Universidad de San Andrés

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Maestría en Economía

Time-varying risk premia forecasts and a trading algorithm

Autor: Nicolás Andrés Dennehy

DNI: 37552029

Mentora de Tesis: Magdalena Cornejo

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Nicolás Andrés Dennehy
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Mentor: Magdalena Cornejo

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Tesis de Maestría en Economía de

Nicolás Andrés Dennehy

"Primas de riesgo variantes en el tiempo y un algoritmo de inversión"

Resumen

La presente tesis analiza la siguiente pregunta: asumiendo que los factores de prima de riesgo predicen retornos de acciones, como lo hacen en el modelo de tres factores de Fama-French, y que dichos factores son variantes en el tiempo, ¿pueden estos factores ser pronosticados por modelos de series de tiempo? Además, si estos factores pueden ser pronosticados, ¿pueden dichas predicciones proveer información para pronosticar los retornos de las acciones? Usando series de tiempo mensuales de retornos y factores de riesgo desde 1926 a 2018, puede diseñarse un algoritmo de dicha naturaleza basado en un modelo VAR parsimonioso cuyas predicciones a su vez se utilizan en regresiones de Fama-French. Luego, el algoritmo elige el portafolio con el retorno esperado más alto. Los resultados muestran que el desempeño fuera de la muestra del algoritmo, aún luego de corregir por su volatilidad, es más alto que el promedio del mercado.

Palabras clave: *mercados financieros, factores de riesgo, algoritmo de inversión, eficiencia de mercado*

"Time-varying risk premia forecasts and a trading algorithm"

Abstract

This thesis analyzes the following question: assuming that risk premia factors predict returns on stocks, as specified in models such as the Fama-French three factor model, and that these premia are time-varying, can risk factors be forecasted through time series models? Furthermore, if these risk factors can be forecasted, can these predictions provide useful information to forecast returns? Using monthly portfolio return and the three factor time series ranging from 1926 to 2018, an iterative algorithm of such nature based on a parsimonious VAR model is set up to forecast factors, which are then inputted into Fama-French regressions. Then, the algorithm picks a long position into the portfolio with the highest expected return. Results show that the out-of-sample returns performance from the algorithm -even after correcting for its volatility- is much higher than average market returns

in the long term, although the link becomes weaker in the short term.
Keywords: *financial markets, trading algorithm, risk premia, market efficiency*

Códigos JEL: G11, G14, G17

Abstract

This thesis analyzes the following question: assuming that risk premia factors predict returns on stocks, as specified in models such as the Fama-French three factor model, and that these premia are time-varying, can risk factors be forecasted through time series models? Furthermore, if these risk factors can be forecasted, can this predictions provide useful information to forecast *returns*? Using monthly portfolio return and the three factor time series ranging from 1926 to 2018, an iterative algorithm of such nature based on a parsimonious VAR model is set up to forecast factors, which are then inputted into Fama-French regressions. Then, the algorithm picks a long position into the portfolio with the highest expected return. Results show that the out-of-sample returns performance from the algorithm -even after correcting for its volatility- is much higher than average market returns in the long term, although the link becomes weaker in the short term.

I'm convinced that there is much inefficiency in the market. These Graham-and-Doddsville investors have successfully exploited gaps between price and value. When the price of a stock can be influenced by a "herd" on Wall Street with prices set at the margin by the most emotional person, or the greediest person, or the most depressed person, it is hard to argue that the market always prices rationally. In fact, market prices are frequently nonsensical.

Warren Buffett

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Chapter 1

Introduction

1.1 Motivation

A vast literature examines the question on which risk factors explain observed returns on financial assets. In fact, hundreds of risk premia for equity, bonds, interest rates, commodities and currencies have been identified in the past decades by economic literature and are actually exploited by investors in their respective markets to diversify their portfolios and maximize returns.

So far, however, the risk factor literature has focused on *what* investors explicitly or implicitly consider as risk measures on which to value financial assets and, therefore, their returns. Most famously, Fama and French (1992) and Carhart (1997) proposed three and four-factor specifications of cross-section return regressions, based merely on empirical observation. The factors found by these authors seem to have a relatively high explaining power on cross-sectional returns and have proved robust by empirical examination.

1.2 Contribution

If it is indeed true that certain risk factors explain cross-section returns -that is, certain risk premia contemporarily explain returns-, can these factors be *forecasted* so as to estimate, in some way, future returns? If they do explain returns at any moment t, and assuming there is some model to forecast how these risk premia will evolve in future periods, is it possible to actually predict future returns by exploiting risk premia forecasts? And furthermore, is it possible to pick a portfolio which delivers greater returns by tracking which risk premia will drive market returns in future periods?

This thesis contributes to this question in at least two ways. First, it argues that this forecasting power does indeed exist and can be examined through a recursive algorithm that iteratively predicts risk premia at each point of time. Although other models of this nature exist, this thesis demonstrates that a parsimonious VAR(1) model is enough to achieve large long term return differentials.

Second, and in contrast with other attempts to solve this question based on choosing between industry portfolios, this thesis makes use of 25 portfolios sorted on size and book-to-market ratios quintiles as classified by Professor Kenneth French. This ordering seems more natural, considering the fact that size and book-to-market ratios constitute Fama-French factors which drive cross-sectional returns according to their model. Although the relation becomes weaker and volatile in the short term, as will be seen, there appears to be a long term outperformance of portfolios with respect to the market average chosen by this algorithm.

The rest of this thesis is organized as follows. Chapter 2 reviews the literature on risk premia and cross sectional returns, upon which this thesis is based and which will guide the models specified in later sections. Chapter 3 discusses the data which is to be used in the algorithm set forward in Chapter 4, which discusses the main question on whether the three Fama and French factors, presumed to explain stock market returns, can effectively be forecasted to predict future stock returns. A VAR specification for the three factors time series is examined, as well as the cross-sectional return regressions to be performed by a prediction algorithm. Chapter 5 displays the results of this forecasting strategy, showing significantly higher returns than the average stock market portfolio in the long term, and finally Chapter 6 discusses the robustness and long-term permanence of these results.

Chapter 2

Literature review

This chapter summarizes the existing literature on risk premia and cross sectional returns. It is structured as follows: first, in section 2.1, the CAPM (Capital Asset Pricing Model) is introduced as a simplification of the modern portfolio theory. Section 2.2 explains how CAPM suffers from empirical shortcomings and describes further specifications -most specifically, the Fama-French three factor model- that try to overcome CAMP's difficulties. The Fama-French model will be the foundation for the following sections. Lastly, section 2.3 reviews shortly the efficient market hypothesis and its implications for this thesis.

2.1 CAPM

In equity markets, the first risk premium clearly identified by the literature is the market risk premium, measuring excess returns on equity relative to the risk free rate due to the volatile nature of stock markets. Sharpe (1964), Treynor (1961), Lintner (1965) and Mossin (1966) independently arrived to different variations of what is now known as the CAPM model based on this logic, building on the portfolio theory work of Markowitz (1952). The CAPM model builds on the assumption that the only risk factor perceived by investors in equity is the stock market return in excess over the risk-free rate, amplified by how sensitive individual stocks are to aggregate market returns -in other words, the determining factor on individual stock returns is their β. Under this model, stocks with higher βs are more sensitive to aggregate market movements, both upward and downward, thus under equilibrium requiring a higher return rate.

More specifically, suppose two kind of assets exist - a risk-free asset other assets which involve different degrees of risk. Under the assumption of no transaction costs, universal agreement on the distribution of returns and investors who hold the same risk-variance preferences, and unlimited risk-free borrowing and lending, all market agents will hold the same portfolio which will satisfy the following equation for each asset $i = 1, 2, ..., N$ (also called the Sharpe-Lintner CAPM equation):

$$\mathbb{E}(R_i) = R_f + [\mathbb{E}(R_M) - R_f]\beta_{iM} \tag{2.1}$$

where $\mathbb{E}(R_i)$ is the expected return for asset i, R_f is the risk-free rate at which there is unlimited lending and borrowing, $\mathbb{E}(R_M)$ is the expected equity market return and β_{iM} is the slope coefficient representing asset i's sensitivity to the equity market premium. Asset βs are, in fact, usually obtained in practice through this simple linear regression using 3-year or 5-year historical data.

Black (1972) further explores the possibility of a CAPM model without a riskless asset. Criticizing the assumption of no unlimited lending and borrowing at the risk-free rate, he proposes a CAPM assuming only that a portfolio z having a β as described above equal to zero is needed. In his version of the CAPM model, where no riskless rate borrowing or lending is available, the equation now becomes

$$\mathbb{E}(R_i) = \mathbb{E}(R_z) + [\mathbb{E}(R_M) - \mathbb{E}(R_z)]\beta_i \tag{2.2}$$

where z indexes any other portfolio with zero correlation to the market portfolio, that is, a zero-beta portfolio.

The approach of the CAPM model has been questioned and criticized fiercely by empirical examination, which has found other factors explaining returns with more power than only an equity risk premium modified by individual βs, and even complete failure to explain returns by the equity risk premium alone. First, it can be easily seen that βs can be wrongly measured and this might imply measurement errors. To arrive at more precise βs, subsequent studies such as Blume and Friend (1973) and Jensen et al. (1972) perform regressions based on portfolios sorted by β percentiles rather than individual stocks, thus reducing measurement errors. This has been the approach in later studies such as Fama and French (1992), on which this paper is based.

Equation (2.1) implies that the expected value of an asset's return, $\mathbb{E}(R_i - R_f)$ is completely explained by its risk premium and thus the intercept (α_i) in the equation

$$\mathbb{E}(R_i - R_f) = \alpha_i + [\mathbb{E}(R_M) - R_f]\beta_{iM} \tag{2.3}$$

is zero. This result has been disproved empirically in early tests of the CAPM model such as Douglas (1968), Jensen et al. (1972) and Blume and Friend (1973). These tests find that there is a positive relation between β and average returns, but the actual slopes are too "flat" to be compatible with CAPM; indeed, their estimations show that the intercept α_i is statistically higher than the risk-free rate - in other words, predicted returns for low beta portfolios are too high while the ones for high beta portfolios are too low.

2.2 Fama-French and the three factor model

The CAPM model took further hits in further decades, after other factors than market betas were found to considerably explain returns. Basu (1977) showed that there is an outperformance of low P/E ratio (Price to Earnings ratio, that is, the common stock price divided by earnings) common stocks compared to high P/E ratio stocks in terms of risk-adjusted returns. Banz (1981) also documents a considerable size effect; that is, he found that a CAPM specification with an added size independent variable resulted in a significant and high β for size. Also, when analyzing Japanese stocks, Chan et al. (1991) found a strong relation between book-to-market ratio (book equity divided by market capitalization) and their returns. This proved hard to reconcile with CAPM, where all portfolios are supposed to be mean-variance efficient - that is, variance among portfolio returns are to be explained only by market βs and other variables should not add any value to the expected returns.

Van Dijk (2011) debates whether the size premium, which proved persistent in the 1980's, is still valid today. There seems to be some evidence for a "fading out" of the size effect, although he argues that it is contradictory and subject to definitions of "small" and "big" portfolios. He finds further evidence of international size premiums, some amounting to very large spreads between small and big companies' returns.

Fama and French (1992) aggregate the literature's criticism of CAPM during the 1980's and early 1990's in what is now known as the Fama-French three factor model. More specifically, these three stock market risk factors are (a) the known CAPM market risk premium, measuring the excess return of stock equity over the US Treasury risk-free rate (b) the size premium, comprising the difference between the returns on a small and big-stock portfolios as found by Banz and (c) the book value to market value (market capitalization) ratio. Significant explaining power was found by the authors on cross-sectional returns for the period 1963-1990 when using the three factors; and most importantly, β alone did a poor job at explaining returns or did not explain them at all. Thus, the following equation reflects the three factor model:

$$R = R_f + \beta_m(R_m - R_f) + \beta_s SMB + \beta_v HML \tag{2.4}$$

where R is the return of stocks, R_f is the risk-free rate, R_m is the market return, and SMB and HML are the outperformances of small minus big portfolios and high book-to-market minus low book-to-market portfolios, respectively (section 3.1 describes this in further detail).

The same authors found in 1993 further risk premia for bond markets, including the term spread for government securities and default spread for corporate bonds. For matters of simplicity, this paper focuses on the Fama-French stock risk premia. A further expansion of this framework comes from Carhart (1997), who shows that a fourth factor, momentum -described as the recent tendency in returns for a certain stock-, can be added to explain mutual fund returns persistence.

The three-factor model by Fama and French has spurred discussion on whether an empirical association between risk premia and observed stock market returns should be acceptable as it is based on merely an observational correlation. Since the work of Fama and French (and Carhart), hundreds of risk factors have been identified and published. As Harvey et al. (2016) show, this includes many factors which have no clear economic explanation. Their research finds up to 300 factor identification papers in top journals since 1973, including financial, macroeconomic, accounting, behavioral and microstructure factors, with this literature expanding most rapidly in the 2000's.

The question of whether risk factors have any forecasting potential on themselves and on future returns arose naturally, although the existing literature is very limited. When applied to the problem of capital budgeting by corporations, Hu (2003) finds that short-term forecasts of Fama-French factors has significant explaining power on the cost of capital in the short term -that is, months in advance-, although this relation disappears in the long term, especially after a year and more abruptly after a couple of years. Regressing 17 industry portfolios on the three Fama-French factors and business cycle variables, he finds that it is indeed possible to forecast the factors and achieve higher returns by holding a long position in the highest expected return industry and shorting the one with the lowest expected return.

Panopoulou and Plastira (2014) set up an autorregresive distributed lag (ARDL) model to test whether there is forecasting ability for Fama-French factors altogether with momentum -as set in Carhart (1997)-, long-term reversal and short-term reversal. They find that there is considerable predictability in the factors, especially when enriching with business cycle variables such as the 1-month Treasury bill, term spread and default spread.

This paper argues for the use of the classical Fama and French three-factor model due to the fact that the size premium and the book-to-market ratio premium follow a microeconomic logic. Small companies, relative to large ones, are naturally riskier in nature due to their failure rate and volatility when compared to larger, mature companies. Book-to-market ratios show as well how "optimistic" investors with respect to future earnings. Research shows that also risk might be increasing in the book-to-market ratio. Donnelly (2014) shows that the price of low book-to-market ratio companies -that is, with a high market capitalization relative to its book value- is more sensitive to earnings disappointments than high book-to-market companies.

Petkova (2006) finds that the SMB and HML -big minus small and high book-to-market minus low book-to-market outperformances- are correlated with innovations in state variables that describe investment opportunities. Specifically, she finds that HML proxies for term spread surprises (innovations) in returns, and SMB proxies for a default spread factor. Thus, it could be argued that HML and SMB are enough to capture all relevant business cycle information. Furthermore, wide risk premia and portfolio data availability makes it easier to use the Fama-French factors, although the approach laid in this paper can naturally be easily expanded to more factors with due data.

2.3 The efficient market hypothesis (EMH)

As the following sections will explain, and as mentioned in the introduction, the main subject of this thesis is to test whether predictability of future returns is possible by using a time series model to forecast risk factors.

Fama (1965) famously posited in his groundbreaking dissertation that individual stock market prices follow a random walk, so that past values of any stock do not provide any useful information in forecasting future prices. In other words, the postulation that prices follow a random walk -in what was later known as the efficient market hypothesis (EMH)- implied that successive price changes were independent and identically distributed. This would imply that

$$\mathbb{E}(r_{j,t+1}|\phi_t) = \mathbb{E}(r_{j,t+1}) \tag{2.5}$$

where $\mathbb{E}(r_{j,t+1})$ is the expected return for any common stock j for period $t+1$ and ϕ_t is the set of all available information at period t. The key assumption in efficient market hypothesis is, then, that *all available information* is reflected on the price at each moment t, so that no piece of information can be exploited to achieve higher returns consistently in time more than the average market would.

Malkiel and Fama (1970) further classified the EMH into three forms, according to the strictness of their assumptions. The weak form is concerned with just the past history of prices; most of the evidence for this form of the EMH comes from proving that prices follow a random walk and that condition (2.5) is satisfied. The semi-strong form of the EMH implies that all public information released into the market -such as stock splits, earnings reports, new securities issues and so on- is quickly incorporated into the stock's price, thus it being impossible to earn any profit on public information. The strongest form of the hypothesis, the strong EMH, is concerned with whether even *private* information is reflected on prices or not. Most of the literature is focused on the weak and semi-strong forms of the EMH. Malkiel and Fama postulate that there is strong evidence in favor of the EMH through empirical examination.

Naturally, the EMH has been met with both much criticism and support. Behavioral economists such as Kahneman, Tversky, Thaler and Slovic have criticized the EMH, postulating on the contrary that prices are subject to human cognitive biases and are not rationally priced according to all available information. Empirical evidence is mixed, with some studies supporting EMH and some providing degrees of empirical refutations of the hypothesis.

Thus, according to the EMH it would be impossible to achieve higher returns than the market by exploiting past information to predict future returns. The rest of this thesis, as commented in the introduction, will investigate a trading algorithm with the purpose of finding whether higher returns are possible by forecasting the Fama-French risk factors, which would imply some kind of contradiction with the EMH.

Chapter 3

Data and inputs

This chapter describes the data that this thesis will exploit for estimating and testing the algorithm strategy set forward in chapter 4. First, section 3.1 describes the factor and portfolio return databases and the methodology used in their estimation. Sections 3.2 and 3.3 analyze and summarize the factors databases, and display the results of unit root tests for these series.

3.1 Databases

All databases used in this paper were collected from Professor Kenneth French's data library at Dartmouth College[1] and comprise the period starting in July 1926 and ending in July 2018, a time lapse of 92 years or 1105 observations.

The first database includes the three Fama-French factors as described in Fama and French (1993). To calculate the excess returns corresponding to each factor, six portfolios were constructed at the end of June of each year of the database based on the intersection of two portfolios based on size (market equity) and three portfolios based on value (BE/ME). The breakpoint for size portfolios is based on the median of NYSE market equity in June of each year. The value portfolios are built based on the 30th and 70th percentiles of NYSE BE/ME ratios. Thus, the intersection of both break points results in six portfolios as described in Table

[1] http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html

1. The market risk premium and both SMB and HML are constructed as follows.

- Equity risk premium (*Mkt.RF*): the excess return of the market calculated as the value-weighted return on all firms incorporated in the US and listed on the NYSE, AMEX or NASDAQ with existing share and price data at the beginning of month t minus the one-month Treasury bill rate.

- Size premium (*SMB*): Small Minus Big is the average excess return on the average of the three small portfolios compared to the average of the three big portfolios:

SMB = 1/3 (Small Value + Small Neutral + Small Growth) - 1/3 (Big Value + Big Neutral + Big Growth)

- High value minus low value (*HML*): excess return on the average of small value and small growth portfolios minus the average of big value and big growth portfolios

HML = 1/2 (Small Value + Small Growth) - 1/2 (Big Value + Big Growth)

Table 3.1: Six portfolios by size and value ratio

	Small ME	Big ME
70+ BE/ME	Small Value	Big Value
30-70 BE/ME	Small Neutral	Big Neutral
30- BE/ME	Small Growth	Big Growth

3.2 Summary statistics

The summary statistics for the factors series show that the market risk premium is the largest and most volatile of the three, having a minimum value of -29.13 percent in September 1931, a month with widespread losses across all portfolios in the midst of the Great Depression. The maximum values for the three factors, between 35.4 and 38.5 percent, were met between July 1932 and July 1933 when a short large recovery in markets took place. Starting in the post-war period, less volatility is seen; although some volatility is witnessed in the early 2000's for the size factor, no such volatility as large as the Great Depression's was observed again.

Table 3.2: Summary statistics of Fama-French factors, July 1926 to July 2018

Statistic	N	Mean	St. Dev.	Min	Pctl(25)	Pctl(75)	Max
Mkt.RF	1,105	0.66	5.33	−29.13	−1.97	3.63	38.85
SMB	1,105	0.21	3.19	−16.87	−1.56	1.73	36.70
HML	1,105	0.37	3.48	−13.28	−1.31	1.74	35.46

Figure 3.1: Time series plot of Fama-French market risk factor

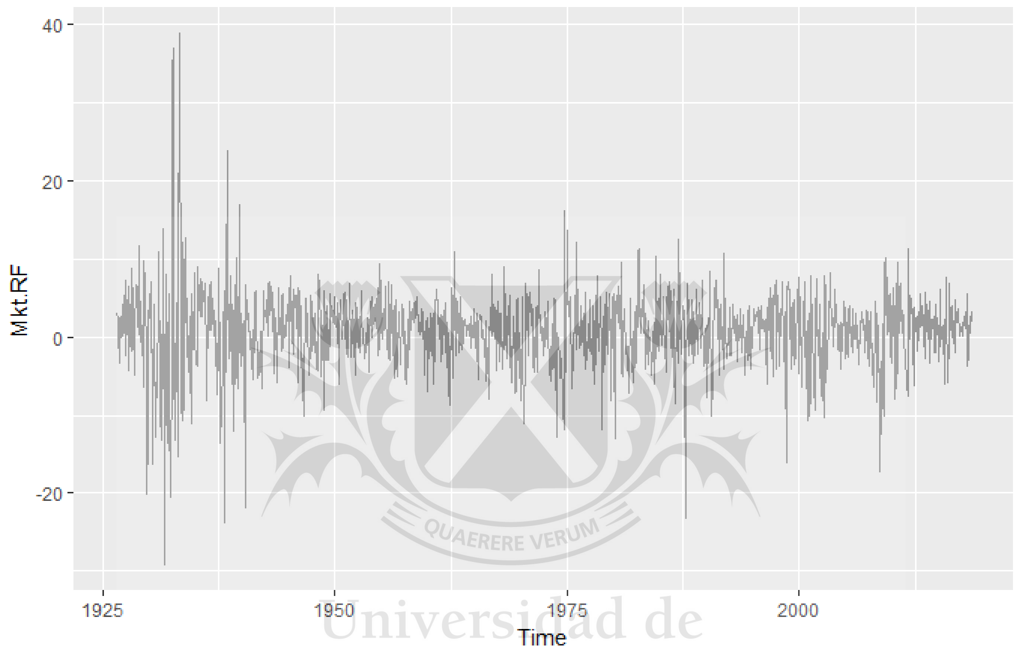


Figure 3.2: Time series plot of Fama-French SMB factor

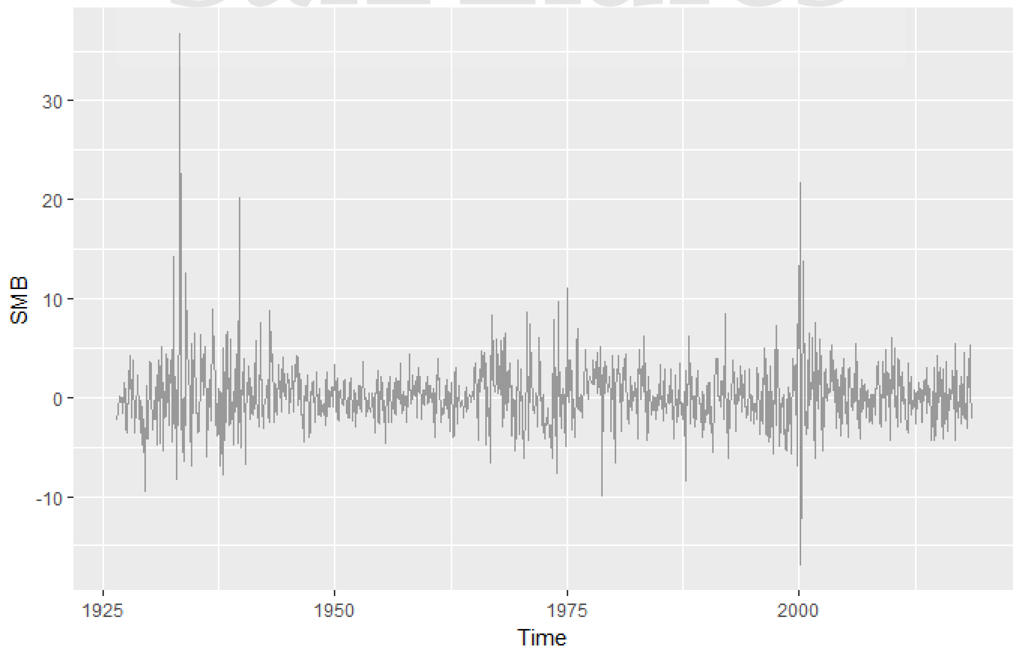
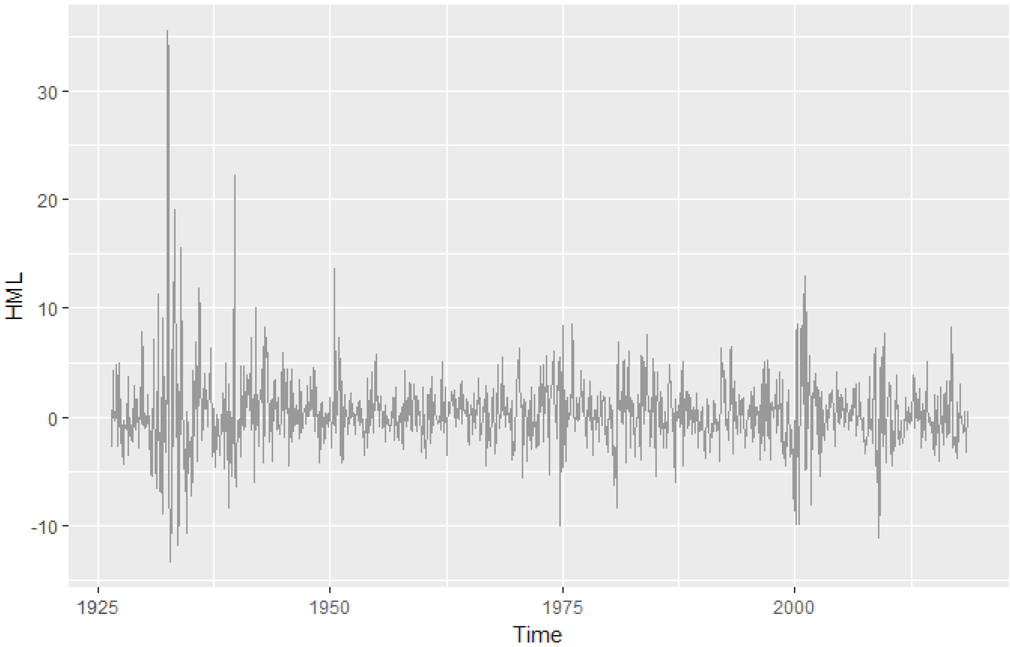


Figure 3.3: Time series plot of Fama-French HML factor



3.3 Unit root analysis

Furthermore, unit root tests were performed on the three factor variables. As seen on Table 3, all three risk premia series reject the null hypothesis of a unit root significantly with a confidence level of 99%, indicating that they are stationary as expected.

Table 3.3: Unit root tests for factor variables

Statistic	ADF	PP	KPSS
Mkt.RF	-9.14***	-934.57***	0.05***
SMB	-8.85***	-1026.00***	0.07***
HML	-9.11***	-786.3***	0.11***

p-value: * $<$ 0.10, ** $<$ 0.05, *** $<$ 0.01

ADF tests use a 10 lag order

PP and KPSS tests use a 7 lag truncation parameter

To replicate Fama and French's time-series regression, portfolio data were also obtained from the Data Library at Dartmouth. The monthly return series for 25 portfolios sorted on size and book-to-market ratio quintiles was used for this paper for matters of simplicity, but several different portfolio divisions are possible and can certainly be increased in complexity as smaller percentiles are used to divide them. Again, these portfolios divided on size and BE/ME quintiles contain all NYSE, AMEX and NASDAQ stocks for which there is available market equity and book equity data. In the rest of this paper, the portfolios will be indexed as follows:

Table 3.4: Portfolio indices

		BE/ME quintiles				
		1	2	3	4	5
Size quintiles	1	1	2	3	4	5
	2	6	7	8	9	10
	3	11	12	13	14	15
	4	16	17	18	19	20
	5	21	22	23	24	25

Chapter 4

Iterative forecasting algorithm

This chapter will introduce the algorithm trading strategy which this thesis is based on. Section 4.1 describes the assumptions which the estimations and testing of the algorithm imply. Section 4.2 specifies the Fama-French regressions needed to estimate three factor βs. Section 4.3 describes how factor forecasts are obtained for each period through a parsimonious VAR(1) model and inputted into Fama-French regressions. Also, this section explains the GARCH specification used to model the variance due to heteroskedasticity and serial correlation the VAR model presents. Finally, section 4.4 explains how returns are to be computed in the testing of the algorithm.

4.1 Assumptions

The algorithm devised in this paper is set so that for each month t a portfolio from the set of 25 quintile-divided portfolios is picked and held until the next month $t+1$. The decision, therefore, is which portfolio to hold in a long position each month so that it will maximize expected returns. As will be seen below, if it is then true that Fama-French factors significantly explain stock returns, these factors could be forecasted through a time series model to arrive at *expected* returns in the following period for each month t.

There are two implicit assumptions that need to be pointed out. First, it is assumed that there are no transaction costs. Although, as will be seen below, the algorithm tends to not switch between portfolios many times -and is biased towards specific portfolios-, when it does so no transaction cost is paid. A real approach to no transaction costs could be, theoretically, investing in an ETF (Exchange Trading Fund) with portfolio holdings of the specified size and book-to-market ratio quintiles. Secondly, it is assumed that a long position of stocks is always held; that is, there are no other kind of financial assets in the optimal portfolio, nor derivatives, not short positions. As Hu (2003) points out, it is possible to devise a strategy where both a long portfolio with maximum expected return is held *at the same time* as a short portfolio of the minimum expected return.

The following steps involve two separate estimation procedures done for each period $t = 763, 764, 765, ..., 1104$ - that is, from January 1990 onwards until one period before the end of the database period (June 2018). Both estimations naturally use an observation window $0, 1, 2, ..., t$ at each period t to reflect available information at each point of time.

4.2 Fama-French three factor regressions

First, cross-section regressions to obtain the Fama-French βs are performed. Here, the independent variable in each regression is the monthly returns on each one of the 25 sorted portfolios while the dependent variables are the three Fama-French factors. Indexing the portfolios with $p = 1, 2, ..., 25$, the following models are estimated:

$$r^p = \beta_0^p + \beta_1^p Mkt.RF + \beta_2^p SMB + \beta_3^p HML \qquad \forall p = 1, 2, ..., 25 \tag{4.1}$$

The resulting βs are the corresponding parameters that will be used to predict returns in $t+1$. βs have been found to be relatively stable for most portfolios and are indeed significantly different between portfolios [1].

[1] To save space, these results are available upon request.

4.3 Factor forecasts

To arrive at forecasts for the three factor series, a VAR (vector autoregressive) model will be used. VAR models were popularized by Sims (1980) and are a natural generalization of univariate time series models such as ARMA (autoregressive/moving average models). VAR models allow the modeling of many time series at once by assuming all of them are endogenous, thus exploiting the fact that lags of some variables might impact contemporary values for the rest of them. McNees (1986) showed that VAR models tend to produce better forecasts that univariate ones.

Factor forecasts for $t + 1$ are obtained through a VAR model comprising the three factor variables $Mkt.RF$, SMB and HML with observations $0, 1, ..., t$. VAR order selection criteria are calculated and are shown to be minimized for one lag with Hannan-Quinn and Schwarz while optimal ten lags are picked with the Akaike and final prediction error information criteria. For matter of simplicity and parameter estimation, a one lag VAR model was chosen following the Hannan-Quinn and Schwarz criteria.

Table 4.1: VAR order selection criteria

Criteria	1	2	3	4	5	6	7	8	9	10
AIC(n)	7.93	7.93	7.93	7.93	7.92	7.91	7.92	7.92	7.89	7.88∗
HQ(n)	7.95∗	7.97	7.98	7.99	8.00	8.01	8.03	8.05	8.03	8.04
SC(n)	7.98∗	8.03	8.07	8.10	8.14	8.17	8.22	8.26	8.27	8.30
FPE(n)	2,773.78	2,778.71	2,779.14	2,766.70	2,746.44	2,731.38	2,743.87	2,740.65	2,664.11	2,631.88∗

* = minimum value

Therefore, the following model is estimated by ordinary least squares (OLS) in each period:

$$\begin{pmatrix} Mkt.RF_t \\ SMB_t \\ HML_t \end{pmatrix} = \begin{pmatrix} \alpha_1 \\ \alpha_2 \\ \alpha_3 \end{pmatrix} + \begin{pmatrix} \phi_{11} & \phi_{12} & \phi_{13} \\ \phi_{21} & \phi_{22} & \phi_{23} \\ \phi_{31} & \phi_{32} & \phi_{33} \end{pmatrix} \begin{pmatrix} Mkt.RF_{t-1} \\ SMB_{t-1} \\ HML_{t-1} \end{pmatrix} + \begin{pmatrix} \epsilon_{t,1} \\ \epsilon_{t,2} \\ \epsilon_{t,3} \end{pmatrix} \tag{4.2}$$

Where $\begin{pmatrix} \alpha_1 & \alpha_2 & \alpha_3 \end{pmatrix}$ is a vector of constants and $\begin{pmatrix} \epsilon_{t,1} & \epsilon_{t,2} & \epsilon_{t,3} \end{pmatrix}$ is a vector of zero mean serially uncorrelated white noise processes.

Appendix A.1 shows the full sample VAR estimation results. As can be seen, $Mkt.RF$ lags are significative when explaining $Mkt.RF$ and SMB, but not HML. Similarly, SMB and HML lags are significative when explaining HML, but not $Mkt.RF$. F statistic joint tests show statistical significance.

Furthermore, the residuals of the full-sample VAR were studied and tested for serial correlation. Both Breusch-Godfrey and an asymptotic Portmanteau tests reject the null hypothesis of no serial correlation in residuals. The VAR model was also tested for heteroskedasticity, normality in residuals, ARCH effects and specification errors. Appendix B shows the results of these tests and VAR estimations with dummies.

Due to these problems in robustness, the VAR model was expanded to model variance through a GARCH(1,1) dynamic conditional correlation (DCC) specification. GARCH (generalized autoregressive conditional heteroskedasticity) models are a generalization of ARCH (autoregressive conditional heteroskedasticity) models. ARCH/GARCH models allow for modeling of a time series process' variance when heteroskedasticity is present, in an analogous way to ARMA/VAR models. This kind of behavior in variance is typical of financial series, which show higher and lower volatility in different points of time; in fact, the $Mkt.RF$, SMB and HML series do show higher variance in certain periods of time such as the Great Depression and the early 2000's.

A basic ARCH(1) -that is, an ARCH model with one lag- model has the form:

$$\begin{aligned} Y_t = u_t \sim N(0, \sigma_t^2) \\ \sigma_t^2 = \omega + \alpha Y_{t-1}^2 \end{aligned} \tag{4.3}$$

Bollerslev (1990) proposed a further generalized ARCH model (GARCH) to allow for conditional variance to depend on its own lagged values. A simple GARCH(1,1) takes the form:

$$
\begin{aligned}
Y_t &= u_t \sim N(0, \sigma_t^2) \\
\sigma_t^2 &= \omega + \alpha Y_{t-1}^2 + \beta \sigma_{t-1}^2
\end{aligned}
\tag{4.4}
$$

Thus, an ARCH model would be a special case in which $\beta = 0$.

Naturally, GARCH models were subsequentially expanded to their multivariate versions. Especially in the field of finance, covariances are important to model due to possible volatility spillovers between different assets. Bollerslev et al. (1988) proposed the VECH model, which proved difficult to use in practice because of the large number of parameters to be estimated and because of the fact that it did not imply a positive definite variance-covariance matrix. Engle and Kroner (1995) solve this problem by creating the BEKK model (named after Baba, Engle, Kraft and Kroner).

Bollerslev (1990), however, also considered the case of a constant conditional correlation (CCC) model to generalize the GARCH(1,1) model in equation (4.4) to a multivariate environment. In the special case where the covariance $\rho_{12,t}$ in the variance-covariance matrix between any two variables 1 and 2 of the model in moment t is constant in time, that is

$$
\Sigma_t = \begin{bmatrix} \sigma_{11,t} & \sigma_{12,t} \\ \sigma_{21,t} & \sigma_{22,t} \end{bmatrix} = \begin{bmatrix} \sigma_{11} & \sigma_{12} \\ \sigma_{21} & \sigma_{22} \end{bmatrix}
\tag{4.5}
$$

then the following reduced GARCH model for n variables can be estimated:

$$
\Sigma_t^* = \omega + \alpha u_{t-1}^2 + \beta \Sigma_{t-1}^*
\tag{4.6}
$$

where $\Sigma *_t$ is a one column matrix containing the model's variances $\sigma_{1,2...n,t}$, $u_{t-1}^2 = (u_{1,t-1}^2, u_{2,t-1}^2, ..., u_{n,t-1}^2)'$ and ω is a vector of intercepts.

Engle (2002) eliminated the need to assume that conditional covariances were constant in time by postulating the dynamic conditional correlation (DCC) model. This was the chosen specification to model variance in the factor forecasts VAR. Estimation results for this model are shown in Appendix B. As can be seen, every parameter in the GARCH model, including the elements of the vector of constants ω, the vector of αs and the vector of βs proved individually statistically significant. Also, there seems to be joint significance for the vector of αs and the vector of βs, as shown by the joint DCC α and DCC β lines.

After the VAR estimation, one step ahead forecasts $\mathbb{E}\left(Mkt.RF_{t+1}\right)$, $\mathbb{E}\left(SMB_{t+1}\right)$ and $\mathbb{E}\left(HML_{t+1}\right)$ are obtained. These forecasts are inputted into each one of the portfolio regression models in equation (1). Expected returns $\mathbb{E}(r^p_{t+1})$ for each portfolio, thus, are obtained through

$$\mathbb{E}(r^p_{t+1}) = \beta_0 + \beta^p_1\,\mathbb{E}(Mkt.RF_{t+1}) + \beta^p_2\,\mathbb{E}(SMB_{t+1}) + \beta^p_3\,\mathbb{E}(HML_{t+1}) \tag{4.7}$$

for each $p = 1, 2, ..., 25$. Portfolio p which yields the largest $\mathbb{E}(r^p_{t+1})$ is finally chosen for each t.

4.4 Computing of returns

Third, an out-of-sample window period needs to be set to test the algorithm. Although return rates for every month since January 1990 are computed, two starting periods are set to display results more clearly and separate two different scenarios: one starting in January 1990 (that is, with observations $t = 763$ to $t = 1105$ - $t = 0$ being July 1926), and the other in July 2007 ($t = 973$), at market highs just before the subprime financial crisis. To compute returns, a simulated portfolio S with value equal to 1 is created at $t = 763$ and $t = 973$.

The portfolio p with maximum expected return as calculated on equation (3) is picked and its actual return in the period $t + 1$ is computed and compounded to the simulated portfolio S:

$$S_t = S_{t-1}(1 + r^p_t) \tag{4.8}$$

For benchmarking purposes, a risk-free portfolio (S_{rf}) with monthly Treasury bills returns and an equity portfolio (S_e) with the compounded returns on total market equity are computed for the same testing window. Also, the compounded returns of each portfolio p are calculated. This will let answer the question of whether there is a size and book-to-market quintile that has consistently higher return than other portfolios and whether keeping a long position in this portfolio yields higher returns than forecasting and switching between portfolios.

Chapter 5

Results

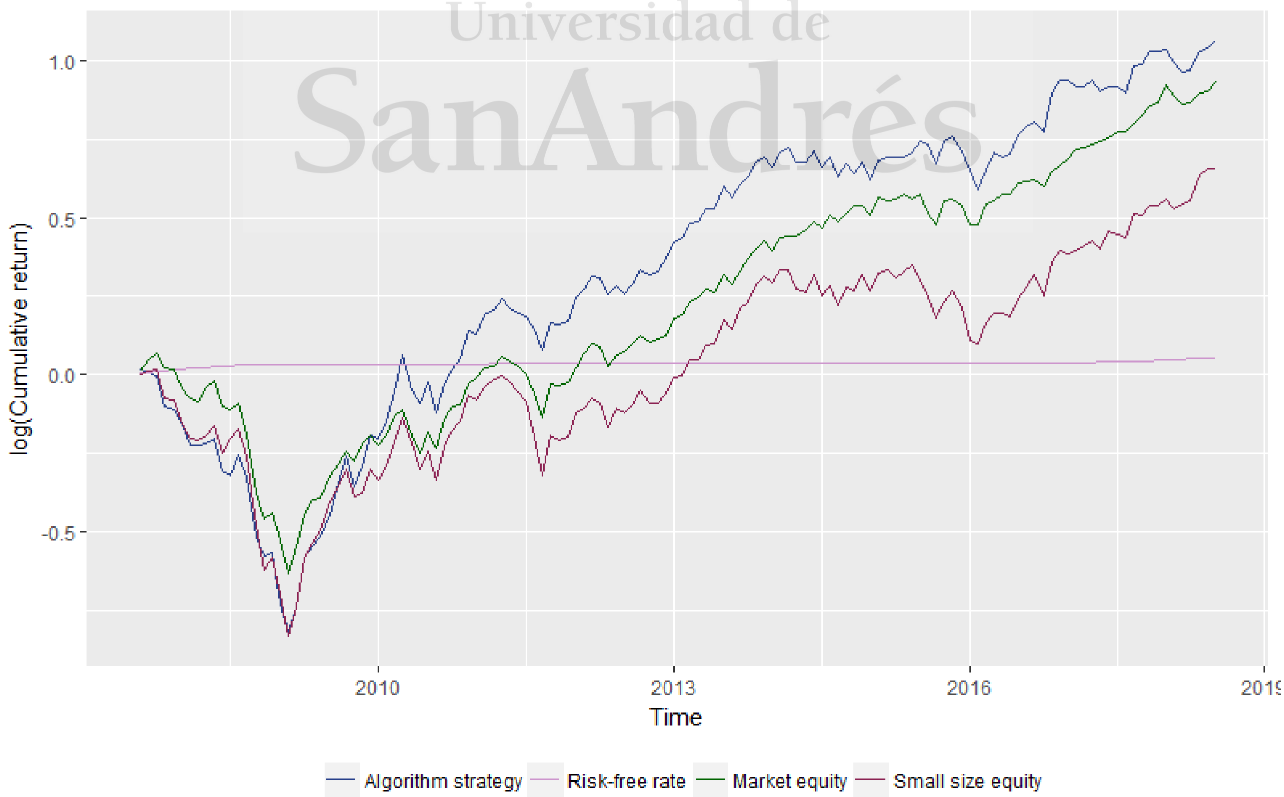
5.1 Returns

Although any starting point could be used to test the algorithm, two starting points were set as mere examples, as described in the previous chapter: $t = 763$ (January 1990) and $t = 973$ (July 2007).

Simulated out-of-sample returns show that, when setting the initial point of time at each said month, the long position returns for the strategy were larger than wide-market returns as measured by the market, value-weighted return of all firms incorporated in the US and listed in the NYSE, AMEX or NASDAQ with available data (which, in the database, is equal to $Mkt.RF + RF$ - see section 2) at the end of the testing period. That is, $S_t > S_e > S_{rf}$ at $t = 1105$ (July 2018). In terms of magnitude, simulated portfolios S_t, S_e and S_{rf} set to an index equal to 1 in January 1990 ($t = 763$) were 75.56, 16.57 and 2.15 respectively in July 2018. Figure 4 shows the log returns of these portfolios. In the second case, starting in July 2007 ($t = 973$), these indices are 2.89 for the algorithm strategy, 2.54 for the average market and 1.05 for the risk free portfolio.

Figure 5.1: Time series plot of cumulative returns by portfolio, start in January 1990

Figure 5.2: Time series plot of cumulative returns by portfolio, start in July 2007



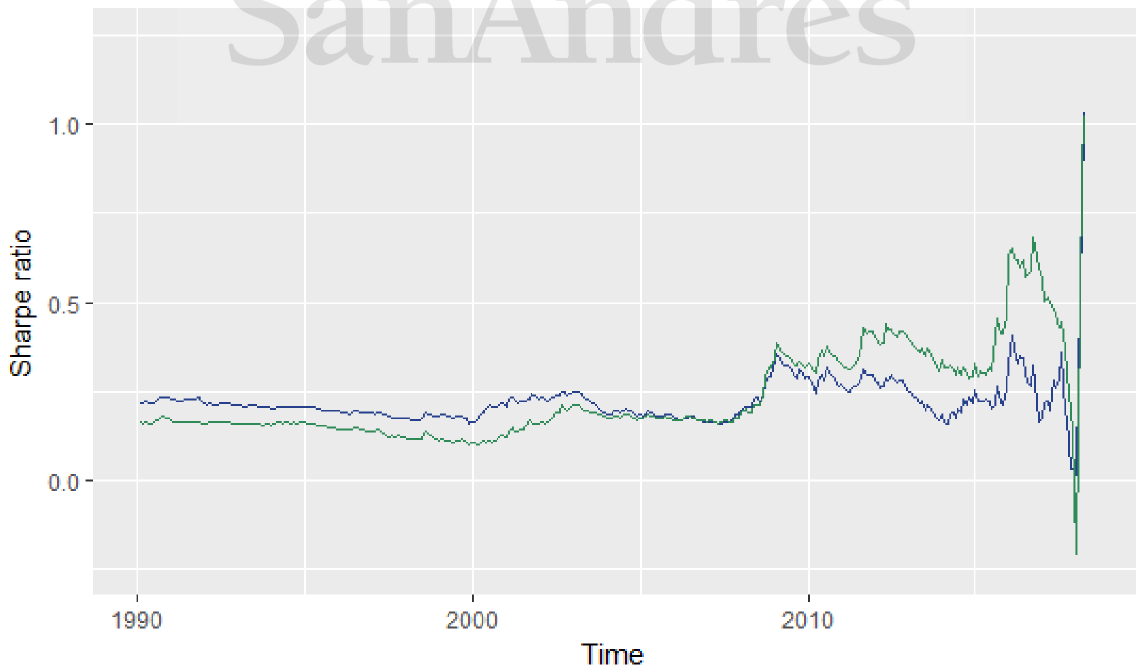
It could indeed be argued that these higher returns arise -as will be discussed below in section 5.3- from the fact that the algorithm has a tendency towards long positions in risky portfolios, more so than the wide market average. Riskier portfolios, under any rational portfolio theory, imply a higher rate of return to compensate for its volatility. For that purpose, Sharpe ratios are computed for both the algorithm strategy and the average market return. The Sharpe ratio for any asset return a is defined as in Sharpe (1994):

$$S_a = \frac{\mathbb{E}(R_a - R_f)}{\sigma(R_a)} \tag{5.1}$$

where R_a is the return of an asset a, $\mathbb{E}(R_a - R_f)$ is the expectation of the excess return of a over the risk-free rate, and $\sigma(R_a)$ is the standard deviation of returns of asset a. The Sharpe ratio proves useful as a means to adjust returns for volatility.

Sharpe ratios were computed for each starting period from January 1990 up to the end of the database (July 2018). In this case, $\mathbb{E}(R_a - R_f)$ was computed as the simple arithmetic average rate of return for both the algorithm and average market over the risk-free rate.

Figure 5.3: Sharpe ratio by starting period



blue = algorithm, green = market average

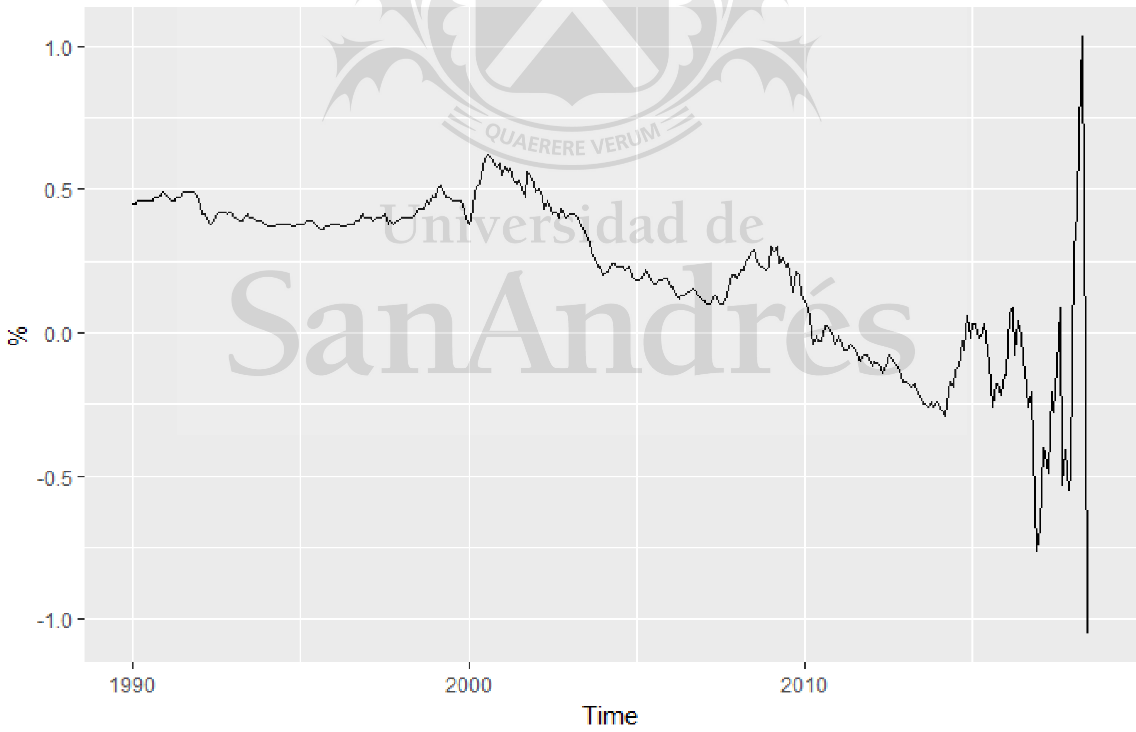
It can be observed in Figure 5.3 that the algorithm strategy, even after correcting for its higher volatility, displays a higher rate of return than the average market up to the mid 2000's. The relation breaks up in the short term, especially since 2008-2009. This "short term fading" effect will be discussed in the next sections.

Figure 5.4 shows the excess return on the forecasting strategy when compared to wide market returns as a function of the starting point in time for the algorithm, that is:

$$excr_t = S_T^{\frac{1}{T-t}} - 1 \tag{5.2}$$

where S_T is the portfolio value at period T previously normalized with value 1 in period t, T being the terminal period ($t = 1105$ or July 2018).

Figure 5.4: Geometric average return of forecasting strategy minus market return



As can be seen, the excess return is decreasing as the testing period shortens when setting the final period in July 2018 and moving the initial period by one month ahead in time. The strategy seems to outrun diversified portfolios in the long term - and the longer the term, the larger the returns. Excess returns become more volatile as the testing window shortens as it becomes more representative of short-term, volatile returns than long-term, stable mean returns. Figure 5.4 shows the fact that, were the strategy to be tested after 2010, the excess return would have vanished. Figure 5.1 and 5.2 are also useful in showing this relation: the excess return of the algorithm strategy in Figure 5.1 over the market return is much larger than the excess return on Figure 5.2, which features a much shorter time span. Thus, the relation or excess return of the algorithm strategy becomes less clear in the short term.

5.2 Possible explanations

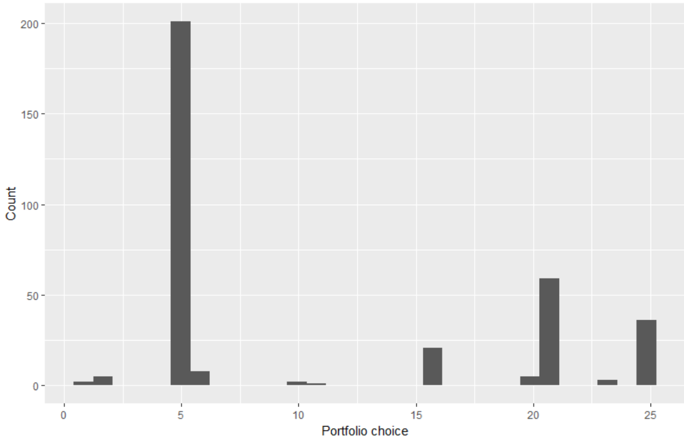
A few things can be pointed out from the simulation results. First and foremost, an interesting pattern can be observed in the frequency of optimal portfolios: portfolio 5, the smallest size quintile with the highest book-to-market ratio quintile (see Table 4 for portfolio indexing), is the most common portfolio chosen by the algorithm. In fact, it is chosen up to 75% of the time, this ratio growing as t approaches the terminal period. As will be discussed in the next section, this does not strike as very surprising due to portfolio 5's nature.

On the other hand, another important result is the fact that the relation between returns of the different portfolios is indeed not "strictly dominating". In other words, returns for the algorithm strategy are *not* larger in each period of time than diversified portfolio returns. As can be seen, this relation in the short term is not clear - as exemplified in Figure 5.1 and 5.2, no advantage is given to the factor forecasting strategy during the early 90's when starting in January 1990 nor until 2010 when starting in July 2007. The algorithm strategy seems to outperform the market *in the long term*, but this relation fades for short periods of time.

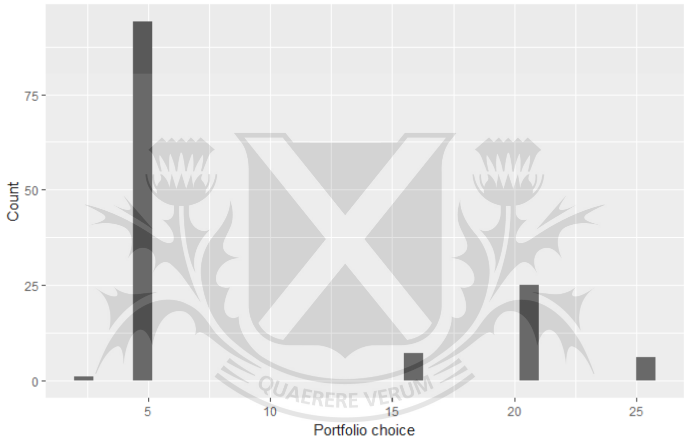
5.3 Remarks

Two effects should be taken in account when analyzing these results. It is not definite whether excess returns from this strategy are obtained through long-term returns of small companies, to which the optimal choice seems to be biased (see Figure 5), and whether the "fading out" of the excess return is linked to the seemingly disappearing size effect (see Van Dijk (2011) for the discussion on the size effect in recent decades). However, excess returns do not seem

Figure 5.5: Histograms of portfolio choices



t = 763 (January 1990)



t = 972 (July 2007)

to be paired with small size effects. In fact, the average small market capitalization portfolio doesn't have more returns than average market equity returns as shown in Figure 2 and Figure 3; furthermore, the size effect seems to have empirically faded since the 1980's, but the forecasting strategy remarkably outperforms market equity during the 90's and early 2000's. Therefore, the fading of the size effect should not appear to have effects on the strategy.

A clue might arise from the fact that portfolio 5, the most chosen portfolio by the algorithm, is the lowest size quintile but also the highest book-to-market ratio quintile (see Table 4 for portfolio indexing). Thus, the excess returns of the strategy might arise from a "value investing" feature - i.e., it might be biased towards small companies acquired at a discount due to its low market value relative to its book value. Figure C.1 and C.2 in Appendix C show the percentage of times portfolio 5 is chosen from the two example starting points of the algorithm, which amounts to 70% in the past decade in both cases.

However, if it were true that the highest return arises from this portfolio -high book-to-market ratio, small market capitalization stocks-, an abnormal high return would be observed for this portfolio, were it to be held in a long position for the totality of the testing period. Although that is the case and higher than average market returns are found for this strategy, it does in no period perform better than the forecasting strategy. As Figures 5.1 and 5.2 show, when starting in January 1990 cumulative returns of holding portfolio 5 consistently up to July 2018 are 4100% while the algorithm's are an astounding 7500%. A simple computation of these results for any other starting point shows that the algorithm outperforms the smallest-cap, highest book-to-market ratio portfolio for all long-term periods.

Chapter 6

Conclusions

6.1 Summary of thesis achievements

To sum up, a recursive algorithm was set up to test whether (1) Fama-French factors can be accurately forecasted with an autoregressive vector model, and (2) these forecasts can be used to boost returns by using them as inputs in a three-factor model with a market risk premium, SMB and HML independent variables.

First, portfolio βs as modeled in Fama and French (1992) are estimated through linear least squares using observations available at each period t for which the algorithm is set. Significant and time-varying βs are found. Second, a parsimonious VAR(1) mean-forecasting model is estimated, again using all available observations, to forecast risk factors for $t + 1$ and these predictions are inputted into the former model to arrive at expected returns for each portfolio. The algorithm then picks the optimal portfolio that maximizes expected returns.

Excess returns in relation to the average market are found when simulating the actual returns of the picked portfolio in the long term, but seems to have either disappeared in the last decade or loses power in the short term. This brings a discussion on what drives this weakening of the outstanding algorithm's performance in short term periods. There does seem to be a joint effect of both size premiums and value premiums when considering the algorithm is heavily biased towards a smallest-sized, high-discount portfolio, which it chooses around 70% of the time.

6.2 Future lines of research

So far, this thesis has argued and demonstrated that a parsimonious VAR(1) model inputted into Fama-French regressions can provide significant information when choosing long positions in portfolios sorted by size and book-to-market ratio quintiles.

However, many questions are still in place. First and most important, are there other factors that describe cross-sectional returns with more explaining power? Specifications such as Carhart (1997) might help in better predicting returns. In fact, hundreds of factors could be used and tested in an algorithm as described in this thesis - whether any factor specification achieves better return results is a question to still be answered.

The second question is whether there is any portfolio arrangement by any other variable that could prove better at achieving higher returns. This thesis was based on size and book-to-market ratios, but could any other orderings based on other factors or even industry portfolios be better at achieving higher returns?

Third, can any other time series specification perform better at forecasting factors? A VAR(1) model was used here, but could any other specification achieve better predictions? This is an open question that will depend critically, also, on the factor variables chosen in analysis.

Finally, further research will be needed to answer the question on whether such an algorithm can be exploited in practice, and whether it still stands valid today. This thesis found that, were the algorithm to be started in recent years, no outperformance would have been achieved. In fact, even less returns than the market average could be obtained. The most important line for future research will prove to be whether the fading out of the algorithm's outperformance is due to its long-term nature, or to the fact that market fundamentals have changed in recent years.

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Appendix A

Full sample VAR estimation results

Table A.1: VAR estimation results, full sample

	Dependent variable:		
	y		
	Mkt.RF	SMB	HML
	(1)	(2)	(3)
Mkt.RF.l1	0.10***	0.14***	0.03
	(0.03)	(0.02)	(0.02)
SMB.l1	−0.02	−0.02	−0.08**
	(0.05)	(0.03)	(0.03)
HML.l1	0.09*	0.04	0.20***
	(0.05)	(0.03)	(0.03)
const	0.57***	0.11	0.29***
	(0.16)	(0.09)	(0.10)
Observations	1,104	1,104	1,104
R^2	0.02	0.06	0.05
Residual Std. Error (df = 1100)	5.30	3.10	3.40
F Statistic (df = 3; 1100)	5.70***	23.55***	18.41***
Standard errors reported in parenthesis	*p<0.1; **p<0.05; ***p<0.01		

Appendix B

Robustness tests

Table B.1: Robustness tests for base VAR model

Test	Statistic	df	p-value
Portmanteau	343.14	135	<0.01
Breusch-Godfrey	86.29	45	<0.01
White (heteroskedasticity)	2231.84	756	<0.01
ARCH effects	967.16	180	<0.01
Jarque-Bera	6042.00	6	<0.01
RESET test (Mkt.RF equation)	8.10	2 / 1017	<0.01
RESET test (SMB equation)	29.08	2 / 1017	<0.01
RESET test (HML equation)	49.95	2 / 1017	<0.01

Table B.2: VAR-GARCH estimation results, full sample

	Estimate	Std. Error	t value	Pr(>|t|)
ω_{MktRF}	0.61	0.24	2.55	0.01
α_{MktRF}	0.13	0.02	5.86	0.00
β_{MktRF}	0.85	0.02	38.48	0.00
ω_{SMB}	0.21	0.13	1.69	0.09
α_{SMB}	0.15	0.05	3.22	0.00
β_{SMB}	0.84	0.04	19.05	0.00
ω_{HML}	0.39	0.17	2.30	0.02
α_{HML}	0.17	0.03	4.96	0.00
β_{HML}	0.80	0.03	26.18	0.00
Joint DCC α	0.04	0.01	4.65	0.00
Joint DCC β	0.94	0.01	68.60	0.00
Number of parameters		26		
Number of observations		1105		
Log likelihood		-8457.65		
Av. log likelihood		-7.65		

Appendix C

Portfolio 5 optimal choices

Figure C.1: Percentage of portfolio 5 optimal choices, start = January 1990

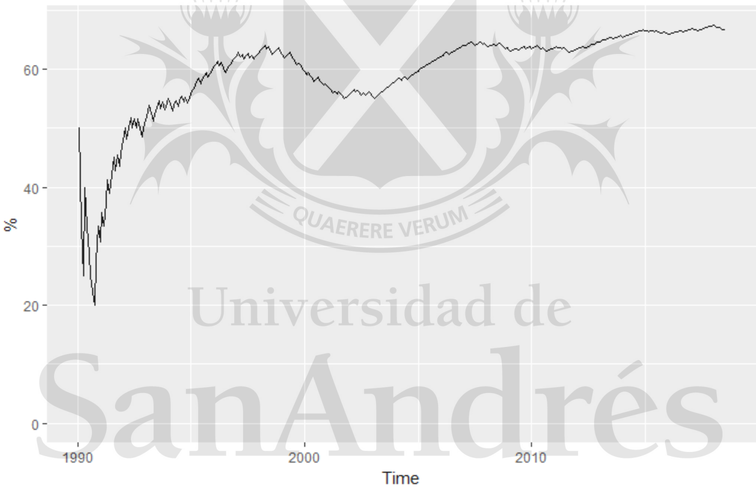

Figure C.2: Percentage of portfolio 5 optimal choices, start = July 2007