a good signal she still believes that the firm could do well or bad). I also refer to the introduction to get an interpretation of "pessimistic" behavior.

In any case this is the assumption used in the original work by Geanakoplos to model boundedly rational players in games. I study the implications of this different information structure for the formation of quotes.

### 3.2. Equilibrium analysis

I also suppose that assumption 1 given in the last section is true. The conditions characterizing an equilibrium are the same as in the last section and as in the standard sequential trade models. The ask in either state is


Note again that in equilibrium the market maker does not differentiate between $V=1$ and $V=2$. That is why the quotes must be the same regardless of the realization of the true value.

We compute the conditional probabilities in the usual fashion.

$$
\begin{aligned}
& P[V=1 \mid \text { buy }] \\
= & \frac{P[\text { buy } \mid V=1] P[V=1]}{P[\text { buy } \mid V=1] P[V=1]+P[\text { buy } \mid V=2] P[V=2]}
\end{aligned}
$$

$$
\begin{aligned}
& P[V=2 \mid \text { buy }] \\
= & \frac{P[b u y \mid V=2] P[V=2]}{P[b u y \mid V=1] P[V=1]+P[b u y \mid V=2] P[V=2]}
\end{aligned}
$$

We consider again the two possible realizations $V=1$ and $V=2$ separately.

### 3.2.1. Case $V=1$

If this is the case then the insider knows exactly that $V=1$. Then the value of $Z$ as defined in section 2 is just $Z=1$. Note that the ask $A$ is strictly greater than 1 and strictly below 2 . Therefore if $V=1$ is true then the insider will not buy the asset. Then

$$
P[b u y \mid V=1]=\rho \varepsilon
$$

### 3.2.2. Case $V=2$

In this case the insider does not know the true state. Therefore the value of $Z$ under this assumption is

$$
Z^{(2)}=q_{1}+2 q_{2}
$$

I claim that there is an equilibrium where the insider does not buy, given that whenever $Z^{(2)}=A$ then the insider decides not to buy the security. Suppose that this is the case. Therefore

$$
\begin{aligned}
P[b u y \mid V=2] & =\rho \varepsilon \\
& =P[b u y \mid V=1]
\end{aligned}
$$

If this is the case, then the conditional probabilities are given by

$$
\begin{aligned}
& P[V=1 \mid \text { buy }]=q_{1} \\
& P[V=2 \mid \text { buy }]=q_{2}
\end{aligned}
$$

Then this implies that the ask is

$$
\begin{aligned}
A & =q_{1}+2 q_{2} \\
& =Z^{(2)}
\end{aligned}
$$

Therefore this is an equilibrium value for the ask.
The bids are given by

$$
\begin{aligned}
B & =E[V \mid \text { sell }] \\
& =P[V=1 \mid \text { sell }]+2[V=2 \mid \text { sell }]
\end{aligned}
$$

Note again that $B \in(1,2)$.
This implies that if $V=1$ then the insider will sell the security since $Z^{(1)}=1$. Therefore

$$
P[\text { sell } \mid V=1]=\rho(1-\varepsilon)+(1-\rho)
$$

If $V=2$ we again have the problem to know if the insider will sell or not. We guess that there is an equilibrium where the insider does not sell if $V=2$. Suppose that this is the true value. Then the insider has $Z^{(2)}=q_{1}+2 q_{2}$ as the expected value of the asset. If the insider does not sell is because $Z^{(2)}>B$. Suppose this is true. Therefore

$$
P[\text { sell } \mid V=2]=\rho(1-\varepsilon)
$$

This implies the following expressions for the conditional probabilities are:

$$
\begin{aligned}
& P[V=1 \mid \text { sell }]=\frac{q_{1}[\rho(1-\varepsilon)+(1-\rho)]}{q_{1}[\rho(1-\varepsilon)+(1-\rho)]+q_{2} \rho(1-\varepsilon)} \\
& P[V=2 \mid \text { buy }]=\frac{q_{2} \rho(1-\varepsilon)}{q_{1}[\rho(1-\varepsilon)+(1-\rho)]+q_{2} \rho(1-\varepsilon)}
\end{aligned}
$$

and the bid is

$$
B=\frac{q_{1}[\rho(1-\varepsilon)+(1-\rho)]+2 q_{2} \rho(1-\varepsilon)}{q_{1}[\rho(1-\varepsilon)+(1-\rho)]+q_{2} \rho(1-\varepsilon)}
$$

Therefore we must have that

$$
q_{1}+2 q_{2}>\frac{q_{1}[\rho(1-\varepsilon)+(1-\rho)]+2 q_{2} \rho(1-\varepsilon)}{q_{1}[\rho(1-\varepsilon)+(1-\rho)]+q_{2} \rho(1-\varepsilon)}
$$

which is true if and only if:

$$
\begin{equation*}
q_{1}[\rho(1-\varepsilon)+(1-\rho)]\left[q_{1}+2 q_{2}-1\right]+q_{2} \rho(1-\varepsilon)\left[q_{1}+2 q_{2}-2\right]>0 \tag{*}
\end{equation*}
$$

This is the condition to have an equilibrium where the insider only sells in the event of $V=1$. Note that this is true when $\rho=0$. By continuity we can say that this equilibrium exists as long as the number of liquidity traders is small enough (although positive).

Note that this also implies that the presence of the usual bid-ask spread. However this model predicts that the insider would only sell the asset as long as she knows that the true value is low (or that the firm is going bankrupt). However she will never buy it (provided that inaction is true whenever the insider is indifferent). On the other hand the bid-ask spread exists regardless of the realization of $V$. This is relevant in the sense that when the realization is $V=2$ there is no adverse selection problem. However the type of information sets faced by the insider implies the presence of the spread even in the absence of asymmetric information under $V=2$.

We summarize this result in the following proposition.
Proposition 3.1. If condition (*) is true then there is an equilibrium where the quotes are given by:

$$
\begin{aligned}
A^{*} & =q_{1}+2 q_{2} \\
B^{*} & =\frac{q_{1}[\rho(1-\varepsilon)+(1-\rho)]+2 q_{2} \rho(1-\varepsilon)}{q_{1}[\rho(1-\varepsilon)+(1-\rho)]+q_{2} \rho(1-\varepsilon)}
\end{aligned}
$$

In this equilibrium there is only a speculative sell if $V=1$, but there is no speculative buy.

## 4. Comment about the two examples.

The first example gives some clarity about the sensitivity of the equilibrium outcomes to the informational assumptions. This may indicate a potential problem of the sequential trade models. In the standard framework ([1]) the assumption is that the insider has perfect information about the true value while the market maker does not have any extra signal. The dealer only infers some of the information through trade. What this first example suggests is that, whenever the usual assumption is not true, then many other possibilities could arise, depending upon how the information partitions observed by each agent are. In a sense people who do not like these types of models could argue against them saying that the equilibrium is "too" dependent on the information available. On the other hand people who work with these models could use this same example to defend this framework. This example could be used to say that this type of models is "flexible" enough so that, by changing the information set in a suitable way, it is possible to generate different possible equilibrium outcomes. Moreover these models might be useful to infer the information available to the traders by looking at alternative equilibrium outcomes and contrasting them with the data. I refer to further potential developments in the last section.

In the second example we get that the speculative trading is just relegated to a sell if the true value is low. There is no kind of buy. This second example could be used to start a line of research towards more general models of "pessimistic behavior" in trading. As already mentioned, this type of information sets (which are non partitional) may be interpreted as a simple construction reflecting more complicated behavior such as pessimistic or optimistic. Although I do not plan to cover this case, it is natural to think about "optimistic" behavior from the part of the insider if the information sets look like the following:


This implies that, even though the true realization is $V=1$, the insider still considers as a possible value the high one.

There is a strong problem with this interpretation though. In the second example we see that whenever the true value is $V=1$, the insider still considers as possible both $V=1$ and $V=2$. However the priors do not change when the true realization is $V=1$. Consider the story told at the beginning of the section 3. Under this interpretation the insider receives some "secret" information from some person about the performance of the firm. Whenever this performance is good, the person informs the insider. Since this is "pessimistic" she still considers possible both realizations of the firm. However, the priors of the insider are not changed even though this person gives to the insider good news about the performance of the firm. This is shown by the fact that, whenever the insider calculates the conditional expected value when
$V=2$ she uses the original priors. This may have an non-very intuitive appeal. One could argue that, despite the fact that a trader may be pessimistic, she should use the information at least to update the priors in some way. Instead, in this case not only the insider still believes possible that the bad outcome is possible even though the true value is high, but also she still it is possible under the same priors as before the realization of the high value takes place. This represents an actual problem for the second example. Nevertheless I still believe that it may constitute a first attempt of modelling boundedly rational traders and also pessimistic behavior.

## 5. Concluding Remarks and Future Research

The main conclusion we get from these two examples is that the assumption about the information availability for each trader is not trivial at all whenever one has to use the sequential trade framework. The first example shows clearly this, by changing the information partitions available to each of the traders. The trade pattern as well as the bid-ask spread are clearly different across different information assumptions. The second example also shows this. In addition the second example gives a first attempt of modelling boundedly rational behavior using sequential trade models by using the "non-partitional information" device used for the first time by Geanakoplos[5].

There is an important remark about assumption 1. As the reader may have already noticed, this assumption is crucial for most of the results, since I use it whenever I justify a case of no-trade. It would be important to explore how the results are changed whenever assumption 1 is not true. In particular, assuming that under indifference the insider trades may lead to very different equilibrium outcomes. I leave this to further developments. The important point here is that some type of behavioral supposition is needed by the risk-neutrality assumption.

This paper offers a good set of possible extensions. On the theoretical side these cases must be generalized to include multiperiod economies, as well as the possibility of not having any realization at all. This is important to see how the results given by Easley and O'Hara among others about the characterization of the quotes processes are changed for different information partitions. Another interesting point to study is the possibility of having revelation of information through time. In a very different framework, Kawamura [7] showed some examples of finitely repeated games without partitional information sets where the players were able to learn the true realized state of nature after the first round of play. Ideas like this may be important to see if the problem of "non-robustness" of the equilibria is just a problem of considering only one period or if it is a more relevant issue.

On the empirical side it is obvious that a multiperiod version of this could be tested in the same way as the standard models are tested (see, for example, [3]). Moreover it would be possible to test different information assumptions by solving
for the equilibrium under different informational assumptions and then estimating the parameters under the equilibrium. In this way one could defend the sequential trade models since in this way it is possible to infer the information available to each type of trader by testing the equilibrium under different information partitions using econometric techniques.

## A. Appendix: proof of Proposition 2.3

Suppose first that the true realization is $V=1$. In this case the insider knows perfectly the value due to the information that is given to her. Therefore $Z^{(1)}=1$ under this assumption. By the usual formula for the ask and the bid (similar to the formula given in section 2, subsection 2.2) both are strictly in the interval $(1,3)$. Therefore it is true that $A(1)>Z^{(1)}$ and $B(1)>Z^{(1)}$. Then the following is true

$$
\begin{aligned}
P[\operatorname{sell} \mid V=1] & =\rho(1-\varepsilon)+1-\rho \\
P[b u y \mid V=1] & =\rho \varepsilon
\end{aligned}
$$

On the other hand $V=4$ implies that $A(4)=B(4)=4$. Since the bid and the ask are both $\mathcal{B}$ - measurable, then the insider can infer the true value whenever $V=4$ since the partition faced by the insider (ex-post) is constituted by the intersection of the sets of the original insider's partition and the elements of the partition generated by the bid and the ask, which is just $\mathcal{B}$. Therefore $Z^{(4)}=4$ and by our no-trade assumption there is no incentive to trade.

Suppose now that $V=2$ or $V=3$. In this case $Z^{(j)}=\left[2 q_{2}+3 q_{3}\right] /\left[q_{2}+q_{3}\right]$ for $j=2,3$. Suppose that the market maker knows that with the bid and the ask corresponding to these two values the insider does not sell but she buys the security. Then we have the following conditional probabilities.

$$
\begin{aligned}
P[\text { buy } \mid V=j] & =\rho \varepsilon+1-\rho \\
P[\operatorname{sell} \mid V=j] & =\rho(1-\varepsilon) \\
j & =2,3
\end{aligned}
$$

With this information we can calculate the bid and the ask for $j=1,2,3$. They are given by the conditional expectations in the usual fashion. After some tedious calculations we get that

$$
\begin{aligned}
A(j) & =\frac{q_{1} \rho \varepsilon+[\rho \varepsilon+(1-\rho)]\left[2 q_{2}+3 q_{3}\right]}{q_{1} \rho \varepsilon+[\rho \varepsilon+(1-\rho)]\left[q_{2}+q_{3}\right]} \\
B(j) & =\frac{q_{1}[\rho(1-\varepsilon)+1-\rho]+\rho(1-\varepsilon)\left[2 q_{2}+3 q_{3}\right]}{q_{1}[\rho(1-\varepsilon)+1-\rho]+\rho(1-\varepsilon)\left[q_{2}+q_{3}\right]}
\end{aligned}
$$

If the guesses are true, then it must be the case that for $j=2,3, Z^{(j)}>A^{(j)}$ and $Z^{(j)}>B^{(j)}$. Note that by our assumptions

$$
q_{1} \rho \varepsilon\left(q_{2}+2 q_{3}\right)>0
$$

This implies

$$
\begin{aligned}
& q_{1} \rho \varepsilon\left(q_{2}+2 q_{3}\right)+[\rho \varepsilon+1-\rho]\left[2 q_{2}+3 q_{3}\right]\left[q_{2}+q_{3}\right] \\
> & {[\rho \varepsilon+1-\rho]\left[2 q_{2}+3 q_{3}\right]\left[q_{2}+q_{3}\right] }
\end{aligned}
$$

Then

$$
\begin{aligned}
& q_{1} \rho \varepsilon\left((2-1) q_{2}+(3-1) q_{3}\right)+[\rho \varepsilon+1-\rho]\left[2 q_{2}+3 q_{3}\right]\left[q_{2}+q_{3}\right] \\
>\quad & {[\rho \varepsilon+1-\rho]\left[2 q_{2}+3 q_{3}\right]\left[q_{2}+q_{3}\right] }
\end{aligned}
$$

$$
\begin{aligned}
& q_{1} \rho \varepsilon\left(2 q_{2}+3 q_{3}\right)+[\rho \varepsilon+1-\rho]\left[2 q_{2}+3 q_{3}\right]\left[q_{2}+q_{3}\right] \\
> & q_{1} \rho \varepsilon\left(q_{2}+q_{3}\right)+[\rho \varepsilon+1-\rho]\left[2 q_{2}+3 q_{3}\right]\left[q_{2}+q_{3}\right]
\end{aligned}
$$

$$
\begin{aligned}
& \\
& >\quad
\end{aligned} \begin{aligned}
& {\left[q_{1} \rho \varepsilon+[\rho \varepsilon+1-\rho]\left[q_{2}+q_{3}\right]\right]\left[2 q_{2}+3 q_{3}\right]} \\
& {\left[q_{1} \rho \varepsilon+[\rho \varepsilon+1-\rho]\left[2 q_{2}+3 q_{3}\right]\right]\left[q_{2}+q_{3}\right]}
\end{aligned}
$$

or

$$
\frac{\left[2 q_{2}+3 q_{3}\right]}{\left[q_{2}+q_{3}\right]}>\frac{q_{1} \rho \varepsilon+[\rho \varepsilon+1-\rho]\left[2 q_{2}+3 q_{3}\right]}{q_{1} \rho \varepsilon+[\rho \varepsilon+1-\rho]\left[q_{2}+q_{3}\right]}
$$

This implies that $Z^{(j)}>A(j)$ for $j=2,3$. Therefore the insider will buy the asset, confirming the "guess" we made.

For the bid the proof is similar. By the assumptions it is true that:

$$
q_{1}[\rho(1-\varepsilon)+1-\rho]\left[q_{2}+2 q_{3}\right]>0
$$

which implies

$$
\begin{aligned}
& q_{1}[\rho(1-\varepsilon)+1-\rho]\left[q_{2}+2 q_{3}\right]+\rho(1-\varepsilon)\left(2 q_{2}+3 q_{3}\right)\left(q_{2}+q_{3}\right) \\
> & \rho(1-\varepsilon)\left(2 q_{2}+3 q_{3}\right)\left(q_{2}+q_{3}\right)
\end{aligned}
$$

Then

$$
\begin{aligned}
& q_{1}[\rho(1-\varepsilon)+1-\rho]\left[(2-1) q_{2}+(3-1) q_{3}\right]+\rho(1-\varepsilon)\left(2 q_{2}+3 q_{3}\right)\left(q_{2}+q_{3}\right) \\
> & \rho(1-\varepsilon)\left(2 q_{2}+3 q_{3}\right)\left(q_{2}+q_{3}\right)
\end{aligned}
$$

$$
\begin{aligned}
& q_{1}[\rho(1-\varepsilon)+1-\rho]\left[2 q_{2}+3 q_{3}\right]+\rho(1-\varepsilon)\left(2 q_{2}+3 q_{3}\right)\left(q_{2}+q_{3}\right) \\
> & q_{1}[\rho(1-\varepsilon)+1-\rho]\left[q_{2}+q_{3}\right]+\rho(1-\varepsilon)\left(2 q_{2}+3 q_{3}\right)\left(q_{2}+q_{3}\right)
\end{aligned}
$$

$$
\left[q_{1}[\rho(1-\varepsilon)+1-\rho]+\rho(1-\varepsilon)\left(q_{2}+q_{3}\right)\right]\left(2 q_{2}+3 q_{3}\right)
$$

$$
>\left[q_{1}[\rho(1-\varepsilon)+1-\rho]+\rho(1-\varepsilon)\left(2 q_{2}+3 q_{3}\right)\right]\left(q_{2}+q_{3}\right)
$$

Then

$$
\frac{\left(2 q_{2}+3 q_{3}\right)}{\left(q_{2}+q_{3}\right)}>\frac{\left[q_{1}[\rho(1-\varepsilon)+1-\rho]+\rho(1-\varepsilon)\left(2 q_{2}+3 q_{3}\right)\right]}{\left[q_{1}[\rho(1-\varepsilon)+1-\rho]+\rho(1-\varepsilon)\left(q_{2}+q_{3}\right)\right]}
$$

This implies $Z^{(j)}>B(j)$ for $j=2,3$. Then the insider will not sell under these two realizations. This completes the proof.

## B. Appendix: Proof of proposition 2.4

First, under $V=1$ the bid and the ask is equal to 1 . By measurability again the information sets faced actually by the insider after knowing the partition induced by the bid and the ask plus the original insider's partition imply that the insider is able to infer correctly the state of the world too. Therefore $Z^{(1)}=1$ and then there is no trade under this realization.

On the other hand when $V=4$ the insider directly infers this correctly from her original partition. Then the insider has $Z^{(4)}=4$. By the ask and the bid formula we see that both $A(4)$ and $B(4)$ are in $(2,4)$. Therefore $A(4)<Z^{(4)}$ and $B(4)<Z^{(4)}$. This implies that the insider will buy and not sell whenever $V=4$ is true.

When $V=2$ or $V=3$ again we must make some guesses about speculative trading which must be fulfilled at equilibrium. Suppose that the dealer knows that the insiders do not buy but sell the security whenever $V=2$ or $V=3$. Then the following are the conditional probabilities.

$$
\begin{aligned}
P[b u y \mid V=j] & =\rho \varepsilon \\
P[\text { sell } \mid V=j] & =\rho(1-\varepsilon)+1-\rho \\
j & =2,3
\end{aligned}
$$

If this is the case, then the asks and the bids are given by the formulae in the equilibrium. This is just the result of the typical though boring algebra.

$$
\begin{aligned}
& A(j)=\frac{\left(2 q_{2}+3 q_{3}\right) \rho \varepsilon+4 q_{4}[\rho \varepsilon+1-\rho]}{\left(q_{2}+q_{3}\right) \rho \varepsilon+q_{4}[\rho \varepsilon+1-\rho]} \\
& B(j)=\frac{\left(2 q_{2}+3 q_{3}\right)[\rho(1-\varepsilon)+1-\rho]+4 q_{4} \rho(1-\varepsilon)}{\left(q_{2}+q_{3}\right)[\rho(1-\varepsilon)+1-\rho]+q_{4} \rho(1-\varepsilon)}
\end{aligned}
$$

for $j=2,3,4$. This gives the trading behavior as assumed before. First, by assumptions

$$
q_{4}(\rho \varepsilon+1-\rho)\left[2 q_{2}+q_{3}\right]>0
$$

Therefore

$$
\begin{aligned}
& q_{4}(\rho \varepsilon+1-\rho)\left[2 q_{2}+q_{3}\right]+\left(2 q_{2}+3 q_{3}\right)\left(q_{2}+q_{3}\right) \rho \varepsilon \\
> & \left(2 q_{2}+3 q_{3}\right)\left(q_{2}+q_{3}\right) \rho \varepsilon
\end{aligned}
$$

Then

$$
\begin{aligned}
& q_{4}(\rho \varepsilon+1-\rho)\left[(4-2) q_{2}+(4-3) q_{3}\right]+\left(2 q_{2}+3 q_{3}\right)\left(q_{2}+q_{3}\right) \rho \varepsilon \\
> & \left(2 q_{2}+3 q_{3}\right)\left(q_{2}+q_{3}\right) \rho \varepsilon
\end{aligned}
$$

Thus

$$
\begin{array}{cc} 
& 4 q_{4}(\rho \varepsilon+1-\rho)\left(q_{2}+q_{3}\right)+\left(2 q_{2}+3 q_{3}\right)\left(q_{2}+q_{3}\right) \rho \varepsilon \\
> & q_{4}(\rho \varepsilon+1-\rho)\left[2 q_{2}+3 q_{3}\right]+\left(2 q_{2}+3 q_{3}\right)\left(q_{2}+q_{3}\right) \rho \varepsilon \\
& \\
> & {\left[4 q_{4}(\rho \varepsilon+1-\rho)+\left(2 q_{2}+3 q_{3}\right) \rho \varepsilon\right]\left(q_{2}+q_{3}\right)} \\
& {\left[q_{4}(\rho \varepsilon+1-\rho)+\left(q_{2}+q_{3}\right) \rho \varepsilon\right]\left(2 q_{2}+3 q_{3}\right)}
\end{array}
$$

Then

$$
\frac{\left[4 q_{4}(\rho \varepsilon+1-\rho)+\left(2 q_{2}+3 q_{3}\right) \rho \varepsilon\right]}{\left[q_{4}(\rho \varepsilon+1-\rho)+\left(q_{2}+q_{3}\right) \rho \varepsilon\right]}>\frac{\left(2 q_{2}+3 q_{3}\right)}{\left(q_{2}+q_{3}\right)}
$$

This implies $A(j)>Z^{(j)}$ for $j=2,3$. Then the insider will not buy the security.
On the other hand

$$
q_{4} \rho(1-\varepsilon)\left[2 q_{2}+q_{3}\right]>0
$$

Therefore

$$
\begin{aligned}
& q_{4} \rho(1-\varepsilon)\left[2 q_{2}+q_{3}\right]+\left(2 q_{2}+3 q_{3}\right)\left(q_{2}+q_{3}\right)(\rho(1-\varepsilon)+1-\rho) \\
> & \left(2 q_{2}+3 q_{3}\right)\left(q_{2}+q_{3}\right)(\rho(1-\varepsilon)+1-\rho)
\end{aligned}
$$

Then

$$
\begin{aligned}
& q_{4} \rho(1-\varepsilon)\left[(4-2) q_{2}+(4-3) q_{3}\right]+\left(2 q_{2}+3 q_{3}\right)\left(q_{2}+q_{3}\right)(\rho(1-\varepsilon)+1-\rho) \\
> & \left(2 q_{2}+3 q_{3}\right)\left(q_{2}+q_{3}\right)(\rho(1-\varepsilon)+1-\rho)
\end{aligned}
$$

Thus

$$
\begin{aligned}
& 4 q_{4} \rho(1-\varepsilon)\left(q_{2}+q_{3}\right)+\left(2 q_{2}+3 q_{3}\right)\left(q_{2}+q_{3}\right)(\rho(1-\varepsilon)+1-\rho) \\
> & q_{4} \rho(1-\varepsilon)\left[2 q_{2}+3 q_{3}\right]+\left(2 q_{2}+3 q_{3}\right)\left(q_{2}+q_{3}\right)(\rho(1-\varepsilon)+1-\rho)
\end{aligned}
$$

$$
\begin{aligned}
& {\left[4 q_{4} \rho(1-\varepsilon)+\left(2 q_{2}+3 q_{3}\right)(\rho(1-\varepsilon)+1-\rho)\right]\left(q_{2}+q_{3}\right) } \\
> & {\left[q_{4} \rho(1-\varepsilon)+\left(q_{2}+q_{3}\right)(\rho(1-\varepsilon)+1-\rho)\right]\left(2 q_{2}+3 q_{3}\right) }
\end{aligned}
$$

Then

$$
\frac{\left[4 q_{4} \rho(1-\varepsilon)+\left(2 q_{2}+3 q_{3}\right)(\rho(1-\varepsilon)+1-\rho)\right]}{\left[q_{4} \rho(1-\varepsilon)+\left(q_{2}+q_{3}\right)(\rho(1-\varepsilon)+1-\rho)\right]}>\frac{\left(2 q_{2}+3 q_{3}\right)}{\left(q_{2}+q_{3}\right)}
$$

This implies that $B(j)>Z^{(j)}$. This says that the insider will sell the security under $j=2,3$. This ends the proof of the proposition.

## C. Appendix: Proof of Lemma 2.6

By definition of equilibrium the insider compares bids and asks with the following expression.

$$
Z^{(s)}=E[V \mid H, A, B](s)
$$

The insider conditions the expected value to the observed set $H$ in her own partition and the information about the bid and the ask distributions. Since the information partitions are common knowledge (although not the actual realized value) then the insider knows the distribution of the equilibrium bid and ask across different states. On the other hand it is also common knowledge that the equilibrium quotes
must be measurable with respect to the market maker's partition. Therefore the following is true

$$
\begin{aligned}
& A(y)=A\left(y^{\prime}\right) \quad \text { if } y, y^{\prime} \in D_{i}, \quad \text { same } i \\
& B(y)=B\left(y^{\prime}\right) \quad \text { if } y, y^{\prime} \in D_{i}, \quad \text { same } i \\
& A(y) \neq A\left(y^{\prime}\right) \quad \text { if } y \in D_{i}, y^{\prime} \in D_{j}, i \neq j \\
& B(y) \neq B\left(y^{\prime}\right) \quad \text { if } y \in D_{i}, y^{\prime} \in D_{j}, i \neq j
\end{aligned}
$$

This also generates a partition of the whole state space $\Omega$, which is of course the same partition as the one observed by the specialist. In other words, the partition of the dealer and the partition induced by equilibrium bid and asks must be the same. (Again this must be so because of the measurability condition). Since this partition is then also observed by the insider in equilibrium, then the insider observes both her own partition and the dealer's partition, as claimed in the lemma.

## D. Appendix: Proof of Proposition 2.5

Suppose first that $M(s)=\{s\}$. Then by lemma (2.6) we have


Suppose now that $M(s)$ is according to the second possibility. That means that it is not a singleton and the information sets defining $M(s)$ are such that the element of the insider's partition containing $s$ is included in the information set of the dealer. This trivially implies that $M(s)=D_{i}(s)$, some $i$. Then the value of $Z^{(x)}$ for any $x$ in $M(s)$ is given by the following expression: (dropping the state $s$ from $M(s)$ )

$$
Z=\frac{\sum_{i: s_{i} \in M} s_{i} q_{i}}{\sum_{i: s_{i} \in M} q_{i}}
$$

Suppose now that the dealer thinks that with the bid and the ask she will post the insider will not buy the asset under any $x$ in $M$. Therefore we have

$$
P[b u y \mid V=x]=\rho \varepsilon
$$

Therefore

$$
P[V=x \mid b u y]=\frac{q_{x}}{\sum_{i: s_{i} \in M} q_{i}}
$$

for any $x$ in $M$. Then the ask is equal to

$$
A(x)=\frac{\sum_{i: s_{i} \in M} s_{i} q_{i}}{\sum_{i: s_{i} \in M} q_{i}}
$$

which is equal to $Z$. By assumption 1, the insider does not buy the asset, confirming the belief of the dealer.

Similarly suppose now that the market maker thinks that the insider does not sell under any of the states in $M$. Therefore for any $x$ in $M$

$$
P[\text { sell } \mid V=x]=\rho(1-\varepsilon)
$$

Then

$$
P[V=x \mid \text { sell }]=\frac{q_{x}}{\sum_{i: s_{i} \in M} q_{i}}
$$

for any $x$ in $M$. Then the bid is equal to

$$
B(x)=\frac{\sum_{i: s_{i} \in M} s_{i} q_{i}}{\sum_{i: s_{i} \in M} q_{i}}
$$

which is equal to $Z$. By assumption 1, the insider does not sell the asset, confirming the belief of the dealer.

Moreover the bid and the ask coincide for any $x$ in $M$. Then there is no bid-ask spread, as claimed.

## E. Appendix: Proof of Proposition 2.7.

Suppose that the market maker believes that under the ask and the bid she posts the insider buys the asset under any $x$ in $M$. Therefore

$$
P[b u y \mid V=x]=\rho \varepsilon+(1-\rho)
$$

Note that by assumption, for any $y$ in $D_{i}$ and not in $M$ we have

$$
P[b u y \mid V=y]=\rho \varepsilon
$$

Therefore we have the following conditional probabilities

$$
\begin{aligned}
P[V=x \mid \text { buy }] & =\frac{q_{x}[\rho \varepsilon+1-\rho]}{\left[\sum_{\substack{i: s_{i} \in D \\
s_{i} \in M}} q_{i}\right] \rho \varepsilon+\left[\sum_{j: s_{j} \in M} q_{j}\right][\rho \varepsilon+1-\rho]} \\
x & \in M \\
P[V=y \mid \text { sell }] & =\frac{q_{y} \rho \varepsilon}{\left[\sum_{\substack{i: s_{i} \in D \\
s_{i} \notin M}} q_{i}\right] \rho \varepsilon+\left[\sum_{j: s_{j} \in M} q_{j}\right][\rho \varepsilon+1-\rho]} \\
y & \in D_{i}, y \notin M
\end{aligned}
$$

This gives the following ask:

$$
A=\frac{\left[\sum_{\substack{i: s_{i} \in D \\ s_{i} \in M}} q_{i} s_{i}\right] \rho \varepsilon+\left[\sum_{j: s_{j} \in M} q_{j} s_{j}\right][\rho \varepsilon+1-\rho]}{\left[\sum_{\substack{i: s_{i} \in D \\ s_{i} \notin M}} q_{i}\right] \rho \varepsilon+\left[\sum_{j: s_{j} \in M} q_{j}\right][\rho \varepsilon+1-\rho]}
$$

On the other hand, we know that

$$
Z^{(x)}=\frac{\left[\sum_{j: s_{j} \in M} q_{j} s_{j}\right]}{\left[\sum_{j: s_{j} \in M} q_{j}\right]}
$$

for any $x$ in $M$. This is true by Lemma 2.6. Then the dealer's belief is confirmed if $Z^{(x)}>A(x)$ for all $x$ in $M$.

If the condition stated in the proposition is true then we clearly have

$$
\left[\sum_{\substack{i: s_{i} \in D_{i} \\ s_{i} \notin I_{j}}} q_{i}\right]\left[\sum_{j: s_{j} \in M} q_{j} s_{j}\right]-\left[\sum_{\substack{i: s_{i} \in D_{i} \\ s_{i} \notin I_{j}}} q_{i} s_{i}\right]\left[\sum_{j: s_{j} \in M} q_{j}\right]>0
$$

This gives:

$$
\rho \varepsilon\left\{\left[\sum_{\substack{i: s_{i} \in D_{i} \\ s_{i} \notin I_{j}}} q_{i}\right]\left[\sum_{j: s_{j} \in M} q_{j} s_{j}\right]-\left[\sum_{\substack{i: s_{i} \in D_{i} \\ s_{i} \notin I_{j}}} q_{i} s_{i}\right]\left[\sum_{j: s_{j} \in M} q_{j}\right]\right\}>0
$$

Therefore

$$
\begin{aligned}
& \rho \varepsilon\left[\sum_{\substack{i: s_{i} \in D_{i} \\
s_{i} \notin I_{j}}} q_{i}\right]\left[\sum_{j: s_{j} \in M} q_{j} s_{j}\right]>\rho \varepsilon\left[\sum_{\substack{i: s_{i} \in D_{i} \\
s_{i} \notin I_{j}}} q_{i} s_{i}\right]\left[\sum_{j: s_{j} \in M} q_{j}\right] \\
& \rho \varepsilon\left[\sum_{\substack{i: s_{i} \in D_{i} \\
s_{i} \in I_{j}}} q_{i}\right]\left[\sum_{j: s_{j} \in M} q_{j} s_{j}\right]+[\rho \varepsilon+1-\rho]\left[\sum_{j: s_{j} \in M} q_{j} s_{j}\right]\left[\sum_{j: s_{j} \in M} q_{j}\right] \\
&> \rho \varepsilon\left[\sum_{\substack{i: s_{i} \in D_{i} \\
s_{i} \notin I_{j}}} q_{i} s_{i}\right]\left[\sum_{j: s_{j} \in M} q_{j}\right]+[\rho \varepsilon+1-\rho]\left[\sum_{j: s_{j} \in M} q_{j} s_{j}\right]\left[\sum_{j: s_{j} \in M} q_{j}\right] \\
&>\left\{\begin{array}{l}
\left.\rho \varepsilon\left[\sum_{\substack{i: s_{i} \in D_{i} \\
s_{i} \notin I_{j}}} q_{i}\right]+[\rho \varepsilon+1-\rho]\left[\sum_{j: s_{j} \in M} q_{j}\right]\right\}\left[\sum_{j: s_{j} \in M} q_{j} s_{j}\right] \\
>
\end{array}\right. \\
&\left.\left\{\sum_{\substack{i: s_{i} \in D_{i} \\
s_{i} \notin I_{j}}} q_{i} s_{i}\right]+[\rho \varepsilon+1-\rho]\left[\sum_{j: s_{j} \in M} q_{j} s_{j}\right]\right\}\left[\sum_{j: s_{j} \in M} q_{j}\right]
\end{aligned}
$$

Then

$$
\begin{aligned}
& \frac{\left[\sum_{j: s_{j} \in M} q_{j} s_{j}\right]}{\left[\sum_{j: s_{j} \in M} q_{j}\right]} \\
> & \frac{\left\{\rho \varepsilon\left[\sum_{\substack{i: s_{i} \in D_{i} \\
s_{i} \in I_{j}}} q_{i} s_{i}\right]+[\rho \varepsilon+1-\rho]\left[\sum_{j: s_{j} \in M} q_{j} s_{j}\right]\right\}}{\left\{\rho \varepsilon\left[\sum_{\substack{i: s_{i} \in D_{i} \\
s_{i} \notin I_{j}}} q_{i}\right]+[\rho \varepsilon+1-\rho]\left[\sum_{j: s_{j} \in M} q_{j}\right]\right\}}
\end{aligned}
$$

This is just $Z^{(x)}>A(x)$ for all $x$ in $M$. Then condition (**) is sufficient to have the insider buying the security under any of the realizations in $M$.

On the other hand, let us suppose that the market maker believes that the insider does not sell the security under any realization $x$ in $M$. This implies that for any $x$ in $M$.

$$
P[\text { sell } \mid V=x]=\rho(1-\varepsilon)
$$

We have two subcases. Suppose that for all $y$ in $D_{i}$ and not in $M$ we also have

$$
P[\text { sell } \mid V=y]=\rho(1-\varepsilon)
$$

Then the bid would be

$$
B^{\prime}=\frac{\sum_{i: s_{i} \in M \cup D_{i}} q_{i} s_{i}}{\sum_{i: s_{i} \in M \cup D_{i}} q_{i}}
$$

Note that the value $Z^{(s)}$ is

$$
Z^{(s)}=\frac{\sum_{i: s_{i} \in M} q_{i} s_{i}}{\sum_{i: s_{i} \in M} q_{i}}
$$

for any $s$ in $M$. Therefore we must have (in this subcase) that $Z^{(s)}>B(s)$.
Again, condition (**) implies

This implies

$$
\begin{aligned}
& {\left[\sum_{\substack{i: s_{i} \in D_{i} \\
s_{i} \notin I_{j}}} q_{i}\right]\left[\sum_{j: s_{j} \in M} q_{j} s_{j}\right]>\left[\sum_{\substack{i: s_{s} \in D_{i} \\
s_{i} \not I_{j}}} q_{i} s_{i}\right]\left[\sum_{j: s_{j} \in M} q_{j}\right]} \\
& {\left[\sum_{\substack{i: s_{i} \in D_{i} \\
s_{i} \notin I_{j}}} q_{i}\right]\left[\sum_{j: s_{j} \in M} q_{j} s_{j}\right]+\left[\sum_{j: s_{j} \in M} q_{j}\right]\left[\sum_{j::_{j} \in M} q_{j} s_{j}\right]} \\
& >\left[\sum_{\substack{i: s_{i} \in D_{i} \\
s_{i} \notin I_{j}}} q_{i} s_{i}\right]\left[\sum_{j: s_{j} \in M} q_{j}\right]+\left[\sum_{j: s_{j} \in M} q_{j}\right]\left[\sum_{j: s_{j} \in M} q_{j} s_{j}\right] \\
& \left(\sum_{j: s_{j} \in M} q_{j} s_{j}\right]\left\{\left[\sum_{j: s_{j} \in M} q_{j}\right]+\left[\sum_{\substack{i: s_{s} \in D_{i} \\
s_{i} \in \not I_{j}}} q_{i}\right]\right\} \\
& >\left[\sum_{j: s_{j} \in M} q_{j}\right]\left\{\left[\sum_{\substack{i: s_{i} \in D_{i} \\
s_{i} \notin I_{j}}} q_{i} s_{i}\right]+\left[\sum_{j: s_{j} \in M} q_{j} s_{j}\right]\right\} \\
& {\left[\sum_{j: s_{j} \in M} q_{j} s_{j}\right]\left\{\sum_{i: s_{i} \in M \cup D_{i}} q_{i}\right\}} \\
& >\left[\sum_{j: s_{j} \in M} q_{j}\right]\left\{\sum_{z: s_{i} \in M \cup D_{i}} q_{i} s_{i}\right\}
\end{aligned}
$$

which implies

$$
\frac{\left[\sum_{j: s_{j} \in M} q_{j} s_{j}\right]}{\left[\sum_{j: s_{j} \in M} q_{j}\right]}>\frac{\left\{\sum_{i: s_{i} \in M \cup D_{i}} q_{i} s_{i}\right\}}{\left\{\sum_{i: s_{i} \in M \cup D_{i}} q_{i}\right\}}
$$

which implies that $Z^{(s)}>B(s)$. Therefore the insider sells under the case in which the insider does not sell for realizations in $D_{i} \backslash M$.

Suppose now that the insider sells the security for realizations in $D_{i} \backslash M$. Therefore we have

$$
P[\text { sell } \mid V=x]=\rho(1-\varepsilon)
$$

for $x$ in $M$ and for all $y$ in $D_{i}$ and not in $M$ we have

$$
P[\text { sell } \mid V=\eta]=\rho(1-\varepsilon)+1-\rho
$$

Then the bid would be

$$
B=\frac{[\rho(1-\varepsilon)+1-\rho]\left[\sum_{\substack{i: s_{i} \in D_{i} \\ s_{i} \notin I_{j}}} q_{i} s_{i}\right]+\rho(1-\varepsilon)\left[\sum_{j: s_{j} \in M} q_{j} s_{j}\right]}{[\rho(1-\varepsilon)+1-\rho]\left[\sum_{\substack{i: s_{i} \in D_{i} \\ s_{i} \in I_{j}}} q_{i}\right]+\rho(1-\varepsilon)\left[\sum_{j: s_{j} \in M} q_{j}\right]}
$$

The value for $Z$ is the same as before. Therefore we must also have that $B<Z$ to confirm the dealer's guess.

By condition (**) we get again

$$
\left[\sum_{\substack{i: s_{i} \in D_{i} \\ s_{i} \notin I_{j}}} q_{i}\right]\left[\sum_{j: s_{j} \in M} q_{j} s_{j}\right]-\left[\sum_{\substack{i: s_{i} \in D_{i} \\ s_{i} \notin I_{j}}} q_{i} s_{i}\right]\left[\sum_{j: s_{j} \in M} q_{j}\right]>0
$$

This gives:

$$
[\rho(1-\varepsilon)+1-\rho]\left\{\left[\sum_{\substack{i: s_{i} \in D_{i} \\ s_{i} \notin I_{j}}} q_{i}\right]\left[\sum_{j: s_{j} \in M} q_{j} s_{j}\right]-\left[\sum_{\substack{i: s_{i} \in D_{i} \\ s_{i} \notin I_{j}}} q_{i} s_{i}\right]\left[\sum_{j: s_{j} \in M} q_{j}\right]\right\}>0
$$

Therefore

$$
\begin{aligned}
& {[\rho(1-\varepsilon)+1-\rho]\left[\sum_{\substack{i: s_{i} \in D_{i} \\
s_{i} \notin I_{j}}} q_{i}\right]\left[\sum_{\substack{j: s_{j} \in M}} q_{j} s_{j}\right]>[\rho(1-\varepsilon)+1-\rho]\left[\sum_{\substack{i: s_{i} \in D_{i} \\
s_{i} \notin I_{j}}} q_{i} s_{i}\right]\left[\sum_{j: s_{j} \in M} q_{j}\right] } \\
& {[\rho(1-\varepsilon)+1-\rho]\left[\sum_{\substack{i: s_{i} \in D_{i} \\
s_{i} \in I_{j}}} q_{i}\right]\left[\sum_{j: s_{j} \in M} q_{j} s_{j}\right]+\rho \varepsilon\left[\sum_{j: s_{j} \in M} q_{j} s_{j}\right]\left[\sum_{j: s_{j} \in M} q_{j}\right] } \\
&> {[\rho(1-\varepsilon)+1-\rho]\left[\sum_{\substack{i: s_{i} \in D_{i} \\
s_{i} \in I_{j}}} q_{i} s_{i}\right]\left[\sum_{j: s_{j} \in M} q_{j}\right]+\rho \varepsilon\left[\sum_{j: s_{j} \in M} q_{j} s_{j}\right]\left[\sum_{j: s_{j} \in M} q_{j}\right] } \\
&\left\{[\rho(1-\varepsilon)+1-\rho]\left[\sum_{\substack{i: s_{i} \in D_{i} \\
s_{i} \in I_{j}}} q_{i}\right]+\rho \varepsilon\left[\sum_{j: s_{j} \in M} q_{j}\right]\right\}\left[\sum_{j: s_{j} \in M} q_{j} s_{j}\right] \\
&>\left\{[\rho(1-\varepsilon)+1-\rho]\left[\sum_{\substack{i: s_{i} \in D_{i} \\
s_{i} \notin I_{j}}} q_{i} s_{i}\right]+\rho \varepsilon\left[\sum_{j: s_{j} \in M} q_{j} s_{j}\right]\right\}\left[\sum_{j: s_{j} \in M} q_{j}\right]
\end{aligned}
$$

This gives

$$
\frac{\left[\sum_{j: s_{j} \in M} q_{j} s_{j}\right]}{\left[\sum_{j: s_{j} \in M} q_{j}\right]}>\frac{[\rho(1-\varepsilon)+1-\rho]\left[\sum_{\substack{i: s_{i} \in D_{i} \\ s_{i} \notin I_{j}}} q_{i} s_{i}\right]+\rho \varepsilon\left[\sum_{j: s_{j} \in M} q_{j} s_{j}\right]}{[\rho(1-\varepsilon)+1-\rho]\left[\sum_{\substack{i: s_{i} \in D_{i} \\ s_{i} \notin I_{j}}} q_{i}\right]+\rho \varepsilon\left[\sum_{j: s_{j} \in M} q_{j}\right]}
$$

This implies that the bid is less than the conditional expected value of the asset for the insider under any $s$ in $M$. Then the insider does not sell the security as claimed.

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[^0]:    ${ }^{1}$ As long as I am concerned, this is the first paper that explores this possibility even by means of examples.

[^1]:    ${ }^{2}$ For an comprehensive survey about the microstructure theory literature see O'Hara

