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PRIVATIZING IN A TWO-PERIOD MODEL: A POSITIVE APPROACH AND THE ROLE OF "GOOD" OUTSIDE CONSTRAINTS

by

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Abstract: In this paper we present a positive model to study the characteristics of privatization processes. Using a two-period setup, we show that the selling of public firms will tend to be performed more inefficiently when the desired level of government spending is very high (or the departures from it are highly undesired), when the interest rate increases , when society becomes more able to protect itself against inflation (i.e., after a hyperinflation), and when the government is rationed in the credit market and discounts heavily the future. Furthermore, improvements in the technology to collect taxes improve the selling of public firms. Most importantly, we analyze the negative side-effects that different restrictions imposed from outside forces (IMF) over the evolution of the inflation rate, have over the characteristics of the privatization process.

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I. Introduction.

It has recently been noticed by students of privatization processes, that the goals that give rise to them differ among countries and go often beyond the improvement of public firms' efficiency (see Vickers and Yarrow (1991) and Word Development (1989)). The formalization of this observation, however, is really in its infancy. This paper presents a model to analyze this idea (i.e. that privatization is not only a means to improve the overall economic efficiency, but also a means to finance some other goals that differ among countries. Following Urbiztondo (1992), we develop a model in which the characteristics of a privatization process depend on the capacity that governments have to finance their deficits through other sources. As in the above cited paper, the decision to privatize the public firms has already been taken by the members of society, who delegate this task to a voter-maximizing government. Here, though, the political game among the members of society dictates that a loss function containing deviations from a target level of government spending, inflation rates, and inefficient selling of the public firms, should be minimized.[1] Since the incumbent government is a voter maximizer this is exactly what it will do.[2] In order to pursue this goal, the government may sell its public firms, collect overt and covert (i.e. inflationary) taxes, and borrow from the international or domestic capital

[1] Other objectives usually emphasized in the analysis of privatization are the elimination of deficits incurred by the firms when they are in public hands, and the signal given to foreign creditors and investors that the country is involved in drastic structural changes. Even though these issues are important, we ignore them here since we assume that the decision to privatize has already been taken.

[2]. Different types of loss functions have been extensively used in the literature on government policymaking. See for example, Persson and Tabellini (1989).

market. Furthermore, the instructions given to the government are that it has to sell the public firms within the first half of its mandate, and by the end of its period in office it shouldn't leave any debt burden to the next administration. That is, there is an artificial division of the planning horizon in two periods, and the government must adjust to it. However, it is possible that the lenders do not believe that the members of society dislike government's use of inflationary taxes as a financial source. If, for instance, the country has a long-lasting history of huge rates of inflation correlated with defaults to its credit obligations, the foreign creditors (probably under the coordination of an institution such as the International Monetary Fund - IMF) may condition lending in future periods to the reduction of the inflation rate during the present administration. We study this possibility in the paper, modelling it in terms of two different types of commitment on the inflation rate of the second period.

What we do in this paper is to solve for the characteristics of the privatization process within a two-period model. Besides obtaining some new results, the conclusions that emerge are consistent with those of Urbiztondo (1992), and they are useful to show their robustness. Furthermore, we analyze the effects that different restrictions imposed from outside forces (IMF) over the use of one instrument (inflation) by the government have over the characteristics of the privatization process. This exploration sheds new light on an issue generally not distinguished, such as the side-effects that an apparently good thing (i.e., low inflation rates) has in some other dimensions (i.e. the efficiency in the allocation of resources due to a privatization process).

II. The Model.

As we mentioned in the introduction, the problem faced by the government is to minimize a loss function that, implicitly, maximizes the support of its constituency, as this function is the outcome of a political game among the members of society (or, alternatively, represents the interests of the median voter). Also, the period over which this task has to be performed is divided in two sub-periods. The loss function is then given by

$$L = \frac{\alpha}{2}\left(G_1 - G_1^*\right)^2 + \frac{\theta}{2}\pi_1^2 + \gamma\left(V^* - V(Z)\right) + \\ + \beta\left[\frac{\alpha}{2}\left(G_2 - G_2^*\right)^2 + \frac{\theta}{2}\pi_2^2 + \gamma\left(V^* - V(Z)\right)\right] \tag{1}$$

where G_t is the level of government spending in period t;

G_t^* is the desired level of government spending in period t, as a result of the political process.

π_t is the inflation rate in period t (the target level is implicitly zero);

V(Z) is the social value of the public firms in private hands, a negative function of the amount Z received for it, (i.e., $V_z < 0$). Additionally, we assume $V_{zz} < 0$;[3]

V^* is the maximum social value of the public firms in private hands;[4]

β is the rate of time preference, with $0 < \beta < 1$; and

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[3]. Mathematically, this assumption ensures strict convexity of the objective function to be minimized by the government. In economic terms, the higher the difference between the price the firms will be allowed to charge and the marginal cost, the higher the amount secured by the government through privatization, and consequently, the higher the welfare loss. Furthermore, it implies that the welfare loss associated to departures from marginal cost pricing of the privatized firms, increases at an increasing rate as the receipts from the privatization process increase.

[4] That is, it is the social value that results if the privatization is performed efficiently so that the price charged is equal to the marginal cost, regardless the structure of the market under consideration.

α, θ and γ are positive weighing parameters.

That is, the decision about the characteristics of the privatization affects both periods, whereas, in principle, the government can choose the level of government spending and the rate of inflation independently in each period. However, these decisions are not unrelated, as the constraints faced by the government link its optimal choices in both periods. Assuming that the privatization is to be performed in the first period and that all the borrowing in the first period from the international capital market has to be repaid in the second period, the budget constraints are the following ones:

$$\begin{aligned} G_1 &= T_1 + F + Z + R(\pi_1), \\ G_2 + rF &= T_2 + R(\pi_2), \end{aligned} \tag{2}$$

where $R(\pi_t)$ is the level of the (covert) inflationary tax in period t, with

$R'(\pi_t) <> 0$, $R''(\pi_t) < 0$ and $R(0) = 0$;

T_t is the level of (overt) tax receipts in period t;[5]

F is the level of borrowing in the international capital market in the first period; and

r is the "gross" interest rate (i.e., $r > 1$).

The problem, then, is to choose Z, π_1, F and π_2 to minimize

[5]. It is important to notice two points on ordinary taxes. First, there are no collection costs for them, and second, distortionary taxation does not affect the government's loss function (or, alternatively, ordinary taxes entail no distortions in this economy). These assumptions are only made to simplify the model, but could be easily relaxed and the general point would subsist.

$$L = \frac{\alpha}{2}\left(T_1 + F + Z + R(\pi_1) - G_1^*\right)^2 + \frac{\theta}{2}\pi_1^2 + \gamma\left(V^* - V(Z)\right) +$$
$$\beta\left(\frac{\alpha}{2}\left(T_2 + R(\pi_2) - rF - G_2^*\right)^2 + \frac{\theta}{2}\pi_2^2 + \gamma\left(V^* - V(Z)\right)\right). \quad (3)$$

The first order conditions that characterize the solution conform the following system of equations

$$\begin{aligned} Z&: \alpha\,(G_1 - G_1^*) - \gamma\,(1 + \beta) V_z = 0 \\ \pi_1&: \alpha\,(G_1 - G_1^*)\, R'(\pi_1) + \theta\,\pi_1 = 0 \\ F&: (G_1 - G_1^*) - \beta\,(G_2 - G_2^*)\, r = 0 \\ \pi_2&: \alpha\,(G_2 - G_2^*)\, R'(\pi_2) + \theta\,\pi_2 = 0. \end{aligned} \quad (4)$$

We can see from the first equation that, since $V_z < 0$, $G_1 < G_1^*$, and using this result in the third equation, $G_2 < G_2^*$. Furthermore, if $\pi_t \geq 0$, for $t = 1,2$, the second and fourth equations show that $R'(\pi_t) \geq 0$, that is, that the government chooses to be on the "right side" of the Laffer Curve. The interpretation of these results is straightforward. As revenues from privatization and inflation are costly to obtain, a rational government will find optimal to choose a level of expenditures below the level dictated by the political process. Additionally, as the Laffer curve for the inflation tax implies that any revenue from inflation can be supported by two rates of inflation, the government will choose the lower one, since this minimizes its loss.

The structure of the problem faced by the government can also illustrate a version of the widely known result of Phelps (1972), as an extension of the Ramsey problem in optimal taxation to include the inflation tax. In our model, distortions arise from three sources: the inflation rate in the first period, the inflation rate en the second period, and the distortions in the allocation of resources caused by "bad " privatizations. From the system of first order conditions, we obtain the following expression

$$-\gamma(1+\beta)\,V_z = \frac{\beta\, r\theta\pi_2}{R'(\pi_2)} = \frac{\theta\pi_1}{R'(\pi_1)}. \tag{5}$$

The interpretation is that the government will choose inflation rates in both periods and the revenue from the selling of public firms up to the point where the marginal cost entailed in the use of each one of them (in terms of units of additional income) is equal. That is, the optimal inflation rate in each period takes a positive value, as it does in Phelps (1972).

III. Comparative Statics of the Model.

As usual, using the system conformed by the first order conditions, we perform some exercises of comparative statics. We study the effects of changes in the marginal valuation of privatizations, the technology of tax collection, the interest rate, the intertemporal rate of preference, the technology of inflationary tax collection, and the marginal disutility of inflation for the government. To avoid notation in the text, we submit all the mathematical calculations to the appendix.

1) *Changes in the marginal valuation of privatizations.*

A change in the value for the government of privatizations (in terms of the allocation of resources), can be analyzed as a change in τ in the objective function. The results are

$$\frac{dZ}{d\gamma} < 0, \quad \frac{d\pi_1}{d\gamma} > 0, \quad \frac{dF}{d\gamma} > 0, \quad \frac{d\pi_2}{d\gamma} > 0. \tag{6}$$

The impact of changes in τ on Z is obvious. An increase in τ generates a movement towards "good privatizations". As the government optimally reduces its receipts derived from the sale of public firms, it chooses to change the optimal values of additional debt and inflation rates in both periods in the opposite direction. That is, an increase in τ produces a substitution from privatizations to other forms to finance the flow of government expenditures (debt and inflation in both periods).

2) *Changes in collection technologies.*

An increase (decrease) in the ability to collect both ordinary and covert - i.e., inflationary - taxes will reduce (increase) the receipts from the privatization process. Once again, the government optimally substitutes between the different sources to finance government expenditures. The result is the expected, since the possibility to obtain additional resources from a non-costly source (ordinary taxation or a higher revenue from the inflation tax for each rate of inflation) permits to reduce the loss derived from "bad" privatizations. In particular, we obtain

$$\frac{dZ}{dT_1} < 0, \quad \frac{dZ}{dT_2} < 0, \quad \frac{dZ}{dR'(\pi_1)} < 0, \quad \frac{dZ}{dR'(\pi_2)} < 0.$$

An interesting consideration is that changes which take place in the second period also affect the characteristics of the privatizations in the first period. This result not only generalizes (given the two-periods setup) the results from Urbiztondo (1992), but also provides a useful prediction for countries that are going to adopt a privatization program. If taxes are easier to collect in the second period, the government will incur in additional indebtedness (because the capacity to pay debt is higher) and optimally choose a lower revenue from the selling of public firms (i.e., a greater improvement in resource allocation due to privatizations). Consequently, governments with perspectives of higher tax revenues for the second period will tend to perform privatizations more efficiently.

3) *Changes in the rate of interest.*

Since the marginal cost of additional debt is the rate of interest, an increase in this cost will originate a substitution towards the other financial sources. Specifically, in the appendix we show that dZ/dr has a positive sign. Furthermore, considering expression (5), we get

$$sign\left(\frac{dZ}{dr}\right) = sign\left(\frac{d\pi_1}{dr}\right) = sign\left(\frac{d\pi_2}{dr}\right).$$

Or, in words, increases (decreases) in the interest rate correspond to increases (decreases) in the receipts from privatizations, and the inflation rates in both periods.

4) *Changes in the marginal disutility of inflation.*

A change in the marginal disutility from inflation (modelled as changes in parameter θ), changes the cost of using the inflation tax to finance government expenditures.[6] The result (proved in the appendix) is that as the disutility from inflation increases, privatizations are performed more

[6] This situation can be understood as taking place after a hyperinflationary period, since the money balances held by the public are reduced to its minimum, increasing the inflation rate required to collect a given level of inflationary tax.

inefficiently, given the substitution from inflationary tax revenues to receipts from privatization and additional debt. In other words, the higher the marginal cost of inflation for the government, the lower the benefits (in terms of resource allocation) derived from selling public firms.

5) *Changes in the rate of intertemporal preference.*

The change in ß, which represents a change in the rate of intertemporal preference, has an ambiguous effect on Z. That is, dZ/dß <> 0. The reason is the following: Even though a more forward-looking government gives relatively more importance to the distortion in the allocation of resources resulting from the privatization process, it also puts more weight on the cost incurred in the second period if the level of the debt increases, as its repayment implies a lower government expenditure in the second period, and then it is possible that it optimally chooses to reduce the level of the debt. This effect would tend to produce a more intense use of the privatization as a financial source, contrary to the more apparent intended reduction due to the consideration of the efficiency costs resulting in the second period from a bad privatization, and leaving undetermined the final result. However, if the government cannot finance its expenditures with debt (i.e., it is credit-rationed), the ambiguity in the impact of changes in ß on Z disappears, and the result is that as ß increases, the efficiency in the allocation of resources obtained through the privatization process also increases (i.e., Z decreases).

IV. The Effect of an Absolute Constraint on the Inflation Rate.

In this section we analyze a standard issue in countries engaged in privatization processes, such as the commitment adopted about future actions.

Assume for instance that the government commits itself to a certain level in the inflation rate for the second period, say π^* (and, implicitly, given the money demand function, to a certain inflationary tax). There may be various reasons behind this commitment: promises in the previous political campaign, IMF impositions, etc. Whatever the reason, if such a constraint is imposed, the first order conditions for the program solved by the government become:

$$Z: \quad \alpha(G_1 - G_1^*) - \gamma(1+\beta)V_z = 0,$$
$$\pi_1: \quad [(G_1 - G_1^*)R'(\pi_1) + \theta\pi_1 = 0,$$
$$F: \quad \alpha(G_1 - G_1^*) - \beta\alpha r(G_2 - G_2^*) = 0.$$

Performing comparative statics exercises with the last system of equations, we get three expected results. First, a change in the commitment for the second period is inversely related with the receipts obtained from privatizations. The reason is that a decrease (increase) in the inflation rate in the second period reduces (increases) the inflationary tax revenues, and, thereby, the ability to repay debt in that period. Consequently, the chosen receipts from privatizations in the first period increase (decrease). This is an "unpleasant" result, since it implies that countries which are going to be involved in stabilization programs are expected to conduct "bad" privatization processes.

Secondly, changes in the strength of the commitment lead to changes in the optimal level of the debt in the same direction $dF/d\pi^* > 0$. The reasoning behind of this result is the same we stated above for the case of Z.

Lastly, changes in the level of the commitment lead to changes in the inflation rate in the first period in the opposite direction. The argument is the same one that is behind the two previous results: variations in the level of the commitment induce to an intertemporal substitution between the sources of government financing in each period, and then a fall (rise) in the level of the commitment causes a rise (fall) in both the Z and π_1. More formally, in the appendix we show that

$$sign(\frac{dZ}{d\pi^*}) = sign(\frac{d\pi_1}{d\pi^*}).$$

Finally, we consider the impact that a change in the level of the commitment has on the optimal level of government expenditures. The result is that increases in π^* relax the intertemporal budget constraint faced by the government, and consequently, the government finds optimal to increase its expenditures in both periods. That is, we get $dG_1/d\pi^* > 0$ and $dG_2/d\pi^* > 0$.[7] This is probably another interesting result. For countries subject to fixed revenues from taxation, a commitment to a lower inflation rate for the second period, leads to an optimal reduction of the government expenditures in both periods.

V. The Effect of a Proportional Constraint on the Inflation Rate.

Commitment about the level of the inflation rate on the second period can take place (or be imposed) in different ways. Additionally to the way dealt with in the previous section, the commitment could be respect to the "evolution" of the inflation rate instead of the "level" of the inflation rate in the second period (that is, the inflation rate in the second period is constrained to be a percentage K lower than the rate in the first period). That is, we impose to the original problem, the restriction $\pi_1=\pi_2 + K$, with $K>0$. Thus, the new first

[7]. Notice that from the first order conditions, a linear and positive relationship between G_1 and G_2 is derived. Consequently, obtaining the effect of changes in any parameter on any of the two variables, the effect on the other immediately arise.

order conditions are

$$
\begin{gathered}
Z\colon \alpha(G_1 - G_1^*) - \gamma(1+\beta) V_z = 0, \\
\pi_1\colon \alpha(G_1 - G_1^*) R'(\pi_1) + \theta\,\pi_1 + \mu = 0, \\
F\colon (G_1 - G_1^*) - \beta\, r(G_2 - G_2^*) = 0, \\
\pi_2\colon \alpha(G_2 - G_2^*) R'(\pi_2) + \theta\,\pi_2 - \mu = 0, \\
\mu\colon \pi_1 - \pi_2 - K = 0,
\end{gathered}
$$

where μ is the Lagrange multiplier associated to the restriction of the problem. Once again, the government picks inflation rates in both periods on the upward sloped side of the Laffer Curve. This is easily proved. From the first equation of the system of F.O.C's, the optimal level of public expenditures is below the dictated by the political process. Thus, from the second equation of the system (given the non-negativity of the Lagrange multiplier) we get $R'(\pi_1)>0$. Then $R'(\pi_2)>0$.[8]

The results of the comparative statics are as follows. Regarding inflation rates, a change in the level of the restriction will affect positively the first period rate of inflation, and negatively the second period rate of inflation. This is a natural result. Once again, the government intertemporally substitutes between inflation rates in both periods as sources to finance its expenditures. Regarding the use of debt, changes in the use of debt are negatively related to changes in the level of the constraint. As an increase (decrease) in the level of the commitment reduces (increases) the inflation rate in the second period, the ability to repay government debt in the second

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[8]. The condition of equalization of the marginal costs in the use of each instrument is now

$$-(1+\beta)\, V_z = -\frac{(\theta\,\pi_1 + \mu)}{R'(\pi_1)} = -\beta\, r \frac{(\theta\,\pi_2 - \mu)}{R'(\pi_2)}.$$

period is reduced (increased), and then, the use of debt in the first period is also reduced (increased).

The effect of changes in K on the receipts from privatization is, however, ambiguous. There are two separated forces that interact. On one hand, as it happened in the case of an absolute constraint, an increase in the level of the restriction provokes an intertemporal substitution towards an increase in the value of Z in order to smooth the effect on the reduction of G_1 due to the (optimally chosen) lower amount of additional debt. On the other hand, however, the government may also find optimal to reduce the value of Z, since now there is an additional incentive to increase the first period inflation rate to relax the restriction in the second period, and hence to substitute away the use of the selling of public firms to finance government's spending (i.e., to reduce Z).

To summarize, the comparative statics results are

$$\frac{dZ}{dK} <> 0, \quad \frac{d\pi_1}{dK} > 0, \quad \frac{d\pi_2}{dk} < 0, \quad \frac{dF}{dK} < 0.$$

Note lastly that, as in the case of an absolute constraint, we are able to predict the effect of a change in K on the optimal levels of government expenditure. The result is the same found in the previous section. Since an increase in K tightens the intertemporal budget constraint of the government, the higher the restriction on the inflation rate for the second period, the lower the optimal levels of government expenditure in both periods. That is, as we show in the appendix, $dG_1/dK < 0$ and $dG_2/dK < 0$.

VI. Concluding Remarks.

The question we are after in this paper is not to determine how the privatization of public firms *should* be performed, but instead, how it *will* be carried out in light of the different objectives pursued by a vote-maximizer government. We model the decisions of a rational government from a positive point of view, provided the fact that the privatization decision has been already taken. We conclude that privatizations will be performed more inefficiently when the interest rate is high, the marginal disutility of inflation is high, the marginal value of privatizations in terms of improving general economic efficiency is low, the ability to collect taxes (overt and covert) is low, and when the government is credit-rationed. Also, and since we consider a two-period model, we contemplate the existence of outside impositions on the evolution of certain policy variables. One such imposition can take the form of an international institution (IMF) conditioning credit to the country in the first period depending on the achievement of (or commitment to) a "low" inflation rate in the second period. We show that this apparently desirable constraint may have a negative effect on the efficiency in the allocation of resources that is generally overlooked.

We believe that the message of the paper, i.e., the existence of deviations from "efficient" privatizations due to the effect this process has on the budget constraint of the government, is not only an "unpleasant" product of the paper, but explicitly points out the existing trade-off between privatization as a mean to foster an efficient allocation of resources, and privatization as a source to finance government spending. This point is the core of the paper. For policy purposes, then, privatization is not a simple panacea. That is, even assuming that the government gets the most out of it, the more important it is to collect funds from the selling of public firms, the lower the expected contribution to the improvement of the general economic efficiency that results from privatization.

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MATHEMATICAL APPENDIX

a) The Unconstrained Model.

Differentiating the system of first order conditions (4), letting H denote the Hessian of the system (with positive value by the second order conditions for a minimum), and using Cramer's Rule, we get the following results:

1. Changes in the marginal value of privatizations:

$$\frac{dZ}{d\gamma}=\frac{(1+\beta)\,V_z}{H}\Big((1+\beta r^2)\big(\alpha^2(G_1-G_1^*)R''(\pi_1)+\alpha\theta\big)+\beta r^2 B\big(\alpha^2(G_2-G_2^*)R''(\pi_2+\alpha\theta)\big)\Big)<0$$

$$\frac{d\pi_1}{d\gamma}=-\frac{V_z(1+\beta)\,\alpha r'(\pi_1)}{H}\big(\alpha^2(G_2-G_2^*)R''(\pi_2)+\alpha\theta\big)\beta r^2>0$$

$$\frac{dF}{d\gamma}=-\frac{V_z}{H}\big(A(G_1-G_1^*)R''(\pi_1)+\theta A\big)>0$$

$$\frac{d\pi_2}{d\gamma}=-\frac{(1+\beta)\,V_z\,(r\alpha^2R'(\pi_2))}{H}\big(\alpha(G_1-G_1^*)R''(\pi_1)+\theta\big)>0$$

where

$$A=\alpha R'(\pi_2)^2+\alpha(G_2-G_2^*)R''(\pi_2)+\theta>0,$$
$$B=\alpha R'(\pi_1)^2+\alpha(G_1-G_1^*)R''(\pi_1)+\theta>0.$$

2. Changes in Tax Collection Technologies:

$$\frac{dZ}{dT_1}=-\frac{1}{H}\Big(\alpha(G_1-G_1^*)R''(\pi_1)\big(\alpha(G_2-G_2^*)R''(\pi^2)+\theta\big)+\big(\alpha\theta(G_2-G_2^*)R'(\pi_2)+\theta^2\big)\alpha^2\beta r^2\Big)<0$$

$$\frac{dZ}{dT_2}=-\frac{\beta r\alpha^2}{H}\big(\alpha(G_1-G_1^*)R''(\pi_1)+\theta\big)\big(\alpha(G_2-G_2^*)R''(\pi_2)+\theta\big)<0.$$

3. Changes in the rate of interest:

$$\frac{dZ}{dr} = - \frac{A\left(\alpha^2 (G_1 - G_1^*) R''(\pi_1) + \alpha\theta\right) \alpha\beta (G_2 - G_2^*)}{H} > 0.$$

4. Changes in the rate of intertemporal preference:

$$\frac{dZ}{d\beta} = \frac{\gamma V_z}{H} \left((1 + \beta r^2)(\alpha^2 (G_1 - G_1^*) R''(\pi_1) + \alpha\theta) A + \beta r^2 B (\alpha^2 (G_2 - G_2^*) R''(\pi_2) + \alpha\theta)\right)$$
$$- \frac{\alpha r A (G_2 - G_2^*)}{H} \left(\alpha^2 (G_1 - G_1^*) R''(\pi_1) + \alpha\theta\right) <> 0.$$

However, if the government is unable to finance its expenditures trough debt, we obtain

$$\frac{dZ}{d\beta}\Big|_{\frac{dF}{d\beta}=0} = \frac{\beta r^2 A (G^2 - G_2^*) \alpha r \left(\alpha^2 (G_1 - G_1^*) R''(\pi_1) + \alpha\theta - \gamma B (1 + \beta) V_{zz}\right)}{H}$$
$$- \frac{A (G_2 - G_2^*)(1 + \beta) \alpha \gamma' r B V_{zz}}{H} + \frac{\gamma V_z \beta r^2 B \left(\alpha^2 (G_2 - G_2^*) R''(\pi_2) + \alpha\theta\right)}{H} < 0.$$

5. Changes in Inflation Tax Collection Technology:

$$\frac{dZ}{dR'(\pi_1)} = \frac{\alpha A (G_1 - G_1^*)}{H} \left(\alpha^2 (G_1 - G_1^*) R''(\pi_1) + \alpha\theta\right) < 0$$
$$\frac{dZ}{dR'(\pi_2)} = \frac{\alpha^2 \beta r (G_2 - G_2^*) R'(\pi_2)}{H} \left(\alpha^2 (G_1 - G_1^*) R''(\pi_1) + \alpha\theta\right) < 0.$$

6. Changes in the Marginal Disutility of Inflation:

$$\frac{dZ}{d\theta} = \frac{\beta r \alpha}{H} \left(\pi_1 R'(\pi_1)\left(\alpha^2 (G_2 - G_2^*) R''(\pi_2) + \alpha\theta\right) + R'(\pi_2)\left(\alpha^2 (G_1 - G_1^*) R''(\pi_1) + \alpha\theta\right)\right) > 0.$$

b) The Case of an Absolute Constraint.

Using the set of first order conditions for this problem, we get the changes on π_1, F and Z derived from changes in π^*. These are

$$\frac{dZ}{d\pi^*} = \frac{-\beta a r R'(\pi^*)}{D}\left(a^2 R''(\pi_1)(G_1 - G_1^*) + a\theta\right) < 0$$

$$\frac{d\pi_1}{d\pi^*} = \frac{\beta a r R'(\pi^*)}{D}\left(a R'(\pi_1)\gamma(1+\beta) V_{zz}\right) < 0$$

$$\frac{dF}{d\pi^*} = \frac{\beta a r R'(\pi^*)}{D}\left(a^2 (G_1 - G_1^*) R''(\pi_1) + a\theta - \gamma(1'+\beta) B V_{zz}\right) > 0.$$

where D>0 is the hessian determinant of the system. With the first period budget constraint for the government and the first order condition with respect to F, we obtain the impact of changes in π^* on the value of public expenditures in both periods.

$$\frac{dG_1}{d\pi^*} = -\frac{\beta a r R'(\pi^*)}{H}\left(\gamma(1+\beta) V_{zz}(a(G_1 - G_1^*) R''(\pi_1) + \theta)\right) > 0,$$

$$\frac{dG_2}{d\pi^*} = \frac{1}{\beta r}\left(\frac{dG_1}{d\pi^*}\right) > 0.$$

c) The Case of a Proportional Constraint.

Defining P as the bordered hessian of the problem (with negative value by the second order conditions for a constrained minimum), we obtain the following results on Z and F due to changes in K:

$$\frac{dZ}{dK} = \frac{r R'(\pi_1)}{P}\left(a(G_2 - G_2^*) R''(\pi_2) + \theta\right) - \frac{R'(\pi_2)}{P}\left(a(G_1 - G_1^*) R''(\pi_1) + \theta\right) <> 0,$$

$$\frac{dF}{dK} = \frac{\beta r a R'(\pi_2)}{P}\left(a^2 (G_1 - G_1^*) R''(\pi_1) + a\theta - (1+\beta)\gamma V_{zz} B\right) - \left(\frac{(1+\beta)\gamma V_{zz} a A R'(\pi_1)}{P}\right).$$

The change in the first period inflation rate is:

$$\frac{d\pi_1}{dK} = \frac{\beta r \alpha^2 R'(\pi_2)\gamma(1+\beta) V_{zz} R'(\pi_1) - \alpha^3 r^2 \beta (G_2 - G_2^*) R''(\pi_2) - \alpha^2 \theta \beta r^2}{P} +$$
$$+ \frac{\gamma(1+\beta) V_{zz}}{P}\left(\alpha A + \alpha^3 r^2 \beta (G_2 - G_2^*) R''(\pi_2) + \alpha^2 \theta \beta r^2\right) > 0.$$

The change in the second period inflation rate is:

$$\frac{d\pi_2}{dK} = \left(\alpha\beta r^2 (\alpha^2 (G_1 - G_1^*) R''(\pi_1) + \alpha\theta) - r\alpha^2 \gamma (1+\beta) V_{zz} R'(\pi_1)\right)\frac{1}{P} -$$
$$-\left(\gamma(1+\beta) V_{zz} (\alpha^2 (G_1 - G_1^*) R''(\pi_1) + \alpha\theta + \alpha\beta r^2 B)\right)\frac{1}{P} < 0.$$

As in the section above, we get the effect of changes in K on the optimal level of government expenditures in both periods. For the second period, we get

$$\frac{dG_2}{dK} = -\left(\frac{r\alpha^2 R'(\pi_2)^2}{P}\left(\gamma(1+\beta) V_{zz} R'(\pi_1)\right) + \frac{\gamma(1+\beta) V_{zz} R'(\pi_1) r\alpha A}{P}\right) +$$
$$+ \gamma(1+\beta) V_{zz} R(\pi_2)\left(\alpha^2 (G_1 - G_1^*) R''(\pi_1) + \alpha\theta\right) < 0.$$

Using this expression, for the first period we obtain

$$\frac{dG_1}{dK} = r\beta\left(\frac{dG_2}{dK}\right) < 0.$$