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Expectations in the Basic New Keynesian Model

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"Expectativas en el Modelo Neokeynesiano Básico"

Resumen

El presente trabajo analiza cómo el Modelo Neokeynesiano Básico de Galí (2008) responde a diferentes especificaciones para la forma en que los agentes construyen sus expectativas inflacionarias. Partiendo de una versión recalibrada del modelo original, en el que operan expectativas racionales, y enfocándonos particularmente en la configuración del modelo en la cual la política monetaria es llevada adelante a través de la oferta exógena de dinero, el objetivo es comparar cómo varía el impacto de shocks estocásticos (monetario y tecnológico), a través de diferentes escenarios. Particularmente, la dinámica de equilibrio y la volatilidad reportada por las variables relevantes será analizada en versiones del modelo bajo expectativas racionales, extrapolativas y adaptativas. Al interpretar cómo la introducción de estos últimos dos esquemas modifica la dinámica del modelo, concluiremos que el impacto de ambos es similar en tanto agregan volatilidad en las variables relevantes. En ambos casos, dichas variables describen recorridos oscilantes en sus caminos hacia el equilibrio de estado estacionario, los cuales transitan luego de ocurridos los shocks analizados. Tal oscilación no se percibe en el caso de expectativas racionales. Asimismo, ambas configuraciones alternativas se introducen, por un lado, sólo en la forma en la cual las firmas construyen sus expectativas (modificando exclusivamente la Curva de Phillips Neokeynesiana) y, posteriormente, en la forma en que todos los agentes de la economía lo hacen (modificando, también, la ecuación dinámica IS). Quedará demostrado que, en el Modelo Neokeynesiano Básico bajo oferta exógena de dinero y con ajuste de precios a la Calvo (1983) con el que trabajamos, modificar la formación de expectativas en todos los agentes no introduce consecuencias significativas respecto a hacerlo sólo en las firmas.

Palabras clave: Modelo Neokeynesiano Básico, Curva de Phillips, Expectativas, Inflación

Universidad de San Andrés

"Expectations on the Basic New Keynesian Model"

Abstract

This paper analyses how the Basic New Keynesian Model from Galí (2008) responds to different settings for the way inflation expectations are constructed by agents in the model. Starting from a re-calibrated version of the original model -under rational expectations- and focusing particularly on the case in which monetary policy is run through a money growth rule, the goal is to compare how the impacts from both monetary and technology stochastic shocks mutate across different scenarios for inflation expectations. Particularly, the evolution of equilibrium dynamics and relevant variables' volatility are analyzed and compared when rational, extrapolative and adaptive processes rule the way inflation expectations are built in the economy. Interpreting how introducing extrapolative and adaptive expectations modifies the model output will allow us to conclude that the overall impact of both settings in the dynamic is similar. In summary, higher volatility is reported by the relevant variables in both scenarios. That is reflected on oscillating paths described by the variables post-shock, contrarily to the rational scenario. Moreover, both types of alternative expectations are introduced twice: first, on the way only firms form their inflation expectations (just modifying the New Keynesian Phillips Curve); and, secondly, on the way all agents in the economy estimate inflation (i.e. by also modifying the dynamic IS equation). Regarding this, we will conclude that, for the Basic New Keynesian Model under exogenous money supply with staggered price setting due to Calvo (1983), modifying inflation expectations for all the agents in the economy has no major implications than doing it only for the firms.

Keywords: Basic New Keynesian Model, Phillips Curve, Expectations, Inflation

Códigos JEL: D84, E12, E31

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Contents

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1 Introduction

It is widely accepted, both in literature and practice, that expectations play a significant role in economics.

Finding the reason why expectations are relevant in the practice is straightforward: agents act in the economy based on the expectations they have on how relevant economic variables will develop in the future. Investment decisions are clearly interfered by the expectations that agents have on the returns of those investments; consumption decisions (at least for no necessity goods) depend on agents' expectations of future prices. Therefore, many relevant economic variables such as asset prices, general products and services prices, investment, consumption and output end up being influenced by expectations.

This relevancy expectations have on the real world is reflected in the academic environment. For instance, there is a large literature regarding the role of expectations in asset markets[1] and much work has also been done on inflation expectations' side, particularly on how addressing them through different approaches changes the way inflation develops in certain contexts for New Keynesian Models. Lyziak (2016)[2], is an interesting and specific example of that kind of work.

Starting from the relevance of expectations, the present work aims to analyze how the response to both monetary and technology shock in the Basic New Keynesian model (BNKM) introduced by Galí (2008) varies when different mechanisms such as extrapolation and learning are introduced for the way agents build their inflation expectations. However, that is not the only modification applied on this paper to the original BNKM described by Galí (2008). The original model is re-calibrated taken consumption parameters from Oviedo (2017) and the interest semi-elasticity of money demand from Vallotta (2004).

The re-calibrated version of the original model, in which agents build their inflation expectations rationally, is set in such a way that monetary policy is run through a money growth rule. Later on, it is modified to address the alternative expectations mechanisms we will work with.

Overall, the model is modified four times. First, extrapolation is introduced on the way firms estimate inflation. Then, extrapolation is extended to all the agents in the economy. Finally, the same exercise is done for learning: firms are set to estimate inflation through an adaptive expectations scheme and then that adaptive behavior is extended to all the agents involved in the model's economy.

The impact from both monetary and technology stochastic shocks is compared

[1] Santocono (2019) summarizes some of it.

[2] *Do inflation expectations matter in a stylised New Keynesian model? The case of Poland*, Lyziak (2016)

across these different scenarios. The evolution of equilibrium dynamics and relevant variable's volatility across these five configurations[3] is analyzed.

While applying the same modification to the firms or to all the agents in the model proves not to bring significantly different results, what can also be concluded from our analysis is that the overall impact of both alternative settings (extrapolation and learning) in the dynamics is similar within each other. In sum, higher volatility is reported by the relevant variables in both scenarios and that is reflected on oscillating paths described by the variables post-shock in the impulse-response functions.

Summarizing the agenda: we are starting at ***Section 2***, which introduces the Basic New Keynesian Model from Galí (2008); ***Section 3*** presents our calibration, based on Oviedo (2017) and Vallotta (2004); ***Section 4*** specifies the modifications applied to the base model in order to reach our four alternative inflation expectations scenarios and in ***Section 5*** the equilibrium dynamics are analyzed for an economy in which monetary policy is run through a money growth rule. Finally, ***Conclusion*** section highlights the main learnings that came out of comparing the evolution of the dynamics across the different settings.

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Universidad de
SanAndrés

[3]Rational expectations, extrapolative firms, extrapolative agents, adaptive firms and adaptive agents.

2 The Model

The Basic New Keynesian Model (BNKM) used across this work is a closed economy model introduced by Galí (2008).

The author reaches this model mainly by getting rid of the perfect competition assumption from the classical monetary model, according to which manufacturers take price for their homogeneous product as given. Contrarily, on the BNKM each firm produces a differentiated good and sets its price. However, that set price cannot be freely adjusted by the firms at any moment. Instead, only a fraction of firms can reset their prices in any given period. This constraint is addressed by Galí through adopting a model of staggered price setting due to Calvo (1983) and characterized by random price durations.

In current section we are briefly[4] presenting the problems each sector faces and listing the equations that describe the equilibrium conditions for the BKNM.

2.1 Households

2.1.1 Households Problem

Galí assumes a representative infinitely-lived household that decides its consumption and labor supply levels for each period seeking to maximize

$$E_0 \sum_{t=0}^{\infty} \beta^t U \left[\left(\int_0^1 C_t(i)^{1-\frac{1}{\varepsilon}} \, di \right)^{\frac{\varepsilon}{\varepsilon-1}}, N_t \right]$$

restricted by a period budget constraint such that

$$\int_0^1 P_t(i)\, C_t(i)\, di + Q_t\, B_t \leq B_{t-1} + W_t\, N_t + T_t$$

and by the following solvency constraint

$$\lim_{t\to\infty} E_t(B_t) \geq 0,$$

where $C_t(i)$ stands for the quantity of good i consumed in period t; $P_t(i)$ denotes the price of good i; W_t is the nominal wage; N_t are the hours worked; B_t the one-period bond purchases; Q_t is those bonds price in period t; T_t represents a lump-sum income that household has (i.e. dividends from ownership of firms); β_t accounts for the discount factor and ε is the demand elasticity.

[4]The present work focuses on analyzing how the equilibrium dynamics for the model change across different versions of it. That is why it is relevant to introduce the model, but for a deeper description of the model basements and the equilibrium conditions derivation please see Galí (2008).

Notice as well that consumption decision is done over a continuum of goods represented by the interval $[0, 1]$.

This means that $\int_0^1 P_t(i)\, C_t(i)\, di = P_t\, C_t$, where P_t represents an aggregate price index, and C_t is the consumption index and also implies that the solution for the problem includes demand equations for each $C_t(i)$. Particularly, each demand equation takes the form

$$C_t(i) = \left(\frac{P_t(i)}{P_t}\right)^{-\varepsilon} C_t, \quad \text{for all } i \in [0, 1]. \tag{1}$$

2.1.2 Households Optimality Conditions

Assuming a period utility given by

$$U(C_t, N_t) = \frac{C_t^{1-\sigma}}{1-\sigma} - \frac{N_t^{1+\phi}}{1+\phi},$$

in which $\frac{1}{\sigma}$ represents the elasticity of inter-temporal substitution and ϕ the elasticity of labor supply, the optimal conditions implied by the households' problem are:

$$-\frac{U_{n,t}}{U_{c,t}} = \frac{W_t}{P_t} \qquad \text{and} \qquad Q_t = \beta E_t\left[\frac{U_{c,t+1}}{U_{c,t}} \frac{P_t}{P_{t+1}}\right] \quad ,$$

which can be log-linearized and expressed such that

$$w_t - p_t = \sigma c_t + \phi n_t \tag{2}$$

and

$$c_t = E_t(c_{t+1}) - \frac{1}{\sigma}\big[i_t - E_t(\pi_{t+1}) - \rho\big]. \tag{3}$$

Lowercase letters are used to denote log of the original variable. Also, the short term nominal rate i_t is defined such that $i_t \equiv -\log Q_t$ and $\rho \equiv -\log \beta$ is the discount rate and $\pi_{t+1} \equiv \frac{p_{t+1}}{p_t}$.

Equations (2) and (3) are the optimality conditions for the households' problem, together with the following ad-hoc log-linear money demand:

$$m_t - p_t = y_t - \eta\, i_t\,, \tag{4}$$

in which $\eta \geq 0$ denotes the interest semi-elasticity of money demand.

QUAERERE VERUM
Universidad de
San Andrés

2.2 Firms

The continuum of goods is produced by a continuum of firms that is also represented by the interval $[0, 1]$, as each firm produces and amount $Y_t(i)$ of a single differentiated good.

Still, all goods are produced under the same technology. So, the production function is common to all firms and represented by the following equation:

$$Y_t(i) = A_t N_t(i)^{1-\alpha} \, ,$$

where A_t stands for the exogenous level of technology and $(1-\alpha)$ is the labor share parameter.

All firms face the demand scheduled presented in *Equation* (1) and take aggregate price and consumption indexes (P_t and C_t, respectively) as given. With all this in mind, on any given period each firm may re-optimize its price with $(1-\theta)$ probability. This causes that, on aggregate, only a portion $(1-\theta)$ of producers change their prices each period. Hence, θ represents an index of price stickiness applied by Galí for this model, following Calvo (1983).

2.2.1 Firms Problem

A firm actually being able to re-optimize its price in period t decides P_t^* seeking to maximize

$$\sum_{k=0}^{\infty} \theta^k \, E_t\{Q_{t,t+k} \, [P_t^* \, Y_{t+k|t} - \Psi_{t+k}(Y_{t+k|t})\,]\}$$

subject to

$$Y_{t+k|t} = \left\{ \frac{P_t^*}{P_{t+k}} \right\}^{-\varepsilon} C_{t+k} \quad ,$$

the sequence of demand constraints according to *Equation* (1).

$Y_{t+k|t}$ represents the output in $t+k$ for a firm that re-optimized its price in period t for the last time; $\Psi(\cdot)$ is the cost function and nominal payoffs $\left[P_t^* \, Y_{t+k|t} - \Psi_{t+k}(Y_{t+k|t})\right]$ are discounted using the stochastic discount factor $Q_{t+k|t} \equiv \beta^k (C_{t+k}/C_t)^{-\sigma} (P_t/P_{t+k})$.

2.2.2 Firms Optimality Condition

$$\sum_{k=0}^{\infty} \theta^k E_t \left\{ Q_{t,t+k} \, Y_{t+k|t} \left(\frac{P_t^*}{P_{t-1}} - \Xi \, \mathrm{MC}_{t+k|t} \, \Pi_{t-1,t+k} \right) \right\} = 0$$

is the first order condition associated to the previously presented firms' problem, divided by P_{t-1} and in which new variables were defined.

Particularly, the desired or frictionless markup was defined as $\Xi \equiv \varepsilon/(\varepsilon - 1)$; the real marginal cost in period $t+k$ for a firm that re-optimized its price for the las time in period t as $\mathrm{MC}_{t+k|t} \equiv \Psi'_{t+k}(Y_{t+k|t})/P_{t+k}$ and $\Pi_{t-1,t+k} \equiv P_{t+k}/Pt$.

Linearizing the optimality condition around the zero inflation steady state through a first order Taylor expansion of the above equation around that steady state yields

$$p_t^* - p_{t-1} = (1-\beta\theta)\sum_{k=0}^{\infty}(\beta\theta)^k E_t\{\hat{mc}_{t+k|t} + p_{t+k} - p_{t-1}\} \quad , \tag{5}$$

where $\hat{mc}_{t+k|t} \equiv mc_{t+k|t} - mc$ is the log deviation of marginal cost from its steady state value $mc = -\log \Xi$. Also, for further uses, $\log \Xi \equiv \xi$.

2.3 Aggregate Price Dynamics

From firms' problem presented in *Section 2.2* and, specifically, from the introduction of price stickiness through θ, it emerges that the dynamic followed by the aggregate price is such that

$$\Pi_t^{1-\varepsilon} = \theta + (1-\theta)\left(\frac{P_t^*}{P_{t-1}}\right)^{1-\varepsilon},$$

for which a log-linearized approximation around a zero inflation steady state in which $P_t^* = P_{t-1} = P_t$ yields

$$\pi_t = (1-\theta)(p_t^* - p_{t-1}). \tag{6}$$

2.4 Equilibrium Conditions

2.4.1 Good markets clearing

Each good market clearing condition is

$$Y_t(i) = C_t(i), \quad \text{leading on aggregate to} \quad Y_t = C_t,$$

which combined with *Equation* (3) takes the form:

$$y_t = E_t\{y_{t+1}\} - \frac{1}{\sigma}(i_t - E_t\{\pi_{t+1}\} - \rho).$$

A re-expression of the above equation in terms of the output gap $\tilde{y}_t \equiv y_t - y_t^n$ (where y_t^n represents the natural level of output, defined as the equilibrium level of output under flexible prices) yields

$$\tilde{y}_t = E_t\{\tilde{y}_{t+1}\} - \frac{1}{\sigma}(r_t - r_t^n), \qquad \text{where} \qquad r_t \equiv i_t - E_t\{\pi_{t+1}\} \tag{7}$$

$$\text{and} \quad r_t^n \equiv \rho + \sigma\, E_t\{\Delta\, \tilde{y}_{t+1}^n\} = \rho + \sigma\, \psi_{ya}^n\, E_t\{\Delta\, a_{t+1}\}, \quad \text{with} \quad \psi_{ya}^n \equiv \tfrac{1+\phi}{\sigma(1-\alpha)+\phi+\alpha}.$$

Equation (7) is the dynamic IS equation. It is one of the equations determining the equilibrium path of real variables in the BNKM. The other two equations involved in that process are the New Keynesian Phillips Curve and an equation describing how monetary policy is conducted, introduced in *Section 2.4.3* and *Section 2.4.4*, respectively.

2.4.2 Labor market clearing

The condition that needs to be satisfied in order to reach market clearing in the labor market, $N_t = \int_0^1 N_t(i)\, di$, can be re-expressed as follows

$$N_t = \int_0^1 \left(\frac{Y_t(i)}{A_t}\right)^{\frac{1}{1-\alpha}} di$$

by incorporating the production function introduced in *Section 2.2*.

Furthermore, considering the market clearing condition for each good market $Y_t(i) = C_t(i)$ and, then, replacing $C_t(i)$ with its definition according to the demand equations described by *Equation* (1), the labor market clearing condition yields

$$N_t = \left(\frac{Y_t}{A_t}\right)^{\frac{1}{1-\alpha}} \int_0^1 \left(\frac{P_t(i)}{P_t}\right)^{-\frac{\varepsilon}{1-\alpha}} di\,.$$

Taking logs on the above equation and running a first-order approximation around the zero inflation steady state allows us to obtain the following approximate relation between aggregate output, employment and technology:

$$y_t = a_t + (1-\alpha)\, n_t\,. \tag{8}$$

2.4.3 New Keynesian Phillips Curve

First, an expression for an individual firm's marginal cost in terms of the economy's average real marginal cost is derived.

The economy's average real marginal cost is defined as: $mc_t = (w_t - p_t) - mpn_t$, where mpn_t stands for marginal product of labor.

Hence, mpn_t can be replaced with the log linearized expression for the marginal product of labor that comes out of the product function, i.e. ${Y'_t}_{N_t}$. Doing so leads to the following equation:

$$mc_t = (w_t - p_t) - (a_t - \alpha n_t) - log(1 - \alpha).$$

Moreover, n_t can be replaced according to *Equation* (8) such that:

$$mc_t = (w_t - p_t) - \frac{1}{1-\alpha}(a_t - \alpha y_t) - log(1 - \alpha).$$

Considering the fact that $mc_{t+k|t} = (w_{t+k} - p_{t+k}) - mpn_{t+k|t}$, the above equation can be re-expressed as

$$mc_{t+k|t} = (w_{t+k} - p_{t+k}) - \frac{1}{1-\alpha}(a_{t+k} - \alpha y_{t+k|t}) - log(1 - \alpha).$$

Incorporating the demand schedule from *Equation* (1) and the good markets clearing condition, the equation for $mc_{t+k|t}$ yields

$$mc_{t+k|t} = mc_{t+k} - \frac{\alpha\,\varepsilon}{1-\alpha}(p^*_t - p_{t+k}).$$

Substituting the above equation into *Equation* (5) and rearranging terms, *Equation* (5) can be re-expressed as follows:

$$p^*_t - p_{t-1} = (1-\beta\,\theta)\,\Theta\sum_{k=0}^{\infty}(\beta\,\theta)^k E_t\{\hat{m}c_{t+k}\} + \sum_{k=0}^{\infty}(\beta\,\theta)^k E_t\{\pi_{t+k}\}, \text{ where } \Theta \equiv \frac{1-\alpha}{1-\alpha+\alpha\,\varepsilon} \leq 1.$$

Rewriting as a difference equation yields

$$p^*_t - p_{t-1} = \beta\,\theta\,E_t\{p^*_{t+1} - p_t\} + (1-\beta\,\theta)\,\Theta\,\hat{m}c_t + \pi_t\,,$$

which combined with *Equation* (6) let us get the inflation equation

$$\pi_t = \beta\,E_t\{\pi_{t+1}\} + \lambda\,\hat{m}c_t\,, \tag{9}$$

Universidad de San Andrés

where

$$\lambda \equiv \frac{(1-\theta)(1-\beta\,\theta)}{\theta}\,\Theta\,.$$

Now, let us derive an expression for the economy's real marginal cost in terms of the aggregate economic activity.

Starting, once again, from $mc_t = (w_t - p_t) - mpn_t$ and substituting on it households' optimality condition from *Equation* (2) as well as *Equation* (8) derived from the labor market clearing condition, we reach the equation below:

$$mc_t = \left(\sigma + \frac{\phi+\alpha}{1-\alpha}\right) y_t - \frac{1+\phi}{1-\alpha}\,a_t - \log\left(1-\alpha\right). \tag{10}$$

Moreover, defining the natural level of output y_t^n as the equilibrium level of output under flexible prices and taking into consideration that the real marginal cost is constant and equal to $-\xi$ under flexible prices, as mentioned in *Section 2.2.2*, *Equation* (10) in equilibrium stands

$$mc = \left(\sigma + \frac{\phi+\alpha}{1-\alpha}\right) y_t^n - \frac{1+\phi}{1-\alpha}\,a_t - \log\left(1-\alpha\right) \quad , \tag{11}$$

implying that

$$y_t^n = \psi_{ya}^n\,a_t + \upsilon_y^n\,,$$

$$\text{where} \quad \psi_{ya}^n \equiv \frac{1+\phi}{\sigma(1-\alpha)+\phi+\alpha} \quad \text{and} \quad \upsilon_y^n \equiv -\frac{(1-\alpha)[\xi-\log(1-\alpha)]}{\sigma(1-\alpha)+\phi+\alpha} \quad .$$

Subtracting *Equation* (11) from *Equation* (10) leads to:

$$\hat{mc}_t = \left(\sigma + \frac{\phi+\alpha}{1-\alpha}\right)\left(y_t - y_t^n\right). \tag{12}$$

Finally, combining *Equation* (12) from above with *Equation* (9), the expression for the *New Keynesian Phillips Curve (NKPC)* is derived:

QUAERERE VERUM
Universidad de San Andrés

$$\pi_t = \beta\, E_t\{\pi_{t+1}\} + \kappa\, \tilde{y}_t, \quad \text{where} \quad \kappa \equiv \lambda \left(\sigma + \frac{\phi + \alpha}{1 - \alpha}\right). \tag{13}$$

As indicated earlier, the NKPC from *Equation* (13) joins the DIS equation from *Equation* (7) on describing the BNKM equilibrium.

Since monetary policy is non-neutral on determining the equilibrium path of real variables in the BNKM, the equilibrium description will be completed once the equation that establishes how monetary policy is precisely conducted in the model's economy is introduced in the next section.

2.4.4 Monetary Policy: Exogenous Money Supply

The central bank can run monetary policy either through an interest rate rule or by controlling exogenously the money supply.

As commented previously, in this paper we will focus on the case in which monetary policy is run through a money growth rule. Hence, the equation that will complete the equilibrium conditions for our model will be in line with that assumption[5].

The BNKM could be characterized by an exogenous path for the growth rate of the money supply, denoted as $\Delta\, m_t$. Defining the real money demand l_t as $l_t \equiv m_t - p_t$ implies both that $l_t = y_t - \eta i_t$, as per *Equation* (4), and that money growth can be expressed as

$$\Delta\, m_t = l_t - l_{t-1} + \pi_t. \tag{14}$$

Equation (14) from above is the third equation describing the equilibrium for the BNKM under an exogenous money supply, joining the NKPC from *Equation* (13) and the DIS equation from *Equation* (7).

Furthermore, for equilibrium dynamics analysis, it is assumed that $\Delta\, m_t$ follows an AR(1) process such that $\Delta\, m_t = \rho\, \Delta\, m_{t-1} + \varepsilon_t^m$, where $\rho_m \in [0, 1)$ denotes the persistence of $\Delta\, m$ and ε_t^m is white noise.

[5] *Section 8.3* at the Appendix presents this equation for the interest rate rule case.

Universidad de SanAndrés

3 Calibration

For assigning values to the model's parameters we are partially departing from Galí's (2008) calibration and, instead, Oviedo's (2017) parameterization for Argentina will be followed for some preference parameters such as the inverse of the elasticity of intertemporal substitution (σ), the elasticity of labor supply (ϕ) and the discount factor (β).

The share of labor in the production function (α), a technology-related parameter, is also set in line with Oviedo (2007) while the interest semi-elasticity of money demand (η) is calibrated following Vallotta (2004).

For ε and θ, Galí's (2008) values are maintained, as well as the relevant period, which remains to be a quarter.

Table 1: Calibration of the model.

Parameter	β	σ	ϕ	α	ε	η	θ
Values	0.9708	5	1.455	0.38	6	3.2	2/3

The fact of departing from Galí's (2008) calibration has no straight influence in the main analysis of this paper, since the equilibrium dynamics are compared across different inflation expectations scenarios, but always keeping the same parameterization[6], i.e. the parameterization introduced in this section.

The autocorrelation of money growth rate shock (ρ_m) is calibrated according to Galí (2008), as well as the autocorrelation of the technology shock (ρ_a=0.9).

Table 2: Shock parameters.

Parameter	ρ_m	ρ_a
Values	0.5	0.9

Note that the existence of autocorrelation for the technology shock comes out of assuming that the technology parameter a_t follows an AR(1) process such that

$$a_t = \rho_a \, a_{t-1} + \varepsilon_t^a \, , \tag{15}$$

where $\rho_a \in [0, 1]$ denotes the persistence of the technology parameter across time and ε_t^a is a zero mean white noise process.

[6]Besides adding extra parameters when necessary to shape the mechanism for inflation expectations.

4 Alternative Mechanisms for Inflation Expectations

Before analyzing how the different expectation scenarios affect the model's equilibrium dynamics, let us introduce those alternatives and describe what assuming each means.

4.1 Extrapolative Expectations

An extrapolative behavior indicates that agents build their expectations of the values that relevant variables will take based on previous realizations of those variables.

Particularly, we will introduce perfect extrapolation for inflation. Accordingly, future inflation values will be estimated by agents to be equal to a pre-existent realization of it. Mathematically, perfect extrapolation of inflation expectations implies that, for those agents characterized by that extrapolative mechanism, $E_t\{\pi_{t+1}\} = \pi_{t-1}$.

4.1.1 Only Firms

For introducing extrapolation in the way firms estimate inflation, $E_t\{\pi_{t+1}\}$ is replaced by π_{t-1} just on the NKPC.

Correspondingly, the NKPC for the scenario with extrapolative firms yields

$$\pi_t = \beta\,\pi_{t-1} + \kappa\,\tilde{y}_t\,.$$

4.1.2 All Agents

In order to extend the extrapolative behavior to all the agents in the economy, besides keeping the modification done in the NKPC, the same replacement is done in the DIS equation.

Once $E_t\{\pi_{t+1}\} = \pi_{t-1}$ is applied to the DIS equation as well, it turns into

$$\tilde{y}_t = E_t\{\tilde{y}_{t+1}\} - \frac{1}{\sigma}(i_t - \pi_{t-1} - r_t^n)\,.$$

4.2 Adaptive Expectations

The adaptive expectations approach is characterized by the following equation describing $E_t\{\pi_{t+1}\}$:

$$E_t\{\pi_{t+1}\} = E_{t-1}\{\pi_t\} + ae\,(\pi_t - E_{t-1}\{\pi_t\}).$$

The intuition behind is that agents following this mechanism extrapolate their previous estimation of inflation, through the first term, but also account for the error reported by that previous estimation, through the second one.

Universidad de San Andres

Thus, adaptive expectations are a learning process on which ae[7] stands for the learning or adaptive coefficient, i.e. how much do agents consider their latest estimation error in their new estimation of π. It can be said that agents will to improve their estimation from an accuracy perspective and learning from their previous miscalculations is the way they pursue this goal.

4.2.1 Only Firms

For introducing adaptive expectations in the way firms estimate inflation $E_t\{\pi_{t+1}\}$ is replaced by $E_{t-1}\{\pi_t\} + ae\,(\pi_t - E_{t-1}\{\pi_t\})$ only in the NKPC.

Consequently, the NKPC for the scenario with adaptive firms yields

$$\pi_t = \beta\,[\,E_{t-1}\{\pi_t\} + ae\,(\pi_t - E_{t-1}\{\pi_t\})] + \kappa\,\tilde{y}_t\,.$$

4.2.2 All Agents

In order to extend the adaptive behavior to all the agents in the economy, besides keeping the modification done in the NKPC, the same replacement is done in the DIS equation.

Once $E_{t-1}\{\pi_t\} + ae\,(\pi_t - E_{t-1}\{\pi_t\})$ is applied to the DIS equation as well, it turns into

$$\tilde{y}_t = E_t\{\tilde{y}_{t+1}\} - \frac{1}{\sigma}(i_t - [E_{t-1}\{\pi_t\} + ae\,(\pi_t - E_{t-1}\{\pi_t\})] - r_t^n)\,.$$

[7] ae was estimated through OLS to be 0.4 using Argentina quarterly data for expected and actual inflation from 2005 to 2018.

5 Equilibrium Dynamics

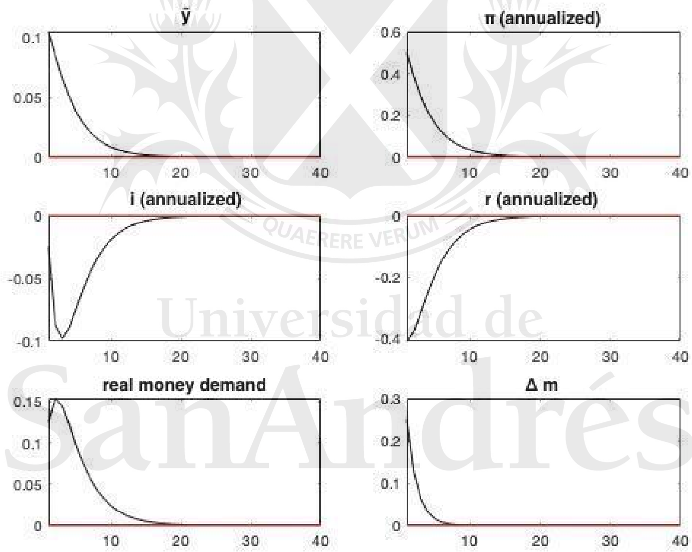
5.1 The Effects of a Monetary Policy Shock

5.1.1 Rational Expectations

Let us go back to the original version of the model, introduced in *Section 2*, and, particularly, consider the alternative in which the central bank runs monetary policy through a money growth rule, as the one from *Section 2.4.4.2*, calibrated accordingly to *Section 3*.

In such model, a positive realization of 25 basis points for ε_t^m (which is equal to a 1 percentage point increase in the annualized rate of money growth), generates responses from the relevant variables as shown in *Figure 1*.

Figure 1: Effects of a Monetary Policy Shock with Rational Agents



It can be seen in *Figure 1* that the increase in the money supply drives a rise in real money demand. The existence of sluggishness in price adjustment in the model causes this, as it avoids the monetary shock to be totally absorbed by inflation. However, from *Equation* (14) we know that real balances rise in our model.

In order for the money market to reach clearing under this increased money demand, output reacts positively and a reduction in the nominal rate takes place. The latter liquidity effect emerges particularly under our calibration since we are setting $\sigma = 5$. Calibrating σ with a lower value, as on Galí (2008) where $\sigma = 1$, prevents the liquidity effect from emerging and, instead, a higher jump in output compensates to clear money market.

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A difference equation for i_t in which the role played by σ on determining i_t is clearly visible can be derived from combining *Equation* (4) with the dynamic IS equation from *Equation* (7).

$$i_t = \frac{\eta}{1+\eta}\, E_t\{i_{t+1}\} + \frac{\rho_m}{1+\eta}\, \Delta\, m_t + \frac{\sigma - 1}{1+\eta}\, E_t\{\Delta\, y_{t+1}\} \tag{16}$$

As $\Delta\, y_{t+1}$ is expected to be negative as part of the process through which the output comes back to its steady state level, when σ is calibrated above one a liquidity effect can emerge, as it is the case with the calibration proposed in this work.

The combination of lower nominal interest rate and rising inflation leads to a decrease in real interest rate, which acts expanding the output gap as per *Equation* (7). Moreover, that output expansion triggers a persistence rise in inflation as denoted by NKPC *Equation* (13).

5.1.2 Extrapolative Expectations

5.1.2.1 Only Firms

Let us now review how the impact caused by the same monetary shock mutates when firms form their inflation expectations through an extrapolative process. For doing so, $E_t\{\pi_{t+1}\}$ is replaced by π_{t-1} only in *Equation* (13), as explained during *Section 4.1.1*.

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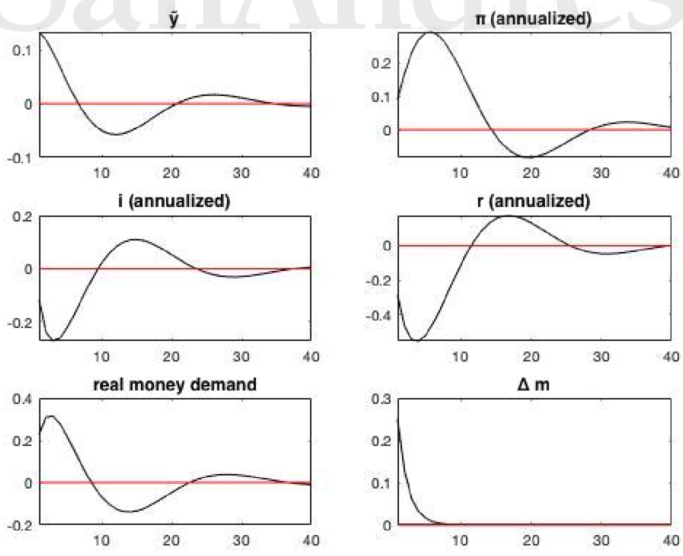
Figure 2: Effects of a Monetary Policy Shock with Extrapolative Firms



Introducing extrapolation has consequences both on the response to the shock and on the path that each variable follows back to its steady state value.

In the first place, it can be noticed in *Figure 2* that due to introducing extrapolation in the NKPC, π_t increases less right after the shock than it did in the rational scenario as π_{t-1}, which now directly determines $E_t(\pi_{t+1})$ and thus indirectly π_t, is equal to zero when the shock occurs. Extrapolating that zero inflation observation drags π_t down.

This lower inflation, always when compared with the case under rational expectations, causes real balances to initially rise even more in response to the monetary shock. The money market equilibrium in this context is accomplished thanks to lower nominal interest rate.

That lower nominal interest rate is not enough to totally offset the lower inflation caused by extrapolation. Hence, the decrease in the real interest rate that takes place right after the shock is now lower. In consequence, r_t is closer to r_t^n than it was in the rational scenario, which pushes output up through the DIS equation. Also, that higher output gap generates some inflation through the second term of the NKPC.

Regarding the path described by each variable on its way back to the equilibrium, there are two main deviations from what occurred in the rational environment: *1)* π rises (and r decreases as a result) during the immediate post-shock quarters and *2)* variables do not have a straight way back to the steady state. Instead, all of them but Δ_m, which follows the same AR(1) process as before, oscillate around the x-axes before reaching the steady state. This graphical note means that, even though the shock had an initial impact making variables to get higher/lower than their steady state values, then, as part of their way back to the equilibrium, all variables reach, respectively, lower/higher levels versus their steady state values.

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This dynamic is similar to that observed by Sturzenegger (1989). The author decomposed Argentinian GDP into permanent (demand) and transitory (supply) shocks, following Blanchard-Quah (1989). One of the outputs from his work is that an oscillating pattern is described by the GPD impulse-response function to a transitory demand shock as well as to a permanent supply shock. The significant and negative first-order serial correlation estimated for δy in that model was the reason explaining that oscillating post-shock behavior from GDP, according to the author.

In our case, the oscillation described and the initial rise in π (and decrease in r) mentioned, are caused by introducing extrapolation in the NKPC. So, let us interpret how it operates.

As commented earlier, right after the shock the first term of the NKPC with extrapolation, $\pi_t = \beta\, \pi_{t-1} + \kappa\, \tilde{y}_t$, is equal to zero since in $t-1$ the steady state held and that implies no inflation. Nonetheless, π_t is positive right after the shock as a response to the positive output gap produced by the referred shock. Accordingly, during the immediate post-shock quarters the first term of the NKPC, π_t is positive and actually higher each time until it reaches its maximum, specifically, when the second term becomes zero as y reaches its steady state value, y^n.

At that point, π starts decreasing and r continues growing as part of their way back to the steady state. This increase in r drags y to values even lower than y^n, impacting π negatively through the second term of the NKPC. Therefore, π double decreasing forces (extrapolation of lower values each time through first term and lower $\tilde{y}$ through the second) lead to negative inflation, consequently decreasing r which recovers y. However, the phase shift caused by basically lagging π_t in the expectations mechanism for firms continues operating and the equilibrium won't be reached yet. The same process is repeated, closer to the steady state each time, until the equilibrium is reached.

In sum, the presence of extrapolation through firms lagging π two periods as per $E_t\{\pi_{t+1}\} \equiv \pi_{t-1}$, extends the impact of the monetary shock, increasing the periods needed by the economy to get back to its steady state, and also makes that path more oscillating as inter-temporal lack of coordination is added by the lagging process ruling firms' inflation expectations.

This higher volatility generally reported by the variables is confirmed by higher standard deviations read in current scenario, versus the rational expectations case (see *Table 3* below).

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Table 3: Exogenous Money Supply - Monetary Shock - Volatility for relevant variables (Standard Deviation)

Variable Name	Rational Expectations	Extrapolative Firms	Extrapolative All Agents	Adaptive Firms	Adaptive All Agents
$\tilde{y}$	0.1685	0.2681	0.2825	0.1032	0.1197
π	0.7642	0.7945	0.8756	0.9460	1.2949
i	0.1976	0.6350	0.7234	0.1824	0.1822
r	0.7453	1.3664	1.3154	0.9536	0.8066

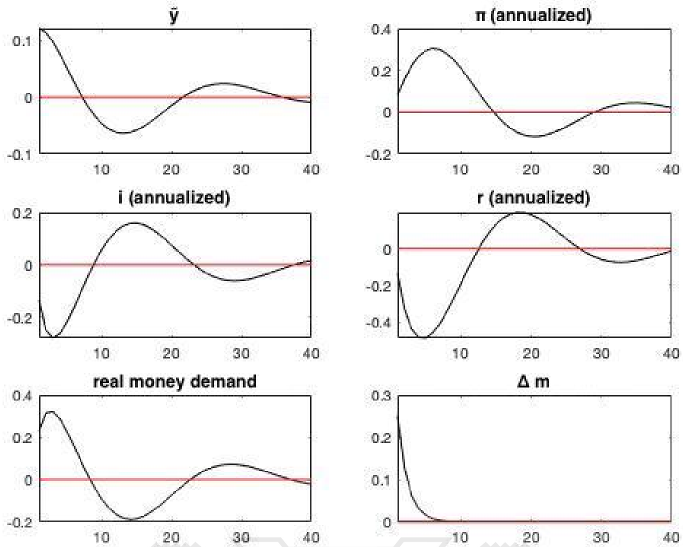
5.1.2.2 All Agents

As depicted by *Table 3*, extending extrapolation to all the agents in the model increases significantly the volatility reported by π, i and $\tilde{y}$.

This is reflected in wider shapes described by the impulse-response functions from *Figure 3*, when strictly compared to those from *Figure 2*[8].

[8]See *Section 8.2* at the Appendix for a Matrix comparing all IRFs.

Figure 3: Effects of a Monetary Policy Shock with Extrapolative Agents



Besides those slightly wider shapes, the path described by the variables during their way back to the steady state remain similar and the analysis done in *Section 5.1.2.1* holds for this case as well. Nevertheless, it is interesting to highlight that adding extrapolation in the way households build their inflation expectations, through the DIS equation, does not modify the equilibrium dynamics significantly.

Consistently with this, introducing extrapolation only on household's side (i.e. only through the DIS equation) generates a similar dynamic than the one described by the rational expectations scenario, as can be seen in *Figure 4*.

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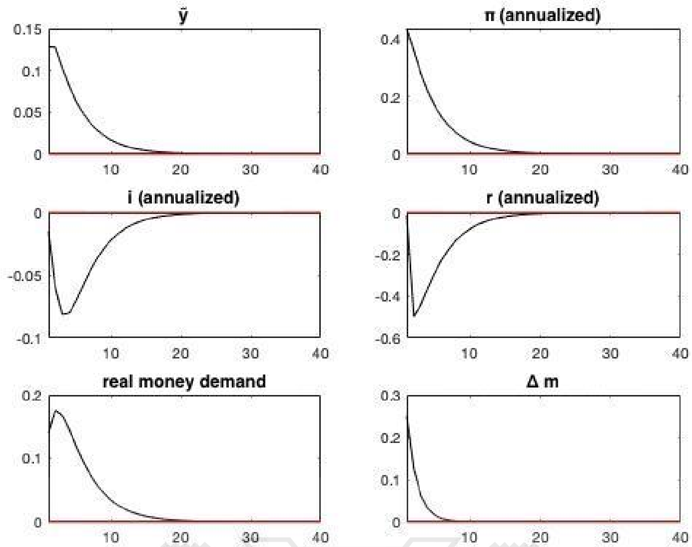
Figure 4: Effects of a Monetary Policy Shock with Extrapolative Households



5.1.3 Adaptive Expectations

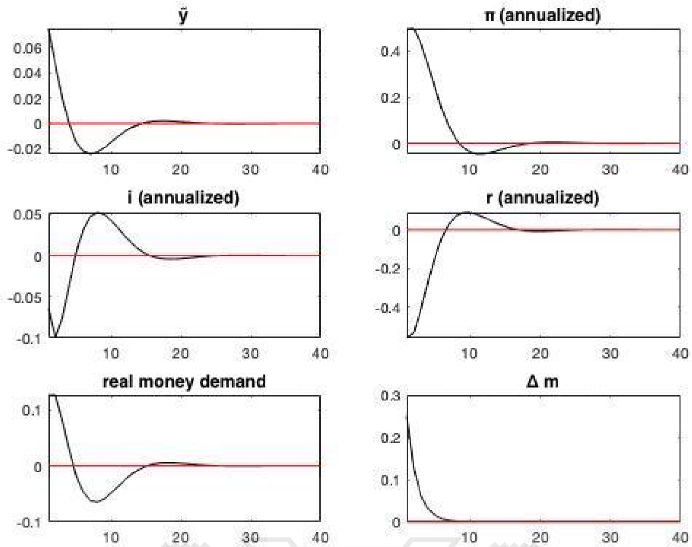
5.1.3.1 Only Firms

Incorporating learning in the NKPC by defining $E_t\{\pi_{t+1}\}$ as described in *Section 4.2* has some remarkable implications that is worth to discuss in this section.

First of all, the initial impact that the shock has on each variable when firms build their inflation expectations through an adaptive process is closer to the one seen for the rational case than to that from the extrapolative scenario.

Adaptive expectations lead to this closer-to-rational initial response to the monetary shock, mainly because now the first term of the NKPC is no longer $\left[\pi_{t-1}\right]$, equal to zero when the shock occurs, but $\left[E_{t-1}\{\pi_t\} + ae\left(\pi_t - E_{t-1}\{\pi_t\}\right)\right]$ instead. This way, inflation rises higher than it did in the extrapolative scenario, absorbing a higher proportion of the increase in Δ_m. Therefore, the positive response from real balances is shortened and the output has to increase less while the nominal rate has to shorter its decrease in order to reach clearing in the money market. The lower drop for the nominal rate, together with a higher estimation of inflation, causes a higher fall in the real interest rate that limits output's positive reaction through the second term of the DIS equation.

Figure 5: Effects of a Monetary Policy Shock with Adaptive Firms



The initial effectiveness of a monetary shock is stronger in the case of extrapolative expectations. The intuition for this rests on the fact that those perfectly backward looking agents who simply extrapolate the previous read of inflation (once again, equal to zero before the shock took place) are unable to react to the shock by modifying their inflation expectations. That is not the case for the rational or adaptive ones, as their $E_t\{\pi_{t+1}\}$ is not necessarily equal to zero when the shock occurs. Indeed, under our calibration the initial impact is quantitatively similar for these last two cases, as mentioned earlier.

The second implication of including learning to the NKPC is that, in their way back to the equilibrium, variables describe an oscillating path similar to the one from the extrapolative scenario.

When adaptive expectations are introduced, previous quarter estimation of inflation for current quarter is lagged and partially fixed according to actual inflation, i.e. according to how far it was from actual inflation. That is how, through the second term of $E_t\{\pi_{t+1}\}$, a learning process develops.3

As that learning process uses previous estimations partially fixed according to current inflation, it reaches an over-adjustment of beliefs at some point. Thus, this time as well (but due to over-adjusting rather than by simply extrapolating old observations that no longer apply), an oscillating path around the equilibrium is described by inflation and, hence, by the relevant variables depending directly or indirectly of it.

Nonetheless, it needs to be underlined that the steady state equilibrium is reached much earlier under this setting than it was in the extrapolative case. Actually, under our calibration the economy is back in its steady state after a similar number of quarters

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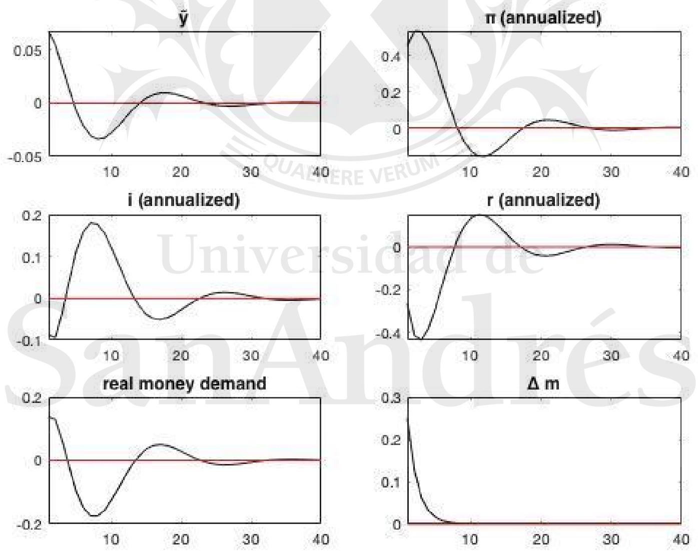
to the ones it took to it doing so in the rational scenario.

This shorter persistence of the shock, together with the minor departure from the steady state it initially causes, is translated into lower volatility for $\tilde{y}$ and i, compared to the extrapolative expectations case. A faster path back to the equilibrium is enough for r to also report a lower volatility in the adaptive scenario than it did in the extrapolative one, despite it initially departs more from the steady state now. Only π, which deviation from the steady state right after the shock was much weaker under extrapolation than it is under adaptive expectations, probed to be even more volatile this round.

5.1.3.2 All Agents

For adaptive expectations, when the learning procedure is introduced in the households' side as well, through the DIS equation, $\tilde{y}$ reported volatility increases, pushing π volatility up through the second term of the NKPC.

Figure 6: Effects of a Monetary Policy Shock with Adaptive Agents



The higher volatility of π and $\tilde{y}$ is reflected on more oscillating paths described by the relevant variables. As a result, a longer amount of quarters is needed for the economy to stabilize back at the zero inflation and natural output steady state equilibrium, compared to previous section's configuration.

Besides the extra volatility and longer persistence of the shock, both associated with the coordination issues linked with the fact of extending the learning process to all the agents in the economy, the equilibrium dynamics are not substantially modified by this assumption, always comparing the results to the case in which only firms behave adaptively.

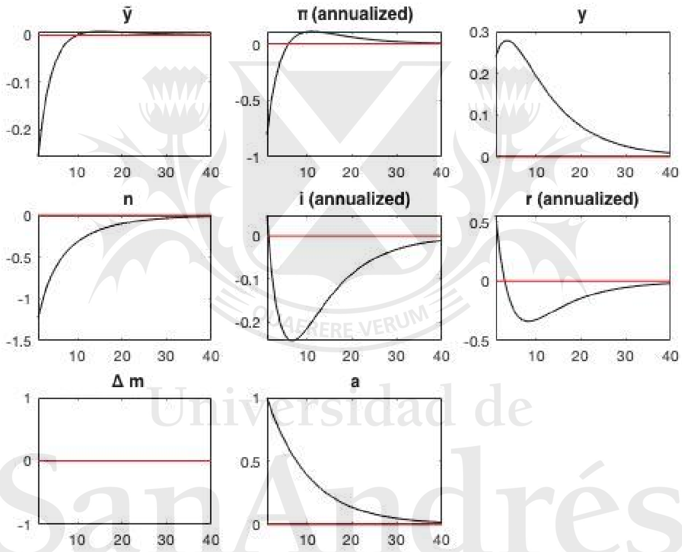
5.2 The Effects of a Technology Shock

5.2.1 Rational Expectations

In order to go through analyzing how our BNKM economy reacts when a unit shock to technology occurs in it, the exogenous money supply assumption will be held. Particularly, and in line with Galí (2008), $\Delta\, m_t$ is assumed to be equal to zero for all t.

Figure 7 shows how the increase in a_t has a positive impact onto the economy's output. However, from *Equation* (11)'s implications we know that the natural level of output also raises when a_t grows, as is the case once the shock has happened.

Figure 7: Effects of a Technology Shock with Rational Agents



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Furthermore, the increase reported by y^n right after the shock is higher than that for y. Therefore, $\tilde{y}$ gets negatively impacted by the shock overall, and so is inflation, consequently. Notice that this is the case even under our calibration, in which setting $\sigma = 5$ limits the increase seen in y^n, compared to calibrating $\sigma = 1$ as in Galí (2008).

However, setting the log utility equal to five has stronger implications when it comes to the nominal interest rate's reaction to the shock, through the third term of *Equation* (16). As can be seen when looking to the impulse response function for the nominal rate, by the time the shock happens the nominal rate increases just slightly. This still leads to an overall increase in the real interest rate, as inflation decreases. That increase in the interest rate is highly contractionary, as the natural real rate for the economy falls, even more than in the original calibration, as a response to the technology shock in order to support the transitory increase in output and consumption.

Regarding the path described by the variables on their way back to the equilibrium, it is clear that, as soon as a decreases over time so does y_n, which reduction closes the gap between natural and actual output. Increasing $\tilde{y}$ is followed by increasing π, as we know from the NKPC.

Focusing on the output, initially it continues to increase while the positive impact from employment recovering (driven, once again, by a decreasing) is higher than the negative straight impact that the reduction of a across quarters has on output. At some point before the fifth quarter, that trend shifts and the first term from *Equation* (8), the one accounting for technology's impact in output is heavier than the second one (i.e. the one accounting for employment's impact in output) and, hence, the output starts decreasing coming back to its steady state level.

Finally, on its way back to the equilibrium, the nominal interest rate describes two stages. First, it declines driven by *Equation* (16)'s third term since $E_t\{\Delta\, y_{t+1}\}$ is more negative each time during the initial quarters, as the described growth in output magnifies the future reduction expected on it. Then, once y starts decreasing and reaches its equilibrium level, the nominal interest rate's equilibrium path starts its second stage, during which it recovers until it reaches the steady state.

The dynamic described in the later paragraph is also followed by the real interest rate. First, both forces defining r push it down (i.e. less negative inflation and lower nominal interest rate) and then, when π is already close to zero, r starts recovering ruled by the increase in i and, ergo, shrinking y.

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5.2.2 Extrapolative Expectations

5.2.2.1 Only Firms

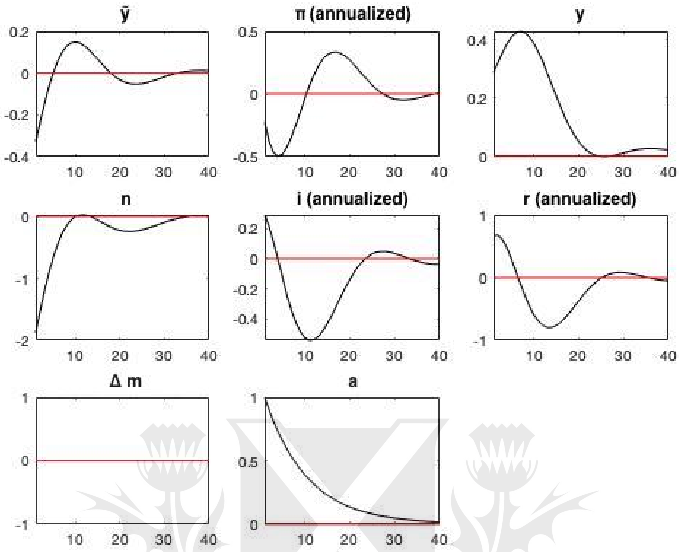
Let us take a look at how the same technology shock impacts our model when extrapolation is introduced for the way firms do their estimation of inflation.

The straight impact that the shock initially has on y and y^n, and correspondingly on $\tilde{y}$ and r gets just barely affected, as the modification introduced does not impact directly those equations defining them neither the values they need to reach in order for the economy to be in equilibrium post-shock.

Nonetheless, setting $E_t\{\,\pi_{t+1}\}$ equal to π_{t-1} has a clear impact on how inflation reacts to the shock. Particularly, as inertia from $t-1$ (when inflation was equal to zero) is now introduced, inflation reacts less to the shock this time. In other words, the first term of the NKPC limits how negative π becomes right after the shock in this extrapolative scenario, than it did under rational expectations.

Considering that the value of r required for reaching the economy's equilibrium post-shock is flat versus the rational scenario, the nominal interest rate's response to the technology shock also differs under current specification since with the lower negative reaction from inflation, a higher nominal rate is needed to reach a similar post-shock real interest rate.

Figure 8: Effects of a Technology Shock with Extrapolative Firms



Regarding the path followed by the variables during their way back to the equilibrium, for π is worth to highlight that, opposite to what happened in the rational scenario, during the first five post-shock quarters it keeps decreasing. This is explained by the fact that each previous realization of negative inflation impacts π now through the first term of the NKPC, joining those negative realizations of $\tilde{y}$ impacting through the second term of it. Right after the fifth quarter, however, inflation starts a growing path, driven by positive realizations of $\tilde{y}$ as the output reaches its maximum by then.

Also starting right after the fifth quarter, and longing until the twentieth, emerges another difference between current configuration's and rational scenario's equilibrium dynamics. Particularly, positive realizations of $\tilde{y}$ take place this time within that period as, once the effect of the technology shock is already fading away, the decreasing real interest rate drives output up. This logic does not apply to y^n as the decreasing a is the only fact impacting it. Moreover, during those quarters $\tilde{y}$ pushes π up, through the second term of the NKPC.

It is also worth to emphasize that, similarly to the case in which a monetary policy shock took place in the economy as in *Section 5.1.2*, when extrapolation leads the way firms set their inflation expectations, most of those variables affected either by π or by $E_t(\pi_{t+1})$ describe oscillating paths around the steady state before stabilizing again around it. These is, again, explained by introducing extrapolation. Extrapolating inflation acceleration (deceleration) perceived in previous quarter, leads at some point to an overshot of the expected increase (decrease) in future inflation that impacts actual inflation through the first terms of the NKPC and, thus, all variables defined by inflation.

Table 4: Exogenous Money Supply - Technology Shock - Volatility for relevant variables (Standard Deviation)

Variable Name	Rational Expectations	Extrapolative Firms	Extrapolative All Agents	Adaptive Firms	Adaptive All Agents
$\tilde{y}$	0.3573	0.5884	0.6340	0.2761	0.3437
π	1.0729	1.4688	1.7011	2.2070	2.9352
i	0.8000	1.6378	1.9598	1.6038	2.3445
r	1.2584	2.6273	2.5567	2.7342	2.6716

Last but not least, when firms in the model have an extrapolative behavior regarding their inflation expectations, the model becomes more volatile (see *Table 4*). When simulating a technology shock, all the relevant variables with no exceptions report a higher volatility versus the rational scenario, same as we saw when analyzing the consequences of a monetary shock in *Section 5.1*.

5.2.2.2 All Agents

As depicted by *Table 4*, extending extrapolation to all the agents in the model increases significantly the volatility reported by π, i and $\tilde{y}$.

That implication, consistent with the results reported when simulating a monetary shock under the same circumstances, is the main consequence of extending the extrapolative behavior to all the economy's agents. As can be concluded from comparing *Figure 9* with *Figure 8*, both the initial impact and the path described by the variables in their way back to the equilibrium report no major modifications, despite the wider shapes associated with the higher volatility.

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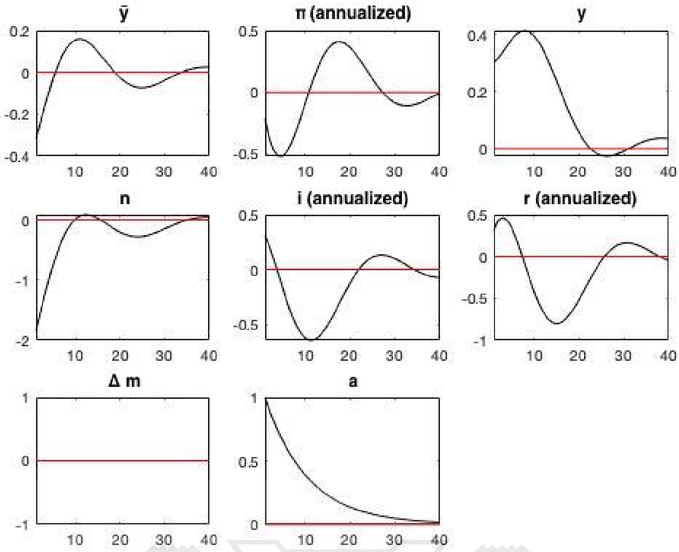
Figure 9: Effects of a Technology Shock with Extrapolative Agents



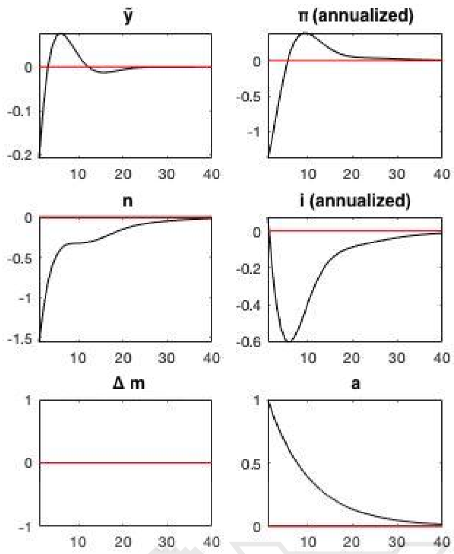
5.2.3 Adaptive Expectations

5.2.3.1 Only Firms

Similar characteristics to those mentioned in *Section 5.1.3.1* can be attributed to the comparison between a technology shock's effect under adaptive expectations and its effect under both rational and extrapolative expectations.

Now, as well as then, the shock's initial impact is quantitatively closer to that from the rational scenario and the variables approach back their steady state much faster than they did when extrapolation was the rule used by firms to set their inflation expectations.

Figure 10: Effects of a Technology Shock with Adaptive Firms



Nonetheless, two particular notes should be addressed for the case of a technology shock taking place in an economy with adaptive firms.

The first note is that while extrapolation had modified significantly the paths described by π, i and r in their way back to the equilibrium, now not only the initial impact that the shock has on them is closer to the one under rationality, as has already been mentioned, but also their equilibrium paths resemble those from the rational version of the model.

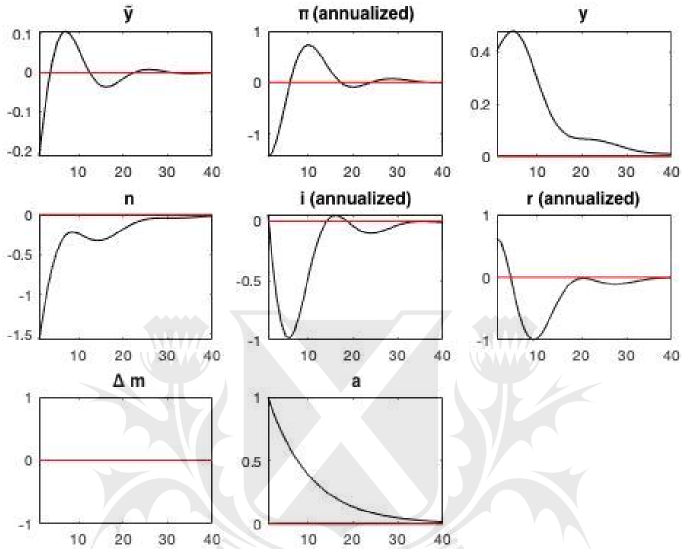
The second one is that the oscillating paths described by variables before stabilizing are not so remarkable now, in comparison with those from the extrapolative case. This is linked with the shorter it takes now for the variables to reach back the steady state, always comparing with the scenario ruled by extrapolation. The learning process is both a source of higher convergence to the steady state (versus the extrapolative process) and a source of higher volatility (versus the rational process). However, only π and r have more pendulous IRFs in this context than they did in the rational version of the model.

Precisely, inflation and real interest rate are the only two variables reporting now their highest volatility among all the analyzed expectations setting for the case in which a technology shock is simulated (see *Table 4*). This higher inflation volatility for the model with adaptive firms is in line to that seen when simulating a monetary shock. On the same page is the lowest volatility reported by the output gap under adaptive firms across both shocks analysis.

5.2.3.2 All Agents

Extending the learning process to all the agents in the economy also has analogous implications in the equilibrium dynamics analysis of a technology shock than it had for the monetary shock.

Figure 11: Effects of a Technology Shock with Adaptive Agents



In particular, and always comparing to the scenario in which only the firms behave adaptively, $\tilde{y}$ reported volatility gets increased here as well, pushing π volatility up (see *Table 4*). Here again, besides this fact, which is reflected on more oscillating paths described by the relevant variables and a longer amount of quarters needed for the economy to stabilize back at the zero inflation equilibrium, the equilibrium dynamics are not radically modified by extending the adaptive assumption.

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6 Conclusion

Along this work we have gone through five different model configurations, analyzing how the impact from both monetary and technology stochastic shocks differ across them. It is time now to recap the insights that came out of that analysis. Some are general features that apply to both shocks. However, specific characteristics related to each shock have also raised and are worth to mention at this stage.

Among those points that are common to both shocks equilibrium dynamics analysis let us start by the fact that when extrapolation or learning are introduced in the process through which firms and households set their inflation expectations, the relevant variables' ways back to the equilibrium become oscillating paths around the steady state. The logic explaining this is that the backward-looking behavior related to both mechanisms leads to an overshot of expectations as previous realization (expectation, in the adaptive case) is totally (partially) indexed in the agents' mind and, consequently, an overshot of those relevant variables that depend, at least partially, on inflation.

Specially in the case of extrapolative expectations, this overshot (relative to the zero inflation steady state) occurs more than once and the process through which variables come back to the equilibrium turns iterative. In the case of adaptive expectations, that oscillation is lower as the backward-looking behavior[9] is softened by the learning or adaptive term, which thus contributes to the fact that variables reach their steady state faster in the adaptive scenarios than they do in the extrapolative ones. Also, the initial impact that the shock has on the economy is closer to that from the rational scenario under adaptive expectations than it is under extrapolative, as in the latter case inflation's reaction to the shock is restricted by the first term of the NKPC being equal to π_{t-1}.

Despite oscillation in the equilibrium dynamics emerges under adaptive expectations for both shocks, a particular note on the path described by π and i (and hence r) in the case of a technology shock must be done. For those variables, the equilibrium dynamics in that case resembles the one they described in the rational scenario, rather than the one from the extrapolative case.

Another fact that can be concluded from analyzing the equilibrium dynamics through our five model configurations for both shocks is that modifying the way in which all the agents in the economy form their inflation expectations does not have major implications than doing so only on firms' side. For the extrapolative and adaptive scenarios, introducing the modification only in the NKPC and, therefore, just on firm's side, had mostly the same consequences as extending it to all the agents in the economy through modifying $E_t(\pi_{t+1})$ in the DIS equation as well. This is related to two facts: first, firms, although they cannot modify their prices every period, are price setters in the BNKM

[9] Besides being based on previous round expectation rather than on a previous actual realization.

we worked with; secondly, holding the rational assumption on household's side implies that households are aware of how the price setters firms build their inflation expectations (i.e. households already internalize the impact of the modification introduced on firms' side).

The last feature common to both shocks that is worth to remark is that π volatility increases consistently across our five scenarios, sorted as we have introduced them. In other words, setting the model such that inflation expectations are extrapolative increases the volatility reported by inflation when either of the shocks is simulated, versus the one it reported under rational expectations. Furthermore, π reported volatility increases even more when an adaptive behavior rules the way inflation expectations are built. Also, extending each behavior to all the agents in the economy increases π volatility within the scenario, compared to the case in which each assumption is introduced only on firms' side, as can be seen in *Table 5* below.

Table 5: π Volatility (Std. Dev.)

Simulated Shock	Rational Expectations	Extrapolative Firms	Extrapolative Agents	Adaptive Firms	Adaptive Agents
Monetary Shock	0.7642	0.7945	0.8756	0.9460	1.2949
Technology Shock	1.0729	1.4688	1.7011	2.2070	2.9352

When looking at the remaining relevant variables' volatility, different statements need to be done for each shock. The output gap, the nominal interest rate and the real interest rate behave differently across shocks and scenarios, in terms of their volatility[10].

For the technology shock, the same statement done for π holds for the three of them with two exceptions: volatility reported by $\tilde{y}$ is lower under adaptive expectations than in the rational scenario and r is more volatile when extrapolation or learning are introduced just on firm's side.

On monetary shock's side, the fact of extrapolative agents leading to higher volatility than rational agents holds but π is indeed the only variable for which introducing adaptive expectations represents a volatility increase versus the scenario ruled by extrapolation. Moreover, adaptive expectations do mean higher volatility than the original scenario just for π and r.

[10]See *Section 8.1* in the Appendix for a table summarizing the standard deviations reported by the relevant variables when simulating both shocks across scenarios.

7 References

Blanchard, O. J. and Quah, D. (1989). "The Dynamic Effets of Aggregate Demand and Supply Disturbances." *The American Economic Review*, Vol. 79, No. 4, pp. 655-673. Available at: *https : //uh.edu/ bsorense/BlanchardQuah1989.pdf*

Calvo, G. (1983). "Staggered Prices in a Utility-Maximizing Framework." *Journal of Monetary Economics*, Vol. 12, pp. 393-398.

D'amato, L. and Garegnani, M.L. (2009). "Short-Run Dynamics of Inflation: Estimating a Hybrid New-Keynesian Phillips Curve for Argentina (1993-2007)." *Banco Central de la República Argentina*, Ensayos Económicos Vol. 55, pp. 33-56.

Galí, J. (2008). "Monetary, Policy, Inflation, and the Business Cycle. An Introduction to the New Keynesian Framework." *Princeton University Press*, pp. 41-70.

Lyziak, T. (2016). "Do inflation expectations matter in a stylised New Keynesian model? The case of Poland." *Narodowy Bank Polski*, Working Paper No. 234.

Oviedo, J. M. (2017). "A Dynamic Stochastic General Equilibrium Model for Argentina. Business Cycle Analysis: 1993-2014." *Instituto de Economía y Finanzas, Facultad de Ciencias Económicas, Universidad de Córdoba.*

Santocono, S. (2019) "Financial Crises: An Approach From Expectations at Asset Markets." *Revista de Economía Política de Buenos Aires*, Vol. 18, pp. 117-154. Available at: *http : //ojs.econ.uba.ar/index.php/REPBA/article/view/*1574.

Sturzenegger, F. (2019) "Explicando las fluctuaciones del producto en la Argentina." *Económica*, Vol. 35, pp. 101-152. Available at: *https : //revistas.unlp.edu.ar/Economica/article/view/5*

Vallotta, A.P. (2004). "Estimaciones de la demanda de dinero en Argentina." *Universidad Nacional de Mar del Plata.* Available at: *http : //www.ricardopanza.com.ar/files/macro2/ Macro_II___24_Demanda_de_Dinero_en_Argentina.pdf*.

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8 Appendix

8.1 Standard Deviation for Relevant Variables

			Exogenous Money Supply	
			Monetary Shock	Technology Shock
		Variable	Std. Dev.	Std. Dev.
Rational Expectations		Output Gap	0.1685	0.3573
Rational Expectations		Inflation	0.7642	1.0729
Rational Expectations		Nominal Rate	0.1976	0.8000
Rational Expectations		Real Rate	0.7453	1.2584
Extrapolative	Only Firms	Output Gap	0.2681	0.5884
Extrapolative	Only Firms	Inflation	0.7945	1.4688
Extrapolative	Only Firms	Nominal Rate	0.6350	1.6378
Extrapolative	Only Firms	Real Rate	1.3664	2.6273
Extrapolative	All Agents	Output Gap	0.2825	0.6340
Extrapolative	All Agents	Inflation	0.8756	1.7011
Extrapolative	All Agents	Nominal Rate	0.7234	1.9598
Extrapolative	All Agents	Real Rate	1.3154	2.5567
Adaptive	Only Firms	Output Gap	0.1032	0.2761
Adaptive	Only Firms	Inflation	0.9460	2.2070
Adaptive	Only Firms	Nominal Rate	0.1824	1.6038
Adaptive	Only Firms	Real Rate	0.9536	2.7342
Adaptive	All Agents	Output Gap	0.1197	0.3437
Adaptive	All Agents	Inflation	1.2949	2.9352
Adaptive	All Agents	Nominal Rate	0.1822	2.3445
Adaptive	All Agents	Real Rate	0.8066	2.6716

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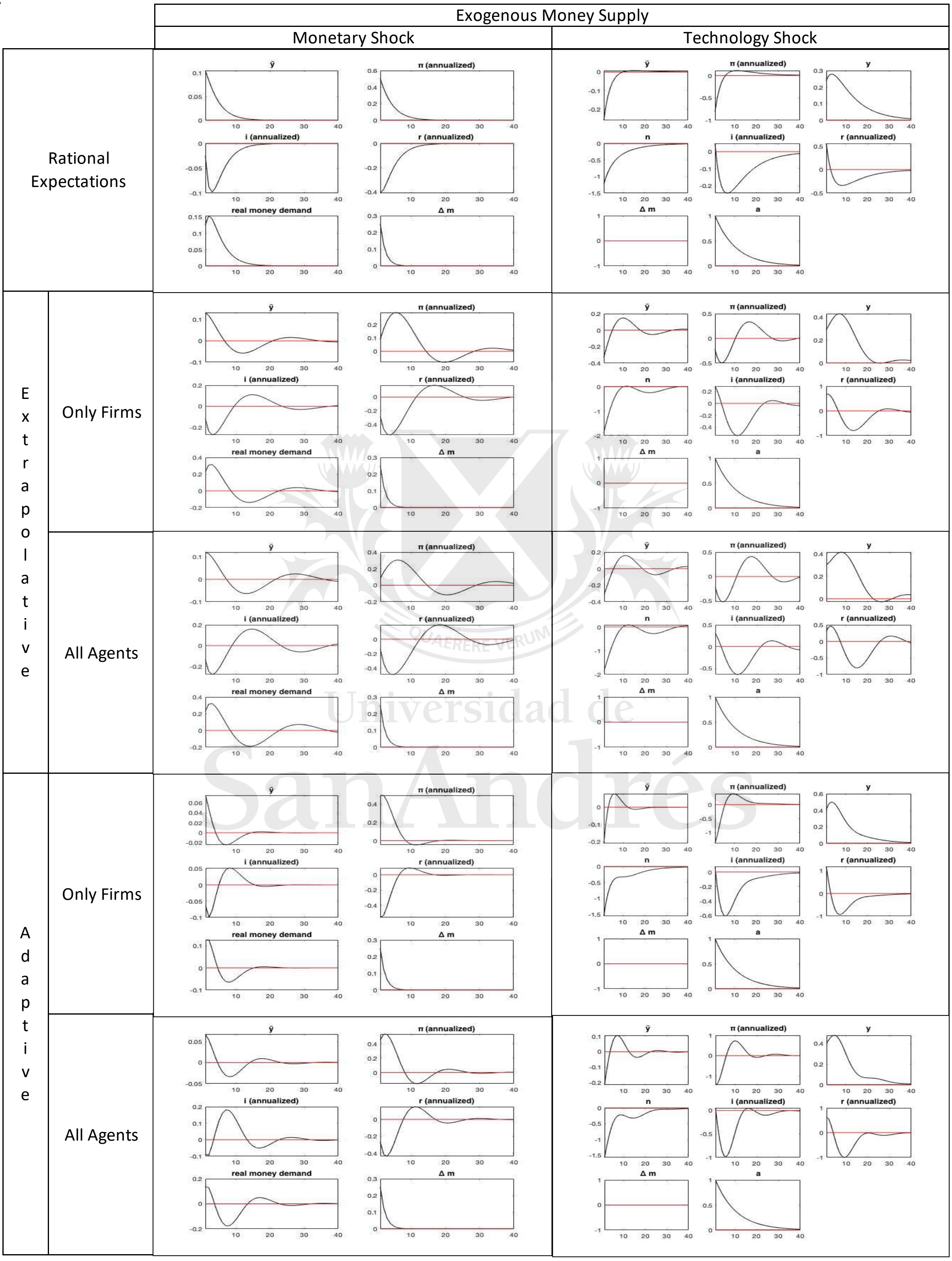
8.2 IRFs Matrix



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8.3 Equilibrium Dynamics under an Interest Rate Rule

Instead of a money growth rule, the central bank may run monetary policy under a simple interest rate rule such that:

$$i_t = \rho + \phi_\pi \, \pi_t + \phi_y \, \tilde{y}_t + \nu_t, \tag{17}$$

where ϕ_π and ϕ_y are non-negative coefficients defined by the monetary authority, ρ is the intercept which choice makes the rule consistent with a zero inflation steady state and ν_t is an exogenous component with zero mean that follows the following AR(1) such that $\nu_t = \rho_\nu \, \nu_{t-1} + \varepsilon_t^\nu$, in which $\rho_\nu \in [0, 1)$ denotes the persistence of ν and ε_t^ν stands for the stochastic shock on the nominal interest rate rule.

Thus, under an interest rate rule, the equilibrium for the BNKM is described by the NKPC from *Equation* (13), the DIS equation from *Equation* (7) and the Interest Rate Rule from *Equation* (16).

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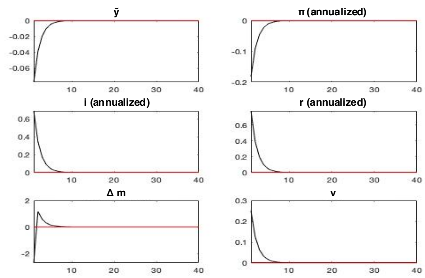
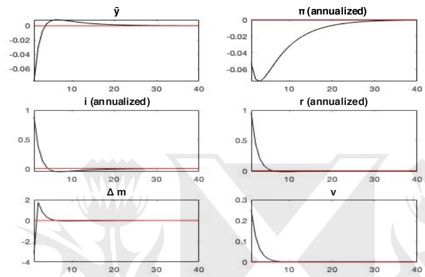
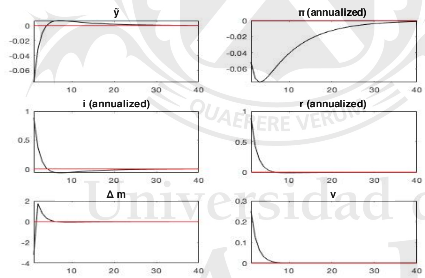
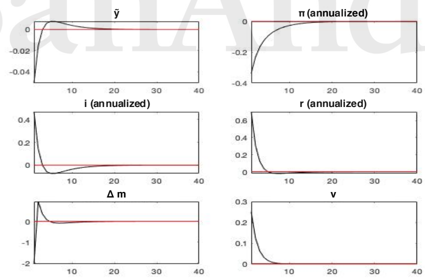
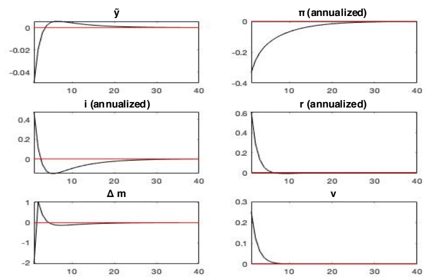
8.3.1 The Effects of a Monetary Policy Shock

		Interest Rate Rule – Monetary Shock: IRFs	Variable	Std. Dev.
Rational Expectations			Output Gap	0.0900
			Inflation	0.2099
			Nominal Rate	0.7984
			Real Rate	0.8998
Extrapolative	Only Firms		Output Gap	0.0863
			Inflation	0.1893
			Nominal Rate	0.9775
			Real Rate	1.0792
Extrapolative	All Agents		Output Gap	0.0847
			Inflation	0.2099
			Nominal Rate	0.9868
			Real Rate	1.0079
Adaptive	Only Firms		Output Gap	0.0559
			Inflation	0.4827
			Nominal Rate	0.5164
			Real Rate	0.7788
Adaptive	All Agents		Output Gap	0.0555
			Inflation	0.5669
			Nominal Rate	0.6235
			Real Rate	0.6924

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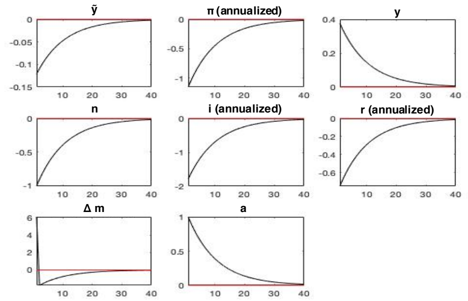
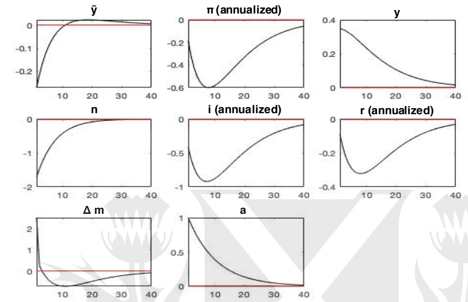
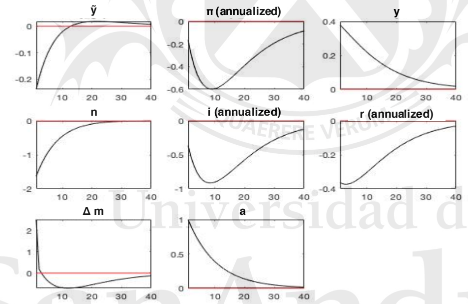
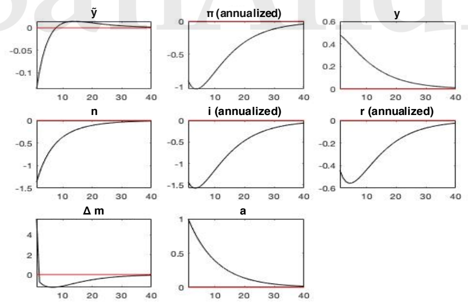
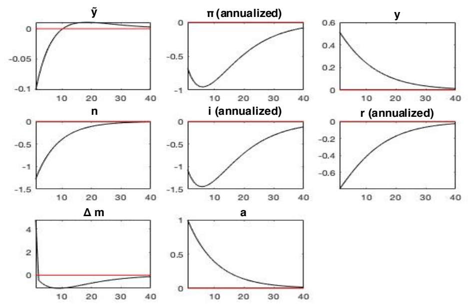
8.3.2 The Effects of a Technology Shock

		Interest Rate Rule		
		Technology Shock		
		IRFs	Variable	Std. Dev.
Rational Expectations			Output Gap	0.2782
			Inflation	2.6451
			Nominal Rate	4.1068
			Real Rate	1.7262
Extrapolative	Only Firms		Output Gap	0.4304
			Inflation	2.2984
			Nominal Rate	3.5157
			Real Rate	1.2299
	All Agents		Output Gap	0.4048
			Inflation	2.4340
			Nominal Rate	3.7186
			Real Rate	1.3753
Adaptive	Only Firms		Output Gap	0.1945
			Inflation	3.4562
			Nominal Rate	5.2253
			Real Rate	1.8964
	All Agents		Output Gap	0.1643
			Inflation	3.6646
			Nominal Rate	5.5291
			Real Rate	2.1405

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