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DEPARTAMENTO DE ECONOMIA

Corruption: Some key elements for the Analysis

Federico Weinschelbaum

QUAERERE VERUM

Universidad de San Andres

Universidad de San Andres

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Federico Weinschelbaum (Department of Economics, UCLA)

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By Federico Weinschelbaum

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1.Introduction

Why is corruption an interesting topic? When we have corruption, decisions are influenced by bribes as well as "official prices" and , as a consequence resources may be misallocated. When corruption is rampant, most economic agents do not follow "official prices" and inefficiencies can become very important.

In addition to the misalignment of incentives, corruption leads to two other sources of inefficiency: the misallocation of resources done in order to avoid corruption (including resources expended in order to detect corruption) and resources expended in order to avoid detection.

Despite the importance of the topic, there is not much theoretical economic literature about corruption. Rose-Ackerman (1988)[1] said " Most work on bribery is descriptive and taxonomic. While this makes for interesting reading and is an important source of background information on the range and diversity of corrupt deals, such research does not systematically examine the economic bases of bribery". On the political science side Robert Klitgaard (1988)[2] asks for a theoretic economic study when he says "... it is fair to say that

[1] Rose-Ackerman, Susan (1988) "Bribery" , The New Palgrave: A Dictionary of Economics, J.Eatwell, M. Milgate and P.Newman, eds. Page 277.
[2] Klitgaard, Robert, (1988) Controlling Corruption , University of California Press. Page 73.

several features of corruption as it is encountered in real life have yet to be incorporated in the economic models ...".

See brief review of the literature.

In section 2 we propose a definition for corruption and we characterize some properties of an economy with corruption. In section 3 we develop a model of corruption. In section 4 we restrict the knowledge of the principal, in section 5 we restrict the knowledge of the agent and the hidden principal , and in section 6 we conclude and propose some possible extensions.

2.A definition of Corruption

What do we mean by "corruption"? Some examples will help suggest what we want to capture in our definition.

Example 1: There is a proposal to build a new airport on an artificial island valued at approximately US$ 1 billion. If this project is undertaken, the decision-maker will have the potential to receive huge bribes from the potential contractors.

Example 2: The office boy goes to a bookstore and offers to buy everything there that the office will need if they give him a receipt for a higher price (getting for himself the difference).

This situation could arise for a buyer-manager or in general for any employee that has decision power over who will be the supplier.

Example 3: The pharmaceutical industry in Argentina used to be strongly regulated. Every medicine had to be authorized by the government in order to be legally sold and the price was fixed by the government. The government is a also big buyer (for hospitals, health insurance, etc.). When there is a private transaction, the decision of what medicine to buy is made by the doctor with, in many cases, the Social Security System as payer. This environment creates strong incentives for the firms to bribe government agencies or doctors to improve their sales.

Let me propose a definition of corruption which can capture the situations described above.

Corruption is present when an agent gets a bribe[3] from another individual or organization in order to benefit his interest, at the expense of the interests of the organization for which he nominally works. An agent receiving a bribe acts not only in his own interest but also in the interest of someone other than his employer.

So corruption involves a principal-agent relationship, since we need a separation between the agent (who may be corruptible) and the organization to which he belongs, they have different interests, and we need a third actor, the potential corrupter. Because the agent is not explicitly working for this actor, I call the corrupter a "hidden principal", although the relationship is not necessarily one of a principal to agent. Corruption can be view as a form of moral hazard, in which a corrupt agent takes actions that are not observed by the principal, but the hazard is not simply low effort and corrupt behavior is encouraged by a

[3] The common type of bribe is an amount of money but it could be in other commodities. The important concept is that a corrupt agent will receive something that will increase his utility.

third party, the corrupter. The main difference between traditional moral hazard and corruption is that in the case of corruption the incentive to take an unwanted action by the principal is naturally endogenous. In contrast, with traditional moral hazard the temptation not to work hard is naturally exogenous, coming from the utility function of the agent. With corruption, agents may have a preference for honesty but we also have to account for the presence of bribes that are determined endogenously. The other important difference comes from the existence of a third actor, introducing the relationship corrupter-corrupt, the issue whether an illegal agreement can be observed and enforced. Since this relationship is an illegal one, resources may be used to detect its presence and also in order to avoid its detection.

What conditions are required to have corruption? In a perfect competitive environment in which all agents know they are perfect competitors, there is no possibility of corruption. But a perfectly competitive equilibrium can be overturned by corruption. So if ex-post the result is a perfectly competitive equilibrium we can affirm that in this economy there is no corruption, but not that there is no place for corruption.

Rose-Ackerman (1975) argues that a corrupt actor has to be in a position of power. While it is true that the corrupter has to believe that the corrupt has power, it is not true that the actor must have the ability to take decisions. In some cases there is a bribe to do something that anyway will be done, the thing is that there is a hidden knowledge between the agent and the hidden principal.

Most of the literature on corruption concentrates on the government. Although government corruption seems widespread[4], it is clear from our definition that we do not need to have government in order to have corruption. So we will try to find an easier model without the complication of modeling the government. Consider for example, a principal-agent relationship on the supply side of the market in which buyers bribe someone to get a commodity. For this to happen, either there must be rationing of the good or the agent is able to sell at a cheaper price. In the later case, the agent, in a certain sense steals the good. Situations such as this are not pure corruption: since corruption is not the unique "anomaly" in the market, it is not clear how much of an effect can be attributed to corruption. Instances of pure corruption in which the principal-agent relationship is on the supply side can occur, but they are not typical. On the other hand, if we look at corruption in the demand side in which the corrupt agent is employed by the buyer, pure corruption is much more natural: typically the agent reports to the principal a price higher than the one actually paid. This is the case considered in this paper.

3.A preliminary model of Corruption

We develop a simple model addressing the problem of corruption in which the agent is an employee of a firm that wants to buy an input. Among the abilities of this manager are not only his knowledge about the market and his negotiation power but also his level of honesty. Corruptibility of such an agent depends on his honesty: if he is not honest, he can be corrupted.

[4]The fact that the principal (the population) has very little control over the agent makes the problem deeper.

The problem of the principal is to maximize its utility subject to the "participation" and "incentive compatibility" constraints of the agent and the seller. We formulate this problem for different levels of information of the players.

We have three players: principal, agent and hidden principal. You can think of the players as the owner of a firm (the principal), the manager (the agent), and the supplier of a commodity or service which the principal has asked the agent to obtain (the hidden principal).

We assume everybody is risk neutral. The principal is assumed to have utility function

$$U_p = It - w_a(\hat{C})$$

where $t \in \{0,1\}$ indicates whether the transaction is done (t=1) or not (t=0). We assume the buyer is trying to buy an input that has no substitute: when the transaction does not occur, the principal cannot sell the output; and when it is done the income is independent of the cost. $w_a(\hat{C})$ is the payment per transaction for the purchase of the input that the principal gives to the agent which cover both the agent payment and the amount he pays to the hidden principal, where $\hat{C}$ is the production cost the agent and the hidden principal announce jointly to the principal.

The agent has utility function

$$U_a = w_a(\hat{C}) - w_{hp}(\hat{C}, C) - h_a b$$

where C is the actual production cost of the input. Assume that C=0 if there is no production, but with production $C=\underline{C}$ with probability q and $C=\overline{C}$ with probability 1-q.

The agent and the hidden principal know the actual cost but the principal does not, he only knows the probability distribution.

w_{hp} ($\hat{C}$,C), the payment that the agent gives to the hidden principal, is a function of the announced cost $\hat{C}$ and the actual cost C, so $w_a(\hat{C}) - w_{hp}(\hat{C}, C)$ is the amount that the agent keeps. $h_a b$ gives the equivalent monetary moral cost of being corrupt where h_a is the level of honesty of the agent and $b \in \{0,1\}$ indicates whether the agent lies (b=1) or not (b=0). Where we define lie when $\hat{C} \neq C$ and no lie when $\hat{C}=C$.

The hidden principal has utility function

$$U_{hp} = w_{hp}(\hat{C}, C) - h_{hp} b - C$$

where h_{hp} is the level of honesty of the hidden principal and $h_{hp}b$ is the monetary moral cost of being a corrupter. Reservation utilities of the agent and the hidden principal are 0; h_a , h_{hp} ≥ 0 and $I > \overline{C} > \underline{C}$. We also assume, as usual, that when actors are indifferent, they announce the truth.

The sequence of the game is as follows:

1) The principal decides the payment system w_a.
2) Nature selects the production cost C of the hidden principal: $\underline{C}$ (with probability q) and $\overline{C}$ (with probability 1-q).

3) The agent announces jointly with the hidden principal the type of the hidden principal (i.e. the value of $\hat{C}$). The agent has the option of refusing the transaction, leading to one "participation " constraint for each state.

Analysis

<u>Theorem</u> 1: without loss of generality we can restrict our analysis to payment schemes for which

$$\overline{C} \geq w_a(\overline{C}) \geq w_a(\underline{C}) \geq \underline{C}$$

Proof: See Appendix 1.

If the actual cost is $\overline{C}$, there is no incentive to lie : if an agent, in collusion with the hidden principal, lies, they will get a lower payment ($w_a(\overline{C}) \geq w_a(\underline{C})$) and both the buyer and the seller will have to pay the moral cost.

<u>Theorem</u> 2: We can restrict our analysis without lost of generality to payment schemes such that either $w_a(\overline{C}) = w_a(\underline{C}) = \underline{C}$ or $w_a(\overline{C}) = \overline{C}$.

Proof: We know from theorem 1 that $\overline{C} \geq w_a(\overline{C})$. For every scheme such that $\overline{C} > w_a(\overline{C})$ we know that the "participation" constraint either of the agent or the hidden principal will not be satisfied when $C = \overline{C}$. So the principal can only get utility when $C = \underline{C}$, and in this case with the scheme $w_a(\overline{C}) = w_a(\underline{C}) = \underline{C}$ the principal gets all the surplus, $\quad_{p} = q(I - \underline{C})$. ■

We can split the schemes in three categories

1) $w_a(\underline{C})=w_a(\overline{C})=\underline{C}$.

2) $w_a(\underline{C})=\underline{C}$, $w_a(\overline{C})=\overline{C}$.

3) $w_a(\underline{C})=\overline{C}-\beta$, $w_a(\overline{C})=\overline{C}$ where $\beta \in [0, \overline{C}-\underline{C})$.

When the actual cost is $\underline{C}$ there could be an incentive to cheat (announce $\overline{C}$) depending on which is the payment scheme.

If we assume that there is no problem of information between the agent and the hidden principal, the agent knows h_{hp} and the hidden principal knows h_a.

Corruption will occur whenever

$$w_a(\overline{C}) - w_a(\underline{C}) > h_a + h_{hp}$$

The left hand side of this inequality is the gain that the coalition of the agent and the hidden principal will have if they announce $\overline{C}$ instead of the actual value $\underline{C}$, the right hand side is the moral cost of the lie.

In this situation the problem of the principal is to[5, 6]:

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[5]We are using the fact that: when the actual cost is $\overline{C}$, there is no incentive to lie and when the actual cost is $\underline{C}$ there will be always transaction since $w_a(\underline{C}) \geq \underline{C}$. You can see the general problem for the case where there is no information problem between the agent and the hidden principal in Appendix 2.

[6]Note that in the construction of the problem we have not made any assumption about the principal's knowledge of the agent's and hidden principal's level of honesty.

$$\underset{w_a(\overline{C}),w_a(\underline{C})}{\text{Max}} \quad U_p = q\{l[I - w_a(\underline{C})] + (1-l)[I - w_a(\overline{C})]\} + (1-q)v[I - w_a(\overline{C})]$$

where l=1 if $w_a(\underline{C}) \geq w_a(\overline{C}) - h_a - h_{hp}$, l=0 otherwise.

v=1 if $w_a(\overline{C}) \geq \overline{C}$, v=0 otherwise

The first term in the addition is the one that corresponds to the utility of the principal when the actual cost is $\underline{C}$, it happens with probability q, l =1 when the announcement will be $\underline{C}$, and l=0 when the announcement will be $\overline{C}$, we know that there always going to be transaction when C=$\underline{C}$. The second part of the equation corresponds to the utility of the principal when C=$\overline{C}$, v=1 when the participation constraint is satisfied.

The easiest game is when everybody knows the level of honesty of everyone else (i.e. h_a,h_{hp}) and an agent who lies has no probability of being caught.

a)If the parameters of the problem are such that the optimal scheme is one where $w_a(\overline{C}) < \overline{C}$, and hence v=0, we know from theorem 2 that the principal will offers $\underline{C}$ regardless of the announcement: $w_a(\overline{C}) = w_a(\underline{C}) = \underline{C}$. We will call this scheme (w_a^1)

b)<u>Lemma</u> 2: *If* $h_a + h_{hp} \geq H_2$ *then* $U_p(\overline{C},\underline{C}) \geq U_p(a,b)$, $\forall a,b$

Where U_p(a,b) means the utility that the principal gets if he use an scheme such that $w_a(\overline{C})$ =a and $w_a(\underline{C})$ = b, and $H_2 = \overline{C} - \underline{C}$.

Proof: When the optimal solution is one where v=1; $w_a(\overline{C}) \geq \overline{C}$, since we know that $w_a(\overline{C}) \leq \overline{C}$ we have that $w_a(\overline{C}) = \overline{C}$. The unique variable that we need to set is $w_a(\underline{C})$, when we set $w_a(\underline{C})$ as low as we can we get the second scheme (w_a^2) where the principal will pay the announced cost:

$$w_a(\underline{C}) = \underline{C},\ w_a(\overline{C}) = \overline{C}$$

When the cost is $\underline{C}$ there is an incentive to lie, if the gain from lying $\overline{C}$ - $\underline{C}$, exceeds the moral cost $h_a + h_{hp}$: They will lie when the gain is greater than the cost. Under this scheme, the utility of the principal is:

$$U_p = q(I-\underline{C}) + (1-q)(I-\overline{C}) \qquad if \quad h_a + h_{hp} \geq \overline{C} - \underline{C} \qquad (l'_1 = 1; l'_2 = 0)$$
$$U_p = (I-\overline{C}) \qquad if \quad h_a + h_{hp} < \overline{C} - \underline{C} \qquad (l'_1 = 0; l'_2 = 1)$$

If $h_a + h_{hp} \geq \overline{C} - \underline{C}$, the true state is revealed , and the principal gets the whole surplus in both states. Therefore this scheme maximizes the utility of the principal when $h_a+h_{hp} \geq \overline{C}-\underline{C}$■.

c) In the third scheme (w_a^3), the principal internalizes the moral cost of lying: when the announcement is $\underline{C}$, the principal pays the amount that made the coalition between the agent and the hidden principal just indifferent between lying and revealing the true cost. Using w_a^2 as a point of comparison, to reduce the incentive to lie the principal has to reduce $w_a(\overline{C})$ - $w_a(\underline{C})$. If $w_a(\overline{C})$ is lower, there will be no transaction when the actual cost is $\overline{C}$ (v'_2=0), so this scheme could be equivalent to w_a^1. If, in the other hand, the principal increases $w_a(\underline{C})$ the value required to avoid the corruption (make l'_1=1) is $w_a(\underline{C}) = \overline{C} - h_a - h_{hp}$, ending to the payment scheme:

$$w_a(\underline{C}) = \overline{C} - h_a - h_{hp}$$
$$w_a(\overline{C}) = \overline{C}$$

The utility of the principal becomes

$$_p = q(I - (\overline{C} - h_a - h_{hp})) + (1-q)(I - \overline{C}) \text{ or equivalently,}$$

$$_p = q(h_a + h_{hp}) + (I - \overline{C})$$

Thus, the principal receives (I - $\overline{C}$) plus a premium equal to $h_a + h_{hp}$ when the real cost is $\underline{C}$, the amount that the coalition is willing to pay in order to avoid lying.

The utility to the principal under this scheme is greater than that obtained under scheme (2) whenever $h_a + h_{hp} < \overline{C}$ - $\underline{C}$. Under scheme (3) the principal "loses" part of the surplus when the actual cost is $\underline{C}$, a payment to the agent and the seller to avoid corruption.

When $h_a + h_{hp} < \overline{C}$ - $\underline{C}$ is costly for the principal avoid corruption but the cost is smaller than the one that would have if it allowed it.

Theorem 3:

1) *If* $h_a + h_{hp} \geq H_2$ *then* $U_p(\overline{C}, \underline{C}) \geq U_p(a,b)$, $\forall a,b$
2) *If* $H_2 > h_a + h_{hp} \geq H_1$ *then* $U_p(\overline{C}, \overline{C} - h_a - h_{hp}) \geq U_p(a,b), \forall a,b$
3) *If* $H_1 > h_a + h_{hp}$ *then* $U_p(\underline{C}, \underline{C}) \geq U_p(a,b), \forall a,b$

Where $H_1 = \max\{I - \underline{C} - (I - \overline{C})/q, 0\}$.

Proof: 1) See lemma 2. 2) and 3), we have seen that when $h_a + h_{hp} < H_2$ scheme (3) is better than (2) so we have to see which is the area where is better to set v=0 and where is better to set v=1, so we have to compare schemes (1) and (3). The principal will use scheme (1) whenever

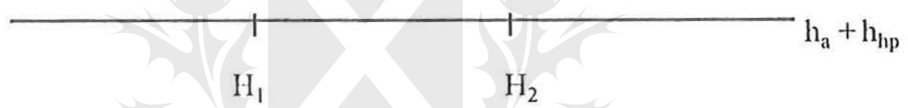
$$q(h_a + h_{hp}) + (I - \overline{C}) < q(I - \underline{C}) \text{ or, equivalently,}$$

$h_a + h_{hp} < I - \underline{C} - (I - \overline{C}) / q \equiv H_1$ as claimed. ■

We conclude in theorem 3 that the optimal scheme, the one that maximizes the utility of the principal, is a function of the total level of honesty of the coalition ($H=h_a+h_{hp}$) that can be graphed as follows:

Payment Schemes

		$w_a^1(\hat{C})$	$w_a^3(\hat{C})$	$w_a^2(\hat{C})$
Announcement	$\overline{C}$	$\underline{C}$	$\overline{C}$	$\overline{C}$
	$\underline{C}$	$\underline{C}$	$\overline{C} - h_a - h_{hp}$	$\underline{C}$
Region		(1)	(3)	(2)



where $H_1 = I - \underline{C} - (I - \overline{C}) / q$ and $H_2 = \overline{C} - \underline{C}$

In this model corruption never occurs, but the reason varies with the level of honesty $H \equiv h_a+h_{hp}$. In region (1)[7] the coalition of agent and hidden principal is so dishonest that the principal , accepting zero profits when the actual cost is $\overline{C}$, would rather pay $\underline{C}$ than an amount sufficient to get them to tell the truth.

In region (3) the principal pays a "premium " sufficient to induce the agent not to lie. In region (2) the coalition is so honest that the gain from lying is smaller than the cost of being dishonest.

[7] The existence of this region depends on $H_1 \geq 0$.

As extreme cases, we can see that when q=0 region (1) will disappear, and scheme (2) and (3) merge since the agent and the hidden principal will always announce $\overline{C}$, the true result. When q=1, region 3 disappears, $H_1=H_2=\overline{C} - \underline{C}$. In both regions (1) and (2) the coalition of agent and hidden principal announce the truth $\underline{C}$, guilding utility (I-$\underline{C}$) for the principal since under scheme (1) $\hat{C}=\underline{C}$ for all the range and under scheme (2) $\hat{C}=\underline{C}$ in region (2) because $h_a+h_{hp} \geq \overline{C} - \underline{C}$.

Note that although there is no corruption, the possibility that it could occur leads to an inefficiency that appears whenever scheme (1) is use since with probability (1-q) the principal obtains no input even though it has a positive "social value"[8] (I - $\overline{C}$).

The no-corruption result is driven by the high degree of knowledge of the principal. If we assume that the principal does not know h_a and h_{hp}, we will have a more realistic world.

Restricting the knowledge of the principal

Assume that the principal knows the distribution of the variable $h_a + h_{hp}$, represented by the distribution function:

$$P(h_a + h_{hp} \leq H) = F(H)$$

The principal can choose what scheme to use by comparing the expected utility that he will get under each one.

[8] I am implicitly assuming (1)either that this game is the whole economic activity of the society or that the resolution of the game will not modify anything else and (2) I is the social value of the production.

For example, let $P(H_1 > h_a + h_{hp}) = \varepsilon_1$ and $P(H_2 > h_a + h_{hp} \geq H_1) = \varepsilon_2$. If $\varepsilon_1, \varepsilon_2$ are small enough, the optimum scheme will be (2), and we will get corruption with probability $\varepsilon_1 + \varepsilon_2$.

Solving the general problem we get as a result that we have corruption with positive probability.

The scheme w_a^1 will give a payoff of $\quad_p = q(I - \underline{C})$, which is independent of the distribution of the honesty level.

Again when the parameters of the problem are such that the optimal scheme is one where $v'_2=0$ this is an optimal scheme.

When the optimal solution is one where v =1 we have to see again two different schemes.

Scheme w_a^3 cannot be implemented since the principal does not know the exact type. But we can calculate which will be the payment for each β that the principal chooses. Where β is the reduction in the payment to the agent that the principal makes when the announcement is $\underline{C}$.

The modified scheme (3) $w_a^{3'}$ is:

$$w_a(\underline{C}) = \overline{C} - \beta$$
$$w_a(\overline{C}) = \overline{C}$$

So the utility of the principal becomes

$$U_p = q(1 - F(\beta))(I - (\overline{C} - \beta)) + [(1-q) + qF(\beta)](I - \overline{C})$$

which can be rewritten as follows

$$U_p = q(1 - F(\beta))\beta + (I - \overline{C})$$

So if the principal chooses the β that maximizes its expected utility we have

$$\frac{\partial Up}{\partial \beta} = q[1 - F(\beta^*) - \beta^* f(\beta^*)] = 0 \qquad \text{(a)}$$

If we call g(H) = 1- F(H) - H f (H) , we can see that g (0) = 1 and g ($\overline{H}$) ≤ 0 (we allow $\overline{H} = \infty$), where $\overline{H}$ is the upper bound of the distribution. If g (H) is continuous (or, equivalently, if F(H) $\in C^2$), we typically have an odd number of solutions to (a) and at least one will satisfy the second order conditions[9]. The principal can compare utilities associated with each local maximum and find which is the global maximum[10], which we call β^*. Note that $\beta^* \leq \overline{C}$ - $\underline{C}$ if not,

$$w_a(\underline{C}) = \overline{C} - \beta^* < \overline{C} - (\overline{C} - \underline{C}) = \underline{C}$$

and there will be no transaction when the actual cost is $\underline{C}$ and the coalition is honest enough to tell the truth. Expected utility is then

$$U_p = [(1 - q) + qF(\beta^*)](I - \overline{C})$$

which is clearly not optimal for the principal (it is even dominated by $\overline{C}$ = $w_a(\underline{C}) = w_a(\overline{C})$).

As with scheme (1), scheme (2) can be implemented as before even though the principal does not know the level of honesty, but the principal's utility will change. It becomes

[9] When (1-F(H))/f(H) is decreasing , which means that the reliability function (1-F(H)) is log-concave we will have a unique solution. This holds for many well known distributions such as Uniform, Normal, Logistic, Extreme Value, Chi-squared, Chi, Exponential, Laplace and any truncation of these distributions. For details see Bagnoli and Bergstrom (1989).

[10] If there is more than one he will be indifferent among them, so we can choose any one.

$_p = q[1-F(\overline{C}-\underline{C})](I-\underline{C})+[(1-q)+qF(\overline{C}-\underline{C})](I-\overline{C})$ or, equivalently,

$_p = q[1-F(\overline{C}-\underline{C})](\overline{C}-\underline{C})+(I-\overline{C})$

The principal gets the premium $\overline{C}-\underline{C}$ when the cost is $\underline{C}$ and the coalition is honest enough to announce the truth.

Choice of the optimal scheme is not as clear as the full information case.

Comparing first scheme (3) with (2), we first find β_0^* without restrictions (i.e. solve (a))

If $(\overline{C}$ - $\underline{C}) \geq \beta_0^*$ so $\beta_0^* = \beta^*$ we know that scheme (3) is at least as good as (2) since

$q[1-F(\beta^*)]\beta^* + (I-\overline{C}) \geq q[1-F(\beta)]\beta + (I-\overline{C}) \quad \forall\beta$
and so, in particular it holds for $\beta = (\overline{C}-\underline{C})$

When $\beta_0^* > (\overline{C}$ - $\underline{C})$ the procedure is to find the optimum level of β^*[11] such that $\beta^* < (\overline{C}$-$\underline{C})$

So we can calculate the optimal comparing the utility of the principal under the three schemes.

$U_p(w_a^1) \equiv U_p(\underline{C},\underline{C}) = q(I-\underline{C})$

$U_p(w_a^2) \equiv U_p(\overline{C},\underline{C}) = q[1-F(\overline{C}-\underline{C})](\overline{C}-\underline{C})+(I-\overline{C})$

$U_p(w_a^{3'}) \equiv U_p(\overline{C},\overline{C}-\beta^*) = q[1-F(\beta^*)]\beta^* + (I-\overline{C}))$

[11]Note that if $\frac{\partial Up}{\partial \beta} > 0, \forall \beta \in [0, \overline{C}-\underline{C})$ we go to the corner solution $\beta^* = (\overline{C}$ - $\underline{C})$ so scheme (2) is better.

The principal will choice the one of the three that gives the greater utility, let's call it w_a*, and it will be a function on the values of the parameters.

When we try to do the comparative statics in order to see how the probability of having corruption changes in response to a change in one of the parameters, all the signs are ambiguous, except for I.

<u>Proposition</u> 1: As I increases, the probability of corruption increases, assuming that the other parameters are chosen randomly.

Proof: We can calculate the probability of corruption as follows.

P(corruption)= $\sum_{i\in\{1,2,3'\}} P(w_a^i = w_a{*})\, P(corruption / w_a^i = w_a{*})$

where P(corruption/w_a^1=w_a*)=0

P(corruption/w_a^2=w_a*) = q F($\overline{C}$ - $\underline{C}$) >0

P(corruption/$w_a^{3'}$=w_a*) = q F(β*) >0

We can easily see that $\frac{\partial \beta^*}{\partial I} = 0$ *and* $\frac{\partial P(corruption / w_a^i)}{\partial I} = 0$ *for* $i \in \{1,2,3'\}$, using this we can write the derivative of the probability of having corruption with respect to income as follows:

$$\frac{\partial \text{p(corruption)}}{\partial \text{I}} = \frac{\partial \text{p}(w_a^2 = w_a^*)}{\partial \text{I}} \text{qF}(\overline{C} - \underline{C}) + \frac{\partial \text{p}(w_a^{3'} = w_a^*)}{\partial \text{I}} \text{qF}(\beta^*)$$

But since $\frac{\partial U_p(w_a^1)}{\partial I} = q < 1 = \frac{\partial U_p(w_a^2)}{\partial I} = \frac{\partial U_p(w_a^{3'})}{\partial I}$ we know that $\frac{\partial P(w_a^2 = w_a^*)}{\partial I}, \frac{\partial P(w_a^{3'} = w_a^*)}{\partial I} > 0$ and $\frac{\partial P(w_a^1 = w_a^*)}{\partial I} = 0$.

Therefore $\frac{\partial P(\text{corruption})}{\partial I} > 0$. ■

In economic terms, this means that when, the income is higher and consequently the total surplus (I - C) is higher in the same magnitude in every state, it is more costly not to produce in some periods, so scheme (1) is less used.

About inefficiency we have that when there is corruption there is inefficiency driven by the way that the "moral costs" are modeled but also when scheme (1) is used we have the inefficiency mentioned in the case where h_a,h_{hp} were known by the principal.

5.Restricting the knowledge of the agent and the hidden principal

Until this point, the "hidden principal", (seller) has played no essential role: we can achieve essentially the same results with only two players, the agent and the principal. We have not yet modeled the relationship agent-hidden principal which, in our view is essential to the phenomenon of corruption. To provide a role for the hidden principal, we need to restrict the knowledge of the agent or the hidden principal. We will assume (1) the agent does not know h_{hp}, although he knows that $P(h_{hp} \leq H_{hp})$ is $G(H_{hp})$, and (2) the hidden principal does not know h_a, he knows that $P(h_a \leq H_a) = A(H_a)$.

Assume the following game sequence

1) The principal decides the payment system $w_a(\hat{C})$.
2) Nature decides the production cost of the hidden principal, $\underline{C}$ (with probability q) and $\overline{C}$ (with probability 1-q).

3.A) The agent makes an offer to the hidden principal, consisting of a payment $w_{hp}(\overline{C},\underline{C})$ and a proposal for the cost that they will announce to the principal.

3.B) The hidden principal accepts or rejects the offer. If the offer is to lie and the hidden principal rejects it, the option of lying is lost.

<u>Lemma</u> 3: If $G(H_{hp})/g(H_{hp})$ is continuous, and we assume that $w_{hp}(\underline{C},\underline{C})$ is independent of the offer that the agent makes. Whenever there is a proposal for cheating $w^*_{hp}(\overline{C},\underline{C})$ is such that the following condition holds.

$$wa(\overline{C}) - wa(\underline{C}) - w_{hp}*(\overline{C},\underline{C}) + w_{hp}(\underline{C},\underline{C}) = \frac{G(w_{hp}*(\overline{C},\underline{C}) - w_{hp}(\underline{C},\underline{C}))}{g(w_{hp}*(\overline{C},\underline{C}) - w_{hp}(\underline{C},\underline{C}))} \qquad \text{(B)}$$

Proof: We know that there will be an incentive to cheat only if the real cost is $\underline{C}$ since theorem 1 still hold.

So when the actual cost is $\underline{C}$ the agent will know that the hidden principal will accept the offer iff

$$w_{hp}(\overline{C},\underline{C}) - \underline{C} - h_{hp} > w_{hp}(\underline{C},\underline{C}) - \underline{C}$$

i.e., the hidden principal's utility when lying is greater than the utility of telling the truth. This can be rewritten as follows

$$w_{hp}(\overline{C},\underline{C}) - w_{hp}(\underline{C},\underline{C}) > h_{hp};$$

i.e., the gain from lying (the left hand side) has to be greater than the cost (the right hand side). This will happen with probability $G(w_{hp}(\overline{C},\underline{C}) - w_{hp}(\underline{C},\underline{C}))$.

The agent will choose the offer that maximizes its utility.

The utility of the agent when the real cost is $\underline{C}$ is [12]

$$\begin{aligned} {}_a &= [wa(\overline{C}) - w_{hp}(\overline{C},\underline{C})]G(w_{hp}(\overline{C},\underline{C}) - w_{hp}(\underline{C},\underline{C})) + \\ &\quad [wa(\underline{C}) - w_{hp}(\underline{C},\underline{C})][1 - G(w_{hp}(\overline{C},\underline{C}) - w_{hp}(\underline{C},\underline{C}))] - h_a \end{aligned}$$

which can be rewritten as follows

$$\begin{aligned} {}_a &= [wa(\overline{C}) - w_{hp}(\overline{C},\underline{C}) - wa(\underline{C}) + w_{hp}(\underline{C},\underline{C})]G(w_{hp}(\overline{C},\underline{C}) - w_{hp}(\underline{C},\underline{C})) \\ &\quad + wa(\underline{C}) - w_{hp}(\underline{C},\underline{C}) - h_a \end{aligned}$$

This utility achieves a maximum when[13]

[12] We will assume that when the agent makes the offer he will pay the moral cost, no matter if the hidden player accepts or not.

[13] Note that in order to be optimal to make an offer (to lie) it has to hold that $[w(\overline{C})-w_{hp}*(\overline{C},\underline{C})-w_a(\underline{C})+w_{hp}(\underline{C},\underline{C})]G(w_{hp}*(\overline{C},\underline{C})-w_{hp}(\underline{C},\underline{C}))-h_a>0$.

$$\frac{\partial U_a}{\partial w_{hp}(\overline{C},\underline{C})} = [g(w_{hp}*(\overline{C},\underline{C}) - w_{hp}(\underline{C},\underline{C}))][w_a(\overline{C}) - w_a(\underline{C}) - w_{hp}*(\overline{C},\underline{C}) + w_{hp}(\underline{C},\underline{C})]$$
$$-G(w_{hp}*(\overline{C},\underline{C}) - w_{hp}(\underline{C},\underline{C}))] = 0$$

But this condition can be rewritten as (B). ■

Proposition 2: If $G(H_{hp})/g(H_{hp})$ is continuous, and we assume that $w_{hp}(\underline{C},\underline{C})$ is independent of the offer that the agent makes. Then $0 > \frac{\partial P(offer)}{\partial h_a}$

Proof: The utility of the agent when there is no offer for cheating is independent of the level the honesty of the agent while the utility of making an offer is decreasing[14]. Therefore the greater h_a the smaller the probability that the utility of making the optimal offer would be greater than the utility of no making an offer. ■

If $G(H_{hp})/g(H_{hp})$ is continuous we know there typically will be an odd number of solutions for equation (B) since when $w_{hp}(\overline{C},\underline{C})=w_{hp}(\underline{C},\underline{C})$ (all the gains from lying are appropriated by the agent) the left hand side is greater than zero and decreasing while the right is zero, when $w_{hp}(\overline{C},\underline{C})=w_{hp}(\underline{C},\underline{C})+w_a(\overline{C})-w_a(\underline{C})$ (all the gains from lying are appropriated by the hidden principal) the left side is zero and the right greater than zero. In particular if we assume that $G(H_{hp})/g(H_{hp})$ is non decreasing[15] we have a unique solution so we can have some results.

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[14]Note we are using the fact that the optimal offer is independent of the level of h_a.

[15]This condition is equivalent to have log-concave c.d.f. function and holds for many well known distributions such as Uniform, Normal, Logistic, Extreme Value, Chi-squared, Chi, Exponential, Laplace, Log Normal, Pareto and any truncation of these distributions. For details see Bagnoli and Bergstrom (1989).

Proposition 3: If $G(H_{hp})/g(H_{hp})$ is continuous, non decreasing, and we assume that $w_{hp}(\underline{C},\underline{C})$ is independent of the offer that the agent makes.

$0 \geq \dfrac{\partial w^{*}_{hp}(\bar{C},\underline{C})}{\partial w_{a}(\underline{C})} \geq -1$ Where it is equal to zero iff $w_{hp}^{*}(\bar{C},\underline{C})=w_{hp}(\underline{C},\underline{C})$, and greater than -1 if $G(H_{hp})/g(H_{hp})$ is increasing. So we have that also the bribe is decreasing.

Proof:

$$\frac{\partial w^{*}_{hp}(\bar{C},\underline{C})}{\partial w_{a}(\underline{C})} = -\frac{1}{\dfrac{d(G(\,)/g(\,))}{dh_{hp}}+1}$$ when there is a proposal to cheat and zero otherwise.

■

Note that the total gains from lying are $w_a(\bar{C})-w_a(\underline{C})$, the amount that is appropriated by the corrupter is $w_{hp}(\bar{C},\underline{C})-w_{hp}(\underline{C},\underline{C})$, while the amount appropriated by the corrupt (i.e. the bribe) is $w_a(\bar{C})-w_a(\underline{C})-w_{hp}(\bar{C},\underline{C})+w_{hp}(\underline{C},\underline{C})$. So proposition 3 is telling us that in case of cheating when the gains are greater, $w_a(\underline{C})$ is smaller, the gains of both the corrupter and the corrupt will be greater, and conversely when the gains are smaller.

Proposition 1': there is a proposition similar to 1 in this game. Proof to be done.

Suppose instead that the hidden principal is a Stackelberg leader

Sequence of the game

1) The principal decides the payment system $w_a(\hat{C})$.

2) Nature decides which is the production cost of the hidden principal, $\underline{C}$ (with probability q) and $\overline{C}$ (with probability 1-q).

3.A) The hidden principal makes an offer to the agent consisting of a payment $w_{hp}(\overline{C},\underline{C})$ and a proposal for the cost that they will announce to the principal.

3.B) The agent accepts or rejects the offer. If the offer is to lie and the agent rejects it, the option of lying is lost.

It continues to be the case that there will be an incentive to cheat only if the real cost is $\underline{C}$.

Lemma 4: If $A(H_a)/a(H_a)$ is continuous, and we assume that $w_{hp}(\underline{C},\underline{C})$ is independent of the offer that the hidden principal makes. Whenever there is a proposal for cheating $w^*_{hp}(\overline{C},\underline{C})$ is such that the following condition holds.

$$w_{hp}^*(\overline{C},\underline{C}) - w_{hp}(\underline{C},\underline{C}) = \frac{A(wa(\overline{C}) - w_{hp}^*(\overline{C},\underline{C}) - wa(\underline{C}) + w_{hp}(\underline{C},\underline{C}))}{a(wa(\overline{C}) - w_{hp}^*(\overline{C},\underline{C}) - wa(\underline{C}) + w_{hp}(\underline{C},\underline{C}))} \qquad \text{(C)}$$

Proof: If the actual cost is $\underline{C}$, the hidden principal will know that the agent will accept the offer iff

$$wa(\overline{C}) - w_{hp}(\overline{C},\underline{C}) - h_a > wa(\underline{C}) - w_{hp}(\underline{C},\underline{C}),$$

i.e., utility when lying is greater than the utility of telling the truth. This can be rewritten as follows

$$wa(\overline{C}) - w_{hp}(\overline{C},\underline{C}) - wa(\underline{C}) + w_{hp}(\underline{C},\underline{C}) > h_a;$$

i.e., the gain from lying (the left hand side) has to be greater than the cost (the right hand side). This will happen with probability equal to $A(w_a(\overline{C}) - w_{hp}(\overline{C},\underline{C}) - w_a(\underline{C}) + w_{hp}(\underline{C},\underline{C}))$.

The hidden principal will choose the offer that maximize its utility[16].

The utility of the hidden principal when the real cost is $\underline{C}$ is equal to

$$
\begin{aligned}
{}_{hp} = {} & w_{hp}(\overline{C},\underline{C})A(w_a(\overline{C}) - w_{hp}(\overline{C},\underline{C}) - w_a(\underline{C})) + w_{hp}(\underline{C},\underline{C})) \\
& + w_{hp}(\underline{C},\underline{C})[1 - A(w_a(\overline{C}) - w_{hp}(\overline{C},\underline{C}) - w_a(\underline{C}))] - \underline{C} - h_{hp}
\end{aligned}
$$

or, equivalently,

$$
\begin{aligned}
{}_{hp} = {} & [w_{hp}(\overline{C},\underline{C}) - w_{hp}(\underline{C},\underline{C})]A(w_a(\overline{C}) - w_{hp}(\overline{C},\underline{C}) - w_a(\underline{C})) + w_{hp}(\underline{C},\underline{C})) \\
& + w_{hp}(\underline{C},\underline{C}) - \underline{C} - h_{hp}
\end{aligned}
$$

It achieves a maximum when[17]

$$
\begin{aligned}
\frac{\partial U_{hp}}{\partial w_{hp}(\overline{C},\underline{C})} = {} & -a(w_a(\overline{C}) - w_{hp}{}^*(\overline{C},\underline{C}) - w_a(\underline{C}) + w_{hp}(\underline{C},\underline{C}))[w_{hp}{}^*(\overline{C},\underline{C}) - w_{hp}(\underline{C},\underline{C}) \\
& + A((w_a(\overline{C}) - w_{hp}{}^*(\overline{C},\underline{C}) - w_a(\underline{C}) + w_{hp}(\underline{C},\underline{C})) = 0
\end{aligned}
$$

But this condition can be rewritten as (C). ■

<u>Proposition</u> 4: If $A(H_{hp})/a(H_{hp})$ is continuous, and we assume that $w_{hp}(\underline{C},\underline{C})$ is independent of the offer that the agent makes. Then $0 > \frac{\partial P(offer)}{\partial h_{hp}}$

Proof: The utility of the hidden principal when there is no offer for cheating is independent of the level the honesty of the agent while the utility of making an offer is decreasing[18].

---

[16] We will assume that the payment without corruption $w_{hp}(\underline{C},\underline{C})$ is independent of the offer.

[17] Note that in order to be optimal to make an offer (to lye) it has to hold that

$$w_{hp}{}^*(\overline{C},\underline{C}) - w_{hp}(\underline{C},\underline{C})]A(w_a(\overline{C}) - w_{hp}{}^*(\overline{C},\underline{C}) - w_a(\underline{C})) + w_{hp}(\underline{C},\underline{C})) - h_{hp} > 0.$$

Therefore the greater h_{hp} the smaller the probability that the utility of making the optimal offer would be greater than the utility of no making an offer. ■

If $A(H_a)/a(H_a)$ is continuous we know, there typically will be an odd number of solutions for equation (C) since when $w_{hp}(\overline{C},\underline{C})=w_{hp}(\underline{C},\underline{C})$ (all the gains from lying are appropriated by the agent). Since the left hand side is zero and increasing while the right is greater than zero, when $w_{hp}(\overline{C},\underline{C})=w_{hp}(\underline{C},\underline{C})+w_a(\overline{C})-w_a(\underline{C})$ (all the gains from lying are appropriated by the hidden principal) the left side is greater than zero and the right is zero. In particular if we assume that $A(H_{hp})/a(H_{hp})$ is non decreasing[19] we have a unique solution so we can have some results.

<u>Proposition</u> 5: If $A(H_{hp})/a(H_{hp})$ is continuous, non decreasing, and we assume that $w_{hp}(\underline{C},\underline{C})$ is independent of the offer that the hidden principal makes.

$0 \geq \dfrac{\partial w^{*}_{hp}(\overline{C},\underline{C})}{\partial w_a(\underline{C})} \geq -1$ Where it is equal to zero iff either $w_{hp}^{*}(\overline{C},\underline{C})=w_{hp}(\underline{C},\underline{C})$ or $A(H_a)/a(H_a)$ is constant and greater than -1 if $A(H_a)/a(H_a)$ is increasing.

Proof:

$$\frac{\partial w^{*}_{hp}(\overline{C},\underline{C})}{\partial w(\underline{C})} = -\frac{\dfrac{d(A(\,)/a(\,))}{dh_{hp}}}{\dfrac{d(A(\,)/a(\,))}{dh_{hp}}+1}$$ when there is a proposal to cheat and zero otherwise.

■

[18] Note we are using the fact that the optimal offer is independent of the level of h_{hp}.

[19] See footnote 13.

Proposition 5 is telling us, in the same way that proposition 3 but for its case , that in case of cheating when the gains are greater, $w_a(\underline{C})$ is smaller, the gains of both the corrupter and the corrupt will be greater, and conversely when the gains are smaller.

Proposition 1": there is a proposition similar to 1 and 1' in this game. Proof to be done.

6. Conclusion and possible extensions

Conclusion to be written.

Next steps:

- An interesting continuation of the model will be internalize the relationship agent hidden principal, this could be done if we assume that the h's are not known (i.e. the agent does not know h_{hp} and the hidden principal does not know h_a). We can also make the distinction between the moral cost of accept an offer of corruption and making the offer.
- Another interesting continuation will be if we allow competition among different hidden principals, and maybe also among agents.
- We can put this model into a repeated game and analyze the differences between a one shot relationship and a durable one.
- We can add a probability of getting caught and maybe make this probability a function of the investing in control by the principal and avoid control by the agent or the hidden principal.

- Assume that the level of honesty is a function of the aggregate level of honesty $h_i = h(H, \theta_i)$.

Appendix 1

Lemma 1:

If $\text{Arg}\max\{U_p(w_a)\} \neq \varnothing$, then $\exists$a payment scheme w'_a such that: $w'_a(\overline{C}) \geq w'_a(\underline{C})$ and $w'_a \in \text{Arg}\max\{U_p(w_a)\}$

Proof: The proof is divided into three exclusive cases which exhaust all the possibilities.

a) Schemes where $\overline{C} \geq w_a(\underline{C}) > w_a(\overline{C})$. When the true cost is $\overline{C}$, it is impossible to satisfy the participation constraint of the agent jointly with that of the hidden principal, unless the moral cost of a lie is zero for the agent and the hidden principal and $\overline{C}= w_a(\underline{C})$. But the principal will never pay $w_a(\overline{C})$. So, letting $w'_a(\overline{C}) = w'_a(\underline{C})= w_a(\underline{C})$ will not decrease the utility of the principal, it will increase whenever in the former scheme there is no production when the real cost is $\overline{C}$ and there is under w'_a (i.e. for positive levels of honesty and $\overline{C}= w_a(\underline{C})$).

b) Schemes where $w_a(\underline{C})>\overline{C} \geq w_a(\overline{C})$. When the real cost is $\overline{C}$, if the scheme is such that $\overline{C} >w_a(\overline{C})$, there will only be production if the moral cost of lie is small enough, whenever the announcement is $\overline{C}$ there will not be production. So doing $w'_a(\underline{C})=w_a(\underline{C})>\overline{C} = w'_a(\overline{C})$ will not decrease the utility of the principal, so we get $w'_a(\underline{C}) >w'_a(\overline{C}) =\overline{C}$. If we reduce $w'_a(\underline{C})$ until $w''_a(\underline{C})= w''_a(\overline{C}) =\overline{C}$, the agent is now willing and able to induce the hidden principal to supply the input if $C=\overline{C}$ or $C=\underline{C}$.

Since the payment by the principal to the agent will not increase the principal's utility under w_a'' must be at least as high as it would be under w_a. If with the former scheme the announcement $\underline{C}$ had positive probability $U_p(w_a'')> U_p(w_a)$.

c) Schemes where $w_a(\underline{C}) > w_a(\overline{C}) > \overline{C}$. If we reduce $w_a(\underline{C})$ until do $w'_a(\underline{C}) = w'_a(\overline{C}) = w_a(\overline{C}) > \overline{C}$ the agent is still willing and able to induce the hidden principal to supply the input for either value of C. Utility of the principal can not go down since the payment will be smaller in the cases were before the announcement were $\underline{C}$, and the same when the announcement were $\overline{C}$. ■

Theorem 1:

If $Arg\max\{U_p(w_a)\} \neq \varnothing$, *then* $\exists$ *a payment scheme* w'_a *such that*:
$\overline{C} \geq w'_a(\overline{C}) \geq w'_a(\underline{C}) \geq \underline{C}$ *and* $w'_a \in Arg\max\{U_p(w_a)\}$

We will split the theorem in three parts in order to do the proof.

i) $w'_a(\overline{C}) \geq w'_a(\underline{C})$

ii) $w'_a(\underline{C}) \geq \underline{C}$

iii) $\overline{C} \geq w'_a(\overline{C})$

Proof:

i) See lemma 1.

ii) Using the result in i) we can split the contradiction of ii) in two cases: a) Schemes where $\underline{C} > w_a(\overline{C}) \geq w_a(\underline{C})$ and b) Schemes where $w_a(\overline{C}) \geq \underline{C} > w_a(\underline{C})$, let's analyze each one .

a) Schemes where $\underline{C} > w_a(\overline{C}) \geq w_a(\underline{C})$ utility of the principal will be 0 because the agent cannot induce the hidden principal to supply the input with a payment which also satisfies the agent participation constraint. Setting $\underline{C} = w_a(\overline{C}) = w_a(\underline{C})$ will increase the utility of the principal because the agent is now willing and able to induce the hidden principal to supply the input when $C = \underline{C}$.

b)Schemes where $w_a(\overline{C}) \geq \underline{C} > w_a(\underline{C})$. If the true cost is $\underline{C}$ there is only production when they lie so the principal will never pay less than $\underline{C}$ so doing $w'_a(\overline{C}) = w_a(\overline{C}) \geq \underline{C} = w'_a(\underline{C})$ will increase the utility of the principal, since we get production when $C=\underline{C}$ no matter which is the level of honesty, and in some cases where we have production before, the principal will pay $\underline{C}$ instead of $w_a(\overline{C})$.

iii) Using the result in i) we can split the contradiction of ii) in two cases: a) Schemes where $w_a(\overline{C}) \geq w_a(\underline{C}) > \overline{C}$ and b) Schemes where $w_a(\overline{C}) > \overline{C} \geq w_a(\underline{C})$, let's analyze each one .

a) Schemes where $w_a(\overline{C}) \geq w_a(\underline{C}) > \overline{C}$. Setting $w'_a(\overline{C}) = w'_a(\underline{C}) = w_a(\underline{C}) > \overline{C}$ the utility of the principal will increase since the agent will still be willing and able to get the hidden principal to supply the input if $C=\underline{C}$ or $C=\overline{C}$ and the payment to the agent is smaller when the announcement was $\overline{C}$. Setting $w''_a(\underline{C}) = w''_a(\overline{C}) = \overline{C}$ will not decrease the utility of the principal since we will continue to have truth revealing scheme and both payments are going down.

b) Schemes where $w_a(\overline{C}) > \overline{C} \geq w_a(\underline{C})$. Setting $w'_a(\overline{C}) = \overline{C} \geq w'_a(\underline{C}) = w_a(\underline{C})$ the agent is now able and willing to induce the hidden principal to supply the input if $C=\underline{C}$ or $C=\overline{C}$, we are using that $w_a(\underline{C}) \geq \underline{C}$, and in the cases where before the announcement where $\overline{C}$ the payment is smaller. ■

Appendix 2

Whenever the agent knows h_{hp} and the hidden principal knows h_a, there will not be coordination problems that make the coalition tells the true when it is optimal to lie. so the problem of the principal is:

$$\underset{w_a(\overline{C}),w_a(\underline{C})}{Max} \quad U_p = q\{l_1[I - w_a(\underline{C})] + l_2[I - w_a(\overline{C})]\} + (1-q)\{v_1[I - w_a(\underline{C})] + v_2[I - w_a(\overline{C})]\}$$

where $l_1=1$ if $w_a(\underline{C}) \geq \sup\{w_a(\overline{C}) - h_a - h_{hp}, \underline{C}\}$, $l_1=0$ otherwise.

$l_2=1$ if $w_a(\overline{C}) - h_a - h_{hp} > w_a(\underline{C})$ *and* $w_a(\overline{C}) - h_a - h_{hp} \geq \underline{C}$, $l_2=0$ otherwise

$v_1=1$ if $w_a(\underline{C}) - h_a - h_{hp} > w_a(\overline{C})$ *and* $w_a(\underline{C}) - h_a - h_{hp} \geq \overline{C}$, $v_1=0$ otherwise.

$v_2=1$ if $w_a(\overline{C}) \geq \sup\{w_a(\underline{C}) - h_a - h_{hp}, \overline{C}\}$, $v_2=0$ otherwise

The first term of the sum is when $C=\underline{C}$ this happen with probability q, $l_1=1$ means that the announcement is $\underline{C}$ so we need to satisfy the incentive compatibility and participation constraint of the coalition between the agent and the hidden principal, $l_2=1$ is when the announcement is $\overline{C}$ this will be when the payment net the moral cost is greater that the payment under $\hat{C} = \underline{C}$ and also no smaller than the cost.

Note that we cannot have $l_1=l_2=1$, but we can have, in principle, $l_1=l_2=0$[20].

The second term is when $C=\overline{C}$, this happens with probability 1-q, $v_2=1$ means that the announcement is $\overline{C}$ so we need to satisfy the incentive compatibility and participation

[20]From theorem 1 we know that $w_a(\underline{C}) \geq \underline{C}$ therefore one will be equal to one and the other to zero.

constraint of the coalition between the agent and the hidden principal, $v_1=1$ is when the announcement is $\underline{C}$ this will be when the payment net the moral cost is greater that the payment under $\hat{C} = \overline{C}$ and also no smaller than the cost.

Note that we cannot have $v_1=v_2=1$, but we can have, $v_1=v_2=0$. From theorem 1 we know that $v_1=0$ and we can rewrite the problem as it is in the text.