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THE PRICE OF GOLD AND THE EXCHANGE RATES

by

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The focus of this paper is on the linkage between the major-currency exchange rates and the international prices of commodities in general and gold in particular. The exchange rate regime that replaced the Bretton Woods system in 1973 has resulted in three major currency blocs—North America, Europe and Japan—with floating exchange rates that have exhibited a great deal of volatility manifested not only in day to day fluctuations but also in sustained real appreciations and depreciations. The "law of one price" assures that these disturbances will be reflected in the price of a commodity such as gold.

For several reasons, gold is an ideal commodity for studying the connection between the major-currency exchange rates and commodity prices. Gold is traded continuously in organized spot and future markets that are closely linked to the foreign exchange markets. Moreover, as annual production (and consumption) of gold is minuscule compared with the global stock, the gold producing countries, whose currencies are typically not traded in organized markets, are unlikely to dominate the world market.

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I. INTRODUCTION AND SUMMARY OF MAIN RESULTS

As the law of one price holds strongly for commodities such as gold, a change in *any* exchange rate requires the price of gold to move *in at least one currency*; moreover, if the major-currency countries possess non-trivial market power over the international price of gold, a shock to the exchange rate between any two major currencies will be reflected in the price (both nominal and real) of gold in *both* currencies. It is in this way that floating exchange rates exacerbate the inherent instability of commodity prices.

That real fluctuations of the U.S. dollar strongly influence (dollar) prices of internationally-traded goods became particularly evident during the intense real depreciation and subsequent appreciation of the dollar from 1975 to mid-1985. In the 1980-85 period when, based on consumer prices, the dollar appreciated 45 per cent in *real* terms against the Deutsche mark, the IMF (dollar-based) commodity price index actually *fell* by 30 per cent, and both import and export (dollar-based) unit values for commodity-exporting countries as a group *fell* by about 14 per cent. All of this occurred despite a 30 percent *rise* in the U.S. consumer prices and a 15 per cent rise in U.S. producer prices. It has been one the objectives of this study to determine the extent to which gold also fits into this pattern.

The minor-currency countries have not escaped the consequences of major currency instability. Since 1973, those countries have, for the most part, tied their currencies in some fashion to a single major currency, to the SDR, or to a currency basket of their own design. Freely floating exchange rates have had very little appeal to policy makers in the minor-currency countries; in recent years, less than one third of the minor currencies have been floating, of which more than half have been "managed" floats.[1] The current

[1] Of the currencies corresponding to the 150 members of the International

system, then, is a mixed one in which the major currency countries pursue money supply rules while most of the minor currency countries adhere to an exchange rate rule or a managed float.

But in a more fundamental sense, *all* currencies—major and minor—have been floating. To attach one currency to another by means of some exchange rate rule is not the same when major currencies are floating as it was with the Bretton Woods fixed exchange rate system; under the current system, fixing one's exchange rate *vis a vis* a single major currency is to *float against the rest*. Thus major-currency exchange rate shocks are felt not only in the major currency countries but also by producers and consumers throughout the world.

Summary of the Main Results

Paradoxically, the *de facto* demonetization of gold in 1973 lead to an spectacular increase in the price of that commodity and the simultaneous floating of the major currencies resulted in great and prolonged fluctuations in the major-currency exchange rates. As the law of one price holds with great vigor in the international gold market, those exchange rate fluctuations have been vividly reflected in both spot and forward gold prices, which are continuously quoted in highly organized markets. This wealth of data has been used in this study to estimate the key parameters relating the price of gold to exchange rates. The main findings, based on an analysis of forecast errors in both the gold and foreign exchange markets for the 1982-90 period, are:

1. The volatility of exchange rates among the major currencies since the

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Monetary Fund as of mid-1990, only 12 are viewed as major currencies (the U.S. dollar, the yen, and the currencies constituting the European Monetary System—the EMS) leaving 138 minor currencies. Fifty five of the minor currencies were attached in some way to a single currency, and 42 were pegged to a basket of currencies (only 7 being pegged to the SDR). Of the remaining, 20 were freely floating and 21 were subject to a managed float. The Swiss franc, of course, should also be treated as a major currency.

dissolution of the Bretton Woods international monetary system has been a major source of price instability in the gold market. Indeed, the instability of *real* exchange rates between major currencies since 1973 has contributed nearly half of the observed volatility in the spot price of gold during the 1982-90 period.

2. With respect to the international gold market, the evidence strongly supports the "efficient market" hypothesis in that unexploited profit opportunities were not systematic during the period under study.

3. While gold is usually denominated in the U.S. dollar, the dollar bloc has but a small influence on the international price of gold.

4. The major gold producers of the world (South Africa, the former USSR, and Australia) appear to have no significant influence on the world price of gold.

5. The world gold market is dominated by the European currency bloc as it possesses approximately two-thirds of the "market power" enjoyed by *all* participants in the world gold market. Accordingly, real appreciations or depreciations of the European currencies have profound effects on the price of gold in all other currencies.

6. Gold appears to be a store of value as it was found that "world" inflation increases the desire to hold gold; it is estimated that the real price of gold rises by about 1.6 per cent in response to a one point increase in the rate of world inflation.

The remainder of the paper is comprised of four sections. The first to follow contains an international pricing model which will be the centerpiece of the empirical analysis. The model predicts that changes in *real* exchange rates among the major currencies will impact on the *real* price of gold in *all* currencies—major and minor alike. In section III we present an analysis of the characteristics of the data and section IV, the core of the paper, contains the empirical findings. Finally, in Section V we describe and quantify the apparent impact of the exchange rate system on the stability of the gold market since the dissolution of the Bretton Woods system in 1973.

II. THE INTERNATIONAL PRICING MODEL

In this section we develop a model that relates the international price of a homogeneous commodity to the major currency exchange rates.[2] We assume that the commodity in question is traded by M countries (or currency blocs) in organized markets that clear continuously.[3] We ignore transport costs, tariffs and other barriers to trade.

We begin the analysis with the law of one price for an unspecified internationally-traded commodity, which states that:

$$P_i = P_j + E_{ij}, \tag{1}$$

where P_i is the (natural logarithm of the) price of that commodity in currency i, and E_{ij} is the (natural logarithm of the) price of currency j in terms of currency i. With no loss of generality, we can set $i = 1$; i.e., the currency of country 1 will be chosen to be the *reference* currency.

The *excess* demand for the commodity in country j, D^j, will be written as a function of its *real* price and a vector, X_j, of all other relevant

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[2] To the best of the author's knowledge, this approach was first used by Ridler and Yandle (1972) to analyze the effects of exchange rate changes on commodity prices.

[3] Our interest is in currency *blocs* rather than countries and hence there is no one-to-one correspondence between countries and currencies. Accordingly, when the analysis involves the Deutsche mark, for example, the reference is to the European bloc (and all other countries that tie their currencies to an EMS currency or the ECU) rather than merely to Germany.

variables (i.e., the "fundamentals") for country j:

$$D^j = D^j\left((P_j - P_j^*),\ X_j\right)$$
$$= D^j\left((P_1 - E_{1j} - P_j^*),\ X_j\right),$$

where P_j^* is the (natural logarithm of the) price level in country j. We explicitly assume exchange rates are not "caused" by the nominal price of the commodity in question, which implies that changes in the X_j variables will affect the price of gold equi-proportionately in *all* currencies.

By adding and subtracting P_1^* to and from $(P_1 - E_{1j} - P_j^*)$, the excess demand for the commodity in country j can be written as a function of the natural logarithm of the ratio of its *real* price in country 1 to the bi-lateral *real* exchange rate between countries 1 and j:

$$D^j = D^j\left((P_1^R - E_{1j}^R),\ X_j\right),$$

where $P_1^R \equiv P_1 - P_1^*$ is the (the natural logarithm of the) *real* price of the commodity in country 1 and $E_{1j}^R \equiv E_{1j} + P_j^* - P_1^*$ is the (natural logarithm of the) bi-lateral *real* exchange rate between countries 1 and j.

Market clearing requires that the M excess demands sum to zero:

$$\sum_j^M D^j\left((P_1^R - E_{1j}^R),\ X_j\right) = 0,^4 \qquad (2)$$

and by differentiating equation (2) totally we obtain, after some rearranging:

$$dP_1^R = \sum_j^M (D_1^j/D_1)\cdot dE_{1j}^R - \sum_j^M (D_2^j/D_1)\cdot dX_j,$$

where $D_1^j \equiv \partial D^j/\partial(P_1^R - E_{1j}^R)$, $D_1 \equiv \sum_j^M D_1^j$, and $D_2^j = \partial D^j/\partial X_j$. A local linear

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[4] For most commodities (e.g., wheat), the excess demand will refer to a *flow* which will be positive (negative) for a net importing (exporting) country. In the case of gold, however, annual production and consumption are minuscule compared to the global stock, so the excess demand functions refer to *stocks* and hence market clearing presumably implies that every excess stock demand is exactly zero.

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approximation relating the *real* price of the commodity to *real* exchange rates is obtained by integration:

$$P_1^R = \sum_j^M \Theta_j \cdot E_{1j}^R + K(X), \tag{3}$$

where:

$$\Theta_j \equiv D_1^j / D_1 .$$

As the D^j are excess demands, they may be either positive or negative, but the D_1^j are non-positive and hence the Θ_j are non-negative fractions that obviously sum to unity.[5]

The term $K(X)$, which is the integral of $-\sum_j^M (D_2^j/D_1) \cdot dX_j$, captures the X_j vectors—the "global" fundamentals—and that term is explicitly assumed to be orthogonal to the E_{1j}^R. As was pointed out above, the fundamentals include all factors (including expectations) that influence the global demand for and supply of the commodity in question other than exchange rates.

The structure of the world market for the commodity in question is completely summarized by the Θ_j as those parameters measure of the *relative market power* possessed by each country that participates in that market. For example, if country S is a *price taker* in that market (i.e., $\Theta_S = 0$), a change in its real exchange rate *vis a vis* reference currency 1 will have no

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[5] The *excess* demand in country j is $D^j \equiv \mathcal{D}^j - \mathcal{S}^j$, where $\mathcal{D}^j$ and $\mathcal{S}^j$ are domestic demand and supply, respectively. The slope of the excess demand function is $D_1^j = (\mathcal{D}^j/P_j^R) \cdot \eta_j - (\mathcal{S}^j/P_j^R) \cdot \varepsilon_j$, where $\eta_j \le 0$ and $\varepsilon_j \ge 0$ are the elasticities of domestic demand and supply, respectively, with respect to the real price of the commodity in country j. The D_1^j, and hence D_1, are clearly non-positive. Note further that, as $E_{11}^R \equiv 0$, Θ_1 does not appear in the summation on the right hand side of equation (3) but, as $\sum_j^M \Theta_j = 1$, Θ_1 can be obtained as $1 - \sum_{j=2}^M \Theta_j$.

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effect on the real price of the commodity in currency 1, and hence the entire impact must fall on that price in its own currency. Country S is, in other words, the classic "small" economy in the context of the world market for the commodity in question. On the other hand, if country D is a *price maker* in that market (i.e., $\Theta_D = 1$), any change in the real exchange rate between countries D and 1 will be reflected in an equi-proportionate change in the real price of the commodity in country 1. In short, country D dominates the world market as the price in that country's currency is invariant with respect to its exchange rate.

To dominate the world market for any commodity, a country must have an extremely elastic excess demand for that commodity. For most commodities, stocks are quite small when compared with annual production and consumption, and hence to dominate the price of a commodity such as wheat or copper, a country must be a major producer and/or a major consumer.[6] The precious metals, however, are unusual in that world *stocks* are very large in comparison with annual production and/or consumption and hence a country can dominate the world market for those metals without being a major consumer or producer—if that country has a high propensity to hoard them.

The "Small" Country Issue

Since the collapse of the Bretton Woods fixed exchange rate system in 1973, exchange rates—both nominal and real—among the major currencies have been highly volatile resulting in large departures from purchasing power

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[6] Again letting D^j and S^j be domestic demand and supply, respectively, in country j for a given commodity, the slope of the *excess* demand in that country is $D^j_1 = (D^j/P^R_j)\cdot\eta_j - (S_j/P^R_j)\cdot\varepsilon_j < 0$, where η_j (ε_j) again is the elasticity of domestic demand (supply) with respect to the real price of the commodity in country j. As Θ_j is proportional to D^j_1, it follows that Θ_j will be greater the larger are D^j and S^j, other things equal.

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parity.[7] These fluctuations in major-currency real exchange rates have exacerbated the inherent instability of commodity prices, an instability that has impacted severely on the minor-currency countries. To illustrate that effect, consider a small country that is unable to exert a significant influence on the world price of the commodity in question. The nominal price, P_S, of the commodity in that country's currency is given by:

$$
\begin{aligned}
P_S &= P_1 + E_{S1} \\
&= (E_{S1} + P_1^{*}) + \sum_{j}^{M} \Theta_j \cdot E_{1j}^{R} + K(X) \\
&= P_S^{*} + E_{S1}^{R} + \sum_{j}^{M} \Theta_j \cdot E_{1j}^{R} + K(X),
\end{aligned}
$$

and the *real* price (which is relevant to producers) is simply:

$$
P_S^{R} = E_{S1}^{R} + \sum_{j}^{M} \Theta_j \cdot E_{1j}^{R} + K(X).
$$

The first term indicates that the domestic real price is proportional to the small country's *real* exchange rate *vis a vis* reference country 1 and hence a real *appreciation* (i.e., $\Delta E_{1S}^{R} > 0$) depresses the domestic real price of the commodity, and *vice versa*. The middle term—which is our main focus—captures the effect of real exchange rates among *third-country* currencies whose volatility has dominated the prices of many commodities since 1973.[8]

Note that the Θ_j, which determine the extent to which third-country real exchange rate adjustments impact on the real price of the commodity in a small country, are independent of the actual trading pattern of that country. A

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[7] See Frenkel (1981) for documentation concerning the large departures from PPP experienced by the major currencies during the 1970s and Edwards (1989) for a massive compilation of real exchange rate data for smaller countries.

[8] See Sjaastad (1990) for the effect of fluctuations in third-country exchange rates on the prices of Australia's major exports (iron, coal, wheat and wool).

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shock to the real exchange rate between, say, the U.S. and Japan might well impact on the producer's incomes in the small country even if their entire output were sold to Europe. What matters is not the actual trading pattern of a small country but rather which countries dominate the international market for the small country's traded goods.

Under the current exchange rate system a small country must either passively accept the consequences of shocks to third-country real exchange rates, or attempt to offset them by manipulating its own real exchange rate, E^R_{s1}. Even if a small country had the power to influence its real exchange rate (which is doubtful) in response to a shock to one or more of the E^R_{1j}, it cannot escape the political issue. As the $\Theta_{X,j}$, where the X subscript refers to any commodity, are not the same for all commodities, a shock to one or more of the E^R_{1j} may well affect the various domestic producers of internationally-traded goods quite differently. Accordingly, a small country may chose not to neutralize an unfavorable shock to, say, the price of gold, as to do so might well worsen the situation of (politically powerful) domestic producers of other internationally-traded commodities.

For the producers of commodity X, however, knowledge of the magnitudes of the $\Theta_{X,j}$ can be very useful. In the first place, it helps to identify the source of the shocks (e.g., internal *versus* external) suffered by those producers. Secondly, to the extent that one can forecast third-country real exchange rates, one can also forecast the effects of movements in those real exchange rates on their own industry. Finally, knowledge of the $\Theta_{X,j}$ can be exploited for portfolio management in import-competing and exporting firms; by denominating their assets and liabilities in foreign currency in accordance with the $\Theta_{X,j}$, they can eliminate (or greatly reduce) the financial impact of shocks to third-country real exchange rates.

The Significance of Forward Markets

With the appropriate time series data, one can readily estimate the Θ_j coefficients of equation (3) and that estimate can be made either in terms of levels or rates of change of the variables.[9] As major currencies and many commodities are traded in highly organized *forward* as well as *spot* markets, we have two sets of prices and exchange rates—spot and forward—and hence two similar but distinct versions of equation (3). Designating the *spot* price of the commodity and *spot* real exchange rates as $P_1^R = P_1^{R,S}$ and $E_{1j}^R = E_{1j}^{R,S}$, respectively, the first version is:

$$P_1^{R,S} = \sum_j^M \Theta_j^S \cdot E_{1j}^{R,S} + K^S(X), \tag{3S}$$

and, for the second, the *forward* price of the commodity and the *forward* real exchange rates are designated $P_1^R = P_1^{R,F}$ and $E_{1j}^R = E_{1j}^{R,F}$, respectively:

$$P_1^{R,F} = \sum_j^M \Theta_j^F \cdot E_{1j}^{R,F} + K^F(X). \tag{3F}$$

As the "thetas" summarize the structure of the world market, we do not expect Θ_j^S to differ from Θ_j^F; indeed, the extremely high correlation between spot and forward prices and exchange rates would be all but impossible if Θ_j^S and Θ_j^F were significantly different. Moreover, for durable commodities or assets with low carrying costs such as gold or foreign exchange, $K^S(X)$ and $K^F(X)$ in equations (3S) and (3F) also will be identical (at a given moment in time); for such commodities or assets, the "fundamentals" captured by those terms will have the same effect on the spot and forward prices.[10]

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[9] When one turns to empirical estimation, it is essential to specify the K(X) variables; industrial production in the OECD countries, for example, has been found to be an important determinant of many commodity prices.

[10] This is evident in the data set to be described below where the simple correlation coefficients between monthly observations on spot and 90-day

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Either version of equation (3) can be estimated directly. There are, however, serious technical difficulties in estimating the Θ_J using either spot or forward prices and exchange rates as those variables usually are non-stationary and subject to high serial correlation. A superior alternative is to utilize *forecast errors* constructed from spot and forward price data. The (*forward*) price of any commodity at time t is for delivery of that commodity some n periods into the future; it is the market's prediction, as of time t, of the spot price that will prevail at time $t+n$. Letting $P^F_{1,t,n}$ and $P^S_{1,t}$ be the natural logarithms of forward and spot prices, respectively, in currency 1 at time t, the forecast error is merely the difference between the realized and predicted spot price:

$$Z_{1,t,n} \equiv P^S_{1,t} - P^F_{1,t-n,n}, \tag{4}$$

and a similar forecast error equation can be written for any asset that is traded in both organized spot and forward markets.

Forecast-error data possess two highly desirable attributes. These data are typically stationary—even in raw form—so no exotic data transformations are required and, if the market is "efficient", the forecast error data will be serially uncorrelated, which eliminates one of the more serious estimation difficulties associated with time series data.[11]

By carefully defining *spot and forward real* prices and exchange rates, one can obtain a simple yet elegant expression for estimating the "thetas" using forecast errors. By adding a t subscript to specify the period and an

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forward prices of gold and the various exchange rates all exceed 0.99.

[11] As an "efficient market" is one in which market agents utilize all available information, serial correlation in forecast errors implies that past forecast errors can predict the current forecast error and hence agents utilizing that information could systematically "beat the market."

S superscript to indicate "spot", the definitions of (the natural logarithms of) the spot real price of the commodity and spot real exchange rates are written as:

$$P^{R,S}_{i,t} \equiv P^{S}_{i,t} - P^{*}_{i,t},$$

and:

$$E^{R,S}_{ij,t} \equiv E^{S}_{ij,t} + P^{*}_{j,t} - P^{*}_{i,t}.$$

The natural logarithms of real forward prices and exchange rates are defined symmetrically as:

$$P^{R,F}_{i,t-n,n} \equiv P^{F}_{i,t-n,n} - P^{*}_{i,t},$$

and:

$$E^{R,F}_{ij,t-n,n} \equiv E^{F}_{ij,t-n,n} + P^{*}_{j,t} - P^{*}_{i,t}.$$ [13]

With these definitions, the realized forecast error [equation (4)] is identical whether expressed in real or nominal terms:

$$\begin{aligned} Z^{R}_{i,t,n} &\equiv P^{R,S}_{i,t} - P^{R,F}_{i,t-n,n} \\ &= P^{S}_{i,t} - P^{F}_{i,t-n,n} \\ &= Z_{i,t,n}, \end{aligned}$$

and the same holds for exchange-rate forecast errors:

$$\begin{aligned} Z^{R}_{E_{ij},t,n} &\equiv E^{R,S}_{ij,t} - E^{R,F}_{ij,t-n,n} \\ &= E^{S}_{ij,t} - E^{F}_{ij,t-n,n} \\ &= Z_{E_{ij},t,n}. \end{aligned}$$

Setting $\Theta^{S}_{j} = \Theta^{F}_{j}$ and $K^{S}(X)_{t} = K^{F}(X)_{t} = K(X)_{t}$, and subtracting equation (3S),

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[13] These definitions invoke the perfect foresight assumption as the real forward prices and forward exchange rates are defined on *future* price levels, which is in keeping with a rational expectations approach.

with an n-period lag, from equation (3F), we obtain the *forecast-error version* of equation (3):

$$Z_{1,t,n} = \sum_{j}^{M} \Theta_j \cdot Z_{E_{1j},t,n} + v_{1t}, \tag{5}$$

where $v_{1t} \equiv K(X)_{t+n} - K(X)_t$ is the change in the fundamentals from time t+n to t. The v_{1t} are likely to be serially correlated, at least when n is "small". Equation (5) will be used in the empirical implementation of the pricing model, which requires the usual assumption that v_{1t} be orthogonal to the $Z_{E_{1j},t,n}$; that is, that forecast errors in the foreign currency market be orthogonal to the first differences of the "fundamentals" that govern the international gold market.

III. THE DATA

The spot gold price data consist of daily observations on the U.S. dollar price of gold for the January 1982—December 1990 period, but the forward price data for gold were obtained only for 90-day contracts which are let at the beginning of each month and hence the gold data set is restricted to 108 monthly observations from January 1982 through December 1990. Data on spot and forward exchange rates between the U.S. dollar and the Deutsche mark, the Swiss franc, the U.K. pound sterling, and the Japanese yen were obtained for the same period to capture the dollar, the European Monetary System (EMS) and the yen blocs.[13] Because the forecast error data require the spot series to *lead* the forward series by three months, the useful data set is limited to 105 overlapping observations from January 1982 to September 1990 or,

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[13] All exchange rate data were obtained from the International Monetary Fund Data Bank. The daily spot quotations for gold are from the London Gold Market and were obtained from Reuters. The forward gold prices were computed from the closing quotations on three-month contracts at the COMEX.

$$(1-L)^d X_t = \Pi(L)\ \varepsilon_t, \tag{6}$$

where ε_t is white noise and L is the usual lag operator. Stationarity is determined by the value of d in equation (6); if $d \in (0.5,1)$, then the X_t series is non-stationary. However, if d lies in the open interval (0,0.5), the series is stationary but exhibits non-periodic, irregular cycles.

The first test to distinguish whether the price of gold and exchange rate series were generated by random walks or (possibly non-stationary) long memory processes utilizes a procedure designed by Diebold (1989) that is based on the properties of the variance-time function. If X_t follows a random walk, then the variance, $\sigma_X^2(k)$, of the k^{th} difference, $\Delta^k X_t$, of that variable is proportional to k:

$$\sigma_X^2(k) = k \cdot \sigma_X^2(1).$$

For a sample whose size and mean are T and μ, respectively, the variance of the k^{th} difference is:

$$\sigma_X^2(k) = \sum_{t=k}^{T} (X_t - X_{t-k} - k \cdot \mu)^2/(T - k + 1).$$

Under the null hypothesis that the series follows a random walk with drift, a simple scalar asymptotic test statistic, R2(k), is calculated as:

$$R2(k) = k \cdot \sigma_X^2(1)/\sigma_X^2(k).$$

Diebold (1989) has calculated the fractiles of the R2(k) test statistic for k = 1, 2, 4, 8, 16, 32 corresponding to the null hypothesis that R2(k) = 1, and also the fractiles of a *joint* test statistic, J2, under the null hypothesis that *all* R2(k) equal unity (again for k = 1, 2, 4, 8, 16, 32).

The five R2 test statistics and the single J2 joint test statistic computed for the 108 spot and forward observations on gold prices and exchange rates are presented in the upper panel of Table 1. Only in the case

Table 1

STATIONARITY TESTS

Diebold Random Walk Test on Gold Prices and Exchange Rates

Test	Gold		Mark		Yen		Pound	
	Spot	Forward	Spot	Forward	Spot	Forward	Spot	Forward
R2(2)	0.98	0.91	1.17	1.19	1.10	1.11	0.90	0.89
R2(4)	0.86	0.76	0.93	0.95	0.87	0.90	0.96	0.93
R2(8)	1.84	2.30	0.65	0.65	0.58	0.61	1.11	1.14
R2(16)	1.42	1.85	0.47	0.45	0.81	0.82	0.81	0.85
R2(32)	1.95	2.06	0.20	0.20	0.34	0.34	0.33	0.34
J2	0.94	1.77	4.96	5.27	2.08	1.97	1.60	1.48

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Maximum Likelihood Estimates of d for Gold, Exchange Rates and Inflation

Price	Gold	Mark	Yen	Pound	Inflation
Spot	0.835	1.021	1.072	1.050	1.082
Forward	0.864	1.078	1.065	1.056	

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Maximum Likelihood Estimates of d for Forecast Errors

Month	Gold	Mark	Yen	Pound
First	-0.074	0.192	0.034	0.102
Second	-0.148	0.152	0.109	0.054
Third	-0.081	0.203	0.033	0.180

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of the Deutsche mark (where the joint test rejects the null hypothesis) can we reject the null hypothesis of a random walk with drift.

Estimation of the fractional parameter d, however, provides a more accurate test of stationarity. Most methods for estimating d are based on the slope of the periodogram; in this study, however, we used Sowell's (1991) maximum likelihood estimate which gives more reliable results, particularly for small samples. The maximum likelihood estimates of d based on the 108 observations on spot and forward gold prices and the original U.S. dollar exchange rate series appear in the middle panel of Table 1. All price and exchange rate series, both spot and forward, appear to be non-stationary as the t tests (not reported) find no estimate of d to be significantly different from unity. Moreover, a model selection procedure based on the Akaike Information and the Schwartz Information criteria indicates that the series have no ARMA component and hence can be treated as random walks.

As the empirical analysis in this paper relies not on stationarity of spot and forward prices but rather on stationarity of forecast errors, we require spot and forward prices to be cointegrated with a cointegration vector [1,-1]. To test that hypothesis, d was re-estimated on each of the three subsets of non-overlapping forecast error data, and the results are presented in the final panel of Table 1.[18] The forecast errors for all four series appear to be stationary as the estimates of d do not differ significantly from zero. The estimate for gold price forecast errors for the second month appears to differ from the others; as there is no clear theoretical explanation for this phenomenon, it is viewed as a curious (but harmless) artifact of the gold price series.

Market Efficiency Tests

The classic market efficiency test is based on an estimate of the following equation:

$$P^{S}_{1,t+n} = \alpha + \beta \cdot P^{F}_{1,t,n}, \tag{7}$$

where $P^{S}_{1,t}$ and $P^{F}_{1,t,n}$ are the (natural logarithms of the) spot and n-period forward prices, respectively, at time t for an unspecified commodity or asset denominated in currency 1 (i.e., the pound sterling). Assuming risk neutrality, a market is said to be weakly efficient if:

$$P^{F}_{1,t} = E(P^{S}_{1,t+n} | I_{t}),$$

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[18] Non-overlapping data are required to avoid the problem of serial correlation and to determine if the estimates are sensitive to the sampling interval.

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where $E(\cdot)$ is the conditional expectation and I_t, the information set at time t, is assumed to include only past values of spot and forward prices. Perfect (weak) market efficiency implies that $\alpha = 0$ and $\beta = 1$.

Chi-Square tests of the restriction $\{\alpha = 0, \ \beta = 1\}$ based on an OLS estimate of equation (7) for each of the three subsets of the non-overlapping gold price and exchange rate data are reported in the upper panel of Table 2.[18] As is evident, that restriction is never rejected at the five percent level of significance. As these results are biased due to the non-stationarity of the spot and forward data that was discovered earlier, a more appropriate test of market efficiency is one based strictly on forecast errors:

$$Z_{1,t,n} = \gamma + \delta \cdot Z_{1,t-1,n}, \tag{8}$$

where $Z_{1,t,n}$ is the forecast error of the price of gold in currency 1. As market efficiency requires that forecast errors be serially uncorrelated, the estimate of δ should not differ significantly from zero. In the middle panel of Table 2 we present the results of twelve F tests of the restriction $\{\gamma = \delta = 0\}$ which are based on OLS estimates of equation (8) for both the price of gold and the exchange rates. In no case is that restriction rejected at the five per cent level of significance.

The final test was for *semi-strong* market efficiency which assumes that the information set includes variables other than past forecast errors on the variable in question. In the context of the model developed in section II, semi-strong market efficiency requires that past gold price *and* exchange-rate

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[18] Estimating equation (7) using the overlapping data results in a severely biased estimate of β. As roughly two-thirds of the innovations in each observation are shared with adjoining observations, the expected value of an estimate of β based on the overlapping data is roughly 0.67. The actual estimates of equation (7) that are reported in the upper panel of Table 2 were made using the White (1980) "robust standard error" routine.

Table 2

TESTS OF WEAK MARKET EFFICIENCY: 1982:1 THROUGH 1990:3

Tests of Joint Restriction on Equation (7):*

VARIABLE	$\chi^2(2)$ STATISTIC			SIGNIFICANCE LEVEL		
	1st	2nd	3rd	1st	2nd	3rd
Gold	3.4755	4.9280	2.4236	0.1759	0.0851	0.2977
DM	1.3577	1.1598	2.8324	0.5072	0.5599	0.2426
Dollar	2.0164	1.9893	1.0710	0.3649	0.3699	0.5854
Yen	2.3722	1.9121	3.6203	0.3054	0.3844	0.1636

Tests of Joint Restriction on Equation (8):

VARIABLE	$F(2,32)$ STATISTIC			SIGNIFICANCE LEVEL		
	1st	2nd	3rd	1st	2nd	3rd
Gold	0.3776	0.0533	0.2921	0.6885	0.9482	0.7487
DM	1.4257	1.2178	3.0304	0.2552	0.3092	0.0623
Dollar	0.4697	0.2790	1.9683	0.6294	0.7584	0.1563
Yen	1.6658	1.4312	2.1174	0.2050	0.2539	0.1369

TESTS OF SEMI-STRONG MARKET EFFICIENCY: 1982:1 THROUGH 1990:3

Tests of Joint Restriction on Equation (9):

VARIABLE	$F(4,29)$ STATISTIC			SIGNIFICANCE LEVEL		
	1st	2nd	3rd	1st	2nd	3rd
Gold	1.4716	0.6864	1.4552	0.2364	0.6072	0.2413
DM	1.0987	0.6199	1.3867	0.3759	0.6519	0.2631
Dollar	0.8552	1.1797	2.3719	0.5022	0.3404	0.0755
Yen	0.6561	0.4733	1.0161	0.6274	0.7549	0.4154

* Estimates of equation (7) were made using robust standard errors, 2 lags.

forecast errors be orthogonal to current gold price *and* exchange-rate forecast errors. In the case of gold, the test for semi-strong market efficiency involves estimation of the following equation:

$$Z_{1,t,n} = \mu + \vartheta_1 \cdot Z_{1,t-1,n} + \sum_{i=2}^{4} \vartheta_i \cdot Z_{E_{1i},t-1,n}, \qquad (9)$$

where $Z_{1,t,n}$ is the forecast error on the price of gold in currency 1 and $Z_{E_{1i},t,n}$ is the forecast error on exchange rates against currency 1. When dealing with the j^{th} exchange rate, the dependent variable $(Z_{1,t,n})$ is

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replaced with $Z_{E_{1j},t,n}$. Semi-strong market efficiency implies that all $\vartheta_i = 0$. The final panel of Table 2 contains the results of twelve F tests on that restriction which are based on OLS estimates of equation (9) and the restriction is never rejected at the five per cent level of significance. In short, *semi-strong* market efficiency cannot be rejected for either the gold or the exchange rate markets.

In summary, both spot and forward exchange rates and the price of gold were found to be non-stationary (integrated of order one) but the forecast error series are stationary and hence require no filtering. In addition, neither weak nor semi-strong market efficiency could be rejected.

IV. ESTIMATES OF THE "THETAS"

As the results of a simple OLS estimate of equation (5) based on the 105 overlapping observations and with standard errors estimated by Hansen-Hodrick (1980) routine indicated that the unit-sum restriction could not be rejected, that restriction was imposed and the results (again with the standard errors estimated by the Hansen-Hodrick technique) are presented in the upper panel of Table 3. The t statistic on the unit-sum restriction is reported in the lower panel of that table; as the restriction is not binding, the results for the restricted and unrestricted regression are very similar apart from an expected increase in the t statistics. These preliminary results suggest that the major gold producers (South Africa, Australia, the former Soviet Union) have no importance in the world gold market; rather, the dollar and EMS blocs dominate that market, with the EMS bloc having the larger weight.

The specification of equation (3) and the regressions reported in Table 3 do not take into account the fact that *all* forecast-error data are overlapping and hence any given innovation will affect several observations; indeed, the partial correlations for all forecast-error data remain quite high for up to

Table 3

RESTRICTED OLS ESTIMATE OF EQUATION (5) FOR GOLD: 1982:1 TO 1990:9

Hansen-Hodrick Standard Errors

$\bar{R}^2 = 0.3137$; SEE = 0.0740; D-W = 0.8870

Q(26) = 63.2603, Significance Level = 0.0001

VARIABLE	LAG	COEFFICIENT	t STATISTIC	SIGNIFICANCE
MARK	0	0.4752	2.0629	0.0417
DOLLAR	0	0.3991	2.7698	0.0067
YEN	0	0.1258	0.6971	0.4874

Tests on Unrestricted Exchange Rate Coefficient Estimates:

Sum:	1.0191
Standard Error of Sum:	0.1472
t Ratio (against zero):	6.9229
Significance Level:	0.0000
t Ratio (against unity):	0.1296
Significance Level:	0.8972

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seven or eight lags. This property of the overlapping data suggests that including lags might be useful—despite the fact that the efficient market hypothesis could not be rejected (on the non-overlapping data). More general forms for equations (3) and (5) that incorporate lags are the following:

$$P^R_{1,t} = \sum_{j=2}^{M} \Theta_j(L) \cdot E^R_{1j,t} + K(X)_t, \qquad (3')$$

$$Z_{1,t,n} = \sum_{j=2}^{M} \Theta_j(L) \cdot Z_{E_{1j},t,n} + v_{1t}, \qquad (5')$$

where L is the usual lag operator.[19] Experimentation with alternative lag structures indicated that once lags were introduced into the dependent variable, lags on the independent variables became redundant. Accordingly, the dependent variable $Z_{1,t,n}$ was replaced with $\alpha(L) \cdot Z_{1,t,n}$ and then

[19] As usual, $\Theta(L)$ is a polynomial of the form $\theta_0 + \theta_1 \cdot L + \theta_2 \cdot L^2 \cdots$.

equation (5′) was reparameterized as follows:

$$\Delta Z_{1,t,n} = \sum_{j=1}^{8} \left(\sum_{k=0}^{j} \alpha_k \right) \cdot \Delta Z_{1,t-j,n} + \alpha(1) \cdot Z_{1,t-9,n} + \sum_{j=2}^{4} \Theta_j \cdot Z_{E_{1j},t,n} + v_{1t}, \qquad (5'')$$

where $\alpha_0 = -1$ and $\alpha(1)$ is the sum of the coefficients of the polynomial $\alpha(L)$, including α_0.

In equation (5″), as the initial effect on the gold price forecast error of a (permanent) shock to the j^{th} real exchange rate forecast error is given by Θ_j, the long run effect is $-\Theta_j/\alpha(1)$ which can be estimated directly when embedded in a nonlinear fashion:

$$\Delta Z_{1,t,n} = \sum_{j=1}^{8} \left(\sum_{k=0}^{j} \alpha_k \right) \cdot \Delta Z_{1,t-j,n} + \alpha(1) \cdot \left(Z_{1,t-9,n} + \sum_{j=2}^{4} \psi_j \cdot Z_{E_{1j},t,n} \right) + v_{1t}, \qquad (10)$$

where $\psi_j = \Theta_j/\alpha(1)$. An unrestricted version of equation (10) was estimated by the RATS nonlinear least squares routine (NLLS) with lags being added until the estimate of $\alpha(1)$ stabilized (which occurred after the eighth lag). The standard errors were then re-estimated by the Hansen-Hodrick technique and, as it was found that the unit-sum restriction on the "thetas" could not be rejected, that restriction was imposed. The restricted estimates are reported in the upper panel of Table 4; in that table, the coefficients reported for each currency are estimates of the ψ_j — the long run values of the Θ_j.

The results employing lags totally dominate those reported in Table 3; despite the first differencing of the dependent variable, the (adjusted) coefficient of determination increased by over 50 per cent and the standard error of estimate declined by nearly one third (to 0.0525 from 0.0740). All estimates of the long run "thetas" are highly significant, and the point estimate of the "theta" for the Deutsche mark (i.e., the EMS bloc), 0.64, is more than three times the estimate of the "theta" for the dollar bloc and about four times that for the yen bloc. The t statistic on the unit-sum

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Table 4

RESTRICTED NLLS ESTIMATE OF EQUATION (10) FOR GOLD: 1982:5 TO 1990:12

Nine Lags on Dependent Variable, Hansen-Hodrick Standard Errors

$\bar{R}^2$ = 0.4753; SEE = 0.0525; D-W = 1.8442

Q(24) = 24.4526, Significance Level = 0.4360

VARIABLE	LAG	COEFFICIENT	t STATISTIC	SIGNIFICANCE
MARK	0	0.6432	6.4099	0.0000
DOLLAR	0	0.1982	3.6813	0.0004
YEN	0	0.1587	2.5733	0.0118
ΔGOLD	1	-0.5851	-10.1367	0.0000
"	2	-0.5199	-8.2146	0.0000
"	3	-0.9704	-16.1864	0.0000
"	4	-0.8594	-12.3395	0.0000
"	5	-0.6173	-8.5860	0.0000
"	6	-1.0047	-15.3503	0.0000
"	7	-0.8530	-11.9477	0.0000
"	8	-0.7040	-9.7-89	0.0000
GOLD	9	-0.7407	-10.8571	0.0000

Tests on Long Run Unrestricted Exchange Rate Coefficient Estimates:

Sum	0.9666
Standard Error:	0.0682
t Ratio (against zero):	14.1688
Significance Level:	0.0000
t Ratio (against unity):	-0.4902
Significance Level:	0.6253

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restriction on the "thetas" is reported in the final panel of Table 4.

Finally, recall that the term $v_{1t} \equiv K(X)_t - K(X)_{t-n}$ in equation (10) captures the change in the "fundamentals" from time t-n to t. An obvious candidate is the world inflation rate which may influence the attraction of gold as a store of value. The world price level, P^*_W, is based on producer prices in the U.S., Europe and Japan with the weights set equal to the estimates of the thetas. Letting $\Pi^W_{t,3}$ be the first difference of $P^*_{W,t}$ over the three-month forecast period, the inflation component of v_{1t} was defined as $[\gamma_0 \cdot (\Delta\Pi^W_{t,3} - \Delta\Pi^W_{t-3,3}) + \gamma(1) \cdot (\Pi^W_{t-1,3} - \Pi^W_{t-4,3})]$ and inserted into equation (10) as follows:[21]

$$\Delta Z_{1,t,n} = \sum_{j=1}^{8}\left(\sum_{k=0}^{j}\alpha_k\right)\cdot\Delta Z_{1,t-j,n} + \gamma_0\cdot(\Delta\Pi^W_{t,3} - \Delta\Pi^W_{t-3,3})$$

$$+ \alpha(1)\cdot\left(Z_{1,t-9,n} + \sum_{j=2}^{4}\psi_j\cdot Z_{E_{1j},t,n} + \Gamma\cdot(\Pi^W_{t-1,3} - \Pi^W_{t-4,3})\right) + v'_{1t}, \qquad (10')$$

where $\Gamma \equiv \gamma(1)/\alpha(1)$ is the long run effect of a (permanent) shock to world inflation on the real spot price of gold.

Equation (10′) was estimated by NLLS using an iterative procedure that sets the weights in the world price level equal to the estimated thetas; again it was found that the unit-sum restriction on the "thetas" could not be rejected. The results of a restricted estimate of that equation, reported in Table 5, are nearly identical with those in Table 4—apart from a minor shift in the theta estimate from Europe and the U.S. towards Japan. With respect to the world inflation variables, the estimate of Γ is highly significant and it suggests that a one point permanent rise in the rate of world inflation increases the real spot price of gold by about 1.6 per cent.[22] The t statistic on the unit-sum restriction on the "thetas" is reported in the final panel of that table.

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[21] The price level used for Europe is a real GDP-weighted average of the price levels in Germany, France, the U.K. and Italy. The GDP weights are averages for the 1982:1—1990:12 period. As producer prices are not available for France and Italy, consumer prices were used for those two countries. As the inflation component of K(X) was defined as $\gamma(L)\cdot\Pi^W_t$ and parameterized as $[\gamma_0\cdot\Pi^W_{t,3} + \gamma_1\cdot\Pi^W_{t-1,3}]$, the inflation component of v_{1t} is $[\gamma_0\cdot(\Pi^W_{t,3} - \Pi^W_{t-3,3}) + \gamma_1\cdot(\Pi^W_{t-1,3} - \Pi^W_{t-4,3})]$ which was reparameterized for equation (10) as $[\gamma_0\cdot(\Delta\Pi^W_{t,3} - \Delta\Pi^W_{t-3,3}) + \gamma(1)\cdot(\Pi^W_{t-1,3} - \Pi^W_{t-4,3})]$, where $\Delta\Pi^W_{t,3} = \Pi^W_{t,3} - \Pi^W_{t-1,3}$.

[22] The order of differentiation, d, was estimated for $\Delta\Pi_{W,t,3} - \Delta\Pi_{W,t-3,3}$ and stationarity could not be rejected; the results are reported in the middle panel of Table 1 in the column labelled "Inflation".

Table 5

RESTRICTED NLLS ESTIMATE OF EQUATION (10') FOR GOLD: 1982:5 TO 1990:12

Nine Lags on Dependent Variable, Hansen-Hodrick Standard Errors

$\bar{R}^2$ = 0.4661; SEE = 0.0529; D-W = 1.8476

Q(24) = 28.2951, Significance Level = 0.2478

VARIABLE	LAG	COEFFICIENT	t STATISTIC	SIGNIFICANCE
MARK	0	0.6305	6.2959	0.0000
DOLLAR	0	0.1908	3.5936	0.0006
YEN	0	0.1788	2.9307	0.0044
$\Delta\Pi_W$	0	0.1382	0.1739	0.8624
Π_W	1	1.6485	2.3907	0.0191
ΔGOLD	1	-0.5900	-10.0191	0.0000
"	2	-0.5223	-8.1060	0.0000
"	3	-0.9655	-16.2916	0.0000
"	4	-0.8449	-12.2300	0.0000
"	5	-0.6065	-8.5027	0.0000
"	6	-1.0015	-15.6559	0.0000
"	7	-0.8426	-11.6916	0.0000
"	8	-0.6841	-9.2966	0.0000
GOLD	9	-0.7375	-11.2365	0.0000

Tests on Long Run Unrestricted Exchange Rate Coefficient Estimates:

Sum	0.9619
Standard Error:	0.0685
t Ratio (against zero):	14.0524
Significance Level:	0.0000
t Ratio (against unity):	-0.5558
Significance Level:	0.5798

It is evident from Tables 4 and 5 that the European countries heavily dominate the international market for gold as movements in the real exchange rates between Europe and the dollar/yen blocs have very strong effects on the U.S. dollar price of gold. A ten per cent real appreciation of the Deutsche mark (against all other currencies) causes the dollar price of gold to *rise* by about 6.3 per cent (and *vice versa*) whereas the same real appreciation of the yen increases the dollar price of gold by less than two per cent. A ten per cent real appreciation of the dollar against all other currencies causes the dollar price of gold to *fall* by more than eight per cent, and *vice versa*.

The results of three simulations of transitory (one month) and permanent shocks to real exchange rates are displayed in the Figure on the following page. These simulations, based on the restricted estimates of equation (10), depict the percentage response of the U.S. dollar price of gold to ten per cent real appreciations (*vis a vis* all other currencies) of the Deutsche mark, the U.S. dollar, and the yen, respectively, occurring during month zero.

V. FLOATING EXCHANGE RATES AND THE STABILITY OF THE GOLD MARKET

There can be little doubt that the instability of the major currency exchange rates has contributed significantly to the violent fluctuations experienced by the price of gold since 1973. There is, of course, no way of divining the behavior of the free market price of gold had the Bretton Woods exchange rate system persisted (but without its link to gold). The monetary instability that has followed the breakdown of that system has undoubtedly increased the attraction—and hence the price—of gold as a store of value, its *de facto* demonetization notwithstanding. To gain some idea of the degree to which the price of gold has been destabilized by exchange rates, an experiment was conducted. Equation (5″) was reparameterized in level form:

$$Z_{1,t,n} = \sum_{k=1}^{9} \alpha_k \cdot Z_{1,t-j,n} + \sum_{j=2}^{4} \Theta_j \cdot Z_{E_{1j},t,n} + v_{1t},$$

and estimated by OLS subject to the unit-sum restriction on the Θ_j.[22] The OLS estimates then were imposed on a reparameterized version of equation (3):

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[22] As reparameterization is merely cosmetic, the restricted OLS estimates of equation (5″) are simple linear combinations of the restricted NLLS estimates of equation (10). For the OLS estimate, the unit-sum restriction on the long run "thetas" takes the form: $\sum_{k=1}^{9} \hat{\alpha}_k + \sum_{j=2}^{4} \hat{\Theta}_j = 1$.

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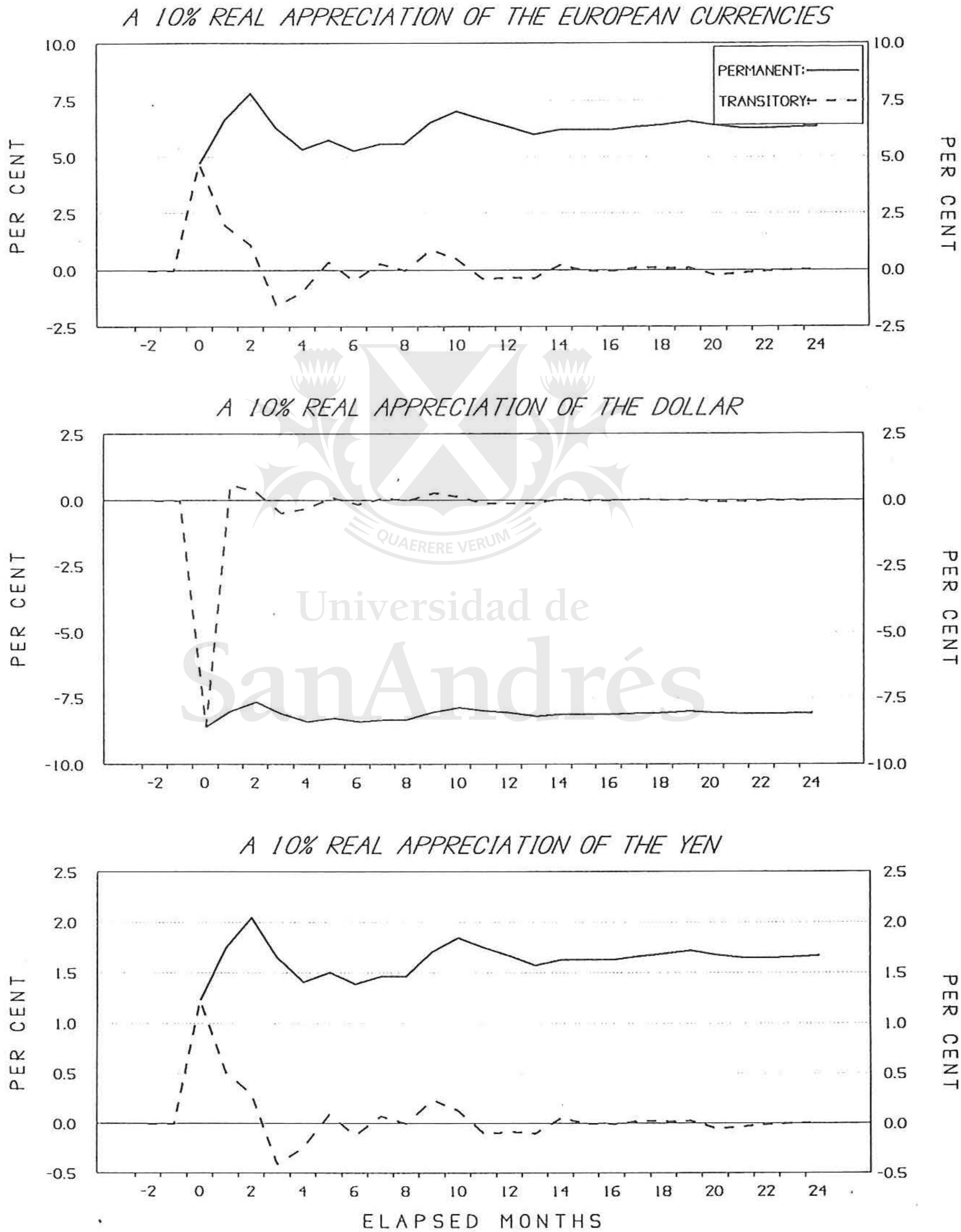
FIGURE: RESPONSE OF U.S. DOLLAR PRICE OF GOLD TO:



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$$P^{R,S}_{1,t} = \sum_{k=1}^{9} \hat{\alpha}_k \cdot P^{R,S}_{1,t-j} + \sum_{j=2}^{4} \hat{\Theta}_j \cdot E^{R,S}_{1j\;,t} + \hat{u}_{1t}, \qquad (3S')$$

and the residuals, $\hat{u}_{1t}$, which reflect all influences on the real spot price of gold *other than the major currency real exchange rates*, were taken as estimates of the "fundamentals" [the $K(X)$ term in equation (3)].

As all variables in equation (3S′) are in natural logarithms, the calculated $\hat{u}_{1t}$ residuals were transformed into arithmetic values (which are dimensionally identical with the real spot price of gold) and then converted into real and nominal "prices" for all four currencies. The coefficients of variation for the transformed residuals and the spot prices of gold appear in Table 6, upper panel.[23] For the period 1982:11—1990:12, the coefficients of variation of the actual spot prices of gold—both nominal and real—are roughly eighty per cent larger than those of the transformed residuals and, apart from the U.K. pound, the differences across the various currencies of denomination are remarkably small. Real exchange rates, then, account for nearly half of the variance in gold prices during the sample period.

One cannot, of course, assume that real exchange rates would have been constant had the Bretton Woods arrangement concerning nominal exchange rates been preserved, and it is impossible to know how they would have behaved under that system over the past twenty years. There is, however, clear evidence that real exchange rates have been far more volatile with the current system than under the Bretton Woods regime.[24] To quantify the effect of the change

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[23] Prior to calculating the standard deviations, both the transformed residuals and spot prices were first-differenced to remove negative trends, which were particularly pronounced in the Deutsche mark and yen series. Sample averages of the spot prices of gold were used to convert all standard deviations into coefficients of variation.

[24] See Mussa (1986) for evidence concerning the stability of those real exchange rates under the Bretton Woods exchange rate regime.

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Table 6

SOURCES OF VARIATION IN THE PRICE OF GOLD

Coefficients of Variation of the:

Currency	Nominal Price of Gold: Actual Price	Nominal Price of Gold: $\hat{u}_{1t}$	Nominal Price of Gold: Ratio	Real Price of Gold: Actual Price	Real Price of Gold: $\hat{u}_{1t}$	Real Price of Gold: Ratio
Pound	5.11	2.43	2.10	5.40	2.53	2.14
Dollar	4.92	2.78	1.77	5.04	2.83	1.78
Mark	5.28	2.88	1.83	5.33	2.91	1.83
Yen	5.68	3.15	1.80	5.63	3.10	1.82

****

Standard Deviations of Quarterly First Differences of Logarithms of Real Exchange Rates Between the U.K. and the U.S., Germany & Japan: 1960-70 and 1973-90, in Per Cent

REAL EXCHANGE RATE DEFINED ON:	GERMANY	U.S.	JAPAN
Consumer Prices			
1960-70:	2.1067	1.7755	2.1934
1973-90:	4.8402	5.4086	5.6491
Ratio:	2.2975	3.0463	2.5754
Producer Prices			
1960-70:	2.0059	1.6974	2.1153
1973-90:	4.6049	5.3005	5.7939
Ratio:	2.2957	3.1227	2.7391

in regime, we calculated the standard deviations of quarterly first differences of the natural logarithms of the real exchange rates between the U.K. and the U.S., Germany and Japan for 1960-70 and 1973-90 and the results are presented in the final panel of Table 6 for both producer and consumer prices.[25] The variability in real exchange rates between the U.K. and the U.S. and Japan have nearly tripled after the abandonment of the Bretton Woods exchange rate regime and, despite the growing monetary integration between the

[25] Quarterly averages of price levels and exchange rates were used in the real exchange rate calculations.

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U.K. and Germany, the variability of that real exchange has more than doubled. While not definitive, these results clearly support the proposition that floating exchange rates among the major currencies have been a major source of the instability of the free market price of gold since 1973.

APPENDIX: FRACTIONAL DIFFERENCING

In Box-Jenkins (1976) terminology, time series data usually are assumed to be integrated of either degree zero or one (and infrequently of degree two); if a variable X_t is integrated of degree zero [i.e., $X_t \sim I(0)$], its variance is finite and innovations have no lasting effect as autocorrelation decays at an exponential rate for distant lags. However, if $X_t \sim I(1)$, the variance of X_t is infinite and, as X_t is the sum of all previous innovations, those innovations have a permanent effect.

A limitation of this approach is that it allows only discrete choices for the degree of integration whereas in principle there exists a range of intermediate values that involve the so-called long memory models. These models stem from Granger (1966) who demonstrated that economic variables have "typical spectral shape", and concluded that "..long term fluctuations in economic variables, if decomposed into frequency components, are such that the amplitudes of the components decrease smoothly with decreasing period". In other words, the spectral density of economic time series is bounded at the origin and the autocorrelation function decreases smoothly as the lag between observations increases. This means that variables tend to display long and irregular cycles or, stated differently, that shocks are persistent.

The correct method for analyzing economic time series, developed in the early 1980s, is known as fractional differencing. The intuition is rather straightforward. Standard time series methodology considers only processes such as ARIMA(p,d,q) where d, the order of differentiation, is assumed to be an integer. Granger and Joyeaux (1980) and Hosking (1981), however, stressed that d is not necessarily an integer; rather, it is a real number. They have suggested a procedure by which d is estimated (which is closely related to unit-root tests) and then a filter based on the estimate of d is applied to preserve the information on persistence. The transformed series can then be analyzed as an ARMA(p,q) process or by traditional time series methods.

The simplest long memory process—the basic building block—is the fractional noise defined as:

$$(1-L)^d X_t = \sum_{j=0}^{\infty} \pi_j \cdot \varepsilon_{t-j}$$

$$= \Pi(L)\ \varepsilon_t,$$

where ε_t is white noise, d is a real number and L is the lag operator. The process is stationary and invertible if $-0.5 < d < 0.5$ and the binomial expansion of $(1-L)^d$ allows one to express π_j as:

$$\pi_j = \frac{\Gamma(j-d)}{\Gamma(j+1)\cdot\Gamma(-d)},$$

which converges in mean square for $-0.5 < d < 0.5$. The fractional noise for $-0.5 < d < 0.5$ has an autoregressive representation:

$$\sum_{j=0}^{\infty} \left[\frac{\Gamma(j-d)}{\Gamma(j+1)\cdot\Gamma(-d)}\right] L^j X_t = \varepsilon_t,$$

and a moving average representation:

$$X_t = \sum_{j=0}^{\infty} \left[\frac{\Gamma(j-d)}{\Gamma(j+1)\cdot\Gamma(d)}\right] L^j \varepsilon_t.$$

More general processes can be obtained from fractional noise; these are usually referred to as ARFIMA (Autoregressive Fractionally Integrated Moving Average) and, in addition to the long memory component, they contain an ARMA

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component that determines the short term movements:

$$(1-L)^d A(L) X_t = B(L) \varepsilon_t.$$

If d lies in the open interval $(0,0.5)$, then the series is stationary but displays non-periodic, irregular cycles. The autocovariance of the series is positive and decays at a geometric rate (compared to the exponential rate of the standard ARIMA models). Alternatively, if $d \in (0.5,1)$ the series is non-stationary; in either case, if d is significantly different from zero, a filter $(1-L)^d$ must be used to obtain a series integrated of degree zero.

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