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ECONOMIC DEVELOPMENT AS A PATTERN FORMATION PROCESS

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1 Introduction

The problem of economic development has been analyzed extensively, in many cases associated to Growth Theory. But, despite the broad and deep insights of development theorists, no significative formal models have been provided. This is more remarkable taking into account the rich stock of growth-theoretic models [1].

The reason for this lack of symbolic models representing developmental phenomena can be found in the very nature of the problem. While in economics *growth* can be understand as a process of quantitative change of variables, *development* means qualitative change [2],[3]. To precise terms, we can resort to a series of definitions provided by J.H.Olivera [4]:

Definition 1
- **Economic growth**: *a time dependent expansion of the social output*
- **Economic development**: *increase in the ratio* current social output-potential social output
- **Economic evolution**: *process of cualitative change of the economic organization in a society*

Potential output, as used above, means the productive capability of a country according its resources and current technology (no technical progress is introduced here). Two alternative but related definitions can be given to this concept:

Definition 2 *Potential output is:*

- *the highest output that can be obtained with the available resources.*

  or

- *the highest output that can be obtained making full utilization of the resources which the country is best endowed with.*

The first definition of potential output has a neoclassical flavor, while the second amounts to considering capital as an internal variable of the system, with given labor and natural resources. Symbolically, if O_p and O_c represent potential output and current output respectively, and calling λ the ratio O_c/O_p it follows from

1. $O_p\lambda = O_c$

   that

2. $Dlog\lambda + DlogO_p = DlogO_c$

The term of concern here is $Dlog\lambda$, the development rate, which depends on the accumulation of capital and on the improvement of the economic organization. Lets note that there are three possible qualitative values (increase, decrease or steady) that can be given to each variation rate in equation 2, but $DlogO_c$ will usually be assumed as non-negative, discarding for simplicity cases like wars and natural catastrophes which destroy human and material capital of a country.

Models of development, which try to provide representations of the behavior of λ, can be *closed* or either *open*, following the Ricardian emphasis on international trade as a condition for economic growth. Otherwise , development problems are usually approached in three different ways: emphasizing more on the allocation of resources; considering the dynamics of great macroeconomic agreggates or assuming a two sector structure of the economy. This last case is very frequent in the development literature. An urban and a rural sector are considered, where the first one shows increasing returns to scale in production, while the second has decreasing or constant returns. Development depends then on which sector leads the economy [5].

Lets note that the notions of economic development and evolution, as given in Definition 1, are not matter of neat distinctions [6]. Moreover, in what follows both terms will be considered as the same, except for the cases in which more precision is required. Then, Olivera's notion of development will be applied. In fact, the variation of λ obeys, as was said, mainly to qualitative changes in the economic organization, like Schumpeter's *innovations* [7].

The goal of this work is to consider a very simple formal model representing the behavior of λ. It departs from considering that development phenomena, as amounting qualitative changes in the structure of an economy, can be better handled in the framework of pattern formation processes [8]. The growth of condensed-matter physics, which tries to explain phenomena of propagation of qualitative changes in disordered media (e.g.magnetization or percolation) required the adequate models to handle them. In fact, the *spin-glasses* model, with its capability to represent features like conflict and cooperation among interactions in a system catalyzed the development of related models. It should be noted that this field of study has been also called *experimental mathematics*, due to the stream of mathematical insights and conjectures it generates. Not much has been proven yet, but it can be said that no pattern formation process seems to fall out of reach of spin-glassy models [9].

In the next section the spin-glassy model will be presented and its behavior will be considered under different parameter settings. Then, it will be discussed its relevance as representation of real world development processes. Finally, prospects for further work will be considered.

2 The model

A spin-glasses model consists mainly in a lattice of points, each one associated to a variable with a restricted (often discrete) range. In each instant of time the value of any variable becomes determined up from the previous value of the variable and the value of its neighbors. A perturbation is introduced in the value of certain variables, chosen at random, and the links among them and their neighbors propagate the perturbation. Depending on the size of the initial perturbation, the original state of the lattice and the parameters of the system (e.g. the number and strength of links) the perturbation will either propagate through the lattice or die off. The different kinds of behavior that such a system can show are notoriously similar to features proper of pattern formation processes. Moreover, no matter how simple a spin-glasses model, its simulation under different "scenarios" provides a gallery of pattern structures appliable as stylized representation of real world structures [10].

The kind of development model to be considered here fits well in the spin-glasses framework. The stylized facts are the following:

- A lattice of n countries will represent a global or regional economy.
- Each country follows a domestic development process leaded by a *key sector*.
- Each key sector will be associated to a *maximum possible output* (which will be considered as the O_p in the second characterization in Definition 2).
- A variable roughly corresponding to O_c will be associated to each country. Assuming that production fulfills demand perfectly and without lags, the agreggate demand of the country's output will be used here as a representative of O_c.
- The links among countries indicate, for a particular country, the degree of participation of all other countries in the demand of its output (in much cases indicating the importance of the output of a country as an input for productive processes in another one).
- Time will be considered in discrete instants.
- The values of the aggregate demand of each country will obtain through a simple partial difference equation, indicating the acumulative influence of the development of neighborhood of countries on its own. This equation shows also the multiplicative effect of changes in output on the proper ouput of a country.

- The functional form of the beforesaid influence will be considered piecewise linear (indicating that this influence obeys different rules according the values of the degree of development of neighbors and its own).

According to these stylized facts the following definitions try to provide a formalization for them:

Definition 3 *Lets consider a complete bidirected graph on a set C of nodes, $|C| = n$. There will be $n(n-1)$ links among different nodes in the graph. Also there will be considered for each node a directed link, with the node being both its head and tail. Therefore, the total number of directed links in the graph will be n^3. Each node i will be associated to a triple (O_p^i, D^i, λ^i) where O_p^i is a constant, D^i a variable. $O_p^i, D^i \in R^+$, with $D^i \leq O_p^i$. $\lambda^i = D^i/O_p^i$. To each directed link in the graph, between nodes i, j there will be associated a parameter $k_{i,j} \in [0,1]$, where i is the source node and j the end node. It will be assumed that $\sum_{i=1}^n k_{i,j} = 1, \forall j \in C$. This means that the matrix $|k_{i,j}|$ will be normalized and asymmetrical. A difference equation will be stated for each node i:*
$D^i(t+1) = f_i(D^i(t) + \sum_{j=1}^n k_{j,i} D^j(t))$ *where*

$$f_i(x) = \begin{cases} O_p^i & \text{if } x \geq O_p^i/\alpha \\ \alpha x & \text{if } x \in [\gamma O_p^i/\alpha, O_p^i/\alpha] \\ \beta x & \text{otherwise} \end{cases}$$

and $\alpha > 1, \beta < 1,$ *and* $\gamma \ll 1$.

3 Discussion

With the the stated model at hand, the problem of determining the alternative development patterns requires to follow the evolution of certain variables when an initial "perturbation" propagates. The precise statement of the problem is given in the next definition:

Definition 4 *Given the initial data $(C, \{O_p^i, \lambda^i(0)\}_i, |k_{i,j}|_{n\times n}, \alpha, \beta, \gamma)$ and a set of initial values of the D^i variables, $\{D^i(0)\}$, determine for each i the set $\{\lambda^i(t)\}_{i=1}^n$ for $t > 0$*

This could be done integrating the difference equations for all nodes in the graph. The goal is to determine the *attractors*, that is the values to which the variables asyntotically approach in the long term. The interrelation of variables requires, as usual in spin-glasses models, numerical experimentation to help conjecturing the long term patterns adopted by the system.

Five different kinds of numerical experiments have been performed on the model:

1. Having fixed $\gamma = 0.5, \alpha = 1.1, \beta = 0.9$ and a set of initial values $D^i(t)$ with a size of one tenth of the corresponding O_p, the behavior of the mean value of λ (i.e. $\sum_{i=1}^{n} \lambda^i/n$) has been followed for $n = 2 \ldots 5$.

2. With fixed $\alpha = 1.1, \beta = 0.9, n = 5$ and a set of initial data of the same order of magnitude as above, the behavior of the mean λ has been considered for $\gamma : 0.2,\ 0.3,\ 0.7,\ 0.8$.

3. With fixed $\gamma = 0.5, \beta = 0.9, n = 5$, and the same initial data, the behavior of the mean λ is followed for $\alpha : 1.01,\ 1.05,\ 1.2,\ 1.3$.

4. With $\gamma = 0.5, \alpha = 1.1, n = 5$, and the same initial data, the behavior of the mean λ has been followed for $\beta : 0.99,\ 0.95,\ 0.8,\ 0.7$.

5. With $\gamma = 0.5, \alpha = 1.1, \beta = 0.9, n = 5$, and varying the initial data as *order of magnitude* of D^i respect to $O_p^i = 1/100,\ 1/10,\ 1$ and a mix of $1/100$ and 1, the evolution of λ has been followed.

The time considered for each experiment above has been $t = 50$. Results are graphically shown in the Appendix. Lets note that numerical experiments have been run on different sets of initial data, different values of $|k_{i,j}|$, and with more wider ranges of parameters without showing remarkable differences with those shown in the Appendix. This means that, although the quantitative values obtained may differ, the patterns of behavior remain the same.

It is obvious that these numerical experiments are not a substitute of a rigourous analysis. The range of values considered is very restricted, not allowing a careless generalization of the observable behaviors. Notwithstanding, a couple of conjectures can be stated and discussed, in relation of known development phenomena, considering, as said above, that the results of experiments 1...5, shown in here are paradigmatic:

Conjecture 1 *It may exist an "optimum" number of countries, in the sense of attaining as a group the maximal possible development. Moreover, the multiplicative effect provided by the external economies atenuates with a greater number of countries, eroding due to the amortiguation effect ($\beta < 1$). Besides, if the number of countries falls short of the "optimum", the group attains also the maximal development but in a slower feedback process, being this an interesting result for the study of cycles. Formally: $\exists n^* \in N$ such that if $n > n^*$, $\forall i \lambda^i(t) < 1$, $\forall t$, or (if with n^*, $\forall i \lambda^i(t) = 1$, for $t > T^*$) if $n << n*$ then if there exists a T such that $\forall i \lambda^i(t) = 1$ for $t > T$, then $T > T^*$.*

Conjecture 2 *The value of γ doesn't matter for the degree of development to be attained, and the same is true for the value of α. This seems to mean that the multiplicative effect and the localization of critical values in the aggregate demand is not related to the final degree of development, being both relevant instead for the* speed *of the development process. Formally: if for γ_1, α_1, $\exists T_1$ such that $\forall i \lambda^i(t) \approx max_{\gamma_1,\alpha_1} \lambda^i$ for $t > T_1$, then if for γ_2, α_2, $\exists T_2$ such that $\forall i \lambda^i(t) \approx max_{\gamma_2,\alpha_2} \lambda^i$, for $t > T_2$, $T_1 \neq T_2$.*

Conjecture 3 *The value of β appears as determinant for the degree of development to which the group of countries gravitates. With lesser values of this parameter, the process not only seems to slow down, but it also decreases showing a rise and then a fall toward a floor degree of development. This means that the amortiguation effect (as a leakage in the multiplicative effect) is notoriously pernicious for the development process. Formally: if $\beta \ll 1$ then $\exists T$ such that $\forall i \lambda^i(t) < 1$ for $t > T$. Moreover, if $\beta \approx 0$ then $\forall i \lambda^i(t) \rightarrow 0$ for $t > T$.*

Conjecture 4 *If some $i \in C$ begins with a high development degree, the mean development approaches the maximum degree. That is, if a country in the group is highly developed, it seems that its development will propagate to the rest. Formally: If there is a $i \in C$ such that $D^i \approx O^i_c$, then $\forall i \lambda^i \rightarrow 1$.*

It is also relevant to show the relations of the model with well known development theories. For instance, a feature of the development process discussed here is the existence of a "ceiling" value for the development, showing a strong analogy with the theory of *disequilibrated development* [11]. This doctrine states that the higher will be the development degree when the initial perturbation persists. So when higher are the leakages in the multiplicative effects, more rapidly will the initial disturbance die off, forcing the system to stay in a less than maximum level of development. Another well known development doctrine, Rostow's theory of *self-sustained take-off*([12]) can be tested in our framework showing that it is not appliable to countries with a low degree of development.

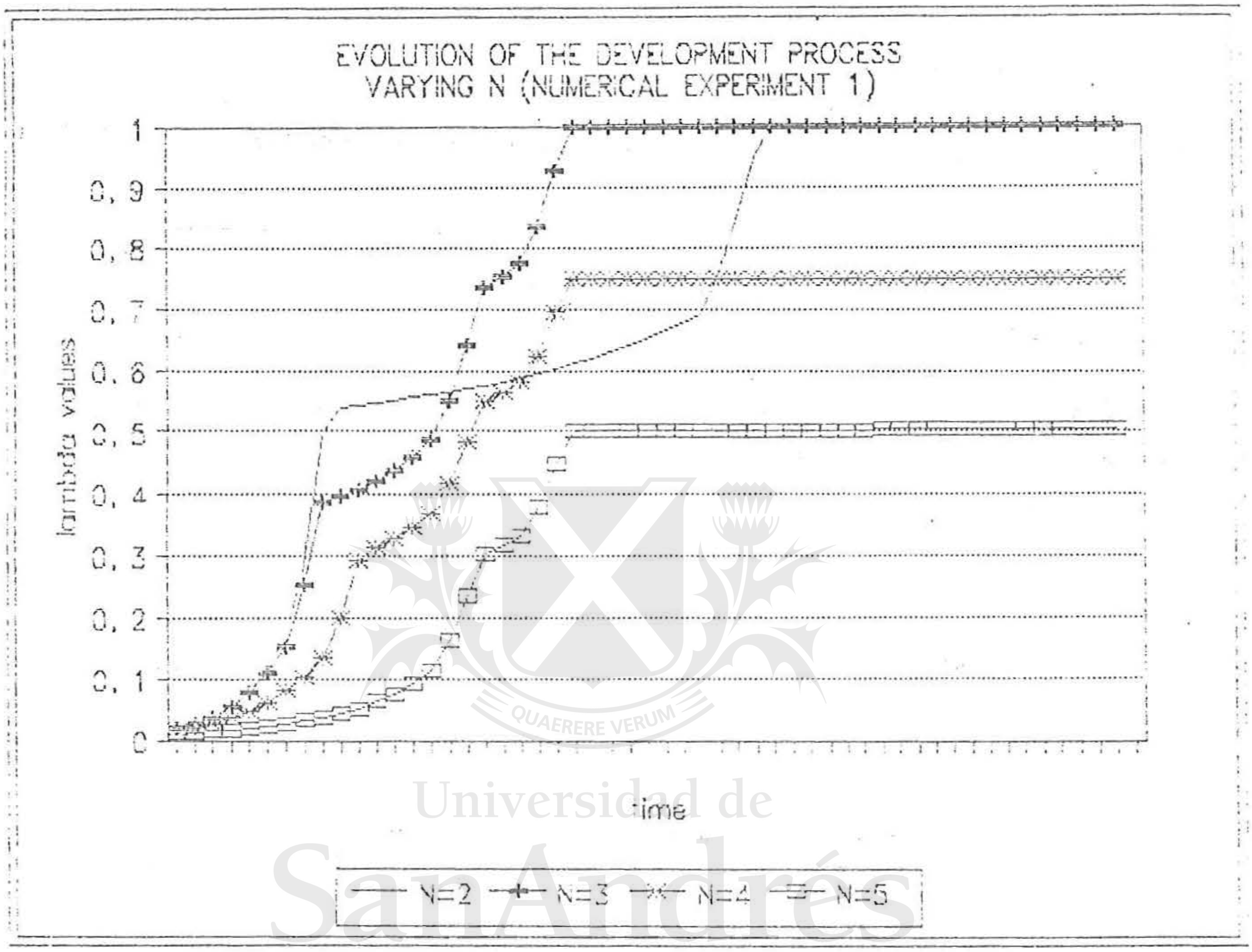
But more than a testbed for other theories, this model, we hope, will be the kernel of a formally stated theory of development, emphasizing the non-linear relations among countries, which lead to the appearance of different patterns of development. Much work is needed for fulfilling this goal. To see what items should be included in a tentative list of further work to be done the following are noteworthy:

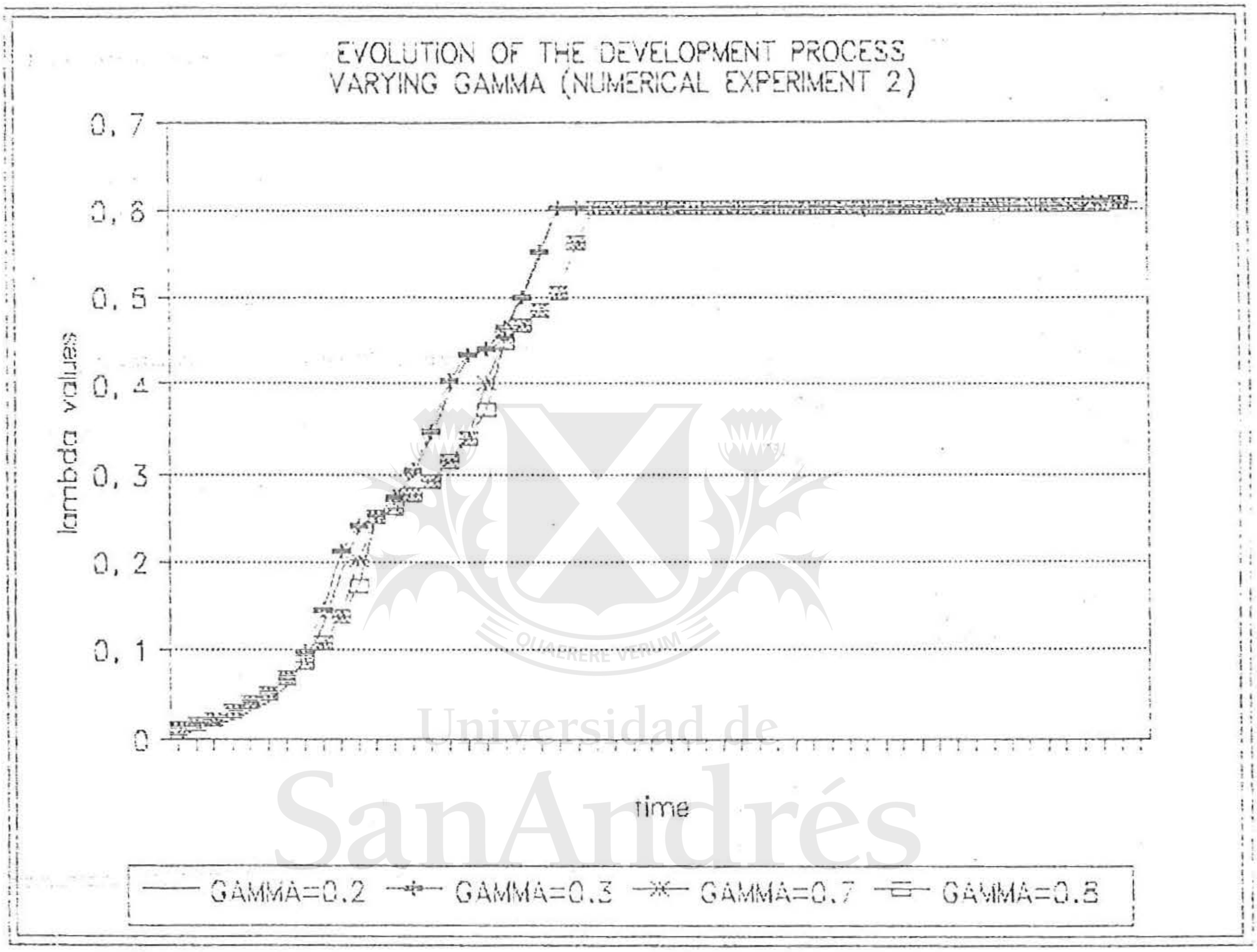
- To provide proofs (or counterexamples) for the conjectures stated above.
- To extend the framework, including a set of *sectors* in each country, allowing a desaggregate analysis of development.
- To consider the more realistic situation in which couplings are not fixed but are variated at random.
- To analyze the appearance of clusters of countries with a similar degree of development.

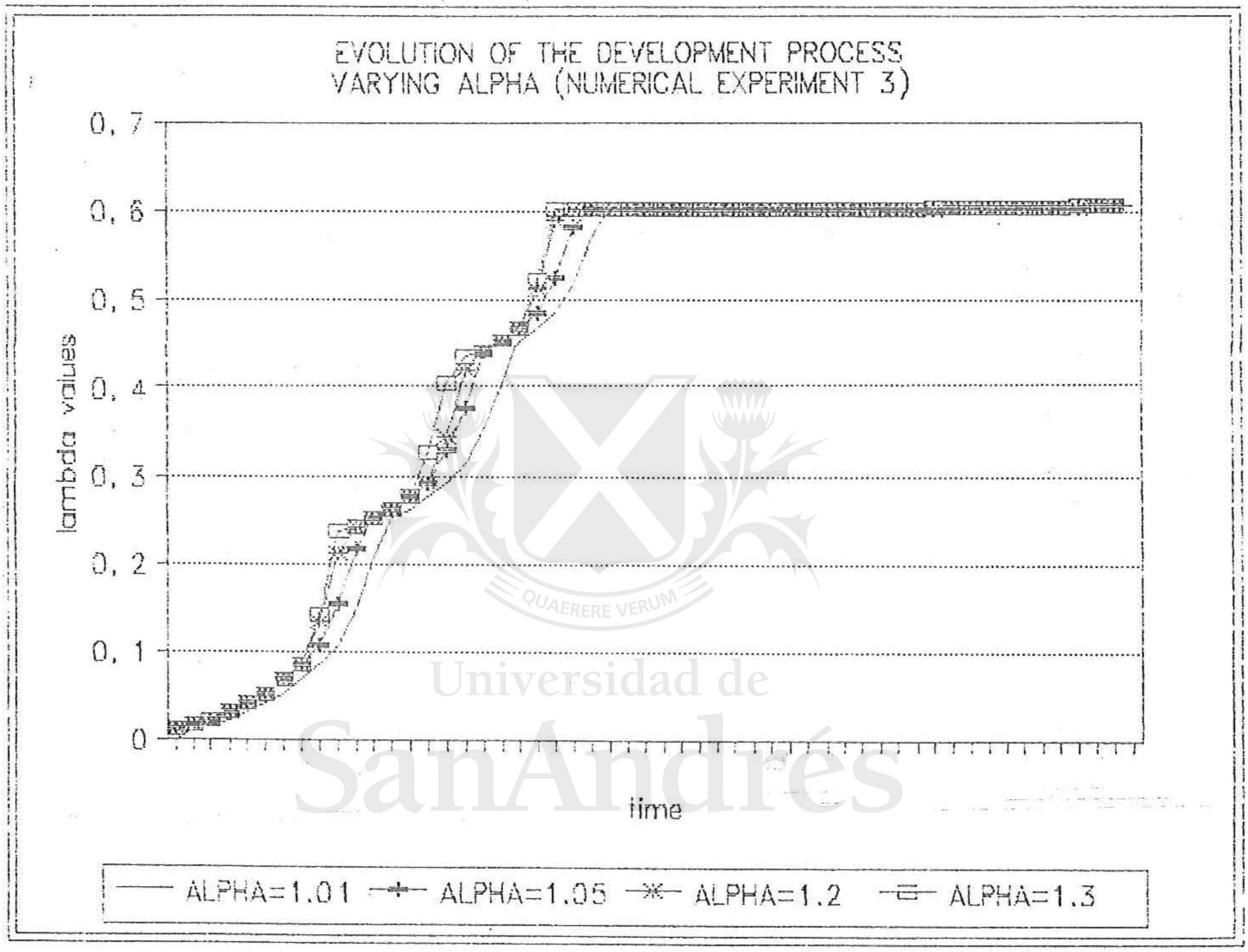
The results of this research should help to provide useful formalizations of development processes making this problem more tractable, allowing to perform reliable policy evaluations.

4 Appendix

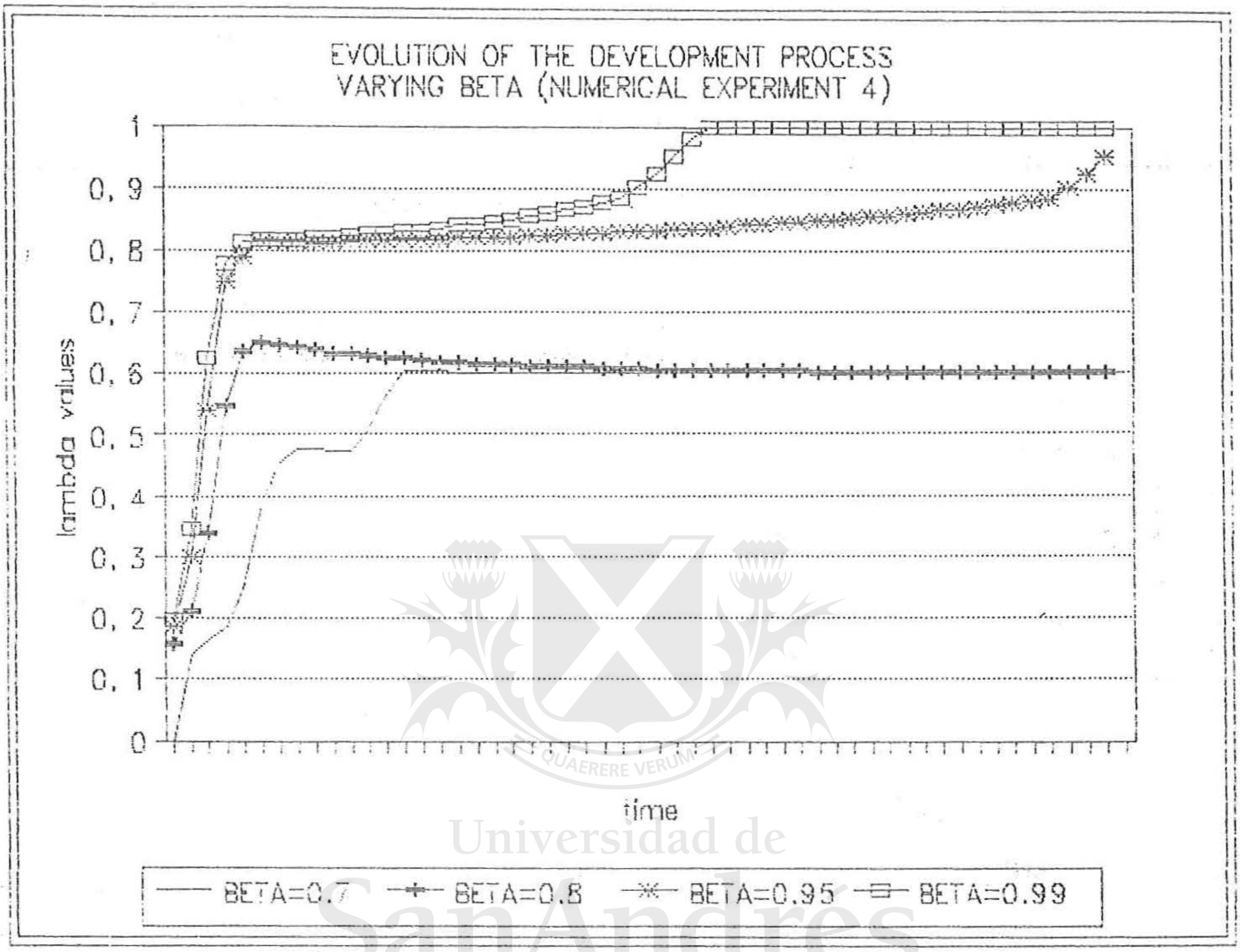
- **Experiment 1** This graph depicts the evolution of the collective development in function of the number of countries. See **Conjecture 1.**
- **Experiment 2** This graph shows the evolution of development varying the "inferior treshold" parameter γ. See **Conjecture 2.**
- **Experiment 3** This graph shows the evolution of development varying the "multiplicative effect" parameter α. See **Conjecture 2.**
- **Experiment 4** This graph depicts the evolution of development varying the "amortiguation effect" parameter β. See **Conjecture 3.**
- **Experiment 5** This graph represents the evolution of development varying the size of initial data $\{D^i(0)\}_{i\in C}$. See **Conjecture 4.**

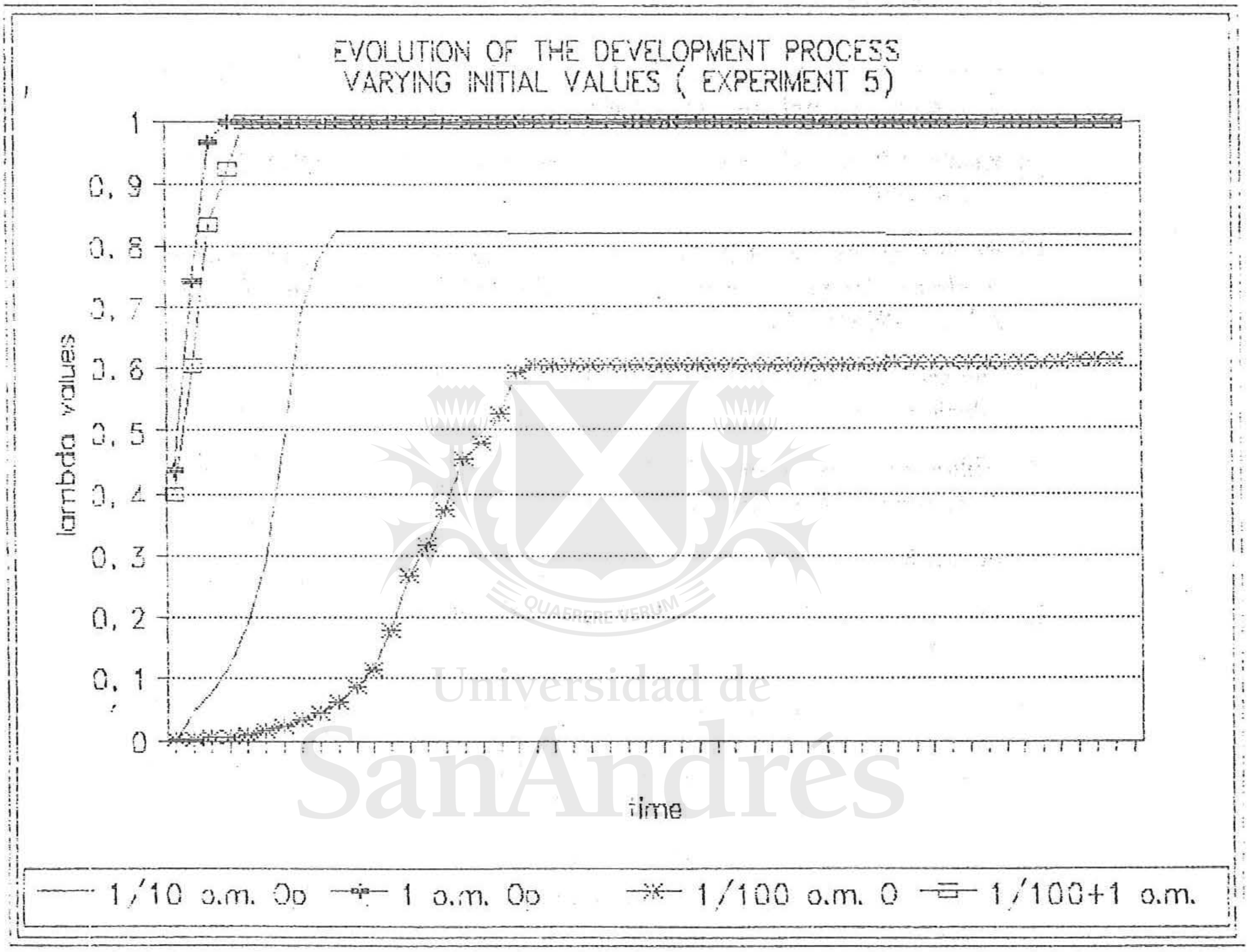

EVOLUTION OF THE DEVELOPMENT PROCESS
VARYING N (NUMERICAL EXPERIMENT 1)
lambda values
time
N=2
N=3
N=4
N=5


EVOLUTION OF THE DEVELOPMENT PROCESS
VARYING GAMMA (NUMERICAL EXPERIMENT 2)
0, 7
0, 6
0, 5
0, 4
0, 3
0, 2
0, 1
0
lambda values
time
GAMMA=0.2
GAMMA=0.3
GAMMA=0.7
GAMMA=0.8



EVOLUTION OF THE DEVELOPMENT PROCESS
VARYING ALPHA (NUMERICAL EXPERIMENT 3)
0, 7
0, 6
0, 5
0, 4
0, 3
0, 2
0, 1
0
lambda values
time
ALPHA=1.01
ALPHA=1.05
ALPHA=1.2
ALPHA=1.3


EVOLUTION OF THE DEVELOPMENT PROCESS
VARYING BETA (NUMERICAL EXPERIMENT 4)
1
0, 9
0, 8
0, 7
0, 6
0, 5
0, 4
0, 3
0, 2
0, 1
0
lambda values
time
BETA=0.7
BETA=0.8
BETA=0.95
BETA=0.99


EVOLUTION OF THE DEVELOPMENT PROCESS
VARYING INITIAL VALUES (EXPERIMENT 5)
1
0, 9
0, 8
0, 7
0, 6
0, 5
0, 4
0, 3
0, 2
0, 1
0
lambda values
time
1/10 o.m. Op
1 o.m. Op
1/100 o.m. 0
1/100+1 o.m.

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