

At the edge, chance is everything.



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The Leverage Game

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*All errors and omissions are mine.

A mis padres.



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The Leverage Game

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Abstract

This study uses individual bank's balance sheet data from Argentina, Brazil, Mexico, South Africa, and Taiwan, to obtain cross-correlations over the time-series leverage ratio. A correlation-based filtering procedure is then implemented to obtain leverage-based networks. Results conclude that topological properties from graph theory describe a surprising similar pattern inside the network between countries. These leverage networks are complex and are characterized by heavy-tailed vertex degree distribution, which reveals the emergence of hubs and suggests the potential scale-free nature of the system. Finally, surrogated Monte Carlo simulations confirm that the complex networks, obtained through the filtering procedure, cannot be explained by chance alone.

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*Creía en infinitas series de tiempos, en una red creciente y vertiginosa de tiempos divergentes,
convergentes y paralelos. Esa trama de tiempos que se aproximan, se bifurcan,
se cortan o que secularmente se ignoran, abarca todas las posibilidades.
No existimos en la mayoría de esos tiempos; en algunos existe usted y no yo;
en otros, yo, no usted; en otros, los dos. En éste, que un favorable azar me depara,
usted ha llegado a mi casa; en otro, usted, al atravesar el jardín, me ha encontrado muerto;
en otro, yo digo estas mismas palabras, pero soy un error, un fantasma.*

J. L. Borges (1941)



1 Introduction

Imagine for a second an auditorium occupied by two hundred people, evenly distributed among men and women, and each holding a tiny remote control. Then imagine the main speaker asking his audience to use that control to enter the number of sexual partners they had the previous year. After a few seconds of awkward silence, the room bursts out laughing. Not deterred, the main speaker, however, asks again, for he is serious about the question. In the end, since the survey is anonymous –and no one has a particular reason to lie–, each individual enters his number. Then the panelist types a couple of times on the keyboard, and the screen pops up the results in the form of the empirical cumulative distribution in log–log binning, which can be seen in Figure 1. The image depicts a Pareto or Zipf’s curve, indicating that only very few people had lots of different sexual partners and that the majority of the people only had a few, which at first glance sounds intuitive (how many girlfriends/boyfriends do *you* have?). This behavior is often referred to as having a power–law distribution.

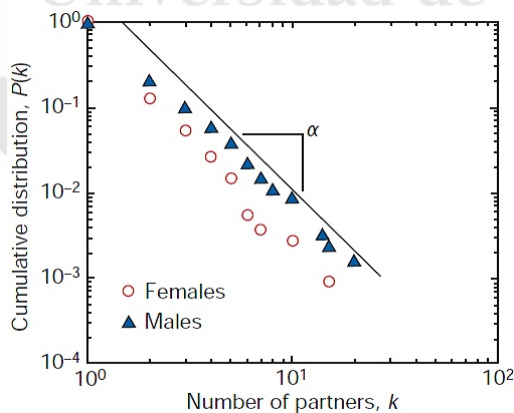


Figure 1: Source: Liljeros *et al.* (2001)

Hold on a second. Although the previous experiment does not exist, the results are in fact real. Actually, Figure 1 was taken from the famous paper by Liljeros *et al.* (2001), who analyzed the number of sexual partners in a random sample of people in

Sweden. However, while this latter study may seem strange at first, it is just an example of the emergence of network research in other disciplines. Originally a development from mathematical graph theory, the methods and topological properties of complex networks have been applied to a great variety of fields such as computer science, biology, neurology, social interactions, physics, the Internet, communication, and more recently finance. For highly recommended surveys on these various developments, see Albert and Barabási (2002), Boccaletti *et al.* (2006), or Dorogovtsev and Mendes (2002).

In finance, and serving as a motivation for this study, Axtell (2001) documents that the size of firms obeys a power-law distribution; Boss *et al.* (2004), De Masi *et al.* (2006), and Bastos e Santos and Cont (2010) showed that the interbank contract size also follows a Zipf's distribution. Following these results, we wonder whether banks' assets –this being the usual proxy for bank size–, liabilities, and equity also obey the same distribution. For this, we use individual financial institutions balance sheet data from five emerging countries: Argentina (2003–2010, quarterly basis, $N = 72$), Brazil (2001–2010, quarterly basis, $N = 70$), Mexico (2004–2010, monthly basis, $N = 26$), South Africa (2003–2010, monthly basis, $N = 27$), and Taiwan (2002–2010, quarterly basis, $N = 29$). In order to assure full observations for every bank, we must account for mergers or acquisitions, and banks with incomplete data are deleted from the sample. It is certainly relevant to notice that the final sample accounts for more than 80% of total bank assets. Before showing any result, we must first recall that not only do we have cross-country balance sheet data, but also time-series data for each of our five countries. Thus a cumulative distribution in a specific time period will only show a “photograph” of the behavior. Hence our purpose in this computation is in fact twofold. First, we take a starting year and examine the distribution; and second, we ask if this distribution is stable with time.

Denote x the quantity of assets (or liabilities) of any bank. Then x obeys a

power-law if its distribution is of the form

$$p(x) \propto x^{-\alpha} \quad (1)$$

where α is the scaling parameter of the distribution. We consider the continuous case only (the discrete form preserves the same intuitions and will be derived in Appendix A). Then the probability distribution of (1) is

$$p(x)dx = P(x \leq X \leq x + dx) = Cx^{-\alpha}dx \quad (2)$$

where C and α are the constant and the exponent, respectively. Since the density diverges as $x \rightarrow 0$, it is usually said that a quantity x follows a power-law for values greater than x_{\min} . Then C is constrained by the fact that (2) must integrate to 1. In particular, after normalizing the constant for the continuous case

$$p(x) = \frac{\alpha - 1}{x_{\min}} \left(\frac{x}{x_{\min}} \right)^{-\alpha} \quad (3)$$

Notice that taking logarithm on both sides of equation (2) yields an important result: if plotted with the appropriate logarithmic binning, the histogram should be a straight line. However, as shown in Goldstein *et al.* (2004) and Newman (2005), this binning procedure can actually be misleading, which is why the CDF or cumulative distribution function is in general a better way to detect power-law distributions, even though in this case the exponent α will differ from the former method. In particular,

$$P(x) = \int_x^{\infty} \frac{\alpha - 1}{x_{\min}} \left(\frac{y}{x_{\min}} \right)^{-\alpha} dy = \left(\frac{x}{x_{\min}} \right)^{-(\alpha-1)} \quad (4)$$

Thus the CDF also follows a power-law, but with a new scaling parameter equal

to $\alpha - 1$. Hence we see why one usually says that $p(x) \propto x^{-\alpha}$ if a straight line fits the CDF plot in logarithmic scales. This tool will be used to assess whether a Pareto distribution is a suitable match for banks' assets, debt and equity.

Figure 2 illustrates the case of Argentina, where in effect a Zipf's law can approximate the behavior of those balance sheet accounts. By using the Kolmogorov–Smirnov test described in Clauset *et al.* (2009) and applied, for example, by Bastos e Santos and Cont (2010) to the interbank exposures in Brazil, we get a *p-value* greater than 0.2, which in this test translates into evidence *in favor* of the power-law fit. The main pitfall in these distribution tests is the low sample size, which reduces the power of the test (Massey, 1951). Nonetheless, is in itself interesting the fact that the power-law pattern is fairly stable with time and across-countries.¹ Appendix B depicts the cases of Brazil, Mexico, South Africa, and Taiwan. The only exception is Taiwan, where the power-law hypothesis is rejected.

¹Plots at intermediate periods show very similar results.

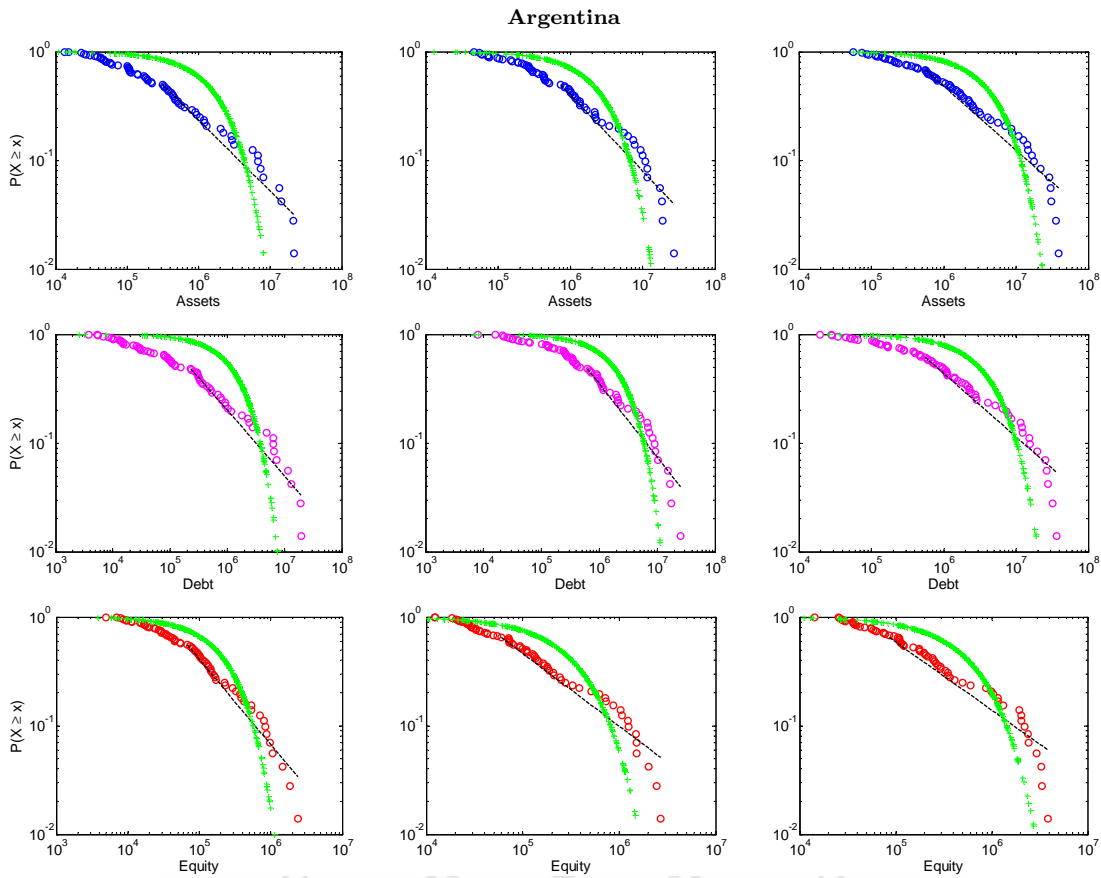


Figure 2: Left Panels show the CDF of Argentinean banks' total assets, liabilities and equity in June 2003; middle Panels same variables for September 2007; and right Panels for December in 2010. MLE yields $\hat{\alpha}$ ranging from 1.5 to 1.7. Results are robust to different period selection. Simulated exponential distribution (in green) with equal expected value as sample data depicts the strong difference in the tails of the distribution.

A typical way to proceed to estimate the scaling parameter α was to estimate a least-squares regression in the log-log plots, the slope being our parameter of interest. However, although this procedure is commonplace in the literature, it has been shown in Goldstein *et al.* (2004) that it often leads to biased estimators; more robust estimates are typically derived from Maximum Likelihood Estimation (MLE) methods, which yield the following estimate

$$\hat{\alpha} = 1 + N \left[\sum_{i=1}^N \ln \left(\frac{x_i}{x_{\min}} \right) \right]^{-1}$$

When applied in Figure 2, MLE yields scaling parameters going from 1.5 to 1.7.

Although to our knowledge there is no specific evidence on overall balance sheet accounts, the values found here are similar to other economic variables documented by the literature (e.g. Axtell, 2001, in the size of firms; Boss *et al.*, 2004, and De Masi *et al.*, 2006, in interbank lending).

It is the purpose of this study to construct a banking network at the country level. With a network in hand, one can analyze its connectivity, structure, even consequences of contagion. One can also compare the banking network among countries and obtain, for example, policy observations regarding shock resilience. There are many algorithms to construct networks. For instance, the seminal works of Mantegna (1999), and Bonanno *et al.* (2001), and later followed, for example, by Bonanno *et al.* (2003, 2004), and Tumminello *et al.* (2007), construct networks based on cross-correlations in time. In that sense, Pearson's correlation coefficient (or a non-linear transformation of it) can be used as a similarity or distance metric (Gower, 1966), which tries to assess how connected the nodes in a network are. However, if one were to construct a network based on similarities in assets, liabilities, or equities, one should first observe that all these variables tend to increase in time. Thus it would appear that bank A and bank B are *herding*, when in fact the increase/decrease in those variables could just be a consequence of a general macroeconomic trend. An intuitive support can be seen in Figure 3, which depicts the evolution of these balance sheets accounts for Argentina and Brazil averaged in each period. They all increase with time.

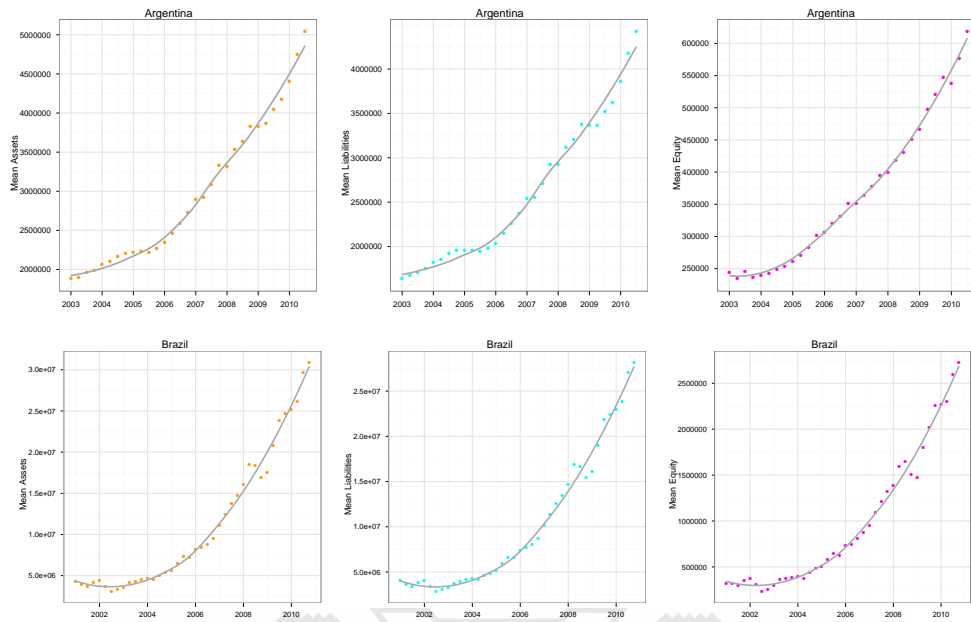


Figure 3: Upper panels depict average total assets, average total liabilities, and average total equity for Argentinean banks in the sample (disregard nominal values). Lower panels depict the same accounts for Brazilian banks (disregard nominal values). A smooth fit from local regression is added as visual guide.

The literature review in the following section will reveal many economic mechanisms which can explain why assets, liabilities, and equity may move similarly across banks. For that reason, this study proposes using as a variable on which to base the network the Liabilities-to-Equity ratio –henceforth the leverage ratio. Leverage appears to be a variable with profound economic meaning in terms of interdependence, but at the same time a robust statistic to the general trend. Further benefits from this ratio will also be discussed in Section 2. A first taste of this hypothesis can be seen in Figure 4, describing that, although the leverage ratio does move within each country, there is no obvious temporal trend which can account for a global correlation between banks.

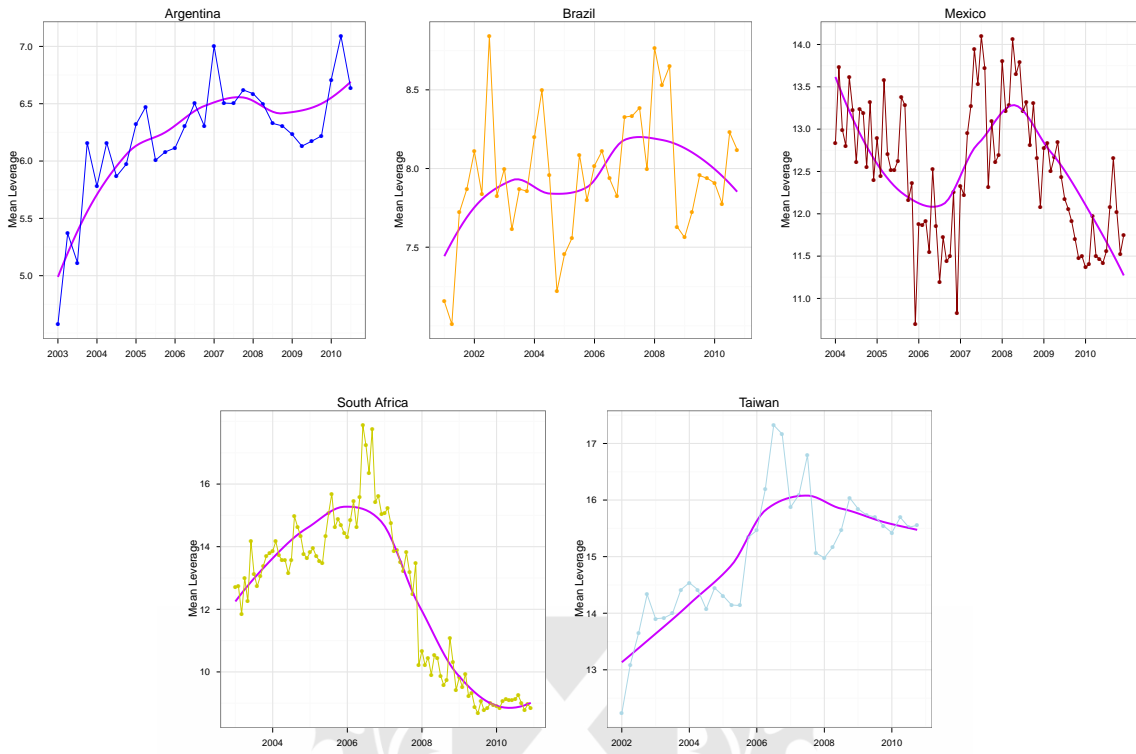


Figure 4: Historical average leverage in the five countries of the sample. No trivial upward or downward trend is experienced in any country. A smooth fit from a local regression is also added as visual guide.

The rest of the thesis is divided as follows. Section 2 reviews the literature and provides a better understanding of the leverage ratio, both at the theoretical and empirical levels. The main Section 3 will get inside the leverage-based network and draw results both within and among countries. Section 4 concludes.

2 The Leverage Ratio

*For it is, so to speak, a game of Snap, of Old Maid, of Musical Chairs – a pastime in which
he is victor who says Snap neither too soon nor too late,
who passes the Old Maid to his neighbour before the game is over,
who secures a chair for himself when the music stops.*

J. M. Keynes (1964)

The literature on the effects of macroeconomic shocks to bank lending or balance sheets is extensive. As quick as a wink, we can review the major mechanisms. For instance, if assets are held in local currency, then a strong devaluation will deteriorate firms' or banks' balance sheets, hence reducing their ability to offer collateral and more lending, respectively. This mechanism is usually referred to as the “balance sheet effect”, and was extensively studied by the literature, going from the classic models of Kiyotaki and Moore (1997), Krugman (1999), and Céspedes *et al.* (2004), and to empiric papers like Tavar (2006). As suggested in Choi and Cook (2004), it is interesting to note that the effect of currency devaluation does not have to be of similar magnitude between financial institutions and private enterprises: if the formers can engage in intense leverage but its assets are fixed in the local currency, then their balance sheets are much more vulnerable to exchange rate fluctuations. Furthermore, Burnside, Eichenbaum, and Rebelo (2001) note that implicit government guarantees on deposits reduces the incentives of the financial institutions to engage in currency hedging, again increasing their exposure.

A second hot topic is that of the “bank–lending channel” (Mishkin, 1995), in which a contractionary monetary policy in the form of open market operations drains reserves and deposits (liabilities) from the banking system, therefore limiting the banks' ability to increase their loans (assets) to the public (Kashyap and Stein, 1995; and Hubbard, 1995). An alternative effect is described in Bernanke and Gertler (1995), where if the Federal

Reserve contracts the money supply and interest rate rises, then it declines the present value of securities, which can be held by banks as capital requirements. Nonetheless, one could argue, as Romer and Romer (1990) do, that a bank's loan supply is neutral from Fed policy, since it can always offset deposit withdrawals by issuing certificates of deposit (CD's). Kashyap and Stein (1994), however, argue that, if bank's balance sheet deteriorates, then the cost of issuing CD's rises. Even more, Kishan and Opiela (2000) and Altunbaş *et al.* (2002) document that, in the event of a monetary contraction, the ability of raising additional funds depends on bank capitalization (leverage ratio) and size (assets); in particular meaning that small undercapitalized banks are the most negatively affected by monetary policy. Additional evidence sustaining the bank lending channel can be found in Bernanke and Blinder (1992), or Kashyap, Stain, and Wilcox (1993), among others.

Another mechanism where banks' liabilities can drop simultaneously is during a bank run. A very familiar example of this situation was experienced in the 2001 Argentine convertibility crisis, where banks were attacked by massive deposit withdrawals, which in turn forced the government to freeze deposits. This particular bank run was intensively analyzed by the literature with the leading documents of Galiani, Heymann, and Tommassi (2003), Calvo, Izquierdo, and Talvi (2003), D' Amato *et al.* (2002), and Gabrielli *et al.* (2003).

A final example of aggregate movements is that of the recent experience of the 2007 subprime crisis. In this case, as pointed out by Cochrane (2010a) the reverse situation occurred in the United States, meaning that overall deposits actually increased, with the exception of Bear Stearns, Lehman Brothers, and Washington Mutual (Morris and Shin, 2008; Brunnermeier, 2009). However, "flight-to-quality" or "flight-to-liquidity" was also a major determinant in the widespread drop of financial assets prices. Therefore, when

banks saw their assets falling, the Federal Reserve took heavy action in flooding the economy with money and ensuring that banks have enough liquidity (Bernanke, 2009a), in part because the traditional funding alternatives were not available, especially after the failure of Lehman Brothers (Gorton and Metrick, 2010; Cochrane, 2010b).

The previous paragraphs tried to review the basic mechanisms widely acknowledged by the literature as ways that monetary policy, macroeconomic shocks, and so, can cause bank's balance sheet to move similarly. However, it is worth noting that the leverage ratio is by no means neutral to macroeconomic shocks. And before explaining why, it is important to note that such a neutral variable, if it existed, wouldn't be suitable either: a variable independent from everything wouldn't provide any economic intuition behind a network. The secret here is to find a variable with economic meaning in terms of bank interconnectedness, but at the same time robust to the economic trend. As Figure 4 showed, plain vanilla leverage happens to be a variable with no obvious trend, but one that is affected by what happens to other banks. Adrian and Shin (2008, 2010, 2011) started a very influential literature on the pro-cyclicality of investment bank leverage ratio. The story goes as follows. Suppose the security dealers and brokers maintain an optimal leverage. If asset prices jump, banks would be holding too much equity, which will lead them to re-leverage by taking on more debt, thus increasing their return on equity. Their leverage would then rise. In downturns, banks are more vulnerable since part of their equity has gone, and they would probably be interested in reducing their exposure. Their leverage would then come down. Adrian and Shin (2011) argue that banks, due to both regulation on banks' capital and the method in which credit rating agencies assign their credit grades, banks are usually interested in targeting a fixed Value-at-Risk (VaR). In particular, they show that leverage can be written as a decreasing function of VaR, which means that if VaR rises in turmoil, banks deleverage. On the other hand, if VaR

decreases on the upside, then banks adjust their leverage upwards. Therefore, targeting a fixed probability of failure causes their leverage ratio to be procyclical, through adjustments in the balance sheet size. This is a very interesting finding, but that doesn't hold for commercial banks –which are present in the sample of this study. Adrian and Shin (2010) show that, although the relation is positive for investment banks, it is neutral for commercial banks. They suggest those banks may be targeting a fixed leverage ratio.

But movements in the leverage ratio can also be a consequence of bank interdependence. Battiston *et al.* (2009) explain that if two banks are involved in interbank lending, a shock to the borrower's assets affecting its debt service probability can also hit on the credit quality of the other bank's loans. In particular, banks will be affected through two distress propagation channels: the first one is the *financial accelerator* (Morris and Shin, 2008). It says that a negative shock to an institution at time t hurts back on $t+1$, deepening the shock. When the borrower is experiencing a hard time, creditors may request its funds back (or deny a debt roll over), forcing the bank to reduce its balance sheet size by selling assets. The second distress propagation effect is referred to as *interdependence* (Battiston *et al.*, 2009). It simply says that if two banks are engaged in interbank lending, a shock to the borrower's assets affecting its debt service probability, can also hurt the market value of the creditor's loans. Thus, bank A's fragility spreads to bank B. In addition, Morris and Shin (2008) and Brunnermeier (2009) explain that leverage is also affected when liquidity dries up in the system. By definition, the ability of a bank to fund itself defines its maximum possible leverage. We already saw that banks may want to increase their leverage on the upside and/or when interest rates are low for at least two reasons: *i*) if moral hazard exists and banks expect an implicit bailout by the Central Bank, they will increase their leverage to maximize the return on equity; *ii*) if other banks start increasing their leverage, competition forces other banks to start doing so. However,

this behavior makes financial institutions very vulnerable to an increase in funding costs: on the downside, haircuts in collateralized borrowing transactions such as repo contracts will typically increase (Gorton and Metrick, 2010). The bank, facing funding shortage, can be forced to sell its assets. Furthermore, if this happens in times of falling prices, fire sales will cause a contraction in equity –leverage rises–, since assets fall by more than liabilities (Chan–Lau, 2010). Therefore fears concerning stability and solvency lead banks to demand higher haircuts or margins in repurchase agreements, which in turn reinforces a generalized tightening in liquidity and interbank lending, again pushing haircuts up, and so on. Brunnermeier and Pedersen (2008) called this contagious phenomenon as the *margin spiral*.

In this regard, given the importance of the leverage ratio but still recalling its high empirical dispersion, a valid question is whether there an optimal fixed leverage ratio. The seminal work of Modigliani and Miller (1958) showed that under certain conditions the value of the firm is irrelevant from the point of view of the capital structure. However, once taxes are added to the equation, things change because interest payments on the debt are tax deductible. Thus, other things equal, the value of the firm is maximized if the leverage ratio is 100%. However, there are some trade–offs involved (Leland, 1994): at some point, as firms leverage up, its bankruptcy risk from a financial distress offsets its gain in firm value through a higher debt interest rate and equity return demanded by debt and equity holders, respectively, and also possibly through a funding shortage. Therefore the cost of capital rises and the value of the institution declines. In addition, the investors' disadvantage from a higher personal income tax can eventually exceed those of corporate taxes, further offsetting the gain from higher debt financing (Miller, 1977). In a nutshell, the optimal leverage is somewhere in between. But the central point here is that *ex-ante* there is no clear indication regarding what the optimal leverage should be.

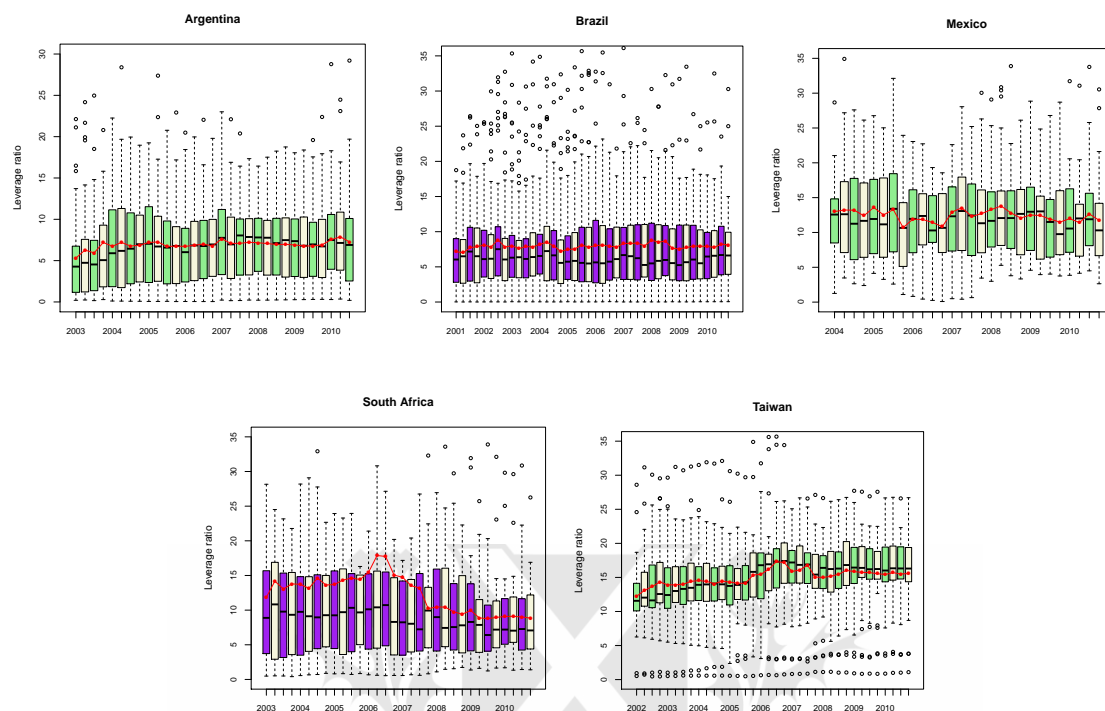


Figure 5: Leverage boxplots on a time basis. Points in red indicate the average in that period. Plots describe an overall relatively stable leverage ratio. The y-axis is limited to 35, and hence greater outliers do not appear.

With these ideas in mind, we can have a detailed look at the evidence. Figure 5 depicts a rather striking dispersion in leverage ratios across banks, a result in itself interesting. One could suggest that size matters: bigger banks may be allowed to sustain more leverage without raising concerns from counterparties. Nonetheless, this hypothesis is not sustained once we separate between big and small banks –size being proxied by total assets. A further step in the analysis of the empirical distribution of leverage is through the CDF. The CDF is plotted in Figure 6 for the last observation of every country –i.e. December 2010. Rather than a power-law –which usually demands data to range more than two decades, and is rejected by the Kolmogorov-Smirnov test–, an alternative distribution is the exponential distribution. Recall that if $X \sim Exp(\lambda)$, then its probability density function is

$$f(x, \lambda) = \begin{cases} \lambda e^{-\lambda x}, & \text{if } x \geq 0 \\ 0 & \text{otherwise} \end{cases} \quad (5)$$

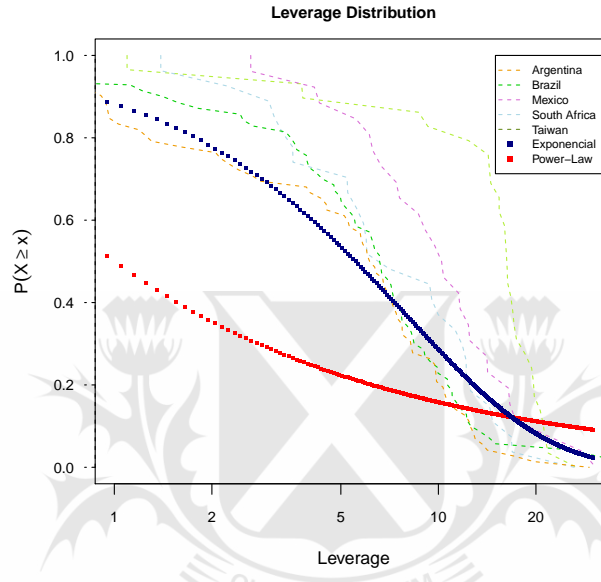


Figure 6: Leverage CDF in December 2010 with x-axis in logs scale. An exponential distribution (in blue) with $\lambda = 1/8$ and a power-law with $\alpha = 1.5$ are added to the plot to assess goodness-of-fit. The plot depicts that the leverage distribution decays more rapidly than both an exponential or power-law distribution.

Applying Maximum Likelihood Estimation in (5) to estimate λ yields $\hat{\lambda} = \frac{1}{\bar{x}}$.

Kolmogorov–Smirnov and Anderson–Darling goodness-of-fit tests (Stephens, 1974) are then applied to test the sample data against the theoretical exponential distribution with the corresponding $\hat{\lambda}$. Results from these tests reject the null hypothesis, further supporting the visual notion from Figure 6 in that the exponential is not a good fit. As observed in Figure 6, the empirical leverage distribution decays more rapidly than a power-law or exponential, therefore suggesting a potential fit with a Log-normal or Gamma distribution.

In summary, this Section has showed the economic importance of the leverage ratio as a robust variable in which a potential network between financial institutions can

be based on. This Section also revealed that, although the absolute level of leverage is very heterogeneous between countries, inside each country the high dispersion and the light-tailed distribution are common findings. The next Section will engage in the construction of a leverage-based network.



3 Leverage Networks

Once begun, a financial crisis can spread unpredictably.

Bernanke (2009b)

Using leverage panel data at individual bank level for Argentina, Brazil, Mexico, South Africa, and Taiwan, the network is constructed as follows. After accounting for mergers and acquisitions, banks with incomplete observations are deleted from the sample—a condition that is necessary for correlation-based networks (Tumminello *et al*, 2007). Using leverage time-series data we calculate all possible cross-correlations according to Pearson’s sample correlation coefficient

$$\rho_{i,j} = \frac{(T^{-1}) \sum_{t=1}^T (x_{i,t} - \bar{x}_i)(x_{j,t} - \bar{x}_j)}{\sqrt{(T^{-1}) \sum_{t=1}^T (x_{i,t} - \bar{x}_i)^2} \sqrt{(T^{-1}) \sum_{t=1}^T (x_{j,t} - \bar{x}_j)^2}}$$

which is the sample version of the population correlation coefficient defined as

$$\tilde{\rho}_{i,j} = \frac{E[x_i x_j] - E[x_i] E[x_j]}{\sqrt{E[x_i^2] - E[x_i]^2} \sqrt{E[x_j^2] - E[x_j]^2}}$$

where $x_{i,t}$ and $x_{j,t}$ are the leverage ratios of banks i and j at time t in the same country.

This procedure results in a correlation matrix $\psi_{N \times N}$. Recall that $\rho_{i,j} \in [-1, 1]$, and that $\rho_{i,j} = 1$ when $i = j$; also that ψ is both square and symmetric, with 1 in the main diagonal.

Therefore the relevant information is stored in $\frac{N(N-1)}{2}$ different pairs, where N denotes the number of banks in the country. In order to measure the similarity between two banks, a threshold on $\rho_{i,j}$ is used as a filtering procedure to distinguish association levels.

A negative correlation between bank i and bank j implies that when bank i increases its leverage, the opposite is true for bank j , and vice versa. And a positive correlation between banks i and j implies that both increase or decrease their debt-to-equity ratio

simultaneously. Notice that the economic mechanisms behind both cases are very different. In the first case, a possible economic rationale would be that of the *fly-to-safety*: in times of worries, people might withdraw their deposits from a small bank, and transfer it to a bigger (perceived as “safer”) bank. Thus liabilities decrease in the small bank, but increase in the other bank. As documented by Oliveira *et al.* (2011), this effect occurred in Brazil in late 2008, with depositors running from smaller banks to larger banks, the latter ones perceived as too-big-to-fail institutions with implicit guarantees from the Central Bank. An alternative possible mechanism comes from what the literature calls the *depositors discipline*: depositors may punish banks with poor performance either by demanding higher interest rates or by withdrawing their deposits. In that regard, Martínez-Peria and Schmukler (2001) find evidence in Argentina, Chile, and Mexico of the depositors discipline mechanism, in particular that bank deposits growth falls as risk exposure increases. In this line of research, Maechler and McDill (2006), and Imai (2006), for example, find that depositors favor big banks, although it is hard to differentiate whether depositors base their decisions on too-big-to-fail sentiments or bank fundamentals. Levy Yeyati *et al.* (2004) also document that during the 2001 Argentine convertibility crisis deposit withdrawals were more pronounced in banks with higher risk taking, thus providing further evidence of negative (but uneven) changes in balance sheet accounts across banks. See Flannery (1998) for a review of the U.S. literature on market discipline. Another possibility, though less plausible, is that since the panel data omits information from banks with incomplete data (such as bankrupt or acquired banks), if assets are falling in the overall economy and bank A acquires bank B, bank A would exhibit stronger financial ratios in a time where the rest are having a hard time. This could explain an inverse movement in the leverage ratios. This situation however is not very likely to occur many times, thus its influence on the correlation coefficient should be secondary.

A very different case arises if the correlation coefficient on the leverage ratio between two banks is positive. Under this scenario a common dynamic affecting their balance sheets is more in line with the theories of leverage procyclicality and/or bank lending channels, both of which were discussed in Section 2.

Thus it is evident that both scenarios (negative and positive $\rho_{i,j}$) should be incorporated in the filtering procedure. This results in two different networks: *i*) we establish that there is a link between bank i and j whenever the sample correlation is *greater* than a certain positive threshold τ^+ ; and *ii*) we establish that there is a link between bank i and j whenever the sample correlation is *lower* than a certain negative threshold τ^- . As we shall see later, this threshold is allowed to vary between τ_{\min}^+ and τ_{\max}^+ in case *i*), and between, τ_{\min}^- and τ_{\max}^- in case *ii*), each of these thresholds leading to a different network. In particular, we set τ to be $[-0.85, -0.1] \cup [0.1, 0.85]$. It is important to emphasize that both intervals are analyzed separately, that is, the procedure considers the negative correlation as a different network from the positive correlation-based network. With the aim of using more friendly names, we shall refer to case *ii*) as the *non-herding* network, and to case *i*) as the *herding* network. In what follows we will continue the construction for the positive case only, which can trivially be translated to the negative case.

Let a be a binary variable such that whenever $\rho_{i,j} > \tau$, $a_{i,j} = 1$, indicating there is an undirected link between the vertices i and j . If not, then $a_{i,j} = 0$. The only exception is if $i = j$, in which case $a_{i,j} = 0$. More succinctly,

$$a_{i,j} = \begin{cases} 1 & \text{if } \rho_{i,j} \geq \tau, i \neq j \\ 0 & \text{if } \rho_{i,j} < \tau, i \neq j \\ 0 & \text{if } i = j \end{cases} \quad (6)$$

Each of these $a_{i,j}$ are stored in an adjacency matrix $A_{N \times N}$, which can fully rep-

represent an undirected graph. Notice that A is also a symmetric matrix, for $a_{i,j} = a_{j,i}$, with 0 in the main diagonal. If $a_{i,j} = 1$, then bank i and bank j (vertices or nodes of the network) are said to be adjacent or neighbours. Notice that the edge is undirected, meaning that we can't say in which direction the link is being established. This is different from interbank lending networks (e.g. Boss *et al.*, 2004; or Iori *et al.*, 2008), where particular information regarding the lender and borrower bank can be used to establish a directed edge following the flow of the money. It is also possible to construct weighted edges (in the previous example, edges can be weighted according to the amount of the interbank loan). For simplicity, this thesis only considers undirected and unweighted edges.

Once $\tau \in [\tau_{\min}^+, \tau_{\max}^+]$ is determined, the network is defined. Thus it seems that the correlation threshold is a key parameter. To see why, think that a very low τ will result in a highly connected network, whereas a high τ results in the opposite. Intuitively, consider the extreme cases: if we impose a very low $\tau = 0.01$, the soft filtering procedure will be assigning a huge number of edges. For instance, two uncorrelated banks, due to mere chance alone, will be connected by a link. On the other hand, if we impose $\tau = 0.99$, it is clear that only very few banks (if any) will be so highly correlated.

This argument can be perceived in Figure 7, which shows the dispersion of the $\frac{N(N-1)}{2}$ cross-correlation pairs using the traditional boxplot tools (Panel A), and the *average vertex degree* $\langle k \rangle$ of the network as a function of the threshold correlation, respectively (Panel B). Where $\langle k \rangle$ is simply defined as

$$\langle k \rangle = \frac{1}{N} \sum_{i=1}^N k_i = \frac{1}{N} \sum_{i \geq j}^N a_{ij}$$

where k_i is the degree of vertex i . In other words, k_i is the number of edges reaching vertex i (one can also think of the number of neighbours), and then $\langle k \rangle$ is the average over all vertices. By definition, there is no edge between vertex i and itself. And

since $k_i \in [0, N - 1]$, then $\langle k \rangle$ also ranges from 0 to $(N - 1)$. With these ideas in mind, one can anticipate that, starting from τ_{\min} , as we *increase* τ the degree of the network should be *decreasing*, for existing links are now being removed.

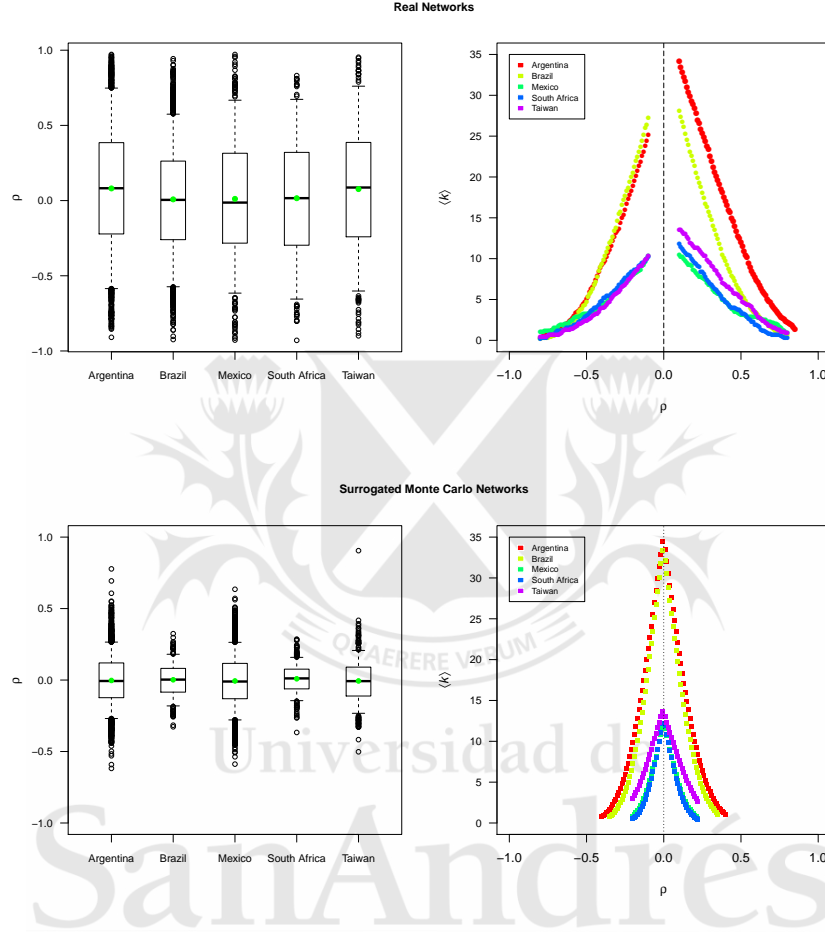


Figure 7: Panel A depicts real cross-correlations boxplots in each country, obtained from all elements above (or below) the main diagonal of the correlation matrix. Panel B shows $\langle k \rangle$ as a function of correlation threshold τ : as we allow a lower (in absolute value) correlation to establish an edge, more links are added to the network, and hence connectivity is increased. Panel C and D repeats for one surrogated realization, and the Monte-Carlo simulated data, respectively. Green dots in boxplots indicate average correlation.

To convince ourselves that results are not driven by spurious correlations, for every calculation using real data, we additionally provide results from shuffled or surrogated data at the country level. This method can be seen as a combination of bootstrap sampling with no replacement at the country-time level and Monte Carlo methods. This serves both as a robustness check as well as a comparison basis: one can compare the real banking

system with what one could expect from a random system. The *random* (or surrogated) case is constructed using the following algorithm. ²

Let $\theta_{x,i}$ be the time-series (ordered) vector of the leverage ratio for bank i from country x . Thus $\theta_{x,i} = [\lambda_{i,1}^x, \lambda_{i,2}^x, \dots, \lambda_{i,T}^x]$, with $t \in [1, T]$ and $i \in [1, N]$. And $\lambda_{i,1}^x$ for example denotes the leverage ratio (Debt-to-Equity) for bank i from country x at time $t = 1$. In our sample, $x = \{\text{Argentina, Brazil, Mexico, South Africa, Taiwan}\}$.

1. Create a new surrogated vector $\theta_{x,i}^*$ by randomly shuffling the observations in the real $\theta_{x,i}$. This step can be seen as a *bootstrap* re-sampling with no replacement. Notice that the observations in $\theta_{x,i}^*$ are exactly the same as in $\theta_{x,i}$, but with a random order.
2. Repeat Step 1 for every bank in country x .
3. Take the surrogated collection $\theta_{x,1}^*, \theta_{x,2}^*, \dots, \theta_{x,N}^*$ and calculate a new correlation matrix ψ_x^* between every two pairs of banks.
4. Apply the above mentioned filtering procedure to ψ_x^* . For every threshold in $[\tau_{\min}^-, \tau_{\max}^-] \cup [\tau_{\min}^+, \tau_{\max}^+]$ a network is obtained, from which one calculates a given metric, e.g. $\langle k \rangle^*$. Since the time-series vector is now random, a softer threshold must be imposed. Typically we set $\tau \in [-0.35, -0.01] \cup [0.01, 0.35]$.
5. Repeat Steps 1-4 R times, and store the metric obtained in each simulation when setting a given τ . In this study the number of simulations $R = 100$.
6. End by Monte Carlo principles: for every metric obtained in Step 4, calculate its average at the threshold level over all simulations. For example, take the collection $\langle k^\tau \rangle^* = \{\langle k_1^\tau \rangle^*, \langle k_2^\tau \rangle^*, \dots, \langle k_R^\tau \rangle^*\}$, where this vector stores the metric $\langle k_r^\tau \rangle^*$ obtained in simulation $r \in [1, R]$ by setting correlation threshold τ . And finally, averaging

²For additional information on Monte Carlo and Bootstrapping methods see Robert and Casella (2004).

one obtains the surrogated Monte Carlo metric $\langle k^\tau \rangle^* = \frac{\sum_{r=1}^R \langle k_r^\tau \rangle^*}{R}$. This is done for every metric showed in this project.

An example of this procedure can be seen in the lower Panels of Figure 7, which confirm two basic notions: that the correlations for the surrogated case are more intensively concentrated around 0, and that it takes a considerable lower correlation threshold for the network to obtain a given $\langle k \rangle$. Kernel-based (Terrel and Scott, 1992) correlation density estimates given in Figure 20 in Appendix C reinforce the difference in the correlation distribution peakedness.

From the above Figures it is clear that thresholds τ are not interchangeable across countries, meaning that in order to compare the networks it is not fair to establish a uniform τ in all countries: a low correlation level in country A might be very high for country B standards, or vice versa. This is similar that what we saw in Panel B of Figure 7, where, for example, it takes a $\rho = 0.4$ in Argentina to obtain a $\langle k \rangle = 10$, whereas it takes a $\rho = 0.1$ in South Africa to reach the same average vertex degree. It is also interesting to note that the network is not symmetric to the correlation threshold: for a given $\langle k \rangle$ the *non-herding* case always needs a softer barrier than the *herding* case. This finding gains more power once we compare it with the shuffled networks: the positive and negative correlation scenarios are a mirror of the other.

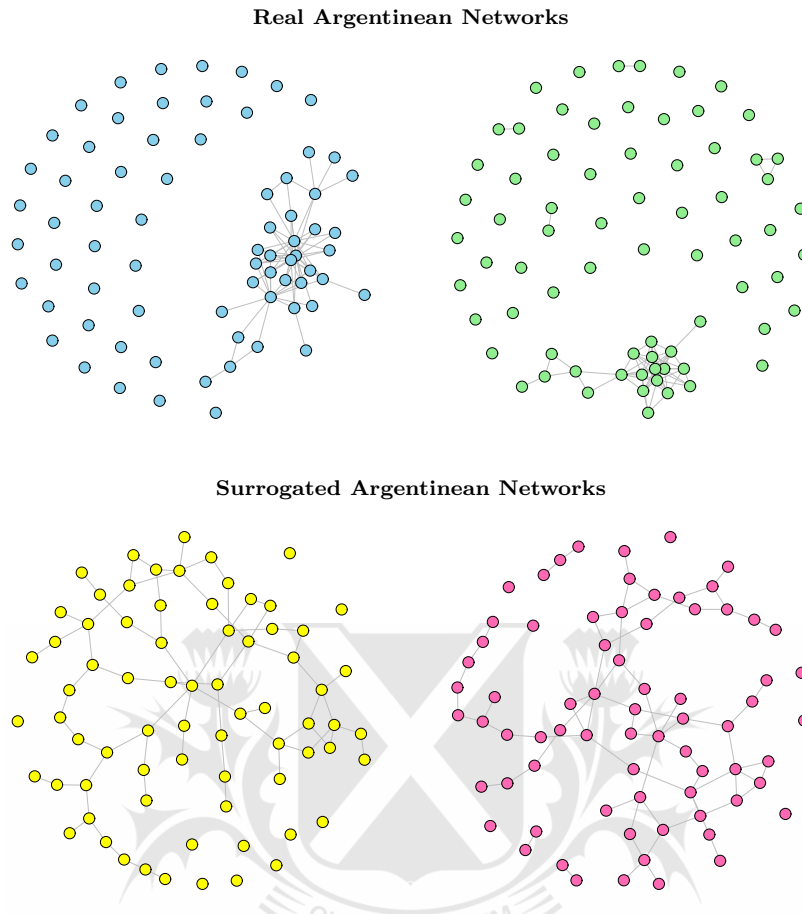


Figure 8: A graphical representation of the Argentinean networks when $\langle k \rangle = 2$. Upper left Panel depicts real network by setting $\tau^- = -0.63$; Upper right Panel the case of $\tau^+ = 0.82$; Lower left and right Panels depict one realization of a surrogated network: $\langle k \rangle = 2$ is achieved by setting $\tau^- = -0.34$ and $\tau^+ = 0.34$.

Figure 8 shows the graphical representation of the undirected Argentinean network with $\langle k \rangle = 2$. For this to happen, the filtering procedure must set $\tau^- = -0.63$ and $\tau^+ = 0.82$ for the negative and the positive case, respectively. It is found a remarkable difference between real and random: in the former networks, the presence of hubs dominate the system, whereas in the random networks vertices tend to have homogeneous degree. These graphs provide evidence that higher connectivity may be desirable to increase risk sharing and reduce hub exposure. In other words, if a low-degree bank is attacked, its influence in the overall network might be negligible, whereas if a hub fails it can cause an entire breakdown in the system. Neither heavy leveraged banks (whose leverage exceeds the 0.8 sample quantile at $t = 1$) nor big banks (whose total assets exceed the 0.8 sample

quantile at $t = 1$) account for high degree vertices. In other words, the emergence of *hubs* (heavily connected nodes) in the system is not directly related to their size or their ability to leverage up over their peers. Hubs are a mixed combination of small and big banks, and strongly and moderately indebted banks. This effect is also found in the other countries to the constraint $\langle k \rangle = 2$. Notice that although Figure 8 might illustrate a similar behavior, the underlying economic dynamic is completely opposite. Recall that in the *non-herding* case there exist a few banks heavily linked that do the *opposite* of the majority of the banks, whereas in the *herding* case banks copy each other. As rational agents there must be some reason behind the polar dynamic. Unfortunately, the data available does not permit to tell which specific activity is triggering the polar relationship between A and B, but it is a finding that should be continued in further studies using interbank financial data.

Once the similarities networks are constructed, we are interested in describe and compare them. For instance, does connectivity increase or decrease systemic risk? As explained in Battiston *et al* (2009) and Minoiu and Reyes (2011), there isn't a monotonic decreasing relationship between systemic risk and connectivity. The economic rationale is that there is a trade-off involved between shock absorption and shock diffusion: as connectivity increases, individual (idiosyncratic) risk declines thanks to risk sharing, but systemic risk may rise due to higher propagation. One could think that a very unconnected network is somewhat sensible when hubs are shocked, and thus greater interconnectedness may be safer. But as more and more edges are drawn in the network, it might reach the point where its degree is so high that a single attack may spread through the entire system.

In order to address these sorts of things it is necessary to describe the topological properties of the networks. The classic metrics are the average path length, clustering

coefficient, assortativity coefficient, degree distribution, entropy, density, among others (Caldarelli, 2007). However, many of these metrics are not neutral to sample size, thus leading to wrong conclusions if one compares same metric between two networks of different size. A very simple way to illustrate that size-dependence is present is by constructing random networks with increasing size. Here we consider two classic models of random networks: Erdős and Rényi (1960), and Barabási and Albert (1999). For a friendly discussion of these (and other) random network models see for example Caldarelli (2007).

But first we must introduce the basic mathematics of graph theory. The number of links connecting vertices i and j is called the *length* of the path. From those possible paths, the shortest one between them is called the *geodesic* path. And its length is the *geodesic distance* $d_{i,j}$. Thus the average geodesic distance is just the average over those $d_{i,j}$. In particular,

$$\ell = \frac{1}{N(N-1)} \sum_{i \neq j} d_{i,j} \quad (7)$$

where $d_{i,j}$ is the shortest distance from vertex i to vertex j . In other words, it is the lowest number of edges (or bridges) that vertex i has to cross to reach vertex j . Notice that formula (7) omits the distance from one vertex to itself. A correction to formula (7) must be done if $d_{i,j} = 0$ with $i = j$ is included in the sum (Newman, 2003). Since $d_{i,j} = \infty$ if there is no actual path between vertices i and j , then ℓ diverges. To circumvent this issue only vertices with existing path are included in the sum. However, in highly disconnected networks this corrected coefficient will be small, thus not reflecting the actual average distance, for it is only including a few vertices. Fortunately, there is an alternative distance measure which derives from the *global efficiency* metric defined by Latora and Marchiori (2001) as

$$E = \frac{1}{N(N-1)} \sum_{i \neq j}^N \frac{1}{d_{i,j}} \quad (8)$$

Since this variable measures how efficiently information is exchanged between vertices, the maximum value occurs when the graph has all the $\frac{N(N-1)}{2}$ possible edges. Since $\frac{1}{d_{i,j}} = 0$ when no path connects vertices i and j , the alternative geodesic distance called the *harmonic mean* (Marchiori and Latora, 2000) is defined as follows

$$h = \frac{1}{E} = \frac{N(N-1)}{\sum_{i \neq j}^N \frac{1}{d_{i,j}}} \quad (9)$$

Another interesting coefficient is that of *betweenness centrality*, which tries to identify the importance of a vertex (Freeman, 1977). If a certain node serves as a bridge to connect two separate clusters, then its function to transfer information inside a network is essential. The betweenness (or load) point centrality of node i is defined as

$$b_i = \sum_{j \neq u}^N \frac{\eta_{j,u}(i)}{\eta_{j,u}} \quad (10)$$

where $\eta_{j,u}$ is the number of shortest paths connecting vertices j and u , and $\eta_{j,u}(i)$ is the number of those paths that pass through node i . Notice that since b_i is a proportion, $0 \leq b_i \leq 1$. Using formula (10) Freeman (1977) also defines a global measure of dominance of the most central point as

$$C'_B = \frac{1}{N-1} \sum_{i=1}^N (b^* - b_i) \quad (11)$$

where b^* is the maximum value of betweenness centrality in the graph.

Another best-seller metric from graph theory is the *clustering coefficient*. There are two well-known definitions. The first variant, proposed by Watts and Strogatz (1998),

starts by defining a local clustering for each vertex as

$$c_i = \frac{N_\Delta(i)}{N_3(i)} \quad (12)$$

where $N_\Delta(i)$ is the number of *triangles* involving vertex i , and $N_3(i)$ is the number of connected triples centered on vertex i . Put differently, since a triangle is simply a set of three vertices fully connected (three edges), then $N_\Delta(i)$ is the number of direct links between neighbours of node i . In particular,

$$N_\Delta(i) = \sum_{j>u} a_{ik}a_{iu}a_{ju}$$

$$N_3(i) = \sum_{j>u} a_{ik}a_{iu} = \frac{k_i(k_i - 1)}{2}$$

Thus C_i is the ratio of actual links between the neighbours of i to possible links between these neighbours, and $0 \leq c_i \leq 1$. By definition, in cases where k_i is either 0 or 1 (causing $c_i = \frac{0}{0}$), $c_i = 0$. The clustering coefficient is obtained by taking the average over these local c_i :

$$\bar{C} = \frac{1}{N} \sum_i^N c_i \quad (13)$$

and again, $0 \leq \bar{C} \leq 1$.

The second clustering coefficient definition originally proposed by Barrat and Weigt (2000) and then extended by Newman (2003) is often referred to as *transitivity*, and is calculated as

$$\tilde{C} = \frac{3N_\Delta}{N_3} \quad (14)$$

where N_{Δ} is the number of triangles in the network, and N_3 is the number of connected triples of vertices. \tilde{C} can be understood as the mean probability that any two vertices (neighbours of the same node) are themselves neighbours. Consider the following example from social networks: if A is friend of B and B is friend of C, then a high \tilde{C} will indicate a high probability that A and C are also friends. The 3 in the numerator of (14) assures that $0 \leq \tilde{C} \leq 1$.

A couple of paragraphs above we said that some of these metrics are size-dependent, and now it's time to show it. In the random model of Erdős-Rényi one sets a number of vertices N and an edge is placed between each pair with a certain probability p . The simulation here is done by increasing N but holding constant the mean degree $\langle k \rangle = p(N - 1)$. We shall see later that degrees in this method end up following a Poisson distribution. The Barabási-Albert model, on the other hand, is based on the ideas of growth and preferential attachment, meaning that, starting with a set of nodes, as new nodes are added to the network, high-degree vertices will establish new links at faster rates than their low-degree peers. The network is constructed as follows: start with a (small) number m_0 of vertices. At each time step, a new node is added with m edges. In other words, this new node is linked to m existing vertices in the system. In order to capture the idea of preferential attachment, the probability that the new vertex will establish a edge with vertex i is linearly proportional to k_i . In particular, $p = \frac{k_i}{\sum_j k_j}$. This step is repeated t times, leading to a random network with $t + m_0$ vertices and mt edges, and a mean degree $\langle k \rangle = 2m$ once the network has stabilized. The latter is true for large t because each new vertex brings together m undirected edges, which count double. One of the attractiveness of the Barabási-Albert model is that the degree distribution ends up being a power-law with exponent $\alpha = 3$. And the network is said to be a scale-free system because the distribution is *time-independent*.

Erdos–Renyi and Barabasi–Albert Random Network Models

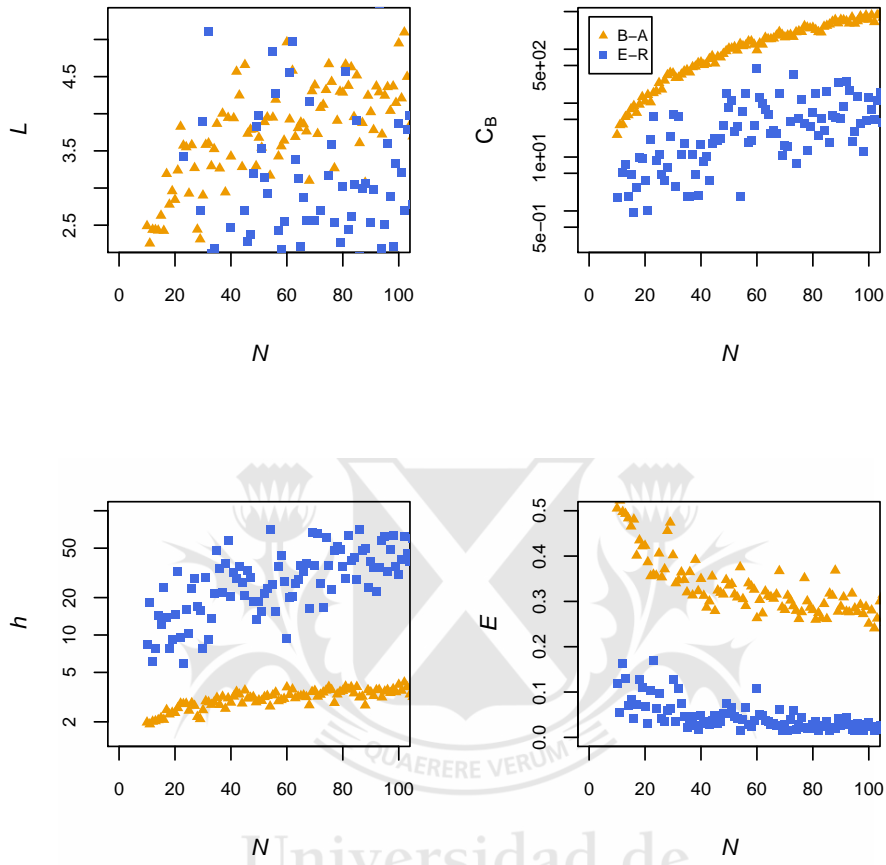


Figure 9: Erdos-Renyi random model (in blue) and Albert and Barabási random model (in orange). From left to right, these panels illustrate that metrics such as L , *Central Point Dominance* (y-axis in log scale), *Harmonic mean* (y-axis in log scale), and *Efficiency* are sensible to network size N .

The above graphs depict that some metrics do depend on network size. For that reason, in this study it is more appropriate to compare Argentina ($N = 72$) and Brazil ($N = 70$) in one side, and Mexico ($N = 26$), South Africa ($N = 27$), and Taiwan ($N = 29$) on the other side.

As we saw a few lines ago, the harmonic mean circumvents the problem when there are many unconnected nodes in the network. Since h is the reciprocal of *global efficiency* (Latora and Marchiori, 2000), an interpretation of Figure 21 in Appendix C is that the efficiency in sending information between any two vertices is very similar in the

negative and positive ρ scenarios. In other words, a network in which nodes connect each other by doing the opposite of their neighbours do not defer in terms of efficiency from a network where nodes *herd* each other. In fact, Figure 21 shows that the efficiency is very similar from simulated networks as well. However, although the harmonic distances may be similar, the vertex degree dispersion measured by its standard deviation is not.

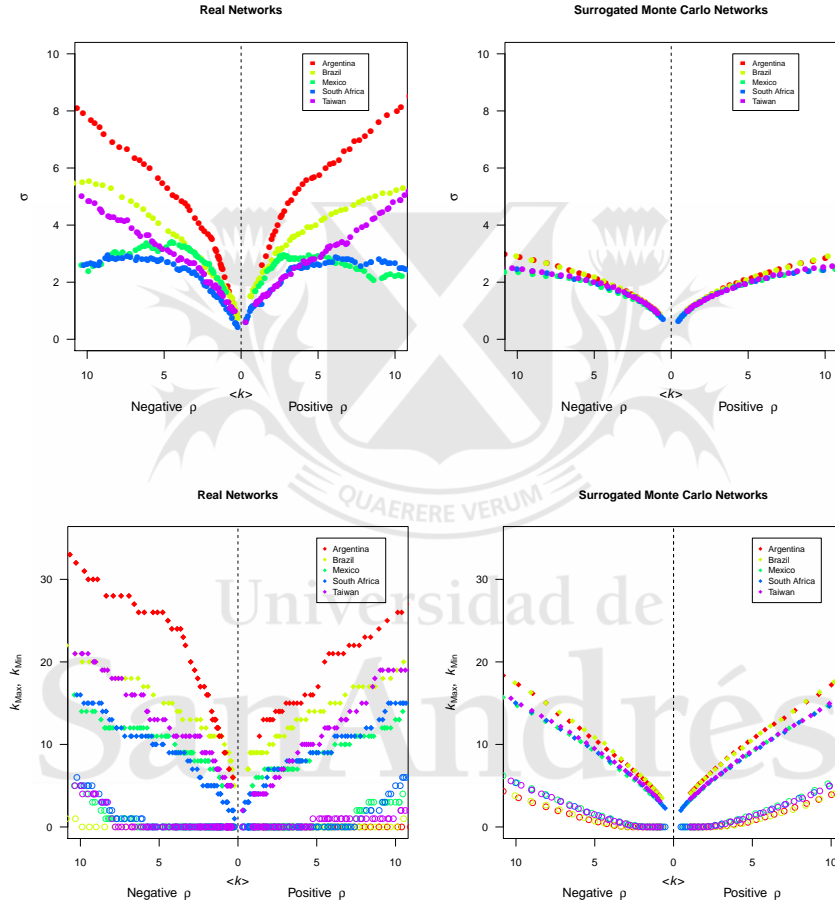


Figure 10: Upper panels depict the standard deviation as a function of $\langle k \rangle$. Real networks exhibit considerable higher dispersion than the simulated networks. Since the lowest degree vertices are similar across countries, lower panels show that the difference in node connectivity is due to hubs. In particular, Argentina stands over Brazil as having a highly uneven bank connectivity; and Taiwan stands over Mexico and South Africa for same reasons.

Upper Panels in Figure 10 illustrate that if a network of this kind were to be a consequence of chance, then it should exhibit a much more stable and lower dispersion. Another rather surprising result is the difference in hubs between Argentina and Brazil:

in spite of having a very similar size N , for a network with given $\langle k \rangle$ the vertex degree is much more uneven in Argentina, with some banks concentrating most of the edges in the system. We know that this difference is not due to different k_{\min} (because they are similar, and close to zero), but because the hubs are stronger in Argentina. A similar pattern appears in Taiwan relative to South Africa and Taiwan. Notice again that all these patterns are lost in a spurious network: actually, it is impossible to disentangle any difference *inside* both subgroups of countries.

Previous findings talk about the parameters of the degree distribution but nothing concrete on the distribution itself. The existence of *hubs* in a system indicate that a heavy-tailed distribution is needed to support the fact that there exists a non-trivial probability of finding nodes intensively linked. One distribution that satisfies that is a power-law or Pareto distribution. This effect has been shown to appear in many situations: in the hyperlink connections in the World Wide Web (Barabási and Albert, 1999), number of sexual partners (Liljeros *et al.*, 2001), number of scientific papers connected by citations (Redner, 1998), science collaboration, where two authors are connected if they have written a paper together (Newman, 2001a, 2001b, 2001c), and even in Hollywood movie networks, where two actors connect if they casted together in the same movie (Watts and Strogatz, 1998; Barabási and Albert, 1999; Albert and Barabási, 2000). A list of other degree power-law examples can be consulted, for example, in Albert and Barabási (2002).

It is interesting to note that although this distribution may emerge in real networks, Erdős and Rényi (1959) showed that the degree k_i of node i in a random network (with no preferential attachment) follows a binomial distribution:

$$p(k_i = k) = \binom{N}{k} p^k (1-p)^{N-k} \quad (15)$$

where N is the number of vertices, and p is the probability of establishing a link between any pair of vertices are connected. By that setting $\langle k \rangle = p(N-1)$ constant, in the limit of large N , $p(k_i = k)$ approximates

$$p(k_i = k) \rightarrow \frac{\langle k \rangle^k e^{-\langle k \rangle}}{k!} \quad (16)$$

which is a Poisson distribution with expected value $\lambda = \langle k \rangle$.

With this literature backup we wonder whether the degree distribution of the leverage-based networks also follows a power-law dynamic. What we know beforehand is that the degree distribution in the surrogated Monte Carlo network should approximate a Poisson distribution. Figures 11-12 depict the CDF of the real networks, along with their corresponding Monte Carlo simulated version. Overplotting in Figure 13 a Poisson distribution with expected value λ equal to sample $\langle k \rangle$ provides robust evidence that banks' degree in the surrogated networks indeed follow a Poisson distribution. In addition, both non-parametric Kolmogorov-Smirnov and Anderson-Darling tests do not reject the hypothesis of the Poisson distribution. What regards to the real network, although a geometric distribution is rejected, Kolmogorov-Smirnov methods for fitting power-law distributions (Clauset *et al.*, 2009; Goldstein *et al.*, 2004; and Newman, 2005) fail to detect a Zipf's law. However, the power-law null hypothesis might be rejected because the test is not taking into consideration finite size effects, a specific problem that should be addressed in further studies with alternative methodologies. Intuitive results in the form of CDF plots for the Argentinean case appear in Figure 13.

Notice that we are fitting a geometric distribution because it is the analogue of

the exponential distribution but for discrete values. The probability density function is given by

$$P(X = k) = (1 - p)^{k-1}p$$

for $k > 0$. But a correction must be made to the usual formula because the degree data can (and certainly does) take $k = 0$, and not taking this into account would be incomplete. The modified PDF is $P(X = k) = (1 - p)^k p$, for $k \geq 0$; and its CDF is $P(X \geq k) = (1 - p)^{k+1}$, for $k \geq 0$, which is what is plotted in Figure 13. Since $E[X] = \frac{1-p}{p}$, p in the simulation is chosen to match the desired mean degree $\langle k \rangle = 2$ and $\langle k \rangle = 8$. It can be observed that the geometric distribution decays faster than the empirical degree CDF, therefore providing evidence of a heavy-tailed degree distribution. This finding further supports both the importance of a potential scale-free nature and that extremely connected vertices can be found with non trivial probability.

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Argentina and Brazil: Degree Distribution for $\langle k \rangle=2$ and $\langle k \rangle=8$

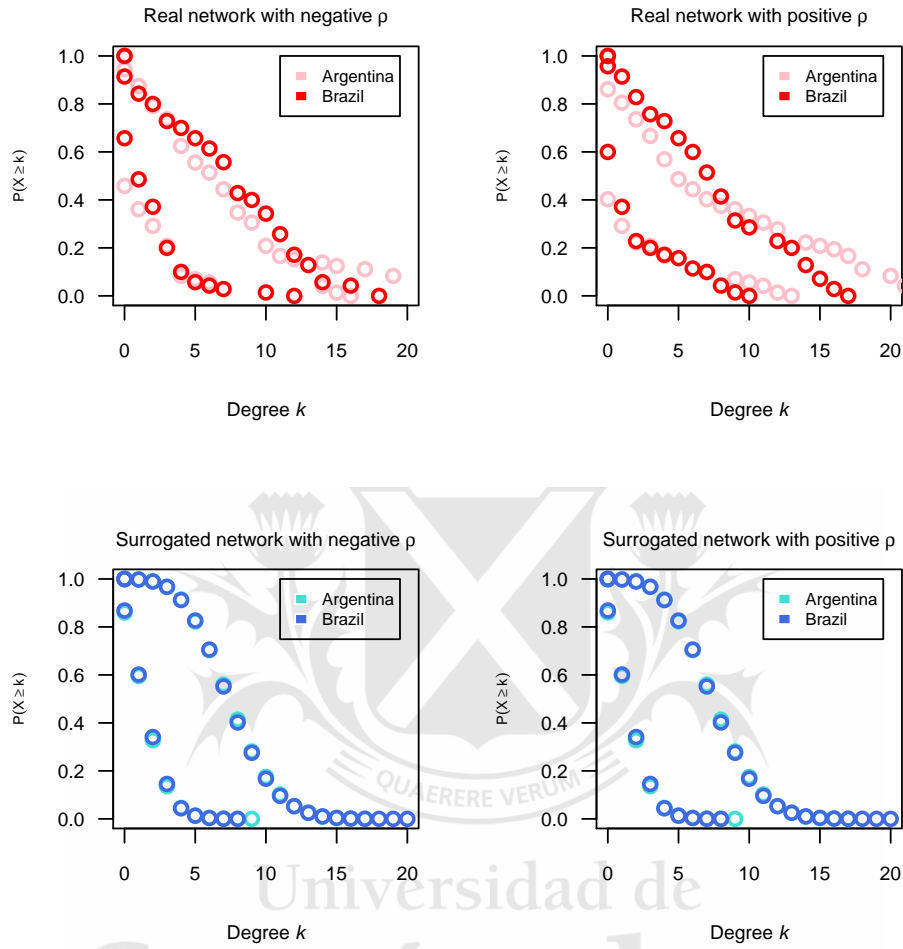


Figure 11: Cumulative distribution function in real (upper Panels) and random (lower Panels) Argentinean and Brazilian networks.

Mexico, South Africa, and Taiwan: Degree Distribution for $\langle k \rangle = 2$ and $\langle k \rangle = 8$

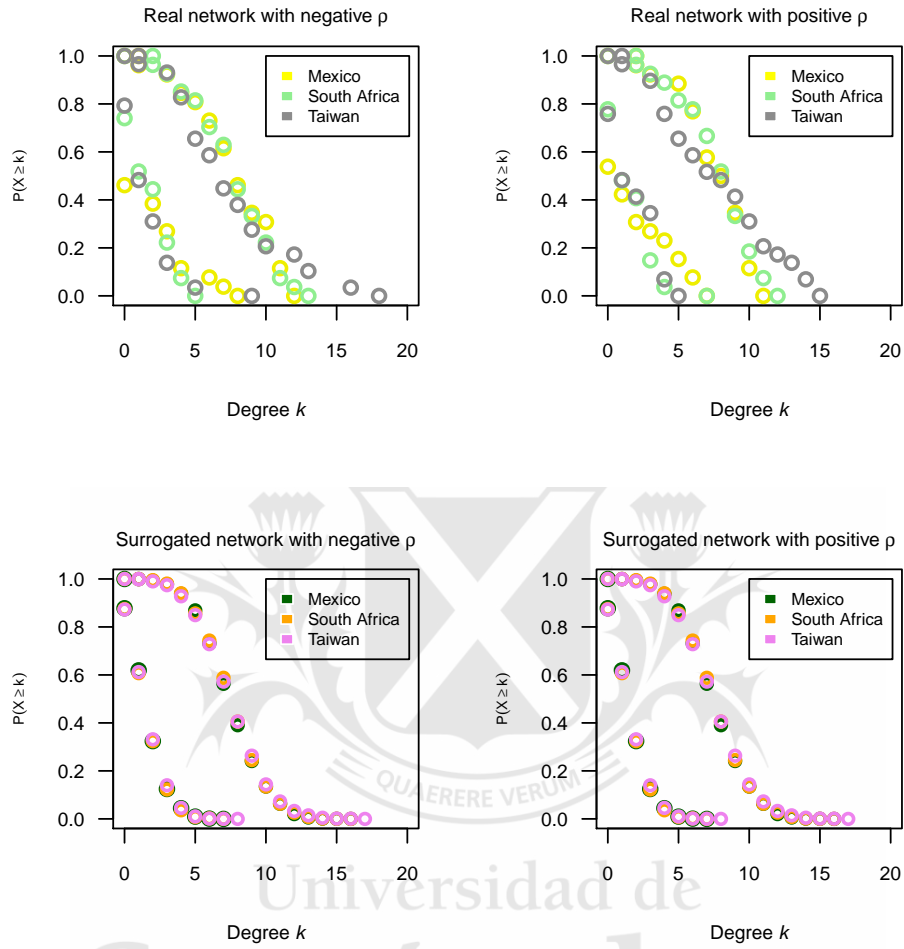


Figure 12: Cumulative distribution function in real (upper Panels) and random (lower Panels) in Mexican, South African, and Taiwanese networks.

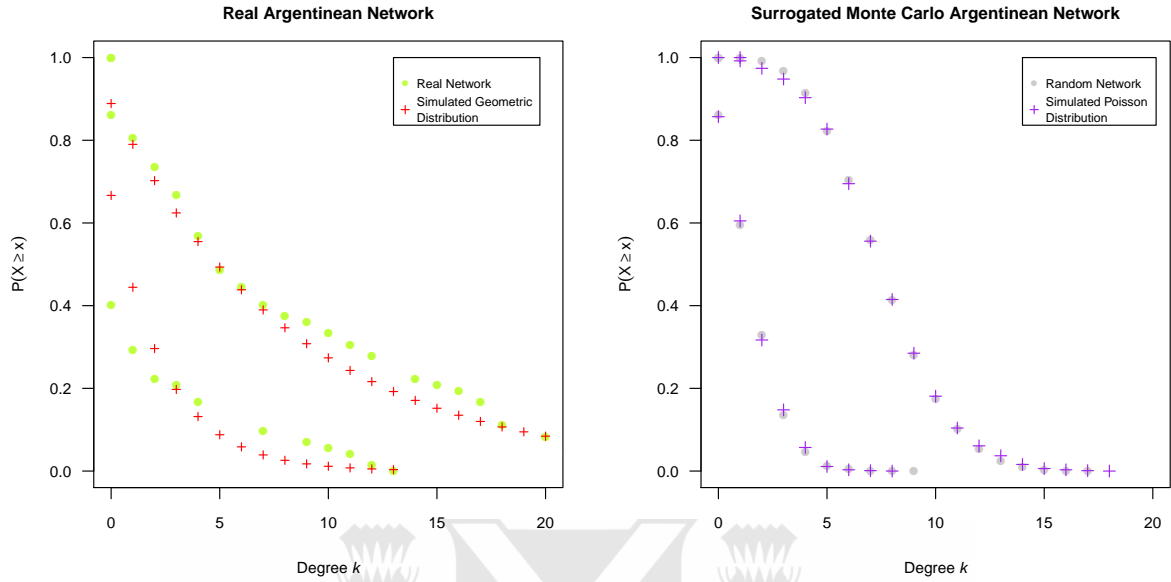


Figure 13: Degree CDF of the Argentinean real (left Panel) and random (right Panel) networks overplotted with a Geometric and Poisson distribution, respectively. Heavy-tail pattern in real networks rejects a Geometric distribution. As predicted, in the random case the Poisson is a perfect fit.

As previously discussed, an interesting measurement is the clustering coefficient of a network, this one being a measure of “cliquishness” (Watts and Strogatz, 1998): the extent to which neighbours of i are themselves neighbours to each other. By definition, recall that the dynamic inside a *non-herding* network is completely different from a *herding* network, and this will be reflected in an asymmetric clustering coefficient: for a given $\langle k \rangle$, $\tilde{C}_{\rho^-} < \tilde{C}_{\rho^+}$. That is, banks j and u linking bank i will have much higher chance of being neighbours in the positive correlation-based network than in the negative correlation scenario. Why is that? Consider the following example: imagine a small network ($N = 3$) constructed with the positive correlation filtering procedure. If bank A and B are both neighbours of C, it must be because $\rho_{A,C}$ and $\rho_{B,C}$ are both greater than τ^+ . But if A and C move together, and B and C also move together, couldn’t we guess that $\rho_{A,B}$ is also somewhat significant? Now imagine the same network but constructed by applying

the negative correlation filtering procedure. If there is a edge connecting banks A and C, and another edge between B and C, then it must be because whenever C leverages up, A and B deleverages, and vice versa. Thus, though it is possible (e.g. provided a very soft threshold is imposed), it is very unlikely that A and B are also connected (for it will be almost “contradictory”). In summary, in terms of the clustering coefficient it simply means that this parameter will be higher in the *herding* network. Figure 14 confirms the previous intuition by examining at the numerical value of the clustering coefficient \tilde{C} as defined by Newman (2003), which corresponds to the one of equation (14). Results remain practically identical if one uses the \bar{C} of equation (13) proposed by Watts and Strogatz (1998).

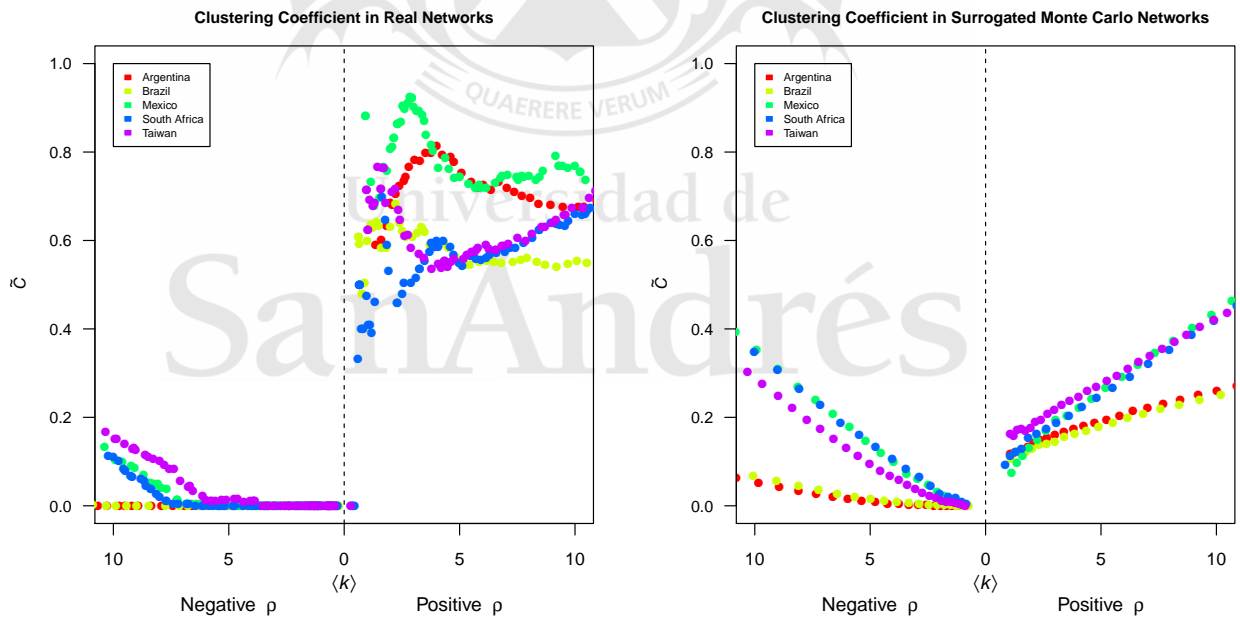


Figure 14: Panel A depicts clustering coefficient in real networks; Panel B depicts clustering coefficient in simulated networks.

From Figure 14 one can extract two interesting results. The first one is the surprising high *transitivity* in the Mexican banking network, as opposed to South Africa and Taiwan. A similar thing occurs in Argentina, whose transitivity is always higher than

Brazil. The second result is that this behavior cannot be explained by chance, because the pattern in the simulated random networks are entirely different. In particular, not only should transitivity be the same for same-size networks, but it should also be considerably *lower*. This comparison thus says that the pattern in the left Panel is indeed providing key information regarding the intrinsic dynamic inside the network. We can't know in which direction causality is going, but we can say that in these compact or condensed networks (in particular Argentina and Mexico) propagation channels should be of policy interest. Identifying critical nodes can be crucial to reduce rates of contagion if a negative shock attacks the *herding* network. We will come back to this once we talk about the implications in terms of financial institution regulation.

The last metric used in this study to describe the financial banking system is the *diameter* of the network. If $d_{i,j}$ is the shortest (or geodesic) distance between vertex i and vertex j , the diameter of the network is simply the longest geodesic of all possible $d_{i,j}$. In order to avoid divergence when $d_{i,j} = \infty$, it is supposed that if a network has unconnected vertices, then its diameter is N (the number of nodes). A very elegant and robust pattern emerges if one compares real networks versus simulated networks. Figure 15 depicts that, for very low $\langle k \rangle$, both real and random networks have same diameter equal to N . However, as the absolute value of the threshold level *decreases*, nodes in the surrogated graph are “activated” randomly, and hence the probability of having no connected nodes decreases. This explains why the diameter decay is very smooth. In contrast, the diameter decay in real networks is extremely violent, mainly reflecting the presence of hubs in the system. As the correlation threshold gets softer, it is more probable that previously connected nodes will establish new links between them than unconnected nodes will establish their first link. These results illustrate not only that the networks proposed in this study capture a similar dynamic in the leverage ratio between countries, but that chance cannot rule these

networks.

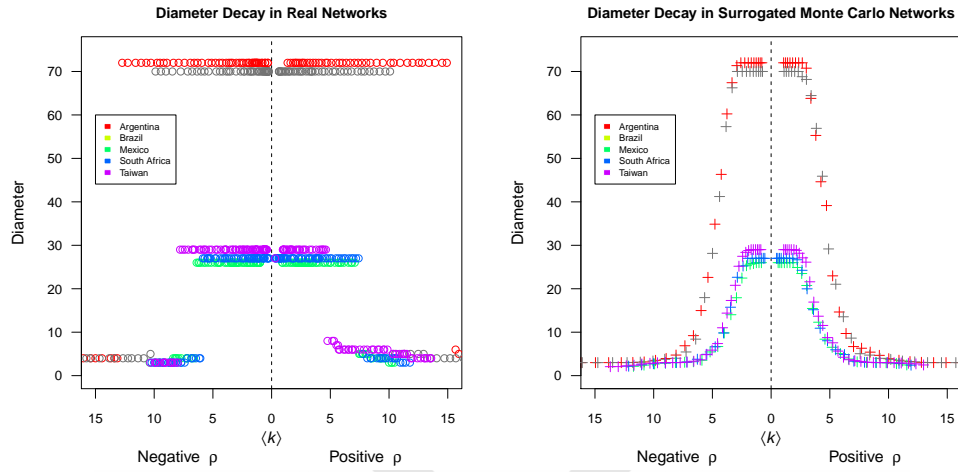


Figure 15: The smooth diameter (longest geodesic) decay in simulated networks differentiates sharply from the violent decay in real leverage-based networks, since nodes are activated randomly. The persistence of the diameter in real networks to stay at the maximum possible value may shed light on the nature of the emergence of hubs.

The next (and final) Section will resume the main findings of this thesis, and will suggest potential policy regulation implications from network theory as well as inconclusive topics that may be continued in further studies.

4 Final thoughts

Remember what we originally tried to accomplish in this study. In the first place, we asked whether we could describe the banking system, and whether its features were too dissimilar across countries. One of the merits of graph theory is that it provides tools which can be used to describe both the nature inside the system and across networks. In addition, it allows to identify patterns of interconnectedness that may not be possible to find with usual econometric methodologies, which will surely be optimal to characterize the mean behavior, but might not be sufficient to study interconnection between agents or institutions. In order to engage in a network study, we introduced the key role of leverage (as defined by Debt-to-Equity). In Section 2, after reviewing the literature on the importance of the leverage ratio as a variable subject to both the business cycle and bank interconnectedness, some findings were outlined regarding the intrinsic empirical nature of leverage. Those results can be summarized in three central things: *i)* in spite of the extensive literature on optimal capital structure, the leverage ratio exhibits a great deal of dispersion within countries; *ii)* bank's leverage do not seem to follow a heavy-tailed distribution (a Gamma or a Log-Normal are reasonable guesses); and *iii)* that after acknowledging the robustness of the Debt-to-Equity ratio to the general macroeconomic trend, there was significant dependence measured by cross-correlations (for some negative, for some positive) in the time-series leverage of individual banks.

Following previous studies, Pearson's correlation coefficient can be interpreted as a measurement of distance, and hence was used to construct a filtering procedure to detect linear associations between banks in a given country. Banks in the network were then linked according to their correlation relative to an adjustable correlation threshold. As the absolute value of the threshold *decreases*, more and more edges are added to the network. We then addressed the topic of whether networks across countries are similar

or not. The main findings suggest that although the topology in the *hearding* and *non-hearding* networks are different, its patterns tend to sustain across countries. For example, although finite size problem might be at the heart of the power-law rejection, the vertex degree in all cases does follow a heavy-tailed distribution, which in turn give place to the critical importance of hubs in the network. Additional methods should be thought and implemented to analyze degree distribution in small-size networks.

Surrogating (shuffling) historical data at the individual bank level and then Monte Carlo methods were instrumented to provide a clear indication of what things would have to be under a pure random scenario. In particular, real complex networks were characterized by high and unstable clustering coefficient, high standard deviation, and sharp diameter decay, which provided ample evidence that the systems obtained by the correlation-based filtering procedure cannot be explained by chance; many complex economic mechanisms must be ruling these networks. For instance, a common finding was the persisting emergence of hubs in the different real networks, an effect completely reversed in the simulated random networks. Identifying the direction of causality on this association will certainly be useful, and should be addressed in further studies by using interbank historical information.

Some observations in terms of policy can still be made. Interestingly, neither individual size nor leverage ratio can predict a hub in the system. This suggests that, in the case an attack or financial shock is made to a (e.g. non-leveraged outlier) hub, the financial system may be much less vulnerable. As we saw in Section 2, the degree of indebtedness and interdependence is fundamental to understand contagion between financial institutions. In a nutshell, it is possible to think in terms of two channels of distress propagation: interdependence (across-banks) and financial accelerator (through-time). In order to reduce risk and increase shock resilience, regulators should recognize a trade-off:

as connectivity and clustering increases, individual (idiosyncratic) risk may decrease due to risk sharing, but global or systemic risk could increase thanks to faster channels of propagation. The evidence in this study suggests that in particular the networks in Argentina, Taiwan, and to some extent Brazil, might be too vulnerable to hubs. And in that regard, in the event of a bank-specific shock one should consider both the *non-hearding* and *hearding* networks, for the interbank implications are entirely different. For instance, Central Bank intervention in attacked hubs may be desirable to prevent a mayor breakdown.

As in J. L. Borges' short story, potential ramifications are no less than infinite. A given network may itself be part of another network, and so on. Therefore a thorough understanding of the financial network is fundamental to both players and regulators of the system. For the regulators the objectives (but not the methods) are clear. But for the players inside the game the question is a bit trickier: when the music stops, will you find your chair, or will you be a ghost?

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Appendix A

From equation (2) we saw that C is a normalizing constant. In the continuous case, it can be derived as follows

$$\int_{x_{\min}}^{\infty} C y^{-\alpha} dy = C \left(\frac{1}{\alpha-1} \right) x_{\min}^{-\alpha+1} = 1$$

Then if $\alpha > 1$, $C = (\alpha - 1) x_{\min}^{\alpha-1}$.

In the discrete case,

$$p(X = x) = C x^{-\alpha} \quad (17)$$

Since this probability distribution also diverges as $x \rightarrow 0$, one says that the power-law behavior starts from lower bound x_{\min} . Then C is constrained by the fact that (17) must sum to 1. In particular, for $x > x_{\min} > 0$,

$$\sum_{x=x_{\min}}^{\infty} C x^{-\alpha} = 1$$

Let $\zeta(\alpha, x_{\min})$ be the Riemann zeta ζ -function. Then $C = \frac{1}{\zeta(\alpha, x_{\min})}$, and

$$p(X = x) = \frac{1}{\zeta(\alpha, x_{\min})} x^{-\alpha}$$

Which finally gives us the cumulative distribution function for the discrete case

$$P(x) = \frac{\varsigma(\alpha, x)}{\zeta(\alpha, x_{\min})}$$

Appendix B

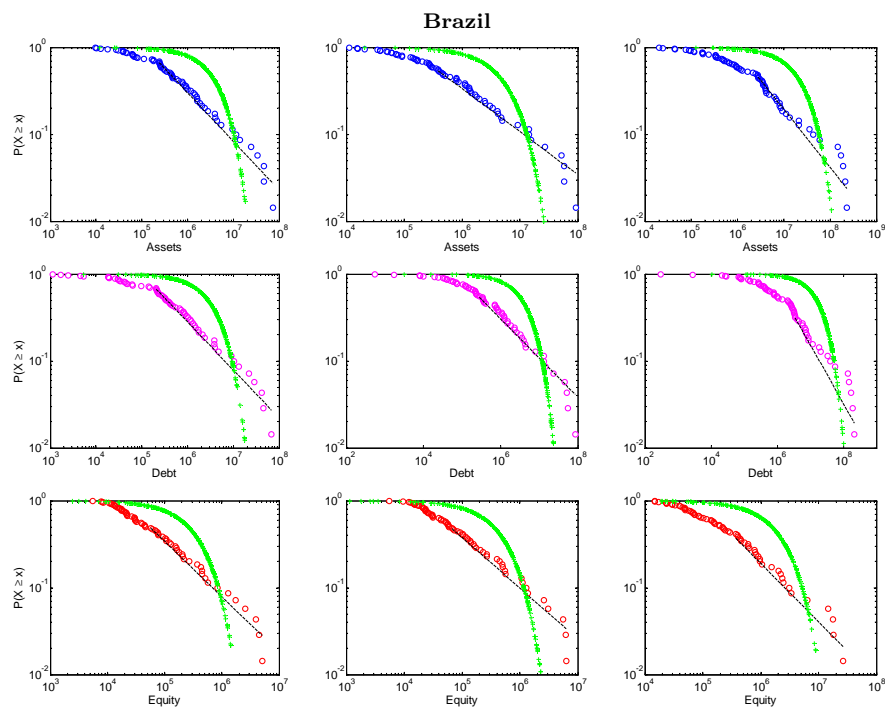


Figure 16: Left Panels show the CDF of Brazilian banks' total assets, liabilities and equity in 2003; middle Panels same variables for 2007; and right Panels in 2010. MLE yields $\hat{\alpha}$ ranging from 1.5 to 1.75. Results are robust to different period selection. Simulated exponential distribution (in green) with equal expected value as sample data depicts the strong difference in the tails of the distribution.

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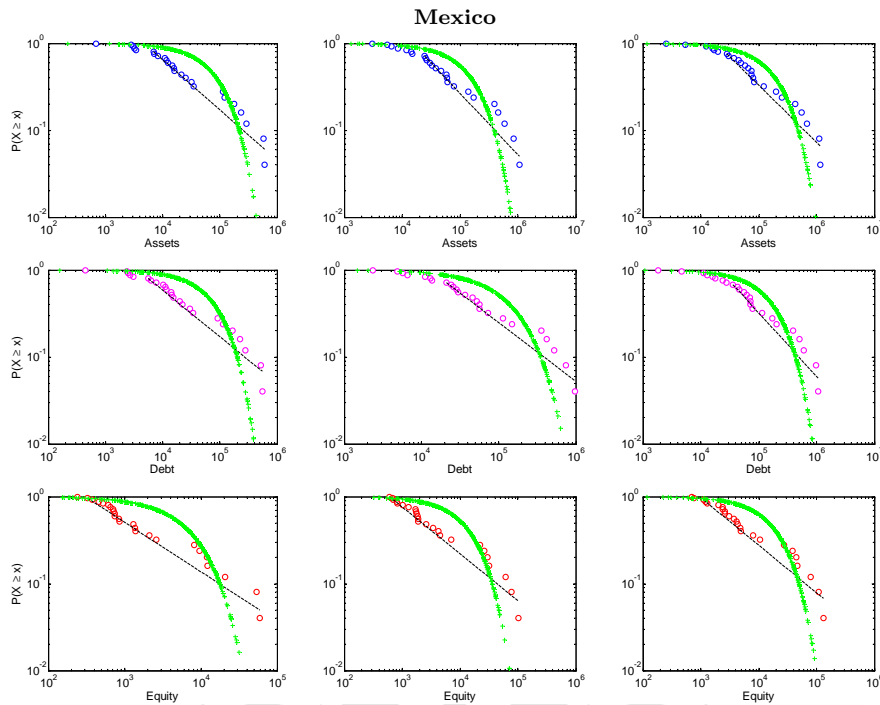


Figure 17: Left Panels show the CDF of Mexican banks' total assets, liabilities and equity in 2004; middle Panels same variables for 2007; and right Panels in 2010. MLE yields $\hat{\alpha}$ ranging from 1.53 to 1.7. Results are robust to different period selection. Simulated exponential distribution (in green) with equal expected value as sample data depicts the strong difference in the tails of the distribution.

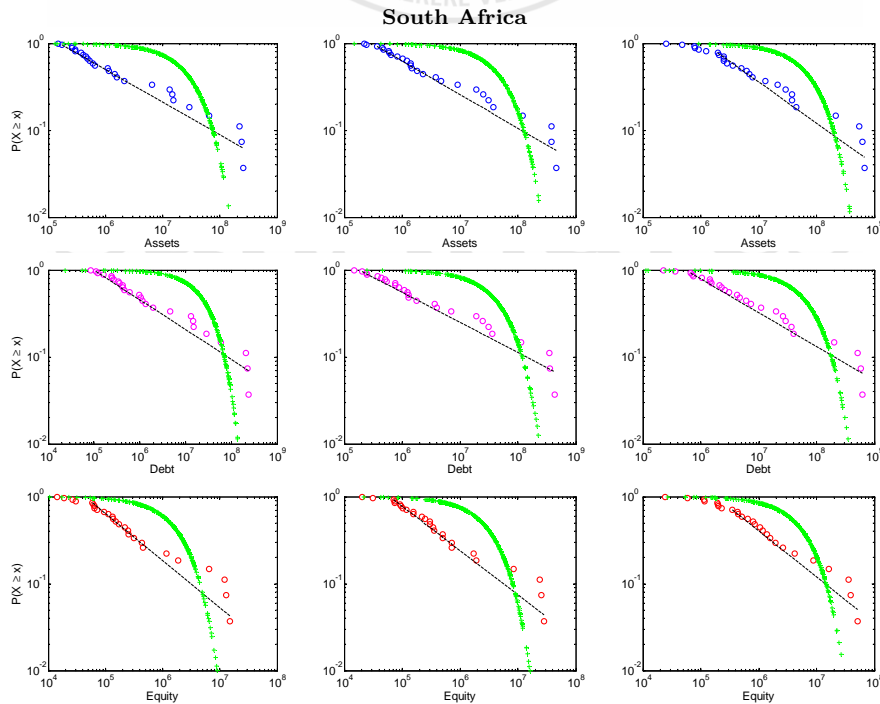


Figure 18: Left Panels show the CDF of South Africa banks' total assets, liabilities and equity in 2003; middle Panels same variables for 2007; and right Panels in 2010. MLE yields $\hat{\alpha}$ ranging from 1.33 to 1.55. Results are robust to different period selection. Simulated exponential distribution (in green) with equal expected value as sample data depicts the strong difference in the tails of the distribution.

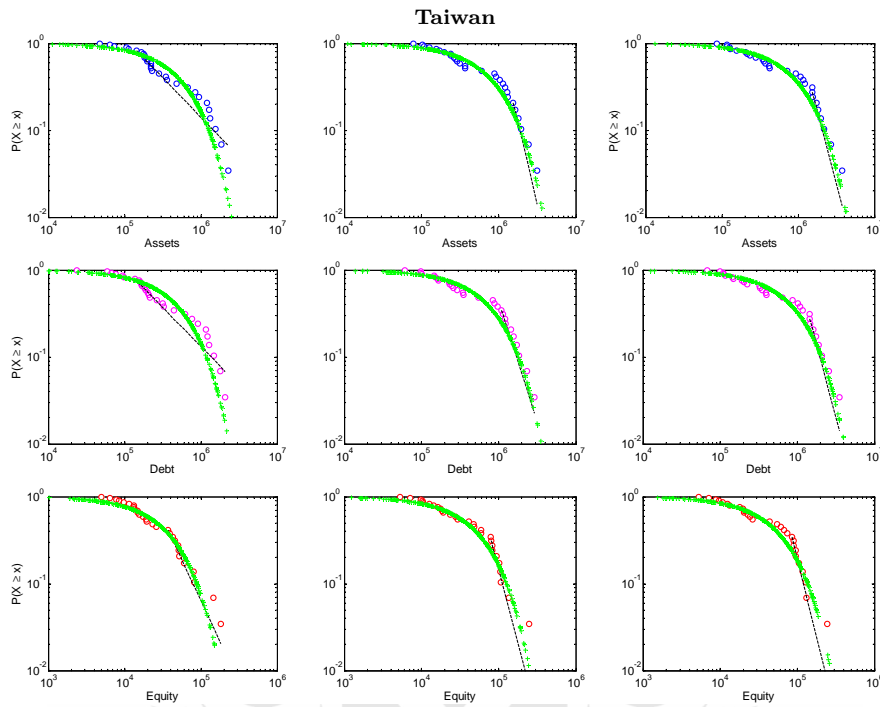


Figure 19: Left Panels show the CDF of South Africa banks' total assets, liabilities and equity in 2003; middle Panels same variables for 2007; and right Panels in 2010. MLE yields $\hat{\alpha}$ ranging from 1.9 to 4.9. Results are robust to different period selection. In this country, simulated exponential distribution (in green) with equal expected value as sample data depicts that size, debt, and equity of Taiwanese banks do not follow a power-law.

Appendix C: Additional Figures

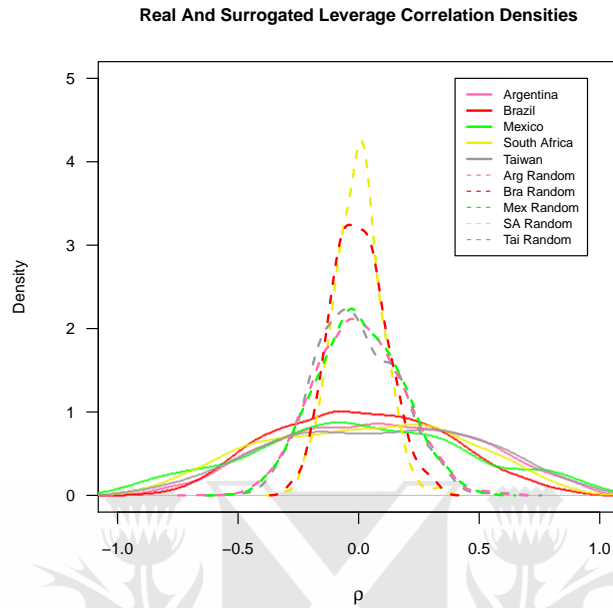


Figure 20: Kernel Densities reveal the difference in correlation concentration for real and simulated networks.

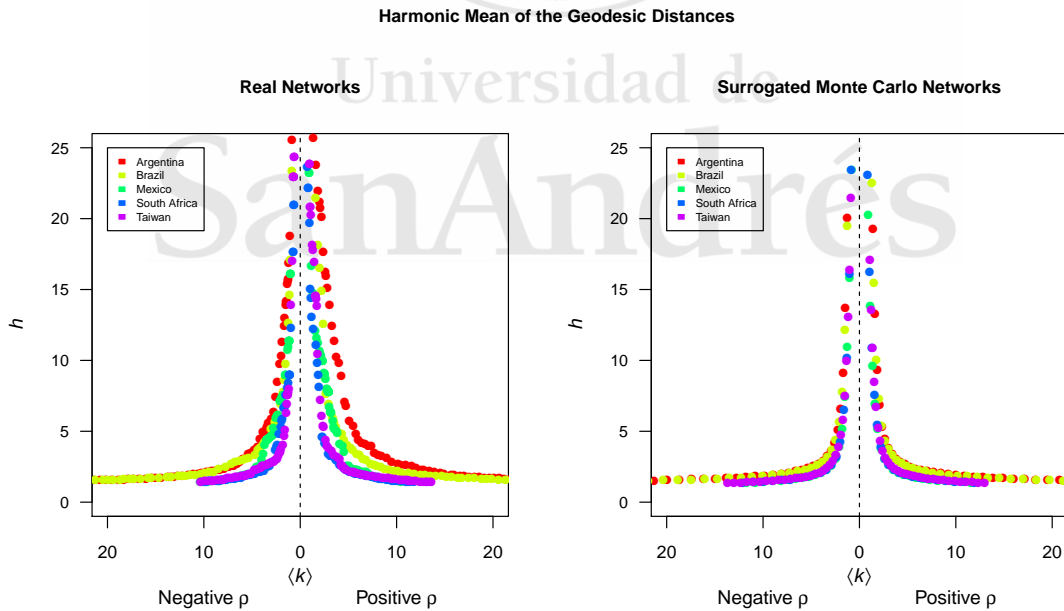


Figure 21: Harmonic mean in real (left Panel) and simulated networks (right Panel)