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Evolutionary Market-Making: Economics vs. Finance

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Abstract

Markets are the basic institutions that lie at the heart of most economic and financial analyses. It is surprising, then, how very little work has been done on the existence of markets themselves. We investigate a model of non-sequential search, where potential buyers and sellers seek each other out. Specifically, we analyze the implications of two paradigms. The first is economic, involving Nash equilibrium and evolutionary game theory. The second is financial and is rooted in the no-arbitrage paradigm. It as well has an evolutionary interpretation. The contrast in the conditions that are required for equilibrium under these two paradigms illuminates some fundamentally different implications between economics and finance for the existence of markets and price dispersion.

Evolutionary Market-Making: Economics vs. Finance

Markets are the basic institutions that lie at the heart of most economic and financial analysis. It is surprising, then, how very little work has been done on the existence of markets themselves. Certainly we must be confident that market exist, otherwise investigations into microstructure, equilibrium, or welfare properties are essentially vacuous. In their investigations as to why agents would expend resources to create markets Gould (1978) and Kormendi (1979) show that the primordial soup of exchange contains both buyers and sellers, each playing two roles. The first of these is to actively trade, and the second, more basic role is to establish the market for the good to be traded.

This dual requirement for the existence of markets can be captured in a model of non-sequential search, where potential buyers and sellers seek each other out. Indeed, in a long-overlooked paper Gould (1980) analyzes what we believe to be the canonical model of market-making. In it, a buyer and a seller act in a Cournot-like fashion in which they search each other out at linear cost, where the probability of a successful search is determined by their joint actions. Results are then given that characterize one-sided and two-sided market-making under assumptions about the substitutability or complementarity of the search technology.

In addition, Gould (1980) leaves the reader with the following questions (among others):

> In what cases and for what commodities will brokers and other middlemen (we call them *arbitrageurs*) be used to make markets?
>
> While retaining the two-sided nature of market-making, what can be said about the situation where there are more than 2 players?
>
> At what point is there a 'perfect' market, in the sense that trade will certainly occur?

A purpose of this paper to provide answers to these questions. The process by which we arrive at these, however, is equally and perhaps more important. It illuminates some fundamentally different implications between economics and finance for the existence of markets and price dispersion.

We begin by creating a Gould-type strategic form search game. Payoffs are defined in terms of a transferable good where the gains of trade are divided between buyer and seller. We then show that Gould's (1980) assumptions about the nature of the search technology are in fact necessary and/or sufficient conditions for the existence of markets. The entire analysis is defined in terms of the standard Nash equilibrium approach to solving games.

Once we have established our benchmark we then explore solutions to the above questions. The notion that the equilibrium concept, and not the game itself, may be altered to check the institutional robustness of a game is one aspect of evolutionary game theory that we employ for this task. Specifically, a relatively new interpretation of evolutionary stable equilibria is used to represent the situation where large numbers of agents interact *locally*.[1] Furthermore, we use Nau and McCardle's (1990,1991,1992) no-arbitrage solution for games of strategy to address the effect of arbitrageurs. As it turns out, the equilibrium concept corresponding to local evolution and that which characterizes no-payoff-arbitrage are one-in-the-same. Namely, correlated equilibrium in the sense of Aumann (1974, 1987). This connection allows us to relieve some of the "discomfort" that Ross (1987) identifies as a symptom of philosophical differences between the Nash concept for games and arbitrage intuition in finance.

The main results of the paper are derived through a comparison of the solutions to the market-making game that correspond to the economic and financial paradigms. We show that the evolutionary Nash equilibrium that corresponds to a long-run interpretation of 'animal spirits' only exists under search complementarity. This outcome, which we associate with the perfectly competitive paradigm in economics, is intuitive in that competitive market-making occurs with relatively little potential for price dispersion. In contrast, a no-arbitrage solution to the same game allows for market-making under search substitutability and a large degree of price disparity. It is the case, however, that price dispersion once again disappears under the financial paradigm with search complements. In this way we demonstrate that, with respect to market-making, conditions exist under which animal spirits and the no-arbitrage criterion are equivalent (search complements) and others where they polar opposites (search substitutes).

The Model

Let there be two asymmetric roles that agents can take in an economy. Role A corresponds to sellers and role B to buyers. Each can search for the other and thereby create a market for the distribution of gains from trade, g_A and g_B, respectively. Search costs are $s_i \geq 0$, $i \in A,B$. The good is transferable with a total gain equal to g. Hence, A's gain is g_A and B's gain is $g_B = g - g_A$, where $g \geq g_A \geq 0$. The probability that they find each other is given by $\pi(s_A,s_B)$, where $\pi(0,0) = 0$ and is strictly positive if $s_i > 0$ for $i \in \{A,B\}$. Due to the completely general specification of the probabilities that are derived from $\pi(\cdot,\cdot)$, the equilibrium is not a function of the intensity of either s_A or s_B. Hence, the discrete strategic form given in Fig.(1) maintains the generality Gould's (1980) model with continuous strategies.

Figure 1: The Market-Making Game

$\downarrow s_A/s_B \rightarrow$	0	$s_B > 0$
0	0,0	$\pi(0,s_B)\cdot g_A$, $\pi(0,s_A)\cdot(g-g_A) - s_B$
$s_A > 0$	$\pi(s_A,0)\cdot g_A - s_A$, $\pi(s_A,0)\cdot(g-g_A)$	$\pi(s_A,s_B)\cdot g_A - s_A$, $\pi(s_A,s_B)\cdot(g-g_A) - s_B$

The asymmetry assumption is a *not* a departure from Gould (1980, p.S170) who asserts that,

> "*In a fundamental sense there are neither buyers nor sellers, simply traders. Each trader is simultaneously a buyer or seller.*"

[1] See Cripps (1991) and Mailath et al (1997).

We agree with Gould on this point. From an evolutionary standpoint, however, both *Gould's* and our game are asymmetric unless we assume that (i) $(g-g_A)=g_A$, (ii) $s_A=s_B$ and (iii) $\pi(s_A,0)=\pi(0,s_B)$. The first condition is a very strong assumption about the bargaining process that occurs after the market is made. We do not wish to make such an assumption. Gould as well finds that it is unlikely to hold. The latter two conditions are only used in the penultimate proposition of the paper. Finally, to foreshadow, the evolutionary solution to asymmetric games is built upon a transformation (the asymmetric contest) in which agents are randomly assigned the role of buyer or seller within our model. This again is in the spirit of Gould's remark.

Consider now the definition and interpretation of a Nash equilibrium in this setting. Let A_i denote the action (strategy) set of player 'i' and the Cartesian product, $A=\times_{i=A,B} A_i$, the set of joint strategies. The symbols a_i and 'a' denote the generic elements of A_i and A, respectively. Finally $a_{-i}\in A_{-i}$ denotes a vector of strategies for all players other than 'i.' It follows that $U_i:A\rightarrow\Re$ is player i's payoff function. A *Nash equilibrium*, (a_i^*,a_{-i}^*), satisfies the following inequality:

(1) $U_i(a_i^*,a_{-i}^*) \geq U_i(a_i,a_{-i}^*)\ \forall\ a_i \in A_i,\ \forall$ i.

The interpretation of Nash equilibrium in this game is that agents act as expected profit maximizers, choosing their optimal search strategy while taking that of the other as given. If $(s_A,s_B)=(0,0)$ no market-making occurs.

We now present and discuss a set of propositions relating to the existence of markets in the above game. All proofs are provided in the appendix.

Proposition 1: Markets fail to exist -- (0,0) is a Nash equilibrium -- iff the gains from search are sufficiently small. Specifically, if $s_A \geq \pi(s_A,0)\cdot g_A$ and $s_B \geq \pi(0,s_B)\cdot(g-g_A)$.

Very little needs to be said with reference to this proposition. Obviously, if the expected gains from trade are less than the search costs then no markets will be created. As such, the proposition is a basic test of consistency of the model. Moreover, it allows us to define Eq.(2) as the primitive or *profitability conditions* for market-making.

(2) $s_A < \pi(s_A,0)\cdot g_A$ and $s_B < \pi(0,s_B)\cdot(g-g_A)$.

Assumption: The profitability conditions given in Eq.(2) hold throughout.

The above assumption and some technological characterizations of the distribution $\pi(\cdot,\cdot)$ are required to establish the existence of market-making. Specifically, search distribution $\pi(\cdot,\cdot)$ exhibits *strategic complementarity* if

(3) $\pi(s_A,s_B) - \pi(s_A,0) > 0$ and $\pi(s_A,s_B) - \pi(0,s_B) > 0$.

On the other hand, search strategies are *strategic substitutes* if:

(4) $\pi(s_A,s_B) - \pi(s_A,0) < 0$ and $\pi(s_A,s_B) - \pi(0,s_B) < 0$.

These technical properties of search enable us to characterize the existence of market-making activity. As compared to Gould (1980), instead of assuming one condition or the other we show how each is required to establish the existence of different types of markets.

Proposition 2: Search substitutability is a sufficient condition for one sided market-making -- $(s_A,0)$ and $(0,s_B)$ -- as Nash equilibria.

Under search substitutability, the market-making game is akin to the game whose proper name is 'Chicken'. This analogy is important because the general lesson that one learns from Chicken is that one party must concede (here, incur a $s_i > 0$) in order create the gains from trade. In this context it is the market-maker that 'concedes' and initiates a costly search in order to distribute the benefits from trade. If neither concedes the market is not made.

From an evolutionary standpoint asymmetric Chicken also has an important implications for market-making. It is meant to represent the case where different players have different 'fighting prowess.' Such a story fits with the necessity of search substitutability for market-making. In essence, the 'first-moving' market-maker is at a disadvantage because she alone is needed in order to create the search externality. From a dynamic perspective, then, there is a danger for market failure as the game becomes a War of Attrition where each side waits for the other to make the market.

A natural conjecture is whether the strategic problems associated with one-sided markets are overcome through the distribution of the gains from trade? In other words, the price (or consumer's surplus) the market-maker receives ought to reflect the costs of market-making. In some sense this is true. Prop.(1) and the profitability condition that follows from it demonstrate that search costs must at least be recuperated. Beyond this, however, there is no indication of how this constraint impinges on the division of the gains from trade. Instead, this is dependent on the specifics of the bargaining process that corresponds to the market (e.g. an auction as in Vickrey 1961). This is not a drawback of our analysis, indeed, it is a verification that buyers and sellers play dual roles in the exchange process. The division of the gains from trade is the other side of the coin and studies of its determinants are too numerous to mention further. It is interesting to note, however, that to our knowledge none of these explicitly considers the question of who made the market as a starting point for analyzing the microstructure of bargaining.

Proposition 3: Search complementaries are necessary and sufficient for joint market-making -- (s_A,s_B) -- as a Nash equilibrium.

Finally, we see that market-making in the presence of complementarities requires the coordination of search activities. There is no coordination *problem*, per se, because $s_i > 0$ is a dominant strategy for each player and (s_A,s_B) is the Pareto efficient outcome. It does not, however, guarantee the existence of perfect markets.

A Necessary Condition for Doing Economics

The previous section characterizes the Nash equilibria for a fairly simple market-making game. It is shown that one-sided or two-sided market-making is intimately related to whether search technology exhibits strategic complementary or substitutes, respectively. These criteria are economic in nature, that is, they can be interpreted as transactions costs. The existence of such transactions costs begs the question as to whether these can lead to unrealized gains from trade? In this section we show that an 'animal spirits' environment -- as interpreted in our game -- results in the nonexistence of markets under search substitutes. Hence, in order to do economics -- that is to say, to assume the existence of animal spirits -- one must assume search complements.

In economic models it is often the case that results that are consistent with 'animal spirits' are derived 'in the limit.' By contrast, instead of appealing to a law of large numbers we employ evolutionary game theory. By definition, the solution concept for evolutionary games – the *evolutionary stable strategy* (ESS) -- embodies an environment where there are many agents of type A and type B. Specifically, it is meant to represent the following dynamic situation. There are a large number of agents of type A and type B who are randomly matched and then play the game. These matchings occur over time and the agents may be boundedly rational in the sense that for long periods they are not in equilibrium. Over time, however, the selection of strategies obeys a *replicator dynamic* property that is Darwinian. It satisfies the criteria that in equilibrium strategies reflect their evolutionary 'fitness.'

A priori it appears that the construction of a dynamic model to capture the process representing the 'survival-of-the-fittest' ought to significantly complicate our model. Fortunately, there exists a long history of biological and economic research that has proven that the dynamic trial and error process of evolution can be summarized with a static characterization.[2] Following Selten (1980), (a_i^*, a_{-i}^*) is an ESS for an asymmetric game iff it satisfies Eq.(5):

(5) $U_i(a_i^*, a_{-i}^*) > U_i(\hat{a}_i, a_{-i}^*) \ \forall\, \hat{a}_i \in A_i, \ \forall\, i.$

Eq.(5) is meant to summarize the concept that, given the large number of players and the random matching between them, there is no '*mutant*' strategy $\hat{a}_i$ – form of market-making behavior – that can invade and perpetuate itself within this environment by doing at least as well as the equilibrium strategy a_i^*. The evolutionary approach to modeling behavior has regained a great deal of attention in economics, "*it is particularly justified when studying generic situations* (where) *there is considerable evidence consistent with social evolution*," (Ellison 1997, p.584). As market-making is one of the most basic of socioeconomic phenomena, an evolutionary approach is certainly appropriate.

We interpret animal spirits to imply the following: *no expected gains from trade will be unexplored.* We conduct our examination of the animal spirits assumption through the introduction of a 'competitive mutant' into the market-making game. Following Prop.(1), all that is required for the consideration of the competitive mutant

[2] For a brief and intuitive introduction to evolutionary theory the reader is guided to Samuelson (1991).

is the satisfaction of the *profitability conditions* in Eq.(2). When these hold, then expected profits are generated through search. As such, our mutant is a market-maker when no other player selects this strategy. The strategic form corresponding to this situation is given in Fig.(2).[3] Specifically, the mutant strategy can be expressed as:

$$M = \begin{cases} s_M > 0 \text{ } if\ matched\ with\ s_A = 0\ or\ s_B = 0. \\ s_M = 0 \text{ } if\ matched\ with\ s_A > 0\ or\ s_B > 0. \end{cases}$$

[Insert Figure 2]

Proposition 4: Search complementarity is a necessary and sufficient condition for evolutionary market-market under 'animal spirits.' If search substitutability is present, then the animal spirits game has no ESS.

What Prop.(4) establishes is that 'animal spirits' is a paradigm under which if one believes that the creation of markets reflects an evolutionary process, then one must assume search complements. Otherwise, the evolutionary process works in such a way that markets do not exist. Yet this aspect of the result is consistent with the idea of uniform pricing, rather than price dispersion, in competitive environments. The market-making game should be thought of as a 'pre-game' or stepping stone to the bargaining process that determines the division of surplus, of which price is the primary determinant. In other words, price is likely to be a function of the market-making outcome, $p=\rho(a_A,a_B)$. Prop.(4) rules out the multiple equilibrium situation of $(s_A,0)$ and $(0,s_B)$ with the price dispersion potential of $\rho(s_A,0)\neq\rho(0,s_B)$. Hence, another interpretation of Prop.(4) is the nonexistence of price dispersion within perfectly competitive markets. This is entirely consistent with the economic paradigm.

Game-Theoretic Interpretations of an Alternative Paradigm

A fundamental question that was left open by Gould (1980) is, is there room for arbitrage in the market-making game? The purpose of this section is to explore the implications of an alternative paradigm; one whose characteristic is that no arbitrage opportunities exist. This is the fundamental paradigm of finance. In this way we are able to restore the missing arbitrage intuition that Ross (1987) cites as a distressing byproduct of the intrusion of game theory in finance.

In a series of papers Nau and McCardle (1990,1991,1992) have recently characterized the no arbitrage condition for games of strategy. The logic of their characterization is as follows. If, given the strategy a_{-i} of others, player 'i' selects a strategy a_i that does not exploit the maximum gains from trade, then there exists another strategy, $\hat{a}_i$, such that the difference $[U_i(a_i,a_{-i})-U_i(\hat{a}_i,a_{-i})]$ is negative. In this case an arbitrageur would be willing to offer the 'stakes factor' $\alpha(i,a_i,\hat{a}_i)\geq 0$ in order to recover a payoff that is proportional to the unrealized gains from trade. This payoff is

(6) $\alpha(i,a_i,\hat{a}_i)\cdot[U_i(\hat{a}_i,a_{-i})- U_i(a_i,a_{-i})]$.

[3] This mutant is inspired by Robson (1995).

Figure 2: Market-Making with 'Animal Spirits'

$\downarrow s_A/s_B \rightarrow$	0	$s_B > 0$	M
0	0,0	$\pi(0,s_B)\cdot g_A,\ \pi(0,s_B)\cdot(g-g_A)-s_B$	$\pi(0,s_B)\cdot g_A,\ \pi(0,s_B)\cdot(g-g_A)-s_B$
$s_A > 0$	$\pi(s_A,0)\cdot g_A-s_A,\ \pi(s_A,0)\cdot(g-g_A)$	$\pi(s_A,s_B)\cdot g_A-s_A,\ \pi(s_A,s_B)\cdot(g-g_A)-s_B$	$\pi(s_A,0)\cdot g_A-s_A,\ \pi(s_A,0)\cdot(g-g_A)$
M	$\pi(s_A,0)\cdot g_A-s_A,\ \pi(s_A,0)\cdot(g-g_A)$	$\pi(0,s_B)\cdot g_A,\ \pi(0,s_B)\cdot(g-g_A)-s_B$	$\pi(s_A,s_B)\cdot g_A-s_A,\ \pi(s_A,s_B)\cdot(g-g_A)-s_B$

Note: by convention we have specified in Fig.(2) that a joint search is conducted for the strategy combination (M,M). Our analysis is not affected if we instead identify the strategy pair (M,M) with the no-search payoff of (0,0).

$$\text{Recall: M} = \begin{cases} s_M > 0 \; if\ matched\ with\ s_A = 0\ or\ s_B = 0. \\ s_M = 0 \; if\ matched\ with\ s_A > 0\ or\ s_B > 0. \end{cases}$$

Moreover, the arbitrageur is not limited to interactions with a single agent, therefore he may also be seeking to profit from unexplored gains from trade with respect to a_j and $\hat{a}_j$ where $j \neq i$. An important implication is that while Nash-type players persist in holding a_{-i} constant in their best reply calculation, the arbitrageur need not. It may be possible to create arbitrage opportunities through a 'series of trades' as defined by the offering of stakes factors to more than one player in the game.[4] Hence, to preclude the creation of arbitrage opportunities (either individually or *jointly*) the players form subjective beliefs over the *joint strategy set*, A.

In other words, the game-theoretic analog of the 'states-of-the-world' is all possible joint strategy combinations in the game. In order that players do not create arbitrage opportunities that are proportional to the payoffs that result from these states, they form subjective beliefs about them. Nau and McCardle (1990) prove that these no-arbitrage beliefs are observationally equivalent to the correlated equilibrium for the game itself. This set of beliefs, $\mu(a_i,a_{-i})$, is characterized by Eqs.(7)-(9).

(7) $\mu(a) \geq 0 \ \forall \ a \in A.$

(8) $\sum_{a \in S} \mu(a) = 1.$

(9) $\sum_{a_{-i} \in A_{-i}} \mu(a_i, a_{-i}) \cdot [U_i(a_i,a_{-i}) - U_i(\hat{a}_i,a_{-i})] \geq , \ \forall \ \hat{a}_i \neq a_i, \ \hat{a}_i \in A_i, \ \forall \ i.$

Given the game in Fig.(1), the Eqs.(7)-(8) become:

(7') $\mu(0,0), \mu(s_A,0), \mu(0,s_B), \mu(s_A,s_B) \geq 0$, and

(8') $\mu(0,0) + \mu(s_A,0) + \mu(0,s_B) + \mu(s_A,s_B) = 1,$

respectively. Eq.(9) is an incentive-compatibility constraint. Referring back to the Nash inequality in Eq.(1), we see that Eq.(9) specifies that the joint strategy (a_i,a_{-i}) is a Nash equilibrium with respect to probability distribution $\mu(a_i,a_{-i})$. Readers familiar with distributions over joint strategies will recognize that Eqs.(7)-(9) are also known as a correlated equilibrium in the sense of Aumann (1974,1977).

The set of no-arbitrage/correlated equilibria for a game contains the set of Nash equilibria and is often quite larger. Moreover, the equilibrium concept itself is a natural one for games in which the financial paradigm of no-arbitrage comes into play. For example, Arce (1997b) employs this criterion in a model of incomplete information to show how market-making can have a large influence over the credibility of macroeconomic policy.

We now turn from the interpretation of correlated equilibrium under the financial paradigm to the issue of correlation and evolution. Consider the case where there is a population of size N_A of players of type A and another of size N_B of type B. Mailath et al (1997) describe a matching process with *local interactions* as one which satisfies properties (A)-(D) below:

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[4] See Arce (1997a) for an example of a fully specified payoff-arbitrage game in strategic form.

(A) There are at least as many players in each population as there are strategies: $|N_A| \geq |A_A|$, and $|N_B| \geq |A_B|$.
(B) Every $j \in N_A$ is associated with some strategy $a_j \in A_A$, and every $k \in N_B$ is associated with some strategy $a_k \in A_B$.
(C) The probability distribution $\mu(j,k)$: $N_A \times N_B \rightarrow [0,1]$ describes the interaction between the populations.
(D) The value $\mu(j,k)$ is interpreted as the probability that, given a meeting, it involves a strategy of type a_j from N_A and strategies of type a_k from N_B.

The idea behind this process is that there are a large number of players who are randomly matched. When 'j' and 'k' are matched they play the game. However, the matching process need not be uniform across the agents in each population, as is *required* in the Nash version of evolutionary stability. For example, fixing $j \in N_A$, the $\mu(j,k)$s represent the different possibilities of a local interaction with any particular member k of population N_B. That is, there is a certain structure to the interaction that they label as *local*. They then prove the following:

Result: Any Nash equilibrium, given local interaction, corresponds to a correlated equilibrium. Moreover, a pattern of local interaction always exists such that the converse also holds.

As a consequence of this result, Mailath et al (1997, p.552) state the following:

> "*The results suggest that when working with matching models, we should be interested in correlated, rather than simply Nash, equilibria.*"

and

> "*The different signals received by the players in a correlated equilibrium appear as different possibilities to for meeting other agents that arise out of the local nature of the interactions.*"

The search model we have been analyzing is certainly an example of matching. In terms of our financial paradigm, one can think of this local matching as occurring between experts as a subgroup of the population. This is consistent with the conventional wisdom that it usually is the experts who are the most likely to exploit arbitrage opportunities. Moreover, the local nature also has its roots in the spacial nature of market-making as described by Hotelling (1929). In addition, Anderlini and Ianni (1997) find that locally interactive search provides a robust rationale for price dispersion. Finally, the signals provided by the correlation mechanism are costless, in the sense that they are payoff-irrelevant. This again is part of the financial paradigm. For example, Franke (1987) associates costless signals with market-making activity.

The point of this section can now be summarized as the following: by examining the correlated equilibrium of the market-making game, we are embedding it within an institutional structure that is both financial and evolutionary.

What about Finance?

A logical conclusion of the preceding section is that the correlated equilibria of the market-making game are worth investigation. In particular, correlation changes the institutional interpretation of the game from one in which agents maximize expected profits to one where no arbitrage opportunities can remain open. Furthermore, it reflects the outcome of an evolutionary process where the matching of agents need not be uniform, as is required by the Nash approach to evolutionary stability.

Of particular interest is whether the resulting distribution on strategies can simultaneously have positive measure over the three trading outcomes: $(s_A,0)$, $(0,s_B)$ and (s_A,s_B)? An affirmative answer is useful for several reasons. First, because if all the probabilities of a successful search, $\pi(s_A,0)$, $\pi(0,s_B)$ and $\pi(s_A,s_B)$, are positive, then this is some sense gets us closer to the notion of perfect markets. No probability of a successful search is 'wasted' in the sense that it is not part of an equilibrium outcome. Second, such a distribution may result in a Pareto-improvement over the Nash market-making outcome. This latter observation is a rationale for correlated equilibrium that dates back to at least Luce and Raiffa (1957).[5] Third, such a distribution implies even greater potential for price dispersion under substitutes, corresponding to the functionals $\rho(s_A,0)$, $\rho(0,s_B)$, *and* $\rho(s_A,s_B)$.

Proposition 5: Consider the case of search substitutes. There exists a no-arbitrage evolutionary equilibrium for the market-making game where $\mu(s_A,0)$, $\mu(0,s_B)$ and $\mu(s_A,s_B)$ are all positive if the following condition holds:

(10) $\mu(s_A,s_B) \le \mu^{max}$, where:

$$(11)\ \mu^{max} = \min\left\{\frac{\mu(s_A,0)[\pi(s_A,0)\cdot g_A - s_A]}{\{\pi(0,s_B)-\pi(s_A.s_B)\}\cdot g_A + s_A}, \frac{\mu(0,s_B)[\pi(0,s_B)\cdot(g-g_A)-s_B]}{\{\pi(s_A,0)-\pi(s_A.s_B)\}\cdot(g-g_A)+s_B}\right\}$$

The profitability condition establishes that the numerator of each of the terms defining μ^{max} is positive. By search substitutes, the denominators are positive. Hence the upper bound on μ^{max} is strictly positive, implying that the theorem is not vacuous. A lower bound on μ^{max} is given in Prop.(6).

In contrast to the economic paradigm, represented by Nash behavior and its evolutionary counterpart, joint market-making can occur under search substitutes. This proposition gets us very close to a 'perfect markets' result. It establishes that, if the conditions of no arbitrage are satisfied, then an equilibrium exists where every avenue for an expected gain from trade is attempted with positive probability.

The intuition of the constraint on joint search, $\mu(s_A,s_B)$, is as follows. The numerator of each argument in Eq.(11) represents the expected payoff of a single-sided search. Recall that Prop.(2) characterized substitutability as a sufficient

[5] The question as to whether (s_A,s_B) occurs as part of a mixed ESS is vacuous because the game is asymmetric. No mixture can satisfy Eq.(5). Cripps (1991) was the first to show that a correlated distribution *can* satisfy Eq.(5) in an evolutionary context.

condition for such equilibrium. Consider now the point of view of player A, which is given in the left-hand term of Eq.(11). Given that A is searching, his single-sided payoff is now discounted by the marginal contribution of B joining this search -- which is the difference $\{\pi(0,s_B)-\pi(s_A,s_B)\}$in the denominator -- *plus* s_A. The latter is the opportunity cost that A "saves" when he is not searching, but B is. The intuition is similar for player B and the left-hand argument in Eq.(11).

Proposition 6: Consider the case of search substitutes under symmetry of search costs and probability of meeting: $s_A=s_B$ and $\pi(s_A,0)=\pi(0,s_B)$. The equilibrium of proposition 5 is a *strict Pareto-improvement* over single-sided search if the following hold:

(12) $\mu(0,s_B)\cdot s_A > \mu(s_A,s_B)\cdot[\pi(s_A,0)-\pi(s_A,s_B)]\cdot g_A$

(13) $\mu(s_A,0)\cdot s_B > \mu(s_A,s_B)\cdot[\pi(0,s_B)-\pi(s_A,s_B)]\cdot(g-g_A)$.

The correlated equilibrium outcome can exceed the expected payoff attributed to single-sided market-making. Attaining a strict Pareto-improvement is a strong rationale for considering the correlated equilibrium of the market-making game. The intuition underlying Eqs.(12)-(13) is as follows. The correlated outcome balances three effects (i) cost saving when the other is searching, (ii) an increased *overall* probability of a successful search through the inclusion of $\pi(s_A,s_B)$, and (iii) the fact that joint search has a lower expected return than one-sided search in a substitutes environment. The first effect is captured in the left-hand side of each equation. It represents the search costs saved when the equilibrium calls for the other party to conduct a one-sided search. These savings must be greater than the tradeoff described in effects (ii) and (iii), which are captured in the right-hand side of Eqs.(12)-(13).

Once again, we emphasize that these 'pre-game' results illustrate the need for further investigation into their effect on bargaining analyses that take the existence of markets as parametric. It is clear that the distribution of market-making costs is significantly different across the three trading outcomes. As such, each pertains to a fundamentally different subgame, where a bargain is held over the distribution of the gains from trade. Any forward-inductive argument would then require a different restriction on the equilibrium price, depending upon who made the market. Hence, while it may be possible to "normalize" a disagreement point at (0,0) *within* a particular bargaining subgame, such a normalization is not legitimate *across* the subgames due to the heterogeneity in market-making costs.

As a consequence, one should expect price dispersion across subgames. Recall from our animal spirits result that when price is a function of the bargaining outcome, $p=\rho(a_A,a_B)$, there was no danger of price dispersion due to multiple equilibria. Props.(5)-(6) now establish the opposite. Under the financial paradigm the price functionals $\rho(s_A,0)$, $\rho(0,s_B)$ and $\rho(s_A,s_B)$ are all generated as part of the no-arbitrage evolutionary equilibrium. There exists an even greater potential for price dispersion than is implied by the multiple equilibrium *Nash* outcome identified in Prop.(2).

Proposition 7: Consider the case of search complements. The correlated equilibrium for this case is $\mu(s_A,s_B)=1$, implying that joint market-making is the unique no-arbitrage outcome.

Prop.(7) tells us that when the search technology is complementary, then the financial paradigm is equivalent to its Nash/evolutionary counterpart. One should expect the same degree of price dispersion as well. In some sense this is to be expected, as incentives are 'aligned' under complements. Moreover, it is the case that under strategic complements joint market-making is a dominant and Pareto-efficient strategy.[6]

Together, Props.(6)-(7) show that the consideration of arbitrage need not lead to inefficiency, as is often suspected arbitrage in search models (e.g. Hogan 1991). Instead, what we find is that the no-arbitrage paradigm leads to a price-dispersion outcome that Pareto-dominates its Nash counterparts.

Conclusion

In this paper we created a model of market-making that is in the spirit of Gould (1980). Buyers and sellers have two roles. The first is to actively trade, and the second is to establish the market for the good to be traded. We focus on this second role through model of simultaneous search between buyers and sellers. The Nash equilibria that generate market-making in this game can be partitioned according to the search technology that results in a successful match. We identify search substitutes as a sufficient condition for one-sided market-making when search costs are symmetric. Conversely, search complementary alone is necessary and sufficient for joint market-making.

We then turn to the issues of market-making when the population of buyers and sellers is large, and the comparison of 'animal spirits' versus the no-arbitrage paradigm. The change in the institutional setting from a 2-player game to one where the number of players is much larger allows us to analyze the game by way of evolutionary game theory. We then analyze a representation of animal spirits, interpreted as the evolutionary Nash equilibrium where no expectedly profitable outcome goes unexplored. We show that the combination of animal spirits and search complements generates two-sided market-making with little potential for price dispersion. This result can be read as market-making justification of the perfectly competitive paradigm in economics.

We subsequently alter our paradigm to satisfy the conditions that no arbitrage opportunities are left open, rather than all expectedly profitable strategies are implemented. It turns out that this too has an evolutionary interpretation, one that relaxes the requirement of uniform matching across agents, as required in Nash equilibrium, and instead allows for local matchings. The equilibrium concept that is appropriate in this environment is correlated equilibrium, as is formally proven by Nau and McCardle (1990, 1991, 1992), Cripps (1991) and Mailath et al (1997). The results under this paradigm are identical with those for animal spirits under search complementarity.

[6] Hence, no proof is provided for Prop.(7).

The agreement between the economic and financial paradigms does not extend to the case of search substitutes. Specifically, in this case market-making does not occur under animal spirits. We interpret this as the nonexistence of price disperse markets under the perfectly competitive paradigm. In contrast, all market-making outcomes that produce some positive probability of a match between buyer and seller are part of the no-arbitrage equilibrium. This multiple equilibrium outcome exhibits the potential for price dispersion. Furthermore, we establish conditions for which this is a strict Pareto-improvement over the Nash outcome. This suggests that middlemen, or arbitrageurs as we call them, play a very important role in the establishment of markets. Moreover, it is the financial paradigm that comes closest to the existence of 'perfect' markets.

Figure 3: Price Dispersion Potential

↓Paradigm/Search→	Substitutes	Complements
Nash	$\rho(s_A,0)$, $\rho(0,s_B)$	$\rho(s_A, s_B)$
Economic/Evolutionary (Animal Spirits)	No Market-making	$\rho(s_A, s_B)$
Financial (No-Arbitrage)	$\rho(s_A,0)$, $\rho(s_A, s_B)$, $\rho(0,s_B)$	$\rho(s_A, s_B)$

In terms of future research these results have important implications for the existence of price dispersion. We show that agents may expend costly resources to make markets. In particular, under one-sided market-making the costs are asymmetric. This sets the stage for the following question: After the market is made, how does the recuperation of this cost impinge on the division of surplus? Thus, if market-making is to be taken seriously, the outcome of this 'pre-game' ought to be explicitly considered as an important microstructure datum in any analysis of bargaining. To our knowledge no game-theoretic analysis of bargaining is set in this environment. However, as Fig.(3) illustrates, if price is indeed a function of the market-making outcome, then the combination of search technologies and market-making environments lead to different price functionals, $\rho(a_A,a_B)$. If search is complementarity, then there is no difference with respect to market existence nor price dispersion. Conversely, under search substitutability the economic and financial paradigms have dramatically different implications for market-making and price dispersion.

Appendix: proofs of the propositions.

Prop.(1): Nash equilibrium requires $s_A \geq \pi(s_A,0)\cdot g_A$ and $s_B \geq \pi(0,s_B)\cdot(g-g_A)$. In other words, the best reply functions are collinear with their respective axes, thus giving the point of intersection at the origin. ■

Prop.(2): *Case A*: Nash equilibrium $(s_A,0)$ occurs under the conditions:

(14) $\pi(s_A,0)\cdot g_A \geq s_A$, which holds under the profitability condition in Eq.(2), and
(15) $\pi(s_A,0)\cdot(g-g_A) \geq \pi(s_A,s_B)\cdot(g-g_A) - s_B$

Eq.(15) can be rewritten as: $[\pi(s_A,0)\cdot - \pi(s_A,s_B)]\cdot(g-g_A) \geq -s_B$. Given that $s_B>0$, a sufficient condition for this inequality to hold is search substitutes, in which case the left-hand side of the inequality is positive. The proof for *Case B*, $(0,s_B)$ as a Nash equilibrium, follows directly. ■

Prop.(3): Mutual best replies requires $U_A(s_A,s_B) \geq U_A(0,s_B) \Rightarrow \pi(s_A,s_B)\cdot \geq \pi(s_A,0)$ and $U_B(s_A,s_B) \geq U_B(s_A,0) \Rightarrow \pi(s_A,s_B) \geq \pi(0,s_A)$. ■

Prop.(4): For the case of substitutability, $(s_A,0)$,(s_A,M), $(0,s_B)$, and (M,s_B) are all Nash equilibria, but none of these are strict. Therefore they are not ESS. For the case of joint search: (s_A,s_B) or (M,M), one needs both $\pi(s_A,s_B)\cdot g_A > \pi(0,s_B)\cdot g_A$ and $\pi(s_A,s_B)\cdot(g-g_A) > \pi(s_A,0)\cdot(g-g_A)$. Obviously, search complementarity is necessary and sufficient for these to hold.

Correlated Equilibria: From the conditions given in Eq.(9) we derive:
(16) $\mu(0,0)\cdot[\ 0 - \pi(s_A,0)\cdot g_A - s_A] + \mu(0,s_B)\cdot[\ \pi(0,s_B)\cdot g_A - \pi(s_A,s_B)\cdot g_A - s_A] \geq 0$
(17) $\mu(s_A,0)\cdot[\ \pi(s_A,0)\cdot g_A - s_A] + \mu(s_A,s_B)\cdot[\ \pi(s_A,s_B)\cdot g_A - s_A - \pi(0,s_B)\cdot g_A] \geq 0$
(18) $\mu(0,0)\cdot[0 - \pi(0,s_A)\cdot(g-g_A) - s_B] + \mu(s_A,0)\cdot[\ \pi(s_A,0)\cdot(g-g_A) - \pi(s_A,s_B)\cdot(g-g_A) - s_B] \geq 0$
(19) $\mu(0,s_B)\cdot[\pi(0,s_B)\cdot(g-g_A) - s_B] + \mu(s_A,s_B)\cdot[\ \pi(s_A,s_B)\cdot(g-g_A) - s_B - \pi(s_A,0)\cdot(g-g_A)] \geq 0$

Simple algebra and rearrangement of terms yields Eqs.(20)-(23):

(20) $\mu(0,0)\cdot[\ s_A - \pi(s_A,0)\cdot g_A] + \mu(0,s_B)\cdot[\{\pi(0,s_B)-\pi(s_A,s_B)\}\cdot g_A - s_A] \geq 0$
(21) $\mu(s_A,0)\cdot[\ \pi(s_A,0)\cdot g_A - s_A] + \mu(s_A,s_B)\cdot[\{\pi(s_A,s_B)-\pi(0,s_B)\}\cdot g_A - s_A] \geq 0$
(22) $\mu(0,0)\cdot[s_B - \pi(0,s_A)\cdot(g-g_A)] + \mu(s_A,0)\cdot[\{\pi(s_A,0)-\pi(s_A,s_B)\}\cdot(g-g_A) - s_B] \geq 0$
(23) $\mu(0,s_B)\cdot[\pi(0,s_A)\cdot(g-g_A) - s_B] + \mu(s_A,s_B)\cdot[\{\pi(s_A,s_B)-\pi(s_A,0)\}\cdot(g-g_A) - s_B] \geq 0$

Prop.(5): If $\mu(0,0)=0$ then market-making occurs and Eqs.(20) and (22) hold for all positive values of $\mu(s_A,0)$ and $\mu(0,s_B)$. To establish $\mu(s_A,s_B) > 0$ we need to examine Eqs.(21) and (22).

Eq.(2) [profitability] and $\mu(s_A,s_B) > 0$ together imply:

(24) $\mu(s_A,0)\cdot[\ \pi(s_A,0)\cdot g_A - s_A] \geq \mu(s_A,s_B)\cdot[\{\pi(0,s_B)-\pi(s_A,s_B)\}\cdot g_A + s_A] > 0$

The immediate left-side of the strict inequality is positive if $\mu(s_A,s_B) > 0$, however, the left-hand side of the weak inequality establishes an upper bound on $\mu(s_A,s_B)$ of:

$$\frac{\mu(s_A,0)[\pi(s_A,0)\cdot g_A - s_A]}{\{\pi(0,s_B) - \pi(s_A.s_B)\}g_A - s_A} \geq \mu(s_A,s_B).$$

Eq.(2) implies and $\mu(s_A,s_B) > 0$ together imply:

(25) $\mu(0,s_B)\cdot[\pi(0,s_B)\cdot(g-g_A) - s_B] \geq \mu(s_A,s_B)\cdot[\{\pi(s_A,0)-\pi(s_A,s_B)\}\cdot(g-g_A) + s_B] > 0$

Again, the left-side of the strict inequality is positive if $\mu(s_A,s_B) > 0$, and the left-hand side of the weak inequality establishes an upper bound on $\mu(s_A,s_B)$ of:

$$\frac{\mu(0,s_B)[\pi(0,s_B)\cdot(g - g_A) - s_B]}{\{\pi(s_A,0) - \pi(s_A.s_B)\}(g - g_A) + s_A} \geq \mu(s_A,s_B).$$

Together, the inequalities yield Eq.(11). ■

Prop.(6): Strict Pareto efficiency requires $U_A(\mu) > U_A(s_A,0)$ and $U_B(\mu) > U_B(0,s_B)$.

Checking first the case of $U_A(\mu) > U_A(s_A,0)$ yields:
$\mu(s_A,s_B)\cdot[\pi(s_A,s_B)\cdot g_A - s_A] + \mu(0,s_B)]\cdot\pi(s_A,0)\cdot g_A > [1-\mu(s_A,0)]\cdot[\pi(s_A,0)\cdot g_A - s_A]$
$\mu(s_A,s_B)\cdot[\pi(s_A,s_B)\cdot g_A - s_A] > [1-\mu(s_A,0)- \mu(0,s_B)]\cdot\pi(s_A,0)\cdot g_A + [\mu(s_A,0)-1]\cdot s_A$
$[1-\mu(s_A,0)- \mu(s_A,s_B)]\cdot s_A > \mu(s_A,s_B)\cdot[\pi(s_A,0)-\pi(s_A,s_B)]\cdot g_A$

It follows that the case of $U_B(\mu) > U_B(0,s_B)$ requires:
$[1-\mu(0,s_B)- \mu(s_A,s_B)]\cdot s_B > \mu(s_A,s_B)\cdot[\pi(0,s_B)-\pi(s_A,s_B)]\cdot(g-g_A)$. ■

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