QUAERERE VERUM

Universidad de San Andres

DEPARTAMENTO DE ECONOMIA

Intraregional inequality, technological diffusion and pattern of international trade

Dr. Maurice Kugler (University of California, Berkeley)

CICLO DE SEMINARIOS 1997

Día: Martes 12 de Julio

9:00 hs.

Intra-Regional Inequality, Technological Diffusion, and the Pattern of International Trade[1]

by

Maurice Kugler
Facultad de Economía, Universidad de los Andes
AA 4976 Santafé de Bogotá, Colombia
mkugler@zeus.uniandes.edu.co

Josef Zweimüller
Department of Economics, University of Zurich
CH-8006 Zurich, Switzerland

Abstract

In this paper we model the determinants of technological innovation and imitation between industrialized and developing regions, and of the pattern of international trade, when preferences are nonhomothetic. By and large, models of the dynamics of North-South trade impose the assumption of unit income elasticity for all consumption goods. We relax this assumption and incorporate the insight from Engel's Law: the budget share allocated to more basic goods falls with income. To account for the impact of income distribution, we introduce preferences where consumers rank goods according to a hierarchy of wants so that the configuration of demand for newer goods depends on the range of affordable consumption. Aggregate demand for different types of goods hence depends on the distribution of wealth within each region. In the model, we assume that the distribution of wealth is unequal in the poor region and even in the prosperous region. This distinction is meant to capture broad modern regional dichotomies of the global North-South type or of the European West-East type. In particular, we are interested in the effect of changes in the distribution of wealth within the poor region on the dynamics of the integrated economy. As often in trade models featuring innovation-imitation dynamics, growth in the integrated system is driven by determinants of the level of activity in the developing region. Specifically, the income distribution characteristics in the poor region determine the aggregate demand for imports and therefore impact the volume of trade.

JEL classification: F12, F15, O11, O31

[1]**Preliminary and incomplete: comments welcome.** The authors wish to thank, subject to the standard proviso, for valuable suggestions Pranab Bardhan, Francois Bourguignon and Paul Romer. This work was begun while both authors were at the Institute for Advanced Studies in Vienna, Austria.

1. Introduction

The dynamics of innovation and imitation between industrialized and less developed regions have been investigated in various contexts. The life-cycle structure of the locational choice for production of newly invented goods over time, where relatively early manufacturing takes place in industrialized countries and gradually shifts to less developed countries (Vernon, 1966), has been formalized in models exploring technology diffusion to emerging economies (e.g. Feenstra, 1996; Helpman, 1993; and, Grossman and Helpman, 1991). By and large, such models impose the assumption of unit income elasticity for all consumption goods.[1] This of course ignores the insight from Engel's Law: the budget share allocated to necessities decreases with income. As observed by Linder(1961), once the difference in expenditure decisions between rich and poor consumers is acknowledged, the trade pattern between industrialized and less developed regions is determined not only by factor endowments and cross-regional income differences, as in the Hecksher-Olin-Samuelson and *intra*-industry (e.g. Helpman and Krugman, 1985) trade models, but also by income distribution within each region. The empirical findings of Francois and Kaplan(1996) indicate the statistical significance of income distribution in determining aggregate expenditure patterns, given nonhomothetic preferences and associated convex expansion paths, and thereby the importance of *intra*-regional income distribution on the volume of international trade. To account for the impact of income distribution, we introduce nonhomothetic preferences in an innovation-imitation model of an integrated world economy.[2] The specification of preferences used is that introduced by Zweimüller(1995) where consumers rank goods according to a hierarchy of wants so that the configuration of demand for newer goods across households depends on the range of affordable consumption. Aggregate demand for different types of goods hence depends

[1]Exising models either suppose that the demand for all goods has the same income elasticity or that there is a representative consumer. Thus, by assumption, they rule out any impact of income distribution on the level and composition of aggregate demand. We take into account the fact that income elasticity with respect to newly available varieties or qualities of goods is larger relative to that with respect to older ones. Then wealth distribution determines aggregate demand.

[2]In contrast to other models which explore the impact of income distribution on growth, e.g. Glass(1994), we do not assume that poor consumers have different preferences than rich consumers by desiring better goods with lower intensity.

on the income distribution in each region. The pattern of trade is determined not only by factor endowments and relative incomes as in standard trade theory but also by *intra*-regional income distributions as pointed out by Linder. In the model, we assume that the distribution of wealth is unequal in the poor region and even in the prosperous region.[3] Hence, our distinction is meant to capture broad modern regional dichotomies of the global North-South or the European East-West type. In particular, we explore the effect of changes in the distribution of wealth within the poor region on the dynamics of the integrated economy.

The inclusion of nonhomothetic preferences in a neo-Schumpeterian model of trade brings about a demand channel through which factor endowment composition affects innovation and trade patterns. For example, a higher wage not only affects the costs of R&D, as well as manufacturing, but also can have a positive impact on profits by boosting demand. This feature raises the possibility of multiple steady states as well as different converging paths even under *common initial conditions.*[4] As often in trade models featuring innovation-imitation dynamics, growth in the integrated economy is driven by the level of activity in the less developed country. This applies to the income distribution characteristics in the poor region in particular. The configuration of trade will be determined by regional demands for different types of goods. In principle, the possibilities of either complete divergence or overtaking could arise. However, these cases would result in autarkic states of the system. In the equilibria, where the economy is integrated, we explore there is weak regional convergence (i.e., in product per capita growth rates but not levels) although regional income differentials persist.

Also, stability implies that trade is mutually beneficial compared to autarchy. However, in the less developed region growth can be inmiserizing for those who derive most of their income from wages. The effect of wealth distribution on growth is ambiguous as well. Since only the rich in the less developed region can afford imported luxurious goods, progressive wealth redistribution may lead to a contraction of trade and thereby lower growth. This would occur for example if the redistribution of wealth is not associated with an attendant rise in demand for relatively new goods. Hence, if the distribution is extremely skewed, even when it is welfare enhancing to shift assets from the rich to the poor, it is not likely

[3] This assumption is consistent with the stylized evidence on distribution and development. See e.g. Benabou(1996) for a review of the empirical as well as theoretical literature on inequality and growth.

[4] While dynamic trade models often display multiple equilibria and path dependance (e.g., Grossman and Helpman, 1992), there is a single trajectory as well as steady state associated with each set of initial conditions.

to spur growth. Such slowdown in production could be compounded, if ensuing the redistribution of wealth, there is a fall in aggregate labor supply in the poor region.

In the model, the main requirement associated with stability is that the rate of time preference be above a lower bound. This means that the loss of innovators due to imitation is less in present value relative to the immediate gain of demand from rich consumers in the less developed region. Also, for a given set of initial conditions, uniqueness is likely to obtain only if impatience is significant so that the aggregate demand positive effect of higher wages on investment incentives is dampened by negative the cost effect. Thus, the existence of stable equilibria in the integrated world economy with trade will tend to imply the existence of multiple balance growth paths, even for a given set of initial conditions.

The paper is organized as follows. Section 2 sets up the primitives of the model: endowments, preferences and technology. Section 3 derives the strategic linkages between innovators and imitators under free entry. Section 4 characterizes the balanced growth equilibria. Section 5 shows some comparative steady state exercises. Section 6 concludes and suggests some further extensions.

2. The basic model

In this section the building blocks of the model are laid out. First, the preferences are specified. We build-in Engel's Law. Second, the endowment structure is characterized. Next, the necessary first-order conditions implied by household optimization are used to write consumption and factor share functions. Finally, we provide the basis for the industrial organization dynamics given by the innovation, imitation and production technologies.

2.1. Preferences

The economy is made up of two countries, A and B populated by L^A and L^B inhabitants respectively. Country A is relatively more prosperous and industrialized than country B. Preferences are defined over consumption goods and leisure. It is assumed that all consumers, independently of their income and their nationality, have the same preferences. Lifetime utility of a type h consumer household in country i is given by

$$U_h^i = \int_0^\infty u\left(C_h^i(t), 1 - l_h^i(t)\right) e^{-\theta t} dt, \tag{2.1}$$

where $u(\cdot,\cdot)$ is the felicity derived at time t whose determinants are $C_h^i(t)$, the level of household consumption, as well as $1-l_h^i(t)$, the fraction of household time devoted to leisure; and where θ is the rate of time preference. We assume that there is a continuum of producible goods, which is potentially infinite. Each good is indexed by $j \in [0,\infty)$. There exists a hierarchy of wants ranking these goods according to their priority. The notion is that individuals consume conveniences only after more basic needs are satisfied. Goods indexed relatively close to zero are basic goods (high priority) while goods with relatively high indices are luxury goods (low priority). We will assume that goods such that $j \geq 1$ yield utility $\frac{1}{j}$ whereas - for simplicity - we assume that the most needed goods $j \in [0,1)$ yield a utility of unity. Further, we suppose that the acquisition of each good is a discrete choice and that no utility from consumption is derived beyond the first unit. If the prices for the various goods are not decreasing in j, each household will consume the goods according to the sequence specified by the hierarchy. Given equal prices, since higher$-j$ goods yield less utility, they are more costly in marginal utility terms than lower-j goods. No good j will ever be demanded by a household until all goods indexed $k < j$ have been consumed. Finally, we assume that the felicity function $u(\cdot,\cdot)$ is separable and symmetric in consumption and leisure. Thus, a household, of type h in country i, purchasing a measure $C_h^i(t) > 1$ of the goods available will consume all $j \in [0, C_h^i(t)]$ at each date $t \in [0,\infty)$ and by devoting a fraction $l_h^i(t)$ of available time to work derives lifetime utility

$$U_h^i = \int_0^\infty \left[\left(1+\ln C_h^i(t)\right) + \ln\left(1-l_h^i(t)\right)\right] e^{-\theta t} dt, \tag{2.2}$$

which is the maximand of a dynamic optimal control problem to be specified below.[5]

2.2. Endowments

Denote the wage of household h in country i by $W_h^i(t)$ and the value of its assets by $V_h^i(t)$. In equilibrium, wealth is held in the form of an asset portfolio with identical proportions of shares, from each producer within the country. We assume that there are no international capital flows. If $r^i(t)$ is the rate of interest in country i, a household earns capital income $r^i(t)\,V_h^i(t)$. Wage income is given by $W_h^i(t)\,l_h^i(t)$,

[5] Although the decision-making criterion has a lexicographical structure, the consumption function is continuous and otherwise well-behaved by construction. Note that there exists a continuum of goods and that the index of the last good consumed corresponds pari passu to the consumption measure.

with $l_h^i(t)$ being the fraction of time the household devotes to work. Current income,

$$Y_h^i(t) = r^i(t) V_h^i(t) + W_h^i(t) l_h^i(t), \tag{2.3}$$

is divided between current consumption and wealth accumulation.

At any moment t, a measure $N(t)$ of goods is available. We assume that these $N(t)$ goods coincide with the interval $[0, N(t)]$ of the hierarchy of wants specified above. A measure $M(t)$ of goods is produced in country B and are relatively cheap. All $j \in [0, M(t)]$ have a common price $p < 1$, which is constant over time. The price of the remaining $N(t) - M(t)$ goods is normalized to unity. These numeraire goods, produced in country A, are relatively expensive.

Intertemporal optimization is characterized by maximization of the utility functional (2.2) above subject to the equation of motion below,

$$\dot{V}_h^i(t) = \begin{cases} Y_h^i(t) - C_h^i(t)\, p & \text{for } C_h^i(t) < M(t) \\ Y_h^i(t) - C_h^i(t) + (1-p)\, M(t) & \text{for } N(t) \geq C_h^i(t) > M(t) \end{cases} \tag{2.4}$$

It will be assumed throughout the paper that workers do not differ with respect to their ability, so within a country each household earns the same wage, i.e. $W_h^i(t) = W^i(t)$. However, we assume that consumers in country B differ with respect to wealth ownership. Wealth is held by households in the form of a portfolio with identical shares of stakes in the profits accruing to each domestically produced good. We will concentrate on a discrete case of asset inequality and assume that there are two groups, the rich R and the poor P, with population shares $1-\beta$ and β, respectively. Denote by $V^B(t)$ the average value of wealth in country B held by each household in the form of a perfectly diversified portfolio. If each poor household owns wealth

$$V_P^B(t) = \alpha(t)\, V^B(t),$$

where $\alpha(t) < 1$, the aggregate wealth owned by the poor is $\beta\alpha(t)\, V^B(t)\, L^B$. Consequently, the rich as a group own $(1-\beta\alpha(t))\, V^B(t)\, L^B$. Therefore, each rich household has assets valued as

$$V_R^B(t) = \frac{1-\beta\alpha(t)}{1-\beta} V^B(t)\, L^B.$$

It follows that at any point of time the parameters β and $\alpha(t)$ are sufficient to characterize the cumulative distribution of wealth.

2.3. Consumption and factor shares

It turns out that the asset distribution is stationary under the present specification of preferences. In particular, the ratio of savings to the value of asset holdings is independent of the level of wealth. Hence, as stated in the *Lemma* below, the share of wealth of each group is fixed.

> **Lemma:** The relative value of the wealth shares of the rich and poor in country B remains constant over time so that $\alpha(t) = \alpha$.
>
> Proof: See appendix.

The solution to the dynamic control problem by each household implies that the ratio of saving to asset value is independent of the level of assets. As the wealth of the rich and the poor grows at the same rate we obtain the conclusion of the *Lemma*.

We will focus in the case where (i) households in the relatively prosperous country A purchase all available varieties, and (ii) the rich but not the poor in the less developed country B can afford imported luxury goods. This means that

$$C^A(t) = N(t) > C_R^B(t) > M(t) > C_P^B(t).$$

Along a balanced growth path where $W^A(t), W^B(t), M(t), N(t), V^A(t), V_R^B(t)$ and $V_P^B(t)$ grow at the same rate g, dynamic optimization by each household leads to the following system of equations.[6] Asset returns are characterized as follows,

$$r = g + \theta = r^A \equiv g^A + \theta = r^B \equiv g^B + \theta. \tag{2.5}$$

Consumer demand for each household type depends on the range of affordable goods. In country A, all goods are consumed and we obtain,

$$C^A(t) = N(t). \tag{2.6}$$

In country B, only the rich households have enough purchasing power to buy, some but not all, luxurious imported goods,

$$C_R^B(t) = \frac{1}{2}\left[W^B(t) + (1-p)M(t) + \theta\frac{1-\beta\alpha}{1-\beta}V^B(t)\right]. \tag{2.7}$$

[6] The rationale for concentrating on balanced growth paths with identical growth rates is that if the two regions grow at different rates (i.e., if $M(t)$ and $N(t)$ either diverge or converge), asymptotically the system is driven to an autarkic state.

The income of the poor in the less developed region is not sufficient to acquire even the whole range of domestically produced goods,

$$C_P^B(t) = \frac{1}{2p}\left[W^B(t) + \theta\alpha V^B(t)\right]. \tag{2.8}$$

Labor supply for each household type depends crucially on the level of desired demand as well as on nonlabor income. In country A, the fraction of time devoted by households to production activities is,

$$l^A(t) = \frac{1}{W^A(t)}\left[N(t) - (1-p)M(t) - \theta V^A(t)\right]. \tag{2.9}$$

In country B, the rich and the poor have analogous decision rules except for the purchasing power advantage afforded by domestic good consumption relative to the acquisition of imports. For rich households, we have,

$$l_R^B(t) = \frac{1}{2W^B(t)}\left[W^B(t) - (1-p)M(t) - \theta\frac{1-\beta\alpha}{1-\beta}V^B(t)\right]. \tag{2.10}$$

While for the poor, we obtain,

$$l_P^B(t) = \frac{1}{2W^B(t)}\left[W^B(t) - \theta\alpha V^B(t)\right]. \tag{2.11}$$

In the appendix, the corresponding Hamiltonians for each household type are set-up and the implied first-order necessary and sufficient conditions are derived.

2.4. Technology

To complete the specification of the primitives of the model, we provide the elements that determine the cost structure of manufacturing in each region. First, in the rich economy, there is a sunk cost stemming from the resource requirement for innovative design. The marginal cost of producing each unit gives the mark-up equation. Second, in the developing economy, there is a fixed cost associated with reverse engineering. Limit pricing together with the variable cost define the mark-up relationship for imitated products. These technical parameters together with the aggregate demand functions determine the free-entry equilibrium conditions in each region.

2.4.1. Invention

Each firm in country A has exclusive use of a blueprint.[7] R&D ventures require deployment costs of $F(t)$ units of labor. Once a design is produced by a firm, it can manufacture final output using $A(t)$ units of labor and acquire a monopoly position for the corresponding good. $A(t)$ is the same for all goods. There is an upper bound on the price to be charged by each incumbent firm. We normalize this limit price to unity. The limit on the price is due to potential production by a competitive fringe. Suppose that - once a good is invented - there are outside firms able to copy this good. The copying technology has a unitary manufacturing requirement of $\frac{1}{W^A(t)}$ units of labor to produce under constant returns, where $\frac{1}{W^A(t)} > A(t)$. Indeed, emulators are less proficient thus allowing a profit margin for the incumbent. With the incumbents' price normalized to unity, under this market structure the wage rate in country A is equal to $W^A(t)$. If an incumbent monopolist tried to bid the wage below that level, the competitive fringe could enter without incurring sunk costs and offer slightly higher wages to attract all the required workers to serve the whole market.[8] It follows that no incumbent will ever pay a wage lower than the reservation level $W^A(t)$. With a wage rate $W^A(t)$ and a price of unity, the profit flow per unit of output sold is $\pi^A(t) = 1 - W^A(t)\,A(t)$.

The following assumption summarizes the evolution of technical opportunities.

Assumption A: (Productivity progress in the innovating country A)

$$F(t) = \frac{F}{N(t)}, A(t) = \frac{a}{N(t)}, and \frac{1}{W^A(t)} = \frac{1}{N(t)\,w^A}.$$

We assume that productivity growth in the relatively prosperous country is driven by innovations. Also, as in most models of endogenous growth we suppose that researchers in future generations build upon experience of previous successes. We adopt the simplest way to capture this idea by assuming that the stock of knowledge in the economy can be proxied by the measure of previous innovations $N(t)$, and the labor input requirement of R&D is inversely related to this measure. Moreover, we assume that the efficiency of final output production, by both incumbents and the competitive fringe, also increases with $N(t)$ which is an

[7]Perfect intellectual property protection prevails in country A. But, entrepreneurs in country B can copy a design without compensating the creator. As will be shown below, the effective life of manufaturing monopolies in country A depends on the dynamics of imitation.

[8]Here, we assume that if copiers and the incumbent offer the same good at the same price, consumers will strictly prefer the product of the incumbent.

index of past manufacturing as well.[9] The labor requirement $A(t)$ of incumbents to produce one unit of output shrinks at the same rate as innovations take place. Finally, we assume that also copiers benefit from a demonstration effect from past production.

On the one hand, this latter assumption implies that the wage rate grows *pari passu* with the measure of previous innovations. On the other hand, if follows that the profit flow per unit sold

$$\pi^A(t) = 1 - W^A(t) A(t) = 1 - w^A a = \pi^A \tag{2.12}$$

is constant over time. (We repeatedly apply the convention that lower case variables for country A are equal to upper case variables normalized by $N(t)$).

2.4.2. Emulation

Firms in the less developed country B do not have access to the innovation technology. To become manufacturers they emulate producers from the innovating country A. Imitation requires set-up costs of $G(t)$ units of labor. After a good has been imitated in country B, imitators can produce at constant marginal cost $W^B(t) B(t)$ where $W^B(t)$ denotes the wage rate and $B(t)$ is the labor input necessary to produce one unit of output using the imitation technology. We will discuss later on the endogenous determination of $W^B(t)$.

Technological change for imitation activities evolves analogously to that in innovating activities.

Assumption B: (Productivity progress in the emulating country B)

$$G(t) = \frac{G}{M(t)} \text{ and } B(t) = \frac{b}{M(t)}.$$

The above characterization of the progress of emulation technologies states that efficiency is determined by the history of imitating activities $M(t)$. Productivity in the blueprint imitation and adaptation process increases as a result of learning from reverse-engineering experience. Successful design copying not

[9] This specification incorporates the insight from the model by (Young, 1993). There learning-by-doing ceases if innovation stops. Experimentation returns are limited by the productive potential of invented designs. Also, the profitability of costly invention depends not only on the spillovers generated by previous R&D activity but also on cumulative learning whic determines production costs.

only adds to the productivity of further imitation but also leads to more efficient production due to the associated increase in manufacturing experience.

In order to be competitive in the world market, country B producers have to underbid country A firms. The lowest price at which country A firms are willing to sell is their marginal cost, $w^A a$. If a country B firm charges a slightly lower price, it can take over the whole world market and drive the country A competitors out of the market. However, the country B firms will only be able to do so if their marginal cost is below that of country A producers. Or equivalently, if $w^B b < w^A a$, where $w^B = \frac{W^B(t)}{M(t)}$ denotes the country B wage rate normalized by the measure of previously imitated goods. We obtain the mark-up for imitating producers by invoking limit pricing. In a similar way we can analyze the equilibrium for imitation. In order to capture the market the imitator has to underbid the price of the current producer. The limit price (i.e., the price which drives the country A firm out of the market) is slightly below the marginal cost of the country A firm, $w^A a$. Since the country B firm has costs $w^B b$, the profits per unit sold are thus,

$$\pi^B = w^A a - w^B b.$$

(Heretofore country-B lower case variables shall denote upper case variables normalized by $M(t)$). Note that a, b and w^A are exogenously given technological parameters. This is not the case for $w^B(t)$. We will see that $w^B(t)$ is endogenously determined by the current account balance condition and is a measure of the terms of trade.

3. Innovation, imitation and trade

In order to describe the steady state we have to describe the implications of our assumptions on preferences and technology for innovation, imitation, and trade. We will assume that in country A there is access to the innovation technology. The equilibrium will be characterized by a situation where the present discounted value of future profits accruing from an innovation is equal to the fixed cost of discovery. Firms in the country B do not have access to the innovation technology, but there are no barriers to entry in imitation activities. The imitation equilibrium characterization is analogous to the free-entry condition for country A innovators.

The value of innovation and imitation success in steady-state equilibrium is characterized under the following conditions. Consumers must have made optimal intertemporal choices on the path of consumption and leisure. There cannot be unexploited profit opportunities in the sense that neither further incentives to innovate nor to imitate with higher intensity exist. Finally, the current account has to balance. We concentrate on balanced growth equilibria where there is weak convergence (i.e. $g^A = g^B$) because otherwise we reach autarchy asymptotically. Hence, from our system of first-order necessary conditions by equation (2.4) characterizing asset returns, there is international interest rate equalization (i.e. $r^A = r^B$) . In steady state, if there is an infinitesimal cost of international transactions, there are no incentives for capital to flow across boundaries.[10] Hence, we assume that all investment projects are financed solely with domestically generated resources. So that trade balance must prevail at all dates $t \in [o, \infty)$.

3.1. Innovation

As mentioned above, we will focus in the case where the rich but not the poor in the less developed country B can afford luxury imports from country A. Let us first consider the incentives of a country A firm. The design of a blueprint costs $w^A F$. Once a new good is on the market it can be sold with a (constant) profit π^A per unit. For a given rate of interest, the value of an innovation depends on how demand develops over time. Figure 1 shows the time path of demand for a country A innovator. At time t_o, the innovation takes place. Starting from t_0 until t_1, only consumers in country A can afford the new good. Along the balanced growth path, incomes increase over time so that at t_1 the rich in country B are able to purchase the good. Later on, at t_2, imitation takes place. At that date the country A innovator is undercut and loses the whole market to the country B emulator.

It is clear that the present value derived from these profit flows depends on the size of the market at each date but this in turn is a function of how fast incomes grow over time. On the one hand, if the growth rate is high, incomes increase quickly and demand rises relatively earlier. On the other hand, also imitation takes place earlier, and the effect of income growth for the value of an innovation is a priori ambiguous.

As above let g^B be the common rate at which the incomes of all groups grow along a given balanced growth path of country B's economy and $N(t_0)$ denote the

[10] However, there are incentives for technology transfer from the industrialized to the developing region, given that in the latter there is no access to the most recent technologies. Technology exchange could be acomplished by introducing either foreign direct investment or trade in intermediate capital inputs. In the present model, we preclude such possibility. We are developing a structure which incorporates technical knowledge flows.

good which has been designed at t_0. Then the length of the time interval (t_0, t_1) during which only consumers in the more prosperous country A can afford the good is given by the condition

$$C_R^B(t_0)\, e^{g^B(t_1-t_0)} = N(t_0).$$

So $(t_1 - t_0)$ is given by,

$$(t_1 - t_0) = -\frac{1}{g^B} \ln \frac{C_R^B(t_0)}{N(t_0)} \equiv -\frac{1}{g^B} \ln c_R^B. \tag{3.1}$$

The total lifetime of the innovator's monopoly is the interval (t_0, t_2). At t_2 ,production ceases to be profitable because imitators can price the good cheaper than the incumbent's marginal cost. In particular, the timing of initial production in country B is implied by

$$M(t_0)\, e^{g^B(t_2-t_0)} = N(t_0),$$

so that,

$$(t_2 - t_0) = -\frac{1}{g^B} \ln \frac{M(t_0)}{N(t_0)} \equiv -\frac{1}{g^B} \ln m. \tag{3.2}$$

The flow profit of the producer of good $N(t_0)$ is $L^A\pi^A$ during the interval (t_1, t_0) . and by $L^A\pi^A + (1-\beta)\, L^B\pi^A$ during (t_2, t_1) . In a steady state with positive growth rates, the costs and benefits of an innovation have to be equal. Otherwise, unexploited profit opportunities would remain. So whenever $g > 0$, the innovation equilibrium condition can be written as

$$w^A F = \frac{\pi^A}{r^A}\left[L^A\left(1 - m^{\frac{r^A}{g^B}}\right) + (1-\beta)\, L^B\left(\left(mc_R^B\right)^{\frac{r^A}{g^B}} - m^{\frac{r^A}{g^B}}\right)\right], \tag{3.3}$$

where the RHS is the present discounted value of the profit flow accruing during the lifetime of the innovating firm.

3.2. Imitation

Just like for an innovation, the value of emulation depends on the development of demand over time. From Figure 1, the date at which good $N(t_0)$ is imitated is

t_2. At this moment the country B imitator crowds the country A innovator out of the market and thus captures the incumbent's whole demand obtaining the immediate flow profit $\left(w^A a - w^B b\right)\left(L^A + (1-\beta) L^B\right)$. When the poor in the country B country are able to afford good $N(t_0)$, at date t_2 in Figure 1, the profit flow jumps to $\left(w^A a - w^B b\right)\left(L^A + L^B\right)$ and remains at that level forever.

The time interval (t_2, t_3) until the poor can afford the imitated good is given by,

$$(t_3 - t_2) = -\frac{1}{g^B} \ln \frac{C_P^B(t_0)}{M(t_0)} \equiv -\frac{1}{g^B} \ln c_P^B. \tag{3.4}$$

In an equilibrium in which imitation is active, we have $g^B > 0$. In that case, the discounted value of the profit flow describes above has to equal the imitation costs. Therefore, the condition

$$w^B G = \frac{\pi^B}{r^B}\left[L^A + (1-\beta) L^B + \beta L^B \left(c_P^B\right)^{\frac{r^B}{g^B}}\right]. \tag{3.5}$$

describes the free entry equilibrium for the manufacturing sector in country B.

3.3. Trade

In the equilibria we consider, there are no incentives for the international flow of capital, implying that neither borrowing nor lending takes place across countries. Thus, current account balance entails that at each instant, imports and exports are identical. As mentioned at the beginning of this section, we will concentrate in the case in which income differences between countries are relatively large, so that the poor in the less developed country cannot afford any imported varieties. Also, at time t, $M(t)$ goods are produced in country B and all these goods are exported to all L^A households in country A. The price of these goods is $w^A a$. So the value of total country A imports (in terms of the numeraire goods produced in country A) is therefore given by $L^A M(t) w^A a$. The demand for exports is given by the number of rich consumers in the country B country. Only this group is assumed to be able to afford imported luxury goods. The level of consumption of this group is $C_R^B(t)$, so the value of exports country B is $(1-\beta) L^B \left(C_R^B(t) - M(t)\right)$. In the steady state, the trade balance condition can therefore be written as,

$$L^A w^A a = (1-\beta) L^B \left(c_R^B - 1\right) \tag{3.6}$$

Notice that on the left hand side only exogenous parameters appear. This means that the volume of trade (in terms of the price of country A goods) is fully determined by population size and by marginal cost of production in country A. The variable which adjusts to balance trade is c_R^B. The dynamics of the adjustment mechanism are the following. If country A 's demand for basic imports exceeds export demand for its luxury goods, there is excess labor demand in country B. This generates upward pressure on w^B, with an ensuing rise in the income of all inhabitants of country B inducing c_R^B to increase and close the gap in the current account. Any disequilibrium is corrected by a tatônnement process whereby w^B moves to equate labor demand and supply. Hence, the endogenous determination of w^B is driven by the pattern of trade.

4. Balanced growth equilibria

In this section we study the dynamics of the integrated system. First, we complete the description of the potential equilibria by finding the resource balance constraint for each region. Second, we find the conditions underlying equilibrium existence and stability. We proceed sequentially by finding initially the stable equilibrium growth rates for the developing economy taking all variables describing economic activity in the industrialized region as predetermined. Then, we check that each found stable equilibria is consistent with a balanced growth path in which free trade prevails. This implies that $0 < m < 1$ asymptotically and therefore that $g^A = g^B$, in steady state. Thus, our last step involves verifying that weak regional convergence is indeed associated with stable dynamics for the whole system.

4.1. Factor markets

In the steady state, this economic system is characterized by the household optimization rules, by the industrial organization among innovators and imitators in equilibrium, and by the balance of trade described in the last section. Finally, we have to determine the labor market equilibrium conditions in each country. Under full employment, this amounts to imposing a binding resource balance constraint as the labor force is the only factor of production.

4.1.1. The industrialized economy

Total labor supply in country A is $L^A l^A$. The optimal supply of labor depends positively upon the wage rate and negatively on asset incomes. As a result, the relatively rich supply less labor than the poor.

There are two sources of labor demand: employment in manufacturing and employment in design acquisition. At any instant, the demand for labor in the R&D sector depends on how many new goods are introduced, that is on $\dot{N}(t)$. Since the labor requirement to develop a new good is equal to $\frac{F}{N(t)}$, the labor demand for the R&D sector in country A is equal to $\frac{\dot{N}(t)}{N(t)}F$. By an analogous argument, we can calculate employment in imitation activities as $\frac{\dot{M}(t)}{M(t)}G$. Employment in the production of final output in country A is given by demand for goods in the range $j \in [M(t), N(t)]$. Since we focus on the case where

$$C_R^A(t) = N(t) > C_R^B(t) > M(t) > C_P^B(t),$$

and as $\frac{a}{N(t)}$ labor units are required to produce one unit of output in country A, labor demand in production is

$$\frac{a}{N(t)}L^A(N(t) - M(t)) + \frac{a}{N(t)}(1-\beta)L^B\left(C_R^B(t) - M(t)\right).$$

Now we can write country A's resource constraint as,

$$L^A l^A = g^A F + a\left[L^A(1-m) + (1-\beta)L^B m\left(c_R^B - 1\right)\right], \tag{4.1}$$

where the following definitions are used:

$$g^A \equiv \frac{\dot{N}(t)}{N(t)}, c_R^B \equiv \frac{C_R^B(t)}{M(t)}, m \equiv \frac{M(t)}{N(t)} \text{ and } c_P^A \equiv \frac{C_P^A}{N}.$$

All these variables are constant in a steady state equilibrium.

4.1.2. The less developed economy

In country B labor supply is given by

$$(1-\beta)L^B l_R^B + \beta L^B l_P^B.$$

The number of consumers which demand all $M(t)$ goods is $L^A + (1-\beta)L^B$, whereas the poor in country B demand only $C_P^B(t) < M(t)$ goods. Using the fact that the unit labor requirement in the production of imitated goods is $\frac{b}{M(t)}$, employment in production in country B is given by

$$\frac{b}{M(t)}L^A M(t) + \frac{b}{M(t)}(1-\beta)L^B M(t) + \frac{b}{M(t)}\beta L^B C_P^B(t).$$

The level of employment in imitation is equal to $\frac{\dot{M}(t)}{M(t)}G$, since to reverse-engineer one design requires $\frac{G}{M(t)}$ labor units and $\dot{M}(t)$ new imitating firms enter at each point in time. In a steady state $m \equiv \frac{M(t)}{N(t)}$ is constant over time, which means that $M(t)$ has to grow at the same rate as $N(t)$. Using the definition for the steady state variables $c_P^B \equiv \frac{C_P^B(t)}{M(t)}$ and $g^B \equiv \frac{\dot{M}(t)}{M(t)}$, the resource constraint the country B can be written as

$$(1-\beta) L^B l_R^B + \beta L^B l_P^B = g^B G + b\left[L^A + (1-\beta) L^B + \beta L^B c_P^B\right]. \qquad (4.2)$$

4.2. The growth rate

To explore the potential balanced growth paths of the system we proceed by steps. First, we characterize the equilibrium dynamics in country B by analyzing a reduced form consisting of two equations (free-entry and resource-balance) with two unknowns (w^B and g^B) taking economic activity in the other country as predetermined. We study the conditions underlying equilibrium existence and stability. Second, we rule out divergence (i.e. $g^A > g^B$ where $m \to 0$) as well as overtaking (i.e. $g^A < g^B$ where $m \to 1$) and concentrate on the nondegenarate case of weak convergence (i.e. $g^A = g^B$ where $m \in [0,1]$).We verify that given a stable growth rate (e.g., $g^B = g^*$) implied by household and entrepreneurial optimization as well as trade balance, the weak convergence condition (i.e., $g^A = g^B = g^*$) is consistent with equilibrium stability for the country A economy and a fortiori for the integrated system.

4.2.1. Equilibria in the less developed economy

In order to solve for the growth rate along a balanced growth path, it turns out to be convenient to rewrite the expression for the optimal level of consumption for the rich in country B, as

$$c_R^B = \frac{1}{2}\left[w^B + \pi^A + \frac{1-\beta\alpha}{1-\beta}\theta v^B\right] \qquad (4.3)$$

where we have defined $v^B \equiv \frac{V^B(t)}{M(t)}$. Using this expression and the trade balance condition, we can solve for θv^B to get

$$\theta v^B = \frac{1}{1-\beta\alpha}\left[\frac{(1-\beta) L^B + p\left(2L^A + (1-\beta) L^B\right)}{L^B} - (1-\beta) w^B\right] \equiv \phi\left(w^B\right).$$

Note that the only endogenous variables in the trade balance condition are w^B and v^B. This is because the demand for imports in country A is fixed by the assumption that all country A consumers purchase all goods produced in country B. The normalized demand for imports of country B goods is exogenously given by the population size in country A and the price of country B products.

We can now use the above equation to express the optimal consumption level for the poor in the less developed country as a function of country B 's wage rate,

$$c_P^B = \frac{1}{2p}\left[w^B + \alpha\phi\left(w^B\right)\right]. \tag{4.4}$$

Substituting the latter two equations into country B 's resource constraint, we obtain an expression in the two endogenous variables g^B and w^B describing the labor market equilibrium in country B,

$$\frac{L^B}{2w^B}\left[w^B - \phi\left(w^B\right) - (1-\beta)\,\pi^A\right] - b\left[L^A + (1-\beta)\,L^B\right] - \frac{b\beta L^B}{2p}\left[w^B + \alpha\phi\left(w^B\right)\right] = g^B G. \tag{4.5}$$

The term on the RHS of the above expression is the demand of labor for reverse engineering activities. The first term on the LHS is the labor supply and the third term is labor demand in the production of final goods.

Using the expressions (4.4) for consumption and $r^B = g^B + \theta$ from (2.5), we can write the free-entry condition in the less developed country as a function of g^B and w^B. Indeed,

$$\left(g^B + \theta\right) w^B G = \pi^B\left[L^A + (1-\beta)\,L^B + \beta L^B\left(\frac{w^B + \alpha\phi\left(w^B\right)}{2w^A}\right)^{\frac{g^B+\theta}{g^B}}\right]. \tag{4.6}$$

This expression for the free-entry condition and the resource balance constraint for country B constitute a reduced form system of two equations with two endogenous variables, g^B and w^B. By inspection it is apparent that the loci implicit in each of the two equations between the endogenous variables g^B and w^B is highly nonlinear. The system is not amenable to an analytical closed-form representation of equilibrium. While the resource balance constraint defines a monotonic locus which is a well defined function for w^B bounded away from zero, the free entry condition locus is a correspondence but not necessarily a function. In order to show that there exists a balanced growth solution to this system, we proceed by finding for each equation, of both labor market and industrial activity equilibria, the bounds of the sleeves (depicted in Figure 2) within which each of the implied growth rates is confined for each $w^B > 0$.

We first find the bounds for the feasible growth rate g^{RC} consistent with a binding resource balance constraint. Define the two functions $g_0^{RC}\left(w^B\right)$ and $g_1^{RC}\left(w^B\right)$ given by,

$$g_0^{RC}\left(w^B\right)G \equiv \frac{L^B\left[w^B - \phi\left(w^B\right) - (1-\beta)\pi^A\right]}{2w^B} - b\left[L^A + (1-\beta)L^B\right], \quad (4.7)$$

and,

$$g_1^{RC}\left(w^B\right)G \equiv \frac{L^B\left[w^B - \phi\left(w^B\right) - (1-\beta)\pi^A\right]}{2w^B} - b\left[L^A + L^B\right]. \quad (4.8)$$

In the above expressions, $g_0^{RC}\left(w^B\right)$ is the growth rate which would be feasible if (i) trade were balanced, (ii) both groups in country B were to supply labor according to the dynamically optimal rules, and (iii) the rich consume all country B varieties (as in equilibrium), but the poor do not consume anything. Similarly, $g_1^{RC}\left(w^B\right)$ is the growth rate where (iii) is changed such that rather than consuming nothing at all the poor consume all available country B goods. Since the poor consume only a limited range of domestically produced goods and do not consume any varieties imported from country A, it is clear that

$$g_1^{RC} = g_0^{RC} - b\beta L^B,$$

and thus the actually feasible growth rate implied by labor market equilibrium, $g^{RC}\left(w^B\right)$ is such that

$$g_0^{RC} > g^{RC} > g_1^{RC} \text{ for each } w^B > 0.$$

Also, since $\phi\left(w^B\right)$ is positive as well as a decreasing linear function of w^B, both $g_0^{RC}\left(w^B\right)$ and $g_1^{RC}\left(w^B\right)$ are strictly increasing in w^B.

We proceed in a similar way concerning the free-entry condition. Denote by $g_0^{FE}\left(w^B\right)$ the normalized free-entry equilibrium level of imitation activities, if the poor did not purchase anything at all, and by $g_1^{FE}\left(w^B\right)$ the corresponding level for the hypothetical case in which the poor were to purchase each variety at the very instant when imitation took place. Then by definition we have,

$$g_0^{FE}\left(w^B\right) \equiv \frac{\pi^B}{w^B G}\left[L^A + (1-\beta) L^B\right] - \theta \tag{4.9}$$

and,

$$g_1^{FE}\left(w^B\right) \equiv \frac{\pi^B}{w^B G}\left[L^A + L^B\right] - \theta \tag{4.10}$$

Using the free-entry condition together with the fact that $0 < c_P^B < 1$, we have that the equilibrium normalized level of instantaneous imitation activity, g^{FE}, has to satisfy

$$g_0^{FE} < g^{FE} < g_1^{FE},$$

for each $w^B > 0$, since

$$g_1^{FE} = g_0^{FE} + \frac{\pi^B \beta}{w^B G} L^B.$$

Also, note that both $g_0^{FE}\left(w^B\right)$ and $g_1^{FE}\left(w^B\right)$ are strictly decreasing in w^B and also that

$$\lim_{w^B \to 0^+} g_0^{FE}\left(w^B\right) = \lim_{w^B \to 0^+} g_1^{FE}\left(w^B\right) = +\infty.$$

Figure 3 shows these functions graphically. For any given $w^B > 0$, $g_0^{RC}\left(w^B\right)$ and $g_1^{RC}\left(w^B\right)$ define the range for the feasible growth rate. For a larger w^B this range shifts upwards. This is because higher wages increase the supply of labor, so a higher imitation rate becomes feasible. In fact, a higher wage also stimulates consumption of the poor. This latter effect is neglected in both curves. However, one can infer that the equilibrium growth rate g^{RC} will lie closer to $g_0^{RC}\left(w^B\right)$ at low levels of w^B and closer to $g_1^{RC}\left(w^B\right)$ at higher wage levels.

Similarly, $g_0^{FE}\left(w^B\right)$ and $g_1^{FE}\left(w^B\right)$ specify the range for the profitable level of imitation activities. For a larger w^B , the range tends to shift downwards. The reason is that a higher wage increases costs. There is an increment in both the sunk investment into reverse engineering and the flow of current expenditures into the production wage bill. As a result, imitation incentives are reduced. The limits $g_0^{FE}\left(w^B\right)$ and $g_1^{FE}\left(w^B\right)$, however, neglect that an increase in w^B not only has a cost effect but it also tends to stimulate profitability. As a result of a higher wage, the poor customer market will develop more quickly, and thus stimulate further imitation. It is therefore unclear how the actual imitation rate g^{FE} behaves within these limits.

From Figure 2, it is obvious that a necessary condition for an equilibrium to exist is that $w_0^{RC} < w_1^{FE}$, where these critical wage levels satisfy

$$g_0^{RC}\left(w_0^{RC}\right) = 0 \text{ and } g_1^{FE}\left(w_1^{FE}\right) = 0.$$

w_0^{RC} is the wage rate necessary to generate enough labor supply to serve all consumers in country A and only rich consumers in the developing country with all goods producible in country B if no imitation takes place. Similarly, w_1^{FE} is the wage rate at which imitation becomes profitable given that the poor were able to purchase all of the imitated goods. By using the expressions for $g_0^{RC}\left(w^B\right)$ and $g_1^{FE}\left(w^B\right)$ we can calculate w_0^{RC} and w_1^{FE} as

$$w_0^{RC} = \frac{(1-\beta)\left(1-\{1-\beta\alpha\}\pi^A\right)L^B + p\left(2L^A + L^B\right)}{L^B(2-\beta-\beta\alpha) - 2b\left[L^A + (1-\beta)L^B\right]} \tag{4.11}$$

and,

$$w_1^{FE} = p\frac{L^A + L^B}{\theta G + b\left[L^A + L^B\right]}. \tag{4.12}$$

From these expressions, it is evident that $w_0^{RC} < w_1^{FE}$ if G and b are sufficiently small (i.e., the average labor cost of country B production is low). As both G and b vanish to negligible levels, w_0^{FE} becomes arbitrarily large whereas w_1^{RC} is bounded. The intuition behind this condition is that if production conditions in country B are sufficiently efficient, then there will be both enough resources to deploy in reverse engineering activities as well as sufficient demand from higher incomes for further imitation to be profitable. Also low production costs endow imitators with sufficient competitiveness to be able to generate a significant volume of exports. Without the latter equilibria for the integrated equilibria would not exist as trade would be unattractive. The following propositions state our findings formally about equilibrium conditions for the developing economy taking as predetermined the behavior of the industrialized economy.

Proposition 1. *(Existence of equilibria): Given a degree of innovation and economic activity in country A, there is a configuration with parameters G and b sufficiently small such that there exists an equilibrium balanced growth path of the less developed country B's economy (i.e., there is at least one pair $\{g^*, w^*\}$ in the positive quadrant such that $g^* = g^{FE}(w^*) = g^{RC}(w^*)$)*

Proof: See Appendix.

The proposition above states that given that the $g^{FE}\left(w^B\right)$ locus is a continuous and monotonic function in the positive quadrant, as long as the $g^{RC}\left(w^B\right)$ locus is a hemi-continuous correspondence, there will be a region of intersection where an equilibrium is contained if the technology in the developing region satisfies a minimal degree of efficiency in reverse engineering and production.

Regarding stability, we note that equilibrium existence requires that the two loci must cross at least once. Given hemicontinuitity of both loci, existence implies that they cross an odd number of times. Of these intersections half of the number of equilibria plus one take place with $g^{RC}\left(w^B\right)$ crossing $g^{FE}\left(w^B\right)$ from below. Then there is excess labor demand before such point and excess labor supply after it. If the path of the economy approaches this point, the wage rises. And if it is beyond it, the wage falls. In these instances, local stability obtains, for the developing economy, since labor market adjustment induces the wage to move so as to return to its original level ensuing slight perturbations.

Proposition 2. *(Stability): If the integrated equilibrium is unique, then it is stable. More generally, there can be multiple equilibria of which there will always be an odd number E, and of these, the number of stable equilibria for the less developed economy is given by $\frac{E+1}{2}$.*

Proof: See Appendix.

Stability for the integrated system is a more complex matter which we analyze next. It is clear that a nonautarkic equilibrium provides us with a restriction for the admissible steady state growth rate for the industrialized economy. However, a stable equilibrium for the developing economy is not necessarily stable for the industrialized region. Therefore, to characterize the stable equilibria for the integrated system, we must study additional dynamic conditions.

4.2.2. Equilibrium stability in the industrialized economy

Note that the above potential equilibria exist independently of the level of $N(t)$ as long as $M(t) < C_R^B(t) < N(t)$ at all dates $t \in [0, \infty)$. In other words in the steady state $g^B = g^* \leq g^A$, where g^A is the balanced growth rate in country A. Hence, we have to rule out regimes where the growth rate in country A falls short of the country B growth rate. Such a scenario would imply that the country B economy catches up, and eventually becomes the leading economy.[11] In contrast, a situation

[11]For instance, it would in principle possible that the innovation technology requires a huge amount of resources relative to imitation. For a given rate of imitation, g^B, it might be that

that would be consistent with the above analysis is divergence: country A grows faster than the less developed country, so that the per capita income differential is asymptotically unbounded. In relative terms, the level of production in country B would become irrelevant. The alternative scenario, to overtaking and divergence, is weak convergence (i.e., $g^* = g^A$). In that case, the whole world economy develops along a common balanced growth path, where $M(t) = mN(t)$, with $0 < m < 1$ as a constant.

In order for a steady state of the world economy to exist, there must be a g^A for which country A's economy is in equilibrium such that $g^A = g^*$, where g^* is an equilibrium balanced growth rate for country B . We have to show that there exists some m, which guarantees that $g^A = g^*$ over the admissible range for m. Since we study an equilibrium where the rich in the poor country can afford some but not all country A varieties, and given the definitions of c_R^B and m, we must have $1 < c_R^B < 1/m$. The trade balance condition

$$w^A a L^A = (1-\beta) L^B \left(c_R^B - 1\right)$$

fixes c_R^B and thus specifies an upper limit for m given by

$$\bar{m} \equiv 1/c_R^B = \frac{(1-\beta) L^B}{(1-\beta) L^B + w^A a L^A}. \tag{4.13}$$

Having found a value of $m < \bar{m}$ such that $g^A = g^*$ is an equilibrium, we must show that the corresponding balanced growth path is stable. That is we have to make sure that after slight perturbations of m or g^A market forces lead the world economy back to its steady state values.

Since, as long as $c_R^B < 1/m$, the imitation rate is independent of the country A growth rate, we can take g^* as given. [12] As g^* can be treated as a predetermined variable, we can express the country A free-entry condition as a function of g^A and m. It turns out convenient to write the country A free-entry condition as,

Northern producers cannot keep track with imitators, the Northern market shrink, and sooner or later innovation activities would become unprofitable altogether, so that the South becomes the leading economy. While this overtaking scenario is an interesting possibility within this model, we do not analyze it. Instead we concentrate the analysis on the case where a steady state for the world economy exists and then study the properties of such an equilibrium. In particular, we are interested in the impact of the distribution of wealth under these conditions.

[12] Note that g^B depends partly on the conditions in the industrialized region, namely on the exogenous variables w^A, a, and L^A. These variables determine the size of the market for country B goods, as well as the price these imitators can charge. However, we can make use of the fact that the innovation technology parameter F does not have an influence on g^B.

$$\left(g^A+\theta\right)w^AF=\pi^AL^A+\pi^Am^{\frac{g^A+\theta}{g^*}}\left(\gamma\left(g^A\right)\right) \tag{4.14}$$

where,

$$\gamma\left(g^A\right)\equiv(1-\beta)\,L^B(c_R^B)^{\frac{g^A+\theta}{g^*}}-\left[L^A+(1-\beta)\,L^B\right].$$

There exists a steady state for the world economy if there is some $m\in(0,\bar{m})$ such that the above equation holds. Figure 3 shows this function graphically. On the horizontal axis we draw m and on the vertical axis we draw g^A. Denote by $\tilde{g}^A\equiv\frac{L^A\pi^A}{w^AF}-\theta$ the equilibrium growth rate of the country A economy when $m=0$, that is under autarchy. The free-entry locus starts at $\tilde{g}^A$ and if $\gamma\left(\tilde{g}^A\right)>0$, then also $\gamma\left(g^A\right)>0$, for all $g^A>\tilde{g}^A$. To see the condition more clearly recall that trade balance requires $w^AaL^A=(1-\beta)\,L^B\left(c_R^B-1\right)$ at all times. Noting that $\pi^A=1-w^Aa$ we can write $\gamma\left(g^A\right)>0$ as,

$$\frac{c_R^B-\pi^A}{1-\pi^A}<\left(c_R^B\right)^{\frac{g^A+\theta}{g}}$$

Since $c_R^B>1$, a higher rate of time preference θ makes the trade-imitation regime more likely. Whether imitation and trade stimulate innovation in country A depends on whether it is better to have a larger market for a short time or to have a small market forever. Also, a higher growth rate means that imitators catch up and profits are destroyed earlier. Moreover, with larger unit profits π^A a longer duration of a small market is more valued, since the resulting profit flow is relatively larger. Finally, the gain in the size of the market, captured by c_R^B, has to be large enough in order to make the trade-imitation regime more likely. In other words, the country A economy can reach a higher growth rate with some imitation rather than under autarchy. So the free entry locus will first increase at g^A close to $\tilde{g}^A$. As g^A becomes larger there are two opposing effects. On the one hand, $\gamma\left(g^A\right)$ continues to increase. On the other hand $m^{\frac{g^A+\theta}{g}}$ becomes smaller with a larger g^A. However, we know that if

$$\gamma\left(\tilde{g}^A\right)>0,\ \text{for all } g^A>\tilde{g}^A \text{ and all } m>0 \tag{4.15}$$

(e.g., if $\frac{L^A\pi^A}{w^AF}$ is large enough), then $\frac{\partial g^A}{\partial m}>0$. Hence, for all m over the admissible range, the trade-imitation regime would lead to a higher innovation rate than under autarchy.

Imitation spurs the innovation rate because a higher m influences both the duration of the profit flow of an innovator and the size of the (future) market for this innovator. The profit flow for innovators is shorter the higher m is since imitators are hot-on-their-heels. That is emulation destroys the rents for country A producers, an effect which evidently limits innovation incentives. However, the mere fact that there is imitation creates trade. If country B has a high share in the country A market, this means that country A imports are high. Since trade is balanced at all times high country A imports go hand-in-hand with a high demand for country A goods in country B. Thus a high m increases the size of the market. This tends to increase the profitability of an innovation.

There is an additional reason why some imitation is beneficial for innovation. If π^A is large, the value of free-trade is very high in terms of increased purchasing power in country A. The advantage of dynamic imitation for the rich country is that by lowering the opportunity cost of acquiring basic goods, active manufacturing in the poor country releases resources to other uses by households. This entails that assets in innovation firms are less lucrative due to stiffer competition but that consumer welfare rises. If the above condition is satisfied, we have $\frac{\partial v^A}{\partial m} < 0$.

The phase diagram of the dynamic relationship between g^A and m is based on the two equations below. The first shows the change over time of g^A,

$$\dot{g}^A = \frac{\partial g^A}{\partial m}\left(g^B - g^A\right). \tag{4.16}$$

If condition (4.15) is satisfied, then $\dot{g}^A > 0$ whenever $g^B > g^A$ and viceversa.

The second equation gives the value of m for each g^* such that $\dot{g}^A = 0$, i.e. $g^* = g^B$, which leads to a stable balanced growth equilibrium with free trade,

$$m^* = \left[\frac{(g^* + \theta)\, w^A F - \pi^A L^A}{\gamma(g^*)}\right]^{\frac{g^*}{g^*+\theta}}. \tag{4.17}$$

To complete the construction of the phase diagram in Figure 3, observe that whenever $g^A > g^B$, by definition it is clear that $\dot{m} < 0$. Also, if $g^A < g^B$, we have that $\dot{m} > 0$.

The stability condition for the existence of an equilibrium with international trade (i.e.,$\frac{\partial g^A}{\partial m} > 0$), entailed by (4.15) requires a sufficiently large θ as explained above. In contrast, the discussion on the growth rate in the less developed economy implies that uniqueness of equilibrium for the integrated economy requires a small θ. Consequently, if the integrated system is stable, it is likely that multiple equilibria coexist.

Finally, note that since by Assumptions A and B, m is a measure for the relative productivity between countries, it can be interpreted as a measure of between-country income differences in the steady state.

5. Comparative steady states

In this section we will analyze the effect of various changes in the primitive parameters of the model. First, we shall perform comparative steady state exercises with respect the most obvious parameters in the model as a sort of consistency check. Hence we expect the finding that the growth rate decreases if the rate of time preference rises and if the fixed cost of invention increases.

Other parametric changes do not have unambiguous effects over the growth rate of the system. For example, changes in the distribution of wealth in the developing region, either through variations in population composition between rich and poor households or in the relative value of assets held by the rich and the poor, affect the equilibrium balanced growth rate in a highly nonlinear fashion.

These and other parametric comparative steady state exercises require simulations. One cannot get a clear perspective of the transitional dynamics or the long-run implications through analytical methods. This is due to the simultaneity of supply and demand effects. For instance a change in w^A affects not only the innovation cost but also may increase the volume of trade if aggregate demand expands. To resolve the ambivalence in results, it is necessary to calibrate the model on the basis of realistic parameter configurations. The impact of changes in the technological parameters a, b and G is also unclear. The innovation-imitation dynamics can also be extremely nonlinear. Sometimes intensity in imitation can spur innovation via a procompetitive effect but sometimes it can stop growth if it is predatory.

We are in the process conducting various comparative steady state analyses by applying numerical methods. The best way to obtain a grasp on the conditions of materialization of each of the multiple equilibria is through simulations. Furthermore, calibration of the model is a way of obtaining further insight into the transitional dynamics in the less developed economy. We can then make use of our stability results to understand the adjustment and equilibrium response to shocks that are temporary (i.e. cyclical perturbations) as well as permanent (i.e. structural changes) .

6. Conclusions and extensions

Although the ambiguity in the results so far is relatively unsatisfactory, it does prove the relevance of incorporating nonhomotheticity in preferences in the dynamic analysis of global trade. As observed by Linder(1961) in his classic study, once the difference in expenditure decisions between rich and poor consumers is acknowledged, we conclude that the trade pattern between industrialized and developing regions is determined not only by factor endowment and cross-regional income differentials, as in the Hecksher-Olin-Samuelson and intra-industry trade models, but also by the income distribution within each region. The incorporation of Engel's Law into the preference structure has dramatic implications regarding the importance of income distribution within regions over both the technology diffusion and trade patterns. This feature introduces an aggregate demand channel which raises the possibility of multiple steady states as well as different converging paths even under *common initial conditions.* As discussed in Section 4, stability of the integrated economy generically implies the existence of multiple equilibria. The latter tend to be Pareto rankable. Equilibria exhibiting high growth in the developing region also display high wages. In spite of the higher production costs entailed by high wages, higher growth is sustainable in view of the demand expansion associated with higher income as well as the ensuing rise in labor supply. The prosperous region should also benefit in view of a higher volume of trade which translates into higher growth.

As pointed out in Section 3, by construction, the model implies balance of the capital account in equilibrium because there is international equalization in rates of return. However, there are incentives for technology transfer, which we rule out by assumption. In order to explore technological diffusion to emerging economies, we characterize the life-cycle structure of the locational choice over time for the production of sophisticated newly invented goods. In the present state of the model, we simply inherit the information exchange structure from dynamic North-South trade models where reverse engineering is the only channel of technological diffusion. In future versions, we shall allow for other mechanisms whereby technical knowledge flows across boundaries. This will enrich our study of the evolution of the trade pattern over time, in the presence of nonhomothetic preferences.

We could treat the stock of technical knowledge as an endowment subject to some type of factor price equalization. When considering technology adoption across boundaries, we must model two types of costs that limit the technological implementation possibilities by late adopters. First, we need to incorporate the resource cost entailed by the required absorptive capacity build-up. Second, we should build-in strategic costs due to intellectual property right protection and nondisclosure clauses that innovators use to limit diffusion and enhance trade secrecy. Hence, whether we model foreign direct investment (FDI) or trade in intermediate goods as the conduits of knowledge, the deployment cost provides a bound on the adoption rate. The conclusions reached should be sensitive to what we assume with regard to each form of information flow. For example, Romer(1994) assumes that intermediates are essential to implement new production methods. Hence, trade barriers to exchange new inputs hamper growth. Feenstra(1996), who considers the impact of trade on growth when knowledge flows are localized, arrives to the same conclusion. However, regarding the impact of FDI, because he assumes that the only benefit to the domestic economy is the generation of low-wage jobs, he concludes that the net effect is domestic industry displacement in the short-run and Dutch disease in the long-run. In contrast, Romer(1993) concludes that FDI is probably the most efficient channel through which less developed countries can bring new technologies and enjoy from their propagation over time due to their nonexcludable nature. Enriching the specification of the technological propagation process will undoubtedly lead to more interesting results, as the considerations to follow suggest.

A technology gap may also persist due to trade secrecy incentives. Beyond the real fixed costs associated with technology transplants, there exists a strategic cost to producers in the industrialized country to the extent that technical knowledge is not fully excludable. Although the benefit of using it in various set-ups stems from the fact that it is nonrival, those possessing technological information will try to erect barriers to its dissemination even if they are only partially successful. The balance of these two effects can be analyzed by studying the impact of intellectual property right (IPR) protection and corporate organization. For instance, Helpman(1993) studies the impact of IPR enforcement in a trade model with innovation-imitation dynamics. To the extent that imitation intensity falls, the monopoly power associated with innovation increases and growth falls. But, Lai(1996) has shown that if FDI is the channel of production transfer the conclusions are exactly reversed. The competitive or predatory impact of imitation on innovation thus depends on the characteristic of the propagation process associated with different conduits of technical knowledge flows.

The main analytical result obtained by introducing a demand channel through which income distribution can affect industrial evolution in a dynamic trade model is the multiplicity of equilibria, even under common initial conditions. This is not just a possible outcome but a highly likely one. Indeed, almost surely multiple equilibria and converging paths obtain because the condition required for the existence of a stable equilibrium is that the rate of time preference be sufficiently high while uniqueness requires a rate of impatience below a very small threshold. This by itself demonstrates the importance of nonhomothetic preferences. These multiple equilibria, arising under *common initial conditions*, are generically Pareto rankable due to the correlation of a high wage with high growth in the developing region and the expanded trade volume for the integrated economy. This suggests a very strong possibility for welfare enhancing policy coordination among the regions which is not present in previous models assuming preference homotheticity. Cooperative arrangements could play a catalytic role not necessarily addressed to overhauling measures meant to change initial conditions but rather targeted to jump starting up the movement toward a better equilibrium.

We are in the process of finding more positive results on the relevance of the *intra*-regional income distribution, through the impact of Engel's Law on demand, in the determination of the dynamic pattern of international trade. To do so, we are calibrating the model and applying numerical methods to simulate realistic scenarios and comparative steady state exercises.

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FIGURE 1A

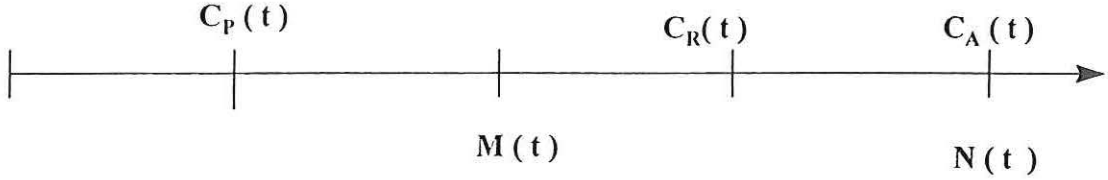

FIGURE 1B

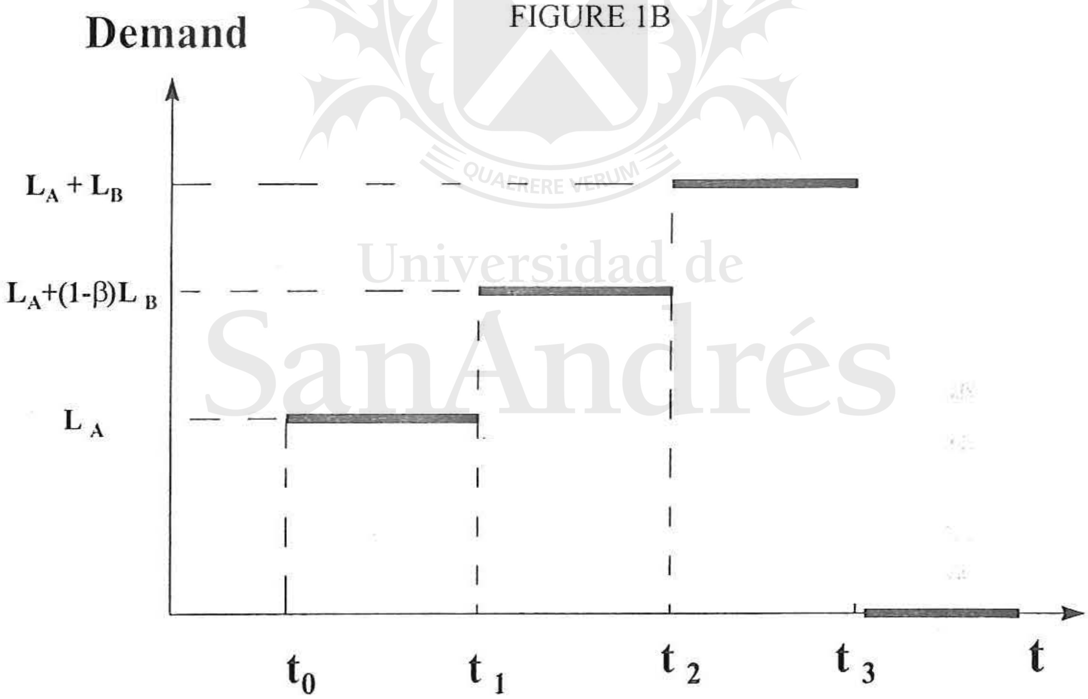

FIGURE 2

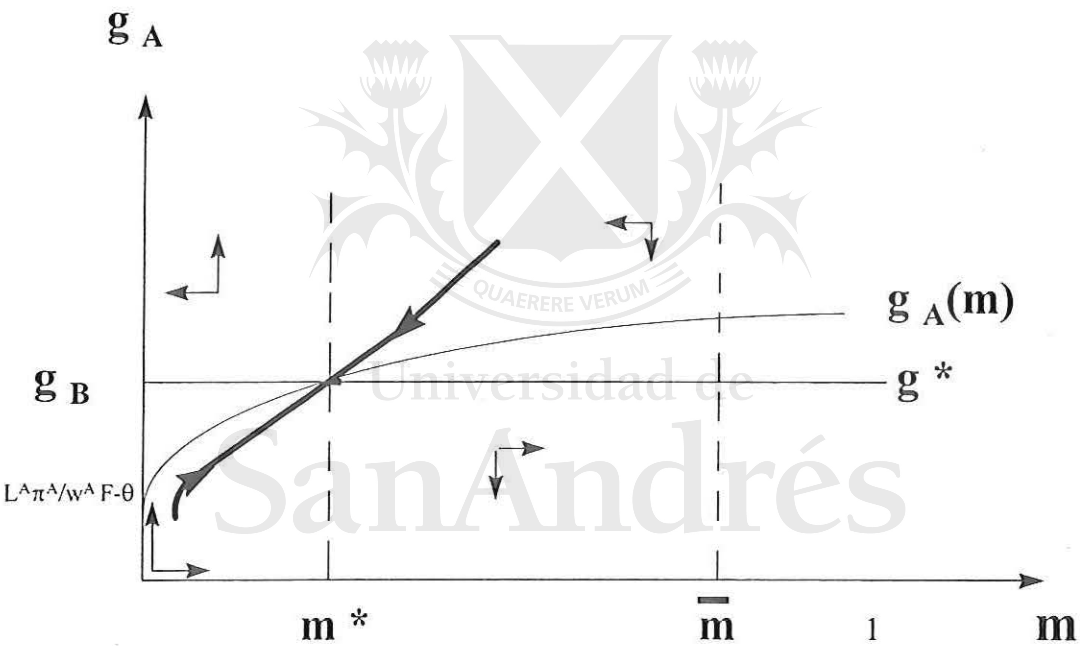

FIGURE 3