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Land-Rich Economies, Education and Economic Development

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Abstract

We analyze the emergence of large-scale education systems in a framework where growth is associated with changes in the configuration of the economy. We model the incentives that the economic elite could have (collectively) to accept taxation destined to finance the education of credit-constrained workers. Contrary to previous work, in our model this incentive does not necessarily arise from a complementarity between physical and human capital in manufacturing. Instead, we emphasize the demand for human-capital-intensive services by high-income groups. Our model seems capable to account for salient features of the development of Latin America in the 19th century, where, in particular, land-rich countries such as Argentina established an extensive public education system and developed a sophisticated service sector before starting significant manufacturing activities.

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1 Introduction

Differences in economic development have been subject to varying interpretations. A traditional, and still relevant literature stressed structural factors, such as the abundance of natural resources, the specialization in activities that offer good opportunities for technical improvements, the existence of high saving propensities, extensive markets or other circumstances that may encourage a faster pace of technological change (see, among others, Chenery and Syrquin, 1975; Di Tella and Zymmelman, 1967; Kuznets, 1965; Nurkse, 1961, and Prebisch, 1951). More recently, the emphasis has shifted to social factors, and especially to the incentive effects of institutions and culture (see, among others, North, 1981; Landes, 1998; and Acemoglu et al., 2005). There is clear evidence that incentives (economic, social and political) and the institutions that mold them matter for development. However, those incentives operate in the concrete environment determined by the economy's configuration and experience. Institutions themselves are influenced by political and economic structures, that is, they are endogenously determined. Thus, a better understanding of the process of economic development requires considering the joint determination of economic structure and social institutions.

Human capital accumulation is a clear example of this interaction between institutional and structural factors. In a world with imperfect capital markets, low-income workers are constrained in their private investment in education. Thus, the nature (and, more starkly, the presence or absence) of a public school system critically determines the extent and the evolution of human capital accumulation. Different societies develop different school systems. The social decisions on education are certainly influenced by broad political factors, but they also respond to economic considerations and, therefore, they depend on the structure of the economy. In turn, changes in a society's levels of schooling and literacy would affect its social structure and, perhaps, the political institutions that determine the educational institutions themselves.

The United States and Canada developed schooling institutions since colonial times. By 1850, every northern state of the US had already enacted a law strongly encouraging or requiring localities to establish "free schools", open to all children and supported by general taxes. The rest of the hemisphere trailed far behind those two countries in education and literacy. Even the most progressive Latin American countries, such as Argentina and Uruguay, lagged more than fifty years behind the U.S. and Canada in

providing primary schooling and attaining high levels of literacy. Most of Latin America was unable to achieve these standards until well into the twentieth century, if then (Mariscal and Sokoloff, 2000).

Why did some countries invest heavily in the education of broad segments of the population while others lagged behind? Galor and Moav (2006) –GM from now on- provide a very interesting explanation: capitalists, as a group, may have incentives to invest in the education of the labor force because the productivity of physical capital in manufacturing production increases with the input of human skills. That is, capitalists can gain from tax-financing the emergence of a public education system in order to raise the return on their assets by increasing the supply of a complementary factor, human capital.

This argument seems relevant to North America (and to Western Europe; see Galor and Moav, 2006), but it would have difficulties explaining the Latin America experience. Galor et al. (2005) extends the analysis in GM by assuming that, although human skills contribute to increase the productivity of industrial capital, they provide no benefits for landlords as such. Then, if landlords have veto power over policies, they would block or delay the growth of public education (see also Bourguignon and Verdier (2000) for a complementary explanation). Certainly, this hypothesis can account for the delay of most Latin American countries, but it still does not rationalize the intermediate cases of the Southern Cone countries (mainly Argentina and Uruguay) and Costa Rica, which started as early as in the second part of the 19th century to develop important schooling systems, with a polity under the dominance of landholders.

In this paper we present a simple model of economic development which could serve to analyze alternative patterns of economic evolution, and to study the emergence of public education systems under different economic conditions. Our main focus, however, is the appearance of public education in land-abundant, open economies, where policies are essentially dictated by the interests of landlords, and which need not engage in the production of manufactured goods, since their demand for these may be wholly satisfied by imports.

The analysis assumes that the skill-intensity of output and consumption baskets increases with income levels, especially because the production of some services requires the input of educated workers. More specifically, the argument is founded on three central elements: First, individual preferences over consumption goods imply changes in the composition of individual spending as income grows, embodied in Engel curves. Second, the

production of sophisticated services (which are non-tradeable goods, in an otherwise open economy) is intensive in human capital. Third, investment in human capital by individual households is constrained by lack of access to credit (see, for example, Banerjee and Newman, 1991; Galor and Zeira, 1993, and Benabou, 1996). We also assume that the quantity and quality of labor are not perfect substitutes. This implies that the number of high-income agents may have strong effects on how many individuals are subsidized to accumulate human capital. Thus, the size of the elite, as the group who demands goods particularly intensive in human capital, may have strong effects on the number of educated workers. This would rationalize a link between historical conditions, especially with regard to the distribution of land, and social choices regarding the scope and the financing of the education system. Education would start earlier in agricultural-based economies when land is highly productive and its property sufficiently distributed as to create a demand for a sizeable number of educated workers. The proposition corresponds with the case made by Engermann and Sokoloff (2000), who indicated that the greater degree of inequality in Latin America, as compared to North America, played an important role in explaining the different behaviors regarding the establishment of educational institutions (see also Mariscal and Sokoloff, 2000).

The experience of Argentina provides an illustration of the argument. In the second half of the 19th century, Argentina became increasingly integrated into the international economy as a large producer and exporter of agricultural goods, and an importer of manufactures. At the same time, the composition of primary output changed significantly, as agriculture expanded over cattle rising activities, a shift that favored less extensive forms of production (see, among others, Adelman, 1994). While the distribution of land and incomes was still more unequal than in North America, where wheat production was mainly based on family farms, it was less concentrated than in other Latin American economies. The expansion of agricultural activities allowed a very substantial growth of the urban population, especially in the city of Buenos Aires. Apart from its administrative functions as the capital of the country, the city developed an increasingly sophisticated, and large, service sector. At the same time, the country experienced what is widely considered one of the key processes in its history: the emergence of the system of public education, associated with the emblematic figure of Domingo Faustino Sarmiento. However, the progress of education was not immediate, and indeed it went along with the development of the economy and, implicitly, with a growing demand for skills (see Martínez Paz, 2003).

It was only in 1875 that the Province of Buenos Aires passed a comprehensive law on public instruction, while the corresponding national instrument was introduced in 1884. However, the large regional differences in rates of scholarization and literacy indicate that, directly or indirectly, spending on education depended very much on the economic configuration of the localities. In 1895, in the city of Buenos Aires, almost 60% of the children of ages 6-14 attended school, doubling the national average while the rate of illiteracy was only 20%, against 57% in the country as a whole (and nearly 80% in poor jurisdictions far from the central agricultural regions, like La Rioja or Neuquén). Furthermore, the type of education provided by the Argentine State seemed to correspond more to the economic incentives perceived at the time than to the vision of the founders of the system. For example, Sarmiento believed that the educational system had to form individuals to work in the agricultural and industrial sectors but instead it tended to qualify individuals for work in the services sector (see Tedesco, 1993).

From a modelling point of view, we use an overlapping generations framework (similar to that in GM) to represent an open economy with a particular specification of the commodity and factor spaces: two tradeable goods (agricultural and industrial) and one non-tradable (services), and four factors (land, physical capital, labor and skills). In this simple model, we focus on the basic properties of comparative advantages, capital accumulation of capital, and diversification of consumption as income increases, while abstracting away issues related to technological change. The starting point in our analysis is a simple agrarian economy, where the capital stock is accumulated by landlords, while the rest of the population is in the subsistence sector. At first, even landlords only consume agricultural goods, although they leave bequests. In such a setting, the first countries where capital accumulation in agriculture reaches the point at which a significant demand for manufactured goods arises would be early-comers to industrialization. Once there is a well-developed international market for manufactured goods, labor-abundant economies may develop industrial activities for the world market, even when their income levels are too low to induce a widespread domestic demand for those goods. These cases (where public education can be rationalized as a result of the interests of industrial capitalists, as in GM) can be represented within the basic framework of our model, as it is briefly presented in the appendix. However, in this paper, our focus is on land-rich economies where the demand for industrial goods is initially satisfied by imports (see Leamer, 1987), and where the accumulation of human capital would only be triggered by the consumption of services.

Thus, while the basic model seems capable of being adapted to analyze different development experiences, we concentrate on that particular pattern and stage of economic growth.

The rest of the paper is organized as follows. The next section describes the setup of the model. In section 3 we analyze the evolution of an agrarian economy and briefly comment on possible alternative paths that may be followed by economies of different structural configurations. Section 4 deals with the case where large-scale educational systems appear in land-rich economies, which have not gone through a previous stage of industrialization. Conclusions are then presented in section 5.

2 Setup of the model

We consider an overlapping generations economy, where agents live for two periods, and there is no population growth (i.e., each agent has a single descendant). At the beginning, there are two kinds of dynasties, landowners and workers, who differ in their factor endowments only (a set of industrial capitalists may emerge if the economy develops a manufacturing sector). The first group has initially an endowment of land, which is not traded in equilibrium, and some physical capital; for simplicity, members of this class do not supply labor. Workers are endowed with a basic set of labor skills, which can be increased by acquiring education.

In every period, the young agent of each dynasty receives a non-negative (but not necessarily strictly positive) bequest from the old agent of his lineage. Those bequests are potentially taxable and, in the model, such taxes fund the spending on public education. The land owned by an individual landlord is automatically transferred to his offspring (this transfer is not included in the definition of bequests). Young agents use their after-tax bequests to accumulate assets: either physical capital for productive activities or, in the case of workers, to acquire human capital which can be purchased by spending on (supplementary) private education.[1] Old agents carry out work and production, they consume, and decide whether and how much to transfer to their offspring as bequests.

In the first period of their lives individuals who receive a nonnegative (after-tax) bequest from the previous generation decide investments on as-

[1] In principle, young agents might use resources to purchase private education. However, in the equilibrium we focus below, and under the assumptions we made, this would not occur.

sets, which generate income in the next period. In the second period of their lives, all individuals allocate their income between consumption and the bequest they leave to the following generation. Workers also participate directly in production and receive a wage in exchange. Young agents do not consume (or, equivalently, their consumption is included in that of their parents).

There are three types of consumable commodities: agricultural and industrial goods, and services. Agricultural and industrial goods may be traded internationally, while services are non-tradable. Agricultural goods can be produced with a subsistence technology, employing unskilled labor only, or by combining physical capital and land. Agricultural goods may be consumed or used as physical capital. Manufacturing production uses labor, capital and skills, and services are provided by skilled (educated) workers.

2.1 Technology and Production

Agricultural goods are denoted, A, industrial goods, I, and services, N. The factors of production are: raw labor, L, land, T (in fixed supply), physical capital, X (which is homogeneous with the agricultural good) and human capital, h. Markets for goods and factors are perfectly competitive, while there are no international capital movements. Therefore, young generations must finance physical investment and education with the bequests transferred by the previous generation. Thus, the trade balance is zero in every period.

2.1.1 The Agricultural Sector

We assume that good A may be consumed or used as physical capital. Agricultural output can be produced with two alternative technologies. The first, that we label "subsistence" production, is a constant returns to scale technology with unskilled labor as its only input, generating output $\widetilde{w}$ per worker. If this technology operates in equilibrium, $\widetilde{w}$ will be the prevailing wage.[2] An agent with income $\widetilde{w}$ consumes agricultural goods only and does not leave bequests, so that the group of subsistence workers does not trade or accumulate assets.

[2]The individuals in this "subsistence" sector do not play an active role in the model, but they provide a reservoir of workforce (a la Lewis) which is not exhausted in the relevant range of variables. The reservation wage could also be interpreted without changing the model as the income required to induce immigration.

The second way of producing agricultural output is with a technology that uses land and capital as inputs. We assume that the aggregate production function of agriculture is $F(\mathbf{T}, \mathbf{X})$, where $\mathbf{T}$ and $\mathbf{X}$ are, respectively, the total surface of land and the aggregate capital used in agriculture. Furthermore, $F(\cdot, \cdot)$ is a constant returns function. If the number of production units (and landlords) in the economy is m, we will suppose, for simplicity, that the distribution of land and capital is uniform. The output of a farm is given by:

$$y_t^A \;=\; \frac{1}{m} F(\mathbf{T}, \mathbf{X}) \;=\; F(\frac{\mathbf{T}}{m}, \frac{\mathbf{X}}{m}) \;=\; f(X_{t-1}^A)$$

where $X_{t-1}^A = \frac{\mathbf{X}}{m}$ is the capital stock of the average production unit used in agriculture (made of goods A), which has been carried over from the previous period. Since land is fixed, we treat $\frac{\mathbf{T}}{m}$ as a constant. Capital fully depreciates in the period. The per-capita production function $f(\cdot)$ has the traditional properties that assure the existence of an interior solution to the profit-maximization problem.

2.1.2 Human Capital and Skill Formation

Human capital (interpreted as skills) is produced through education. For simplicity, the inputs of this activity are assumed to consist solely of good A. The skills of an individual in period $t+1$ are a function of the resources spent on the agent's education (e) in period t:

$$h_{t+1} = h(e_t)$$

with $h' > 0$, $h'' < 0$.

2.1.3 The Manufacturing Sector

Industrial goods are assumed to be produced with labor, capital and human capital:

$$y_t^I = g(L_t^I, X_{t-1}^I, h_t^I)$$

where L^I is the number of workers employed in manufacturing, h^I their average level of skills (accumulated in t-1), and X^I the amount of capital used in the sector, and carried over from the previous period. For the sake of concreteness, we adopt the following Cobb-Douglas specification:

$$y_t^I = z_I (L^I)^{\rho_L} (X^I)^{\rho_X} \varphi(h^I)$$

with $\rho_L + \rho_X = 1$ and $\varphi'(h^I) > 0$. $z_I \varphi(0) > 0$ indicates the productivity coefficient when only unskilled labor is employed. If $w(h)$ is the wage of a worker with skills h, it can be shown that, starting from $h^I = 0$, a firm would not choose to employ workers with marginally higher skills when

$$\rho_L \frac{w'(0)}{w(0)} > \frac{\varphi'(0)}{\varphi(0)}$$

That is, the manufacturing sector will not demand skilled labor when the increase in wages as h increases from zero is steeper than the corresponding increase in productivity. If this is the case, and manufacturing production was to start, industrial labor would be drawn from the pool of subsistence workers, at a fixed wage $\widetilde{w}$. Then, given the elastic supply of labor, and, for a given price p_I, the value of the marginal productivity of capital at the optimal level of benefits when $h = 0$, would be:

$$r_I(p_I, \widetilde{w}) = s[z_I \varphi(0) p_I \widetilde{w}^{-\rho_L}]^{\frac{1}{\rho_X}}$$

where s is a constant that depends on the parameters ρ. Thus, industrial activities are not be initiated in an open economy for an aggregate capital stock X if the marginal product of capital in agriculture exceeds the rate of return in the manufacturing sector:

$$r_I(p_I, \widetilde{w}) < f'(X)$$

In the rest of the paper, it will be assumed that this condition holds for all relevant values of X, implying that the international price p_I is too low and the productivity of capital in agriculture too high to induce industrialization in the late-comer land-rich economy we are considering. The case of the economies that industrialize is schematically addressed in the appendix.

2.1.4 The Service Sector

The third consumable good, N, is interpreted as an urban, relatively sophisticated service, the production of which requires only skilled labor.[3] An

[3]The assumption here is that, if the economy does not develop an industrial sector, the production of consumer services would be the only source of demand for skills. This

individual who demands services hires skilled workers competitively, and consumes the output they produce, which depends on the number of persons who participate in the supply of the services as well as on their average level of human capital:

$$y_t^N(n_t, g(h_t)) = \psi(n_t) \int h_t g(h_t) dh_t$$

where y^N is the volume of services, produced by n workers with a distribution function of skills $g(h)$, which are supplied to a given customer. Given the wage $w(h)$ associated with a level of skills h, the value of services consumed per individual is given by $p_N c_N = n \int w(h) g(h) dh$.

Consumers are assumed to care about the quality of the suppliers of services (and output is assumed to grow unboundedly with h), but to a limited extent on the number of workers who contribute to production. The intuition behind this assumption is simply that, for a wide range of services (from medical attention to entertainment, say), when a certain plateau of suppliers is reached, additional workers make little difference for the utility of the resulting consumption, but this is enhanced with an increase in the skills applied to production. The function ψ would then be strictly concave. Although this condition is sufficient to define the qualitative features of our argument, we make the stronger assumption that the marginal contribution of additional workers goes to zero at a finite number $\widetilde{n}$ of suppliers per customer. Thus, ψ is increasing in n up to a maximum $\widetilde{n}$. For simplicity, we will use the following specification:

$$\psi(n) \;=\; \min[n, \widetilde{n}]$$

The demand for services of an individual consumer is such that, for low levels of spending, both the number of workers and their qualification rise with the value of consumption up to the point where $\widetilde{n}$ workers are employed. All the subsequent increases in consumption result by augmenting the average skills of the pool of suppliers. Moreover, it can be shown that, if the wage function, $w(h)$ is convex, an individual demands only one type of workers, with skills equal to the optimal average as given by the consumption optimum. These statements are summarized in the following proposition.

is clearly a simplification, which disregards other important activities which require the input of skilled workers, such as the public administration and education itself. However, it may be thought that those "intermediate" demands for skills would appear when the economy has reached a stage of sophistication and consumption diversification as to induce a significant "final demand" for skills such as the one emphasized in the model.

Proposition 1 *Given a value of expenditure in services, denoted by $p_N c_N$, and defining the average level of wages and skills of the workers that participate in the production of those services: $\int w(h)g(h)dh = \overline{w}$, $\int hg(h)dh = \overline{h}$, and defining the elasticity of wages with respect to skills: $\epsilon_w(h) = w'(h)\frac{h}{w(h)}$*

- *If the function $w(h)$ is convex, then an agent demands only one quality of labor, with skills $h = \overline{h}$*
- *If there exists h^* such that $\epsilon_w(h^*) = 1$ and $\frac{p_N c_N}{w(h^*)} = n^* \leq \widetilde{n}$, then n^* and $\bar{h} = h^*$ are, respectively, the number of workers hired by the consumer and their level of skills.*
- *Otherwise, if either no h^* verifies $\epsilon_w(h^*) = 1$ or, if there exists such h^*, but $\frac{p_N c_N}{w(h^*)} \geq \widetilde{n}$, then the optimal amount of labor is $\widetilde{n}$ while the average $\bar{h}$ verifies that $\frac{p_N c_N}{\widetilde{n}} = w(\bar{h})$.*

Proof: *The maximal production of services for a given cost solves:*

$$\max \ \psi(n) \int hg(h)dh \quad (*)$$

$$s.t. \ \ p_N c_N = n \int w(h)g(h)dh$$

$$\text{and} \int g(h)dh \ = \ 1$$

The dual problem is,

$$\min \ n \int w(h)g(h)dh \quad (**)$$

$$s.t. \ \bar{y} \ = \ \psi(n) \int hg(h)dh$$

$$\text{and} \int g(h)dh \ = \ 1$$

Since $w(\cdot)$ is convex, $\int w(h)g(h)dh \ > \ w(\int hg(h)dh)$ for every skill distribution $g(h)$. Cost minimization implies that the optimal demand is concentrated at skills $\overline{h}$, so that $g(h)$ is non-zero only at $\overline{h}$. Problem $(*)$ then reduces to:

$$\max \psi(n)\bar{h}, \ s.t. \ p_N c_N = nw(\overline{h}) \text{ and } \overline{h} = \int hg(h)dh$$

The variables of choice are n and $\overline{h}$. The first order conditions are:

$$n: \quad \psi'(n)\overline{h} \;\leq\; \lambda w(\bar{h})$$

$$\overline{h}: \quad \psi(n) \;=\; \lambda n w'(\overline{h})$$

where λ is the Lagrange multiplier of the cost constraint. Then we have that:

$$\epsilon_\psi(n) \;=\; \frac{\psi'(n)}{\psi(n)} n \;\leq\; \frac{w(\overline{h})}{w'(\overline{h})} \frac{1}{\overline{h}} \;=\; \frac{1}{\epsilon_w(\overline{h})} \quad (\mathbf{1})$$

with equality for an interior solution.

Recall that $\epsilon_\psi(n)$ is either 1 (for $n \leq \widetilde{n}$) or 0 (for $n > \widetilde{n}$). Then, according to condition (**1**) *the optimal n must verify that at an interior solution $\epsilon_\psi(n) = 1$ while $\overline{h}$ must be such that $\epsilon_w(\overline{h})$ is also equal to 1. On the other hand, the expenditure on services ($p_N c_N$) is given. Therefore, the amount h^* such that $\epsilon_w(h^*) = 1$ must verify that $\frac{p_N c_N}{w(h^*)} = n^* \leq \widetilde{n}$. If so, the optimal amounts of labor and average human capital are $n = n^*$ and $\overline{h} = h^*$. Otherwise, condition* (**1**) *verifies with strict inequality, and the optimal n is not interior, i.e. $\epsilon_\psi(n) = 0$, implying that $n > \widetilde{n}$. Then, the choice problem of the agent is just to maximize $\widetilde{n}\overline{h}$, subject to $p_N c_N = n w(\overline{h})$. The solution to this problem obtains at $n = \widetilde{n}$ while $\overline{h}$ is takes simply the value h^{**} such that $\frac{p_N c_N}{\widetilde{n}} = w(h^{**})$.* □

2.2 The Agents

Individuals, within as well as across generations, are identical in their preferences and innate abilities.

2.2.1 Preferences

Preferences are defined over consumption-bequest bundles, $c = \langle c_A, c_I, c_N, b^o \rangle$, $c \in \mathcal{R}^4_+$, where c_j is the quantity consumed of good j, while b^o is the bequest (measured in terms of agricultural goods) left to offspring (we reserve the notation b for the bequest received by the agent when young).

These preferences capture the intuition that consumers prefer a diversified bundle, but there are certain consumption thresholds to be reached before adding an additional degree of diversity. We partition the consumption space into four subsets (which may be thought of as "stages"), according

to which thresholds have been exceeded (or equivalently, what degree of consumption diversification has been reached). Within each stage, preferences are described by a (stage-specific) Stone-Geary function. Preferences and the associated demand curves have implicit an ordering of the goods which are part of the consumption basket at different levels of income. There are four stages: i) consumption of agricultural goods (A), only; ii) consumption of A and positive bequests; iii) consumption of A and industrial (I) goods as well as positive bequests; iv) consumption of goods A, I and services (N), and positive bequests.

Specifically, preferences are represented by the following expression:

$$u(c) = \begin{cases} c_A & \text{if } \; c_A \leq \widetilde{c}_{A1} \\ (c_A - \widetilde{c}_{A1})^{1-\beta}(b^o)^{\beta} + k_1 & \text{if } \; c \in \mathcal{C}^2 \\ \left[(c_A - \widetilde{c}_{A2})^{\alpha_{A3}} c_I^{\alpha_{I3}}\right]^{(1-\beta)} (b^o - \widetilde{b}_2^o)^{\beta} + k_2 & \text{if } \; c \in \mathcal{C}^3 \\ \left[(c_A - \widetilde{c}_{A3})^{\alpha_{A4}} (c_I - \widetilde{c}_{I3})^{\alpha_{I4}} c_N^{\alpha_{N4}}\right]^{(1-\beta)} (b^o - \widetilde{b}_3^o)^{\beta} + k_3 & \text{if } \; c \in \mathcal{C}^4 \end{cases}$$

$$\mathcal{C}^i = \{c \in R_+^4 : \widetilde{c}_{J,i-1} < c_J \leq \widetilde{c}_{J,i} \text{ for every } c_J = c_A, c_I, c_N, b^o; i = 2, 3, 4\}$$

$$\alpha_{A3} + \alpha_{I3} = 1 \; ; \alpha_{A4} + \alpha_{I4} + \alpha_{N4} = 1$$

$$k_1 = u(\widetilde{c}_{A1}, 0, 0, 0); k_2 = u(\widetilde{c}_{A2}, 0, 0, \widetilde{b}_2^o); k_3 = u(\widetilde{c}_{A3}, \widetilde{c}_{I3}, 0, \widetilde{b}_3^o)$$

In words, each $\mathcal{C}^i$ represents a range of values of consumption, determined by the income and limited to some kinds of goods. By definition each of them is, geometrically, a "box" in R_+^4, with its frontier defined by the thresholds of consumption $\widetilde{c}$. These boxes are ordered: $\mathcal{C}^j$ lies to the upper-right side of a box $\mathcal{C}^i$ if and only if $i < j$. Under this characterization the preferences are still well behaved.

Proposition 2 *u represents a rational preference ordering.*

Proof: *We have to show that the preference ordering ($\preceq$) defined by $u(c)$ over $\mathcal{C}$ is complete, transitive and monotonic.*

Completeness follows immediately. If two bundles c and $\bar{c}$ belong to $\mathcal{C}$ two cases are possible:

- *$c \in \mathcal{C}^i$ and $\bar{c} \in \mathcal{C}^j$, with $i \neq j$. Then, by definition, if $i > j$, $\bar{c} \prec c$, while if $j > i$, $c \prec \bar{c}$.*

- *$c, \bar{c} \in \mathcal{C}^i$. Then, either $u(c) \leq u(\bar{c})$ and therefore $c \preceq \bar{c}$* or *$u(\bar{c}) \leq u(c)$, i.e. $\bar{c} \preceq c$.*

Similarly for transitivity: consider c, $\bar{c}$, $\bar{\bar{c}} \in \mathcal{C}$, where $c \preceq \bar{c}$ and *$\bar{c} \preceq \bar{\bar{c}}$. Four cases are possible:*

- *c, $\bar{c}$, $\bar{c} \in \mathcal{C}^i$. Then, by definition $u(c) \leq u(\bar{c})$, while $u(\bar{c}) \leq u(\bar{c})$. Therefore $u(c) \leq u(\bar{c})$, i.e. $c \preceq \bar{c}$.*
- *c, $\bar{c} \in \mathcal{C}^i$ and $\bar{c} \in \mathcal{C}^j$, $i \neq j$. By definition, since $\bar{c} \preceq \bar{c}$, $i < j$. Therefore $c \preceq \bar{c}$.*
- *$c \in \mathcal{C}^i$ and $\bar{c}$, $\bar{c} \in \mathcal{C}^j$, $i \neq j$. Since $c \preceq \bar{c}$, $i < j$. Therefore, $c \preceq \bar{c}$.*
- *$c \in \mathcal{C}^i$, $\bar{c} \in \mathcal{C}^j$ and $\bar{c} \in \mathcal{C}^k$, $i \neq j$, $j \neq k$, $i \neq k$. Then, since $c \preceq \bar{c}$, $i < j$, and since $\bar{c} \preceq \bar{c}$, $j < k$. Therefore, $i < k$ and $c \preceq \bar{c}$.*

Finally, to show monotonicity, just consider $c \leq \bar{c}$. Again, we can analyze this by cases:

- *If $c \in \mathcal{C}^i$ and $\bar{c} \in \mathcal{C}^j$, with $i \neq j$, by the definition of $\mathcal{C}^1, \ldots, \mathcal{C}^4$, given $1 \leq k < l \leq 4$, for every $c^k \in \mathcal{C}^k$ there exists $c^l \in \mathcal{C}^l$ such that $c^k \leq c^l$, while for every $c^l \in \mathcal{C}^l$ there is no $c^k \in \mathcal{C}^k$ such that $c^l \leq c^k$. Therefore, if $c \leq \bar{c}$ then $i < j$, i.e. $c \prec \bar{c}$.*
- *If $c, \bar{c} \in \mathcal{C}^i$ then, since each u is monotonic, $u(c) \leq u(\bar{c})$. This implies that $c \preceq \bar{c}$.* $\square$

In all cases, whenever an income threshold is crossed (when an individual can purchase the minimum consumption quantities of a certain stage), the agent prefers the most diversified consumption bundle attainable (that is, consumption corresponds to the highest feasible stage). Regarding the incentive for leaving a bequest, individual welfare varies directly with the amount of resources left to an offspring, independently of the use of that bequest by the next generation. This implies, in particular, that savings depend only on the income of the adult agent, and not on the expected return on assets.[4]

[4]This form of bequest motive (i.e., the "joy of giving") is common in the recent literature on income distribution and growth. The assumption that the rate of return is irrelevant in the decision to leave bequests does not greatly affect the qualitative results

Although this particular formulation of preferences has features that are not altogether appealing (like potential jumps in quantities demanded in the transition from one stage to another, see below), it captures a differentiation between "basic needs" and "luxury goods", and generates a "consumption ladder" where new goods get included into the basket as income grows.

2.2.2 Demand

Demand curves arise from maximizing welfare subject to a budget constraint, which in its most general form is given by:

$$c_A + p_I c_I + p_N c_N + b^o \leq i$$

where i is total income of the old agent. With the particular functional form we have adopted for preferences, optimization will yield threshold income levels, determining the transition from one diversification stage to the following. From standard methods we obtain:

Lemma 1 *The consumption-bequest baskets of an agent are as follows:*

- *In C^1: $c_A \leq \widetilde{c}_{A1} = \widetilde{i}_1$: $c_A = i$.*
- *In $\mathcal{C}^2$:*

$$c_A = (1-\beta)(i - \widetilde{i}_1) + \widetilde{c}_{A1}$$

and

$$b^o = \beta(i - \widetilde{i}_1)$$

- *In $\mathcal{C}^3$:*

$$b^o = \beta(i - \widetilde{i}_2) + \widetilde{b}_2^o$$

emphasized in the paper. However, it may have strong implications in some contexts. In particular, this type of savings function allows the existence of states where the marginal net product of capital is negative. Also, initial differences in endowments may have no effect on steady state consumption, while that would not happen, say, with standard Euler equations if all agents face the same interest rate, since the ratio of marginal utilities of any two agents would be preserved over time.

$$c_A = \widetilde{c}_{A2} + (1-\beta)\alpha_{A3}(i - \widetilde{i}_2)$$

$$p_I c_I = (1-\beta)\alpha_{I3}(i - \widetilde{i}_2).$$

where $\widetilde{i}_2 \;=\; \widetilde{c}_{A2} + \widetilde{b}_2^o$.

- *In* $\mathcal{C}^4$*:*

$$b^o = \beta(i - \widetilde{i}_3(p_I)) + \widetilde{b}_3^o$$

$$c_A = (1-\beta)\alpha_{A4}(i - \widetilde{i}_3(p_I)) + \widetilde{c}_{A3}$$

$$p_I c_I = (1-\beta)\alpha_{I4}(i - \widetilde{i}_3(p_I)) + p_I \widetilde{c}_{I3}$$

$$nw(\bar{h}) = (1-\beta)\alpha_{N4}(i - \widetilde{i}_3(p_I))$$

where $\widetilde{i}_3 \;=\; \widetilde{c}_{A3} + p_I \widetilde{c}_{I3} + \widetilde{b}_3^o$.

Proof: *Immediate, from the maximization of* $u(c)$ *subject to the budget constraint.* □

Notice that while there is no jump in agricultural consumption when making the transition from the first to the second stage, there may be jumps in either consumption or bequests when making any of the subsequent transitions. To fix this idea, consider the case of the transition into industrial consumption. As soon as $i \geq \widetilde{i}_2$, it must be the case that $c_A \geq \widetilde{c}_{A2}$ and $b^o \geq \widetilde{b}_2^o$. However, it could be that, for instance, $\lim_{i \nearrow \widetilde{i}_2} c_A^* > \widetilde{c}_{A2}$ and $\lim_{i \nearrow \widetilde{i}_2} b^o \leq \widetilde{b}_2^o$. This would imply that after crossing the threshold, consumption of agricultural goods has a discrete drop, and bequests go up. The opposite situation, where consumption increases and bequests (hence, capital accumulation) fall after the transition, is also feasible. Since this last case would imply that the economy may get trapped in an oscillation around transitions, we will impose conditions such that this possibility is avoided.

Once having eliminated the possibility of having oscillations at the transition thresholds, the structure of demands can be used to distinguish between two economies with the same amounts of land and aggregate capital but exhibiting different income distributions. More precisely, calling these economies $\epsilon_1(\mathbf{T}, \mathbf{X})$ and $\epsilon_2(\mathbf{T}, \mathbf{X})$, where $\mathbf{T}$ and $\mathbf{X}$ are the given amounts of land and capital, if the number of landlords in them are, respectively, m_1 and m_2, with $m_1 < m_2$ we have:

Lemma 2 *If the consumptions in $\epsilon_1(\mathbf{T}, \mathbf{X})$ and $\epsilon_2(\mathbf{T}, \mathbf{X})$ are c_{ϵ_1} and c_{ϵ_2}, with $c_{\epsilon_1} \in \mathcal{C}^i$ and $c_{\epsilon_2} \in \mathcal{C}^j$, then $i \geq j$.*

Proof: *A representative landlord in ϵ_1 has, according to our characterization of the agricultural sector, an income*

$$i_1 = f(X^A_{t-1}) = \frac{1}{m_1} F(\mathbf{T}, \mathbf{X})$$

while an average landlord in ϵ_2 earns

$$i_2 = f(X^A_{t-1}) = \frac{1}{m_2} F(\mathbf{T}, \mathbf{X}).$$

Since $m_1 < m_2$, $i_1 > i_2$. Then, by the definition of demands, $c_{\epsilon_1} > c_{\epsilon_2}$. Then, if $c_{\epsilon_1} \in \mathcal{C}^i$ and $c_{\epsilon_2} \in \mathcal{C}^j$, either $i = j$ or $i > j$. It can also be noticed that, since bequests increase with income, economy ϵ_1 will reach a given level of the capital stock at an earlier date than the economy where property is more subdivided. □

This means that the landlords in the economy with a more concentrated income distribution will reach earlier a more diversified consumption.

3 Growth and structural evolution

3.1 An agricultural economy

We start by analyzing an economy where only good A is produced and consumed. Here, unskilled individuals work in the subsistence sector, consuming $\widetilde{w}$, and have no other choices available. Landlords invest in physical capital all the bequest they receive, and choose their consumption/bequest bundle in the second period. By assumption, in this stage, only the agricultural

good is produced and consumed. Then, if the initial capital stock (received by the first generation) is X_0, the demand functions discussed before induce the following dynamics:

Proposition 3 *Let $\widetilde{i}_1 = \widetilde{c}_{A1}$, $\widetilde{i}_2 = \widetilde{c}_{A2} + \widetilde{b}_2^o$, assume that $f(0) < \widetilde{i}_1$ and that there is at least a value of X such that $X = \beta(f(X) - \widetilde{i}_1)$. Then:*

- *there are two fixed points for $\beta(f(X) - \widetilde{i}_1)$, $\widehat{X}_1^l$ and $\widehat{X}_1^h$ (low and high, respectively),*
- *the economy reaches a steady state in $\mathcal{C}^2$ (where landlords leave positive bequests and consume only agricultural goods) if and only if $\widetilde{i}_2 > f(X_0) \geq f(\widehat{X}_1^l) > \widetilde{i}_1$ and $f(\widehat{X}_1^h) < \widetilde{i}_2$.*

Proof: *By assumption, $f(0) - \widetilde{i}_1 < 0$ and there exists X such that $X = \beta(f(X) - \widetilde{i}_1)$. Then, since $\beta(f(X) - \widetilde{i}_1)$ is strictly concave, it has two fixed points, $\widehat{X}_1^l$ and $\widehat{X}_1^h$, with $\widehat{X}_1^l \leq \widehat{X}_1^h$. The fixed points are steady states of the dynamics of the capital stock (and, equivalently, bequests). $\widehat{X}_1^l$ is unstable, while $\widehat{X}_1^h$ is stable under the dynamics defined by b_2^o on $\mathcal{C}^2$.*

If X_0 is such that $\widetilde{i}_2 > f(X_0) > f(\widehat{X}_1^l) > \widetilde{i}_1$ and $f(\widehat{X}_1^h) < \widetilde{i}_2$, there is capital accumulation, and the economy is not trapped in a subsistence equilibrium where bequests are zero. Furthermore, capital accumulation stops at $\widehat{X}_1^h$ while the economy is still in $\mathcal{C}^2$. Conversely, if the economy reaches a steady state in $\mathcal{C}^2$, it must be at either one of the fixed points, be it because $X_0 = \widehat{X}_1^l$ or because the dynamics leads to the stable value $\widehat{X}_1^h$. $\square$

3.2 The transition to industrial consumption

If capital accumulation proceeds, landlords would start consuming manufactured goods. Some properties for this stage can be summarized in the following proposition:

Proposition 4 *If $f(\widehat{X}_1^h) > \widetilde{i}_2 = f(\widetilde{X}_2)$ and $p_I < \frac{1}{s^{\rho_X} z_I \varphi(0)} \widetilde{w}^{\rho_L} (f'(X))^{\rho_X}$*

- *the economy enters stage 3 (with consumption of goods and bequests of landlords in $\mathcal{C}^3$),*
- *the manufactured goods consumed by landlords are supplied from imports, and there is no domestic production of industrial goods.*

Proof: *If the capital required for entering stage 3, $\widetilde{X}_2$, is less than $\widehat{X}_1^h$, the economy does not get trapped in stage 2. The expression on p_I just restates the condition for no production of good I discussed in section 2.1.3.* $\square$

This result indicates the conditions for the economy to enter $\mathcal{C}^3$, with consumption of industrial goods, obtained through imports. However, there may be the possibility that in the transition to that stage, consumption rises as much as to lower the bequests (and therefore the capital for the next period) to levels below the transition point, forcing the economy to return to stage 2. A condition to avoid that trap is summarized in the following:[5]

Theorem 1 *A sufficient condition for not returning to stage 2 is that $\beta(\widetilde{i}_2 - \widetilde{i}_1) < \widetilde{b}_2^o$.*

Proof: *The economy enters stage 3 once the capital stock exceeds $\widetilde{X}_2$, with $f(\widehat{X}_1^h) > f(\widetilde{X}_2) > f(\widehat{X}_1^l)$. But in order to ensure that it does not return to $\mathcal{C}^2$, the unstable steady state at $\mathcal{C}^3$, $\widehat{X}_2^l$, must be such that $f(\widehat{X}_2^l) \leq f(\widetilde{X}_2)$.*

The functions that characterize the dynamics of bequests at $\mathcal{C}^2$ and $\mathcal{C}^3$ are:

$$\mathcal{C}^2: \quad b^o = \beta(i - \widetilde{i}_1) = \beta i - \beta\widetilde{i}_1$$

$$\mathcal{C}^3: \quad b^o = \beta(i - \widetilde{i}_2) + \widetilde{b}_2^o = \beta i - (\beta\widetilde{i}_2 - \widetilde{b}_2^o)$$

Since the only variable is i, the latter is a parallel displacement of the former. Then, if we take the difference between them, say b^o in $\mathcal{C}^3$ less b^o in $\mathcal{C}^2$, suppose that

$$-(\beta\widetilde{i}_2 - \widetilde{b}_2^o) + \beta\widetilde{i}_1 > 0 \quad (\mathbf{II})$$

If the sign of this expression is positive, there is an upwards displacement of bequests in going from $\mathcal{C}^2$ to $\mathcal{C}^3$. If so, the fixed points at $\mathcal{C}^2$, $\widehat{X}_1^l \leq \widehat{X}_1^h$, and those at $\mathcal{C}^3$, $\widehat{X}_2^l \leq \widehat{X}_2^h$, will be such that $\widehat{X}_2^l \leq \widehat{X}_1^l$ and $\widehat{X}_1^h \leq \widehat{X}_2^h$.

In fact (**II**) *is equivalent to $\beta(\widetilde{i}_2 - \widetilde{c}_{A1}) < \widetilde{b}_2^o$, in which case we have that:*

[5] A background condition that we assume from now on is that there exists, at each $\mathcal{C}^i$ ($i = 2, 3, 4$), at least one fixed point in the dynamics of bequests.

$$f(\widehat{X}_2^l) \leq f(\widehat{X}_1^l) \leq \ f(\widetilde{X}_2)$$

which ensures that capital will not decrease down from $\widetilde{X}_2$. □

We are interested in modeling a "late comer" economy, where consumption of industrial goods grows significant after other countries have already developed a manufacturing sector and participate in international trade of goods. This means that industrial goods are available for trade. The international price of good I is assumed to be sufficiently low (relative to domestic factor prices) to discourage production. This assumption held also in the previous stage (of pure agricultural consumption), and will continue to hold if the economy enters into the next stage, where landlords consume services. We explore in the appendix the alternative scenarios of "early industrialization", where the economy produces the industrial goods consumed locally, or manufacturing activities get started "for export" before there is a domestic demand for those goods.

4 The rise of public education

4.1 Demand for services in a late-comer economy

As seen in the previous section, some economies may never go through a phase of industrial production, even if their income is such that they include manufactured goods in their consumption bundle. Then the economy may reach a stage with a significant demand for "sophisticated" services before undergoing industrialization. The following proposition describes the dynamics in this stage.

Proposition 5 *Let* $\widetilde{i}_3 = f(\widetilde{X}_3) = \widetilde{c}_{A3} + p_I\widetilde{c}_{I3} + \widetilde{b}_3^o$. *If* $\widehat{X}_2^h > \widetilde{X}_3 > \widehat{X}_2^l$, *the economy enters into stage 4, where* $\widehat{X}_2^h$ *and* $\widehat{X}_2^l$ *are, respectively, the high and low steady states when the dynamics of bequests is determined by dynamics on* $\mathcal{C}^3$.

A sufficient condition for avoiding a downward-jump in bequests once in $\mathcal{C}^4$ *is that* $\beta(\widetilde{i}_3 \ - \ \widetilde{i}_2) \ < \ (\widetilde{b}_3^o - \widetilde{b}_2^o)$.

Proof: *Immediate.* $\widehat{X}_2^h > \widetilde{X}_3 > \widehat{X}_2^l$ *simply establishes that accumulation in* $\mathcal{C}^3$ *has reached the point where the consumption basket diversifies to include services (the condition depends on the price* p_I*) before attaining the stable steady state at that stage. To obtain the sufficient condition for continued accumulation it is enough to recast the proof of Theorem 1 for* $\mathcal{C}^4$*.* $\square$

The evolution of the consumption and production structure need not end there. The possibility that an economy develops an industrial sector after services, for example, may be of special interest. However, we shall not pursue that analysis here, and concentrate on the question of how the supply of educated workers to produce services is generated.

4.2 The Emergence of Human Capital

The demand for services would induce a demand for skills. We assume that the skills required for the provision of services require some kind of formal education, which must be acquired when young. Thus, at any given time the number of (adult) skilled workers is fixed: subsistence workers cannot migrate into the service sector. This implies that different wages can prevail in both sectors, since there is no arbitrage opportunity, and therefore that there are potential gains for a young unskilled worker considering whether to acquire human capital. But young subsistence workers are credit-constrained, and individual landlords do not have incentives to finance the education of young workers who, by assumption, can freely choose their employment and cannot commit to the repayment of potential "education loans". As a consequence, some kind of collective action mechanism might improve the welfare of the elite. As in GM, but through a different channel, young landlords who anticipate their future demand for services might accept to finance the education system by way of a tax on the bequests they receive.

4.2.1 Public education

Adult agents with an income above a certain threshold will demand services. These will be provided by skilled workers. Here, the education of those skilled workers is assumed to be provided by public school system financed by taxes on the bequests received by young landlords. The characteristics of this public system are decided upon by a central authority who internalizes the optimal behavior of the group of young landlords, and

can perfectly enforce tax collection (for simplicity, a balanced budget is assumed). Education is supposed to be convenient for the worker (that is: the wage of skilled labor is higher than $\widetilde{w}$). We assume that the authority can limit the size of the set of individuals who receive education (in practice, this may be done by varying the geographical coverage of the education system, or by determining conditions of schooling such that some groups have preferential access). The planner internalizes the quantity-quality choice of individual consumers (which would be established in the manner discussed above). Here we will assume, for simplicity, that the solution of the policy problem will be such that number of workers per landlord has reached the saturation point $\widetilde{n}$, so that the number of individuals that receive education is determined by that condition, and the margin of decision of the authority is on the level of skills to be supplied.

Then, taking into consideration the consumption behavior of the set of landlords (denoted group A), the authority will establish taxes on bequests (which, in this framework, operate as lump-sum transfers from landlords to the government), and choose a distribution of human capital ($g(h)$) that young workers to be educated receive. This distribution (which, in the optimum will be concentrated on a single point) results in an average level of skills $\bar{h}$. As stated above, skills are produced with (agricultural) inputs according to the function $h = h(e)$.

The results of the optimization of the policy-maker are summarized in the following proposition.

Proposition 6 *The education system will provide a single level of education e to all the group of individuals who receive training. If for some e^*, the function $h(\cdot)$ has an elasticity $\epsilon_h(e^*) = 1$ then the optimal amount of workers that receive education is $n \leq \widetilde{n}$ and the level of education will be e^*. Otherwise, if $\epsilon_h(e) < 1$, $\widetilde{n}$ workers will be educated and e will verify the following condition between the marginal utilities of income (derived from holdings of land and physical capital) and education for service suppliers:*

$$(1-(1-\beta)\alpha_{N4})\widetilde{n}\frac{f'(b-\widetilde{n}e)}{i^A-\widetilde{i}_3} \;=\; (1-\beta)\alpha_{N4}\frac{\bar{h}'(e)}{\bar{h}(e)}$$

Proof: *Let n be the number of workers who receive education per landlord, and $l(e)$ the proportion of those agents who receive an education corresponding to spending e; $\int l(e)de = 1$. Total spending in education is $n\int el(e)de$, and the income of the average landlord, when old, is: $i^A = f(b-n\int el(e)de)$, given that the bequests received when young have been taxed in the amount*

necessary in order to finance education expenditures. Given the demand functions in the stage $\mathcal{C}^4$, the utility of the landlord can be written as (ignoring constants):

$$u^A = (1-(1-\beta)\alpha_{N4})\ln(i^A - \widetilde{i}_3) + (1-\beta)\alpha_{N4}\ln n \int h(e)l(e)de$$

Remembering that $i^A = f(b - n\int el(e)de)$, maximization with respect to $l(e^k)$ subject to the constraint $\int l(e)de = 1$ results in the following, if in the optimum $l(e^k) > 0$:

$$-(1-\gamma)\frac{f'}{i^A - \widetilde{i}_3}ne^k + \gamma\frac{h(e^k)}{\overline{h}} \ - \lambda = 0$$

where $\gamma = (1-\beta)\alpha_{N4}$, λ is the multiplier of the constraint, and the average level of skills is: $\overline{h} = \int h(e)l(e)de$.

Considering the condition for a level of education $e^k + \Delta e$:

$$(1-\gamma)\frac{f'}{i^A - \widetilde{i}_3}n(e^k + \Delta e - e^k) \ = \ \gamma(h(e^k + \Delta e) - h(e^k))\frac{1}{\overline{h}}$$

i.e. $\frac{\Delta h(e^k)}{\Delta e}$ is constant for every e^k with non-zero demand. Therefore, h' must be the same at every optimal level. That is, there exists just one value e^* at which this is true, since h is strictly concave.

Then, $l(e)$ is a degenerate distribution that yields a single value $h(e)$, which, according to the preferences of the A agents has to be $\overline{h}$.

The expression of u^A can be rewritten as:

$$u^A = (1-\gamma)\ln(i^A - \widetilde{i}_3) + \gamma\ln n\overline{h}(e)$$

Maximizing the utility with respect to the single value e results in:

$$(1-\gamma)n\frac{f'}{i^A - \widetilde{i}_3} = \gamma\frac{\overline{h}'(e)}{\overline{h}(e)}$$

while the first order condition for n is:

$$(1-\gamma)\frac{f'}{i^A - \widetilde{i}_3}e \leq \frac{\gamma}{n}$$

with strict equality if the interior solution is such that $n \leq \widetilde{n}$. In that case, the ratio between the two first-order would be e^* such that $\frac{h'(e)}{h(e)}e = 1$. With $e = e^*$, if the optimal value of n for $\psi = n$ was larger than $\widetilde{n}$, then the solution would correspond to the level of e that satisfies the corresponding FOC with $n = \widetilde{n}$; this value would be such that $\epsilon_h(e) < 1$ and $e > e^*$. $\square$

This proposition establishes that, if $\widetilde{e}$ is the level of education such that the elasticity of skills with respect of education is one, as long as the optimal spending in education is $E \leq \widetilde{n}\widetilde{e}$, education per worker will be fixed at $\widetilde{e}$, and the expansion of education will be "extensive", through the increase in n. After that threshold is reached, $n = \widetilde{n}$, and the additional spending will result in proportional increases in e.

In what follows we will assume for simplicity that education has already saturated the level of workers, although the model seems capable of rationalizing the existence of a stage where a growing number of workers receives basic education, followed by another where the size of the educated set stops increasing and the average level of skills rises. Also, if the élite is subdivided, in the sense that the education system is destined to satisfy the demands of groups with different incomes and demands for services, there can be a distribution of people who receive different levels of education; the size of the members of each level would be a function of the size of the set of landlords that demands services requiring those skills. In any case, if all landlords require the services of $\widetilde{n}$ workers, the size of the group of educated workers would be proportionate to the size of that elite.

It may be noticed that the solution implied by the optimization of the welfare of the landlords has a pro-education bias (given that $n = \widetilde{n}$) compared to the case where the planner takes into account the interests of the educated workers: when increasing the supply of skills, all the additional services to be produced will be consumed by landlords, while an increase in the output of agricultural goods will be shared between landlords and service workers, because the latter will benefit from a rise in the value of spending on services.

In the transition where members of group A start to demand services, the number of workers who receive education is bounded at $\widetilde{n}$ per landlord. While we assume this for any economy, the distribution of land ownership affects the aggregate number of educated workers. To see this consider, again as in section 2.2.2, two economies (ϵ_1 and ϵ_2)with the same amounts

of land but different number of landlords , m_1 and m_2, respectively, with $m_1 < m_2$. Then we have:

Corollary 1 *In economy ϵ_2, the start of education would occur later than in ϵ_1, but the number educated individuals who receive education would be larger.*

Proof: *The first part is a immediate consequence of Lemma 2: since in ϵ_1 services will be consumed earlier, the educational system has to be created before than in ϵ_2. On the other hand, according to Proposition 6, once reached the critical number of educated agents, $\widetilde{n}$ per landlord, and maintaining the assumption that only landlords consume services, the number of educated agents will be $m_1\widetilde{n} < m_2\widetilde{n}$, i.e., the number of educated individuals will be larger in ϵ_2.* $\square$

The emergence of taxation to finance education modifies the dynamics of bequests, since these are determined by income, which is reduced by taxation through its effects on capital. However, it can be shown that the condition for accumulation to proceed in the stage with consumption of services is the same as the condition found before, when taxation was not considered.

Proposition 7 *The same conditions on $\widetilde{X}_3$ established before for accumulation to proceed after the transition to stage 4 hold also with an optimal level of taxation on bequests, except if $\widetilde{X}_3 = \widetilde{b}_3 = \widehat{X}_3^l$ with $e > 0$.*

Proof: *For the proposition to hold, we have to consider a new transition value $\widetilde{X}_3' = b - ne$. If $f(\widehat{X}_2^h) > f(\widetilde{X}_3')$ and $f(\widehat{X}_3^l) \leq f(\widetilde{X}_3') \leq f(\widehat{X}_3^h)$, the accumulation process will continue. To see that this can in fact be so, recall that these conditions are verified by $\widetilde{X}_3$. So, if $\widetilde{X}_3'$ is close enough to $\widetilde{X}_3$ we are done. In fact, expenditures on education will start from values near zero, since when $f(\widetilde{X}_3')$ is very close to $\widetilde{i}_3$, $\frac{\bar{h}'(e)}{\bar{h}(e)}$ grows unboundedly large. That means that e is very close to 0. So, the only possible problem arises if $\widetilde{X}_3 = \widehat{X}_3^l > \widetilde{X}_3'$.* $\square$

4.2.2 The price of human capital

We are interested in analyzing the conditions that link the price of human capital to other parameters in the model. A first, and intuitive, result is that

holding international prices constant, all the relevant quantities depend on the level of capital accumulation in the economy, as summarized in the level of bequests, b. Since the capital stock increases with b, and the value of spending in services is proportional to the income of landlords in excess of the threshold $(i - \widetilde{i}_3)$, if the number of service workers remains constant at $\widetilde{n}$ per landlord, the wages of those individuals grows directly with b. Such wages result from the level of skills of the workers and the unit "price of skills". The evolution of that price (or, in other words, the rate of return on education) is described in the following proposition.

Proposition 8 *The wage per unit of skills is given by:*

$$w_h = \frac{w(\bar{h}(e))}{\bar{h}(e)} = \gamma \frac{f(b - \widetilde{n}e) - \widetilde{i}_3}{\widetilde{n}\bar{h}(e)}$$

where, as before: $\gamma = (1 - \beta)\alpha_{N4}$

Let $z = f(b - \widetilde{n}e) - \widetilde{i}$*, and let* $\widehat{f}(x) = \frac{f'(x)}{f(x)}$ *denote the logarithmic derivative of the function* f*. Education and the capital stock vary as a function of* b *according to:*

$$\frac{\partial e}{\partial b} = \frac{\widehat{z'} - \widehat{z}}{\widetilde{n}(\widehat{z'} - \widehat{z}) + (\widehat{\bar{h}'} - \widehat{h})} > 0$$

where $\widehat{z} = \frac{f'(X)}{f(X) - \widetilde{i}_3}$, $\widehat{z'} = \frac{f''(X)}{f'(X)}$, $\widehat{h} = \frac{h'(e)}{h(e)}$ *and* $\widehat{h'} = \frac{h''(e)}{h'(e)}$
and:

$$\frac{\partial X}{\partial b} = 1 - \widetilde{n}\frac{\partial e}{\partial b} = \frac{(\widehat{\bar{h}'} - \widehat{h})}{\widetilde{n}(\widehat{z'} - \widehat{z}) + (\widehat{\bar{h}'} - \widehat{h})} > 0$$

Then, the sign of the change in the unit wage is governed by:

$$sgn(\frac{\partial w_h}{\partial b}) = sgn(\frac{1}{w_h}\frac{\partial w_h}{\partial b}) = sgn((1 - \gamma)\frac{f''}{f'} - \gamma\frac{\bar{h}''}{\bar{h}'})$$

Proof: *The characterization of* w_h *just recasts the demand function of services*

$$\widetilde{n}w(\bar{h})h = \gamma(i^A - \widetilde{i}_3)$$

or

$$w(\bar{h}) = \frac{\gamma}{\widetilde{n}}\frac{z}{h}$$

On the other hand, $\frac{\partial e}{\partial b}$ *obtains by differentiating w.r.t.* b *the semi-elasticity condition of arbitrage between investments:*

$$(1-\gamma)\frac{f'(b-\widetilde{n}e)}{f(b-\widetilde{n}e)-\widetilde{i}_3} = (1-\gamma)\widehat{z} = \gamma\widehat{h} = \gamma\frac{\bar{h}'(e)}{\bar{h}(e)}$$

The characterization of $\frac{\partial X}{\partial b}$ *follows from the specification of* $\frac{\partial e}{\partial b}$.

The condition on the sign of $\frac{\partial w_h}{\partial b}$ *is obtained by differentiating w.r.t.* b *the characterization of* w_h.□

Thus, the evolution of the returns on skills depends on the technological features of the agriculture and education sectors. For example, if the productivity of education falls less quickly than the productivity of investment in sector A then, for a given rise in bequest, investment would be increasingly directed towards education, and at some point the wage per unit skills would decline.[6]

4.3 Moving Ahead: Brief Comments on Subsequent Phases

Certainly, the creation of a large public school system might affect the way in which a late-comer land-rich economy would evolve. Here, we briefly mention some alternative paths that may be followed, and which seem capable of being analyzed using the basic framework of this paper.

Possible Emergence of Manufacturing At some point, the effect of decreasing returns in the primary goods sector and the expansion of the supply of skills could result in the emergence of a manufacturing sector. This may also occur if, for some reason, the international price of industrial goods increases relative to those of agricultural commodities. That is, industrialization may take place through a price shock or, possibly, as a consequence of the accumulation of factors.

In any case, capital would flow to manufacturing. This may change the political economy in several ways. One issue would be whether landlords

[6] As a consequence of the assumption that the production of skills depends only on the input of goods, the result ignores a potentially important effect: education itself can be a (maybe large) source of demand for skills. In the context of the model, this may introduce a wedge between the interests of young landlords (who want education to increase the future supply of services) and the older generation, who would have their consumption opportunities reduced as skilled labor is drawn from the direct production of services to the education sector.

transform themselves into entrepreneurs with interests in both tradable sectors, or whether they are lenders of resources to a new group of industrial entrepreneurs (the division of the elite has been an often emphasized feature of resource-based economies with an incipient industrialization). Also, it is likely that the group of educated workers gains political influence. With the emergence of manufacturing, these agents would have mixed interests as suppliers of services, as workers in the manufacturing sector and, if their income is sufficient, as consumers of industrial goods. In addition, the inflow of workers from the subsistence sector to the (mainly urban) industrial sector may also create a new significant group of influence, with interests in raising the demand for industrial labor. That configuration is likely to raise issues related to industrial protection and the public spending in education.

Diversified Consumption of Skilled Workers If skilled workers get rich enough in the process of economic growth, they would start to diversify their consumption-bequest basket. If educated workers leave bequests to their offspring, the accumulation of resources would take a different form. The specification of the investment options of those workers may vary: if there is an active capital market, they could act as lenders of funds to owners of physical capital, as well as potential purchasers of private education. From the point of view of the landlords, the emergence of a privately financed education sector would represent a positive development, since that would lower the price of skills without taxation on the elite. This development may lead to a crowding out effect on public education.

The appearance of a demand for industrial goods by skilled workers does not vary noticeably the pattern of evolution in the land-rich economy: this will become more open as demand shifts from locally produced agricultural goods to imported manufactures, without much change elsewhere. The case would be different if the wage of skilled workers rises to the point where they also demand services. There would then be a "secondary" demand for skills (and for new skilled workers), on the part of the agents who previously were selling services to landlords/capitalists. Also, the incentives of policymakers would change, since an A planner would recognize that there are no exclusive goods any more, consumed only by landlords. An increment in the supply of skills would benefit the educated workers as consumers. However, the A planner would likewise internalize the fact that a lower supply of skills increases the wage of educated workers, and raises their demand for services, which would crowd out the demand by landlords. The two effects would be

weighted when considering taxation and spending in education.

5 Conclusion

We have presented a model that can rationalize different patterns in the emergence of educational systems, in a way that can be pertinent in accounting for contrasts between the experiences of countries in the American continent in the 19th century. As a representation of economic development, the range of validity of the model is still limited by the fact that we have disregarded phenomena like capital movements and, especially, technical change, which should be central elements in a more general analysis. However, as it is, the model seems useful to highlight different motives for the elite to finance the education of low-income workers, and to point out possible alternative paths of economic evolution.

The model focuses on the demand for human-capital-intensive services of high-income groups. This channel can generate a demand for education, and appears because we adopt a setup with multiple goods, where consumption preferences are non-homothetic and the demand for skill-intensive commodities emerges at comparatively high levels of income. We also assumed that the quantity and quality of labor are not perfect substitutes; consequently, the number of high-income agents may have strong effects on how many individuals are subsidized to accumulate human capital.

Several classes of economies, with different qualitative behaviors, were identified. The first kind is that of early comers to industrialization. These are economies where, in the process of capital accumulation, agricultural productivity is high enough to generate a widespread demand for manufactured goods, which must be produced internally. The growth of the agricultural-industrial economy (with a bias towards industry, due to the higher income elasticity of the demand for the corresponding goods) may lead to the emergence of a demand for skills.

A second class of economies are those which are well endowed with labor, and where agricultural productivity is not enough to trigger industrialization for the domestic market, but can engage in labor intensive manufacturing for exports if and when an international market for those goods develop. In this basic setup, we merely refer to the first steps of industrialization for these economies. Further work should certainly consider with more detail the processes of technical change and the incentives to supply and demand for human capital in production. Preliminary results suggest that in order

for education to emerge as a result of a capitalist-led political choice, the wages of unskilled workers should be sufficiently high (see the Appendix).

Our focus was on land-rich economies where income growth is such that a large demand for industrial goods appears at a time when the supply by early comers is already well developed. Straightforward comparative advantage implies that those economies will import manufactures. If the demand for sophisticated services starts for incomes above a certain threshold, increases in the value of the output of primary goods can imply that, at some point, a demand for skilled labor may appear in order to satisfy that consumption by high-income groups. These groups, then, would not oppose the emergence of public education to increase the skills of a set of workers, the number of which would depend on the number of landlords who demand services. Hence, the diffusion of education would depend on the size of the elite and, indirectly, on the degree of concentration of land ownership. The growth of an educated class can change the political balance, and the incentives to provide public education, by incorporating into the picture a new influential group, and also by giving rise to a population who in some cases may self-finance the acquisition of skills of descendants. A large manufacturing activity may or may not arise spontaneously. Over time, a new political economy of industrial protection is likely to result from the interplay of the interests of landlord, capitalists, skilled workers (at first, mainly occupied in services). Quite different paths seem possible according to how the implicit conflicts are processed.

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7 Appendix

7.1 Early industrialization: a preliminary sketch of an argument

Consider an economy that accumulates agricultural capital *before* there is a significant international supply of industrial goods. If capital accumulation proceeds to the point where landlords start demanding industrial goods, they must be produced locally since the economy is closed for all practical matters. Then, it is clear that production would diversify into manufacturing as a result of the new composition of consumption demands. Thus, if and when the income of landlords reaches the threshold where their consumption diversifies, there would be a shift of capital into manufacturing, and the dynamics would change compared with the agricultural stage. Now, the logic of capital-skills complementarity would apply. With certain technologies, manufacturing skills may be acquired through on-the-job training (apprenticeship). At some point, the provision of public education may be in the interest of capitalists, as in GM.

Once the early-comers to industrialization have engaged in that path, the economies that lag behind in capital accumulation need not follow the same sequence: when there is an active international market where manufactures trade for primary goods, the late-comers may industrialize "prematurely", or alternatively, become producers of services for high-income groups without first developing manufactures. Regarding the first possibility, standard international trade arguments indicate that an economy with suitable factor endowments can produce industrial goods for the world market, independently of domestic consumption. This would be more likely if landlords are sufficiently frugal and entrepreneurial while agriculture is not-too-productive (which speeds up the arrival of the moment where investment in manufacturing becomes profitable at the margin relative to accumulating agricultural capital) and there is a large supply of labor capable of moving from a subsistence sector to manufactures (à la Lewis). Also here the provision of education would be likely to be predicated on a perceived demand for more skilled industrial workers.

7.2 Export-led industrialization

This case would represent an economy which, with suitable factor endowments, produces good I for the world market, even without a widespread

consumption of that good. Given the specification of the production function and the assumption that uneducated labor is supplied elastically at wage $\widetilde{w}$, and assuming that capital is perfectly mobile between sectors:

Proposition 9 *Capital X is allocated to sector I, when workers have no skills acquired through education, if:*

$$f'(X) \ \leq r_I(p_I, \widetilde{w}, h_0)$$

where $r_I = \widetilde{w}\frac{\rho_X}{\rho_L}\frac{L}{X}$ is the return on capital in manufacturing and the (subsistence) wage is $\widetilde{w}$, while labor has the basic (zero-education) level of skills. At $\widetilde{X}_I$, the minimal level of capital accumulation that verifies this condition, the economy will enter into the manufacturing stage. A sufficient condition that ensures that the economy will not return to agricultural stage, once it entered into the manufacturing stage is that $\beta(\widetilde{i}_2 \ - \ \widetilde{c}_{A1}) \ < \ \widetilde{b}_2^o$.

Proof: *Given that $\widetilde{X}_I$, the minimal value that verifies the condition is lower than the stable steady state in the agricultural stage, $\widehat{X}_1^h$, capital will also be invested in manufacturing. The sufficient condition to avoid the return to the agricultural stage is the same as for the economy without production of manufactures, as described by Theorem 1.* □

The condition above will be more easily satisfied with low subsistence wages, not-too-productive agriculture and frugal capitalists. High productivity and/or high prices of good I do as well induce production.

If the workers do not save or educate and only agricultural goods are consumed, the system can be described by:

$$i_K = f(X_A) + (X - X_A)r$$

$$f'(X_A) \ = \ r = r_I(p_I, \widetilde{w}, h_0)$$

$$b^o \ = \ \beta(i_K - \widetilde{i}_1) = \ X \ = \ X_A + X_I$$

The first equation defines the income of the capitalists (agents K, who are at the same time the landlords) as the output of the agricultural sector plus the return on capital invested in manufacturing. The second equation specifies the equilibrium allocation of the capital inherited from the previous generation and the rate of return. The third equation establishes the

bequest, and specifies that it must be used to install future capital in both sectors.

In order to consider the incentives to start an educational system in such an economy, consider now the existence of a social planner representing the capitalist-landlord agents (K). This authority may tax agents K (the agents who leave bequests) in order to finance public education, as a representative of that group. The incentive would be to educate individuals will work in industry I. In principle, it may be the case that the industry hires educated and uneducated workers. Let μ be the fraction of the labor force (L) that is skilled, and let $\bar{h}(e)$ be their average level of human capital, which corresponds to a per-individual expenditure in education denoted by e. Then, total spending in education would be:

$$E \;=\; \mu L e$$

The allocation of resources would be driven by the maximization of the total returns to agents K derived from agriculture and industrial activities. Then, it may be shown that:

Proposition 10 *Spending in education will not start as long as:*

$$f'(b - X_I) \;=\; r_I(p_I, \widetilde{w}, 0) \;>\; \frac{1}{\rho_L}\frac{\varphi'(0)}{\varphi(0)}\bar{h}'(0)\widetilde{w}$$

and this condition can be expressed as:

$$(\frac{p_I}{\widetilde{w}})^{1/\rho_X} > v\frac{\varphi'(0)}{(\varphi(0))^{1+\frac{1}{\rho_X}}}\bar{h}'(0)$$

Proof: *Skilled workers must earn at least $\widetilde{w}$. Otherwise they will not accept employment in manufacturing. Consider now a K planner contemplating an investment in education, starting from a situation where all industrial workers are unskilled. The problem of the planner can be stated as:*

$$\max_{X_I,\mu,L,e} \; f(b - X_I - \mu L e) \;+\; p_I\varphi(\mu\bar{h}(e))L^{\rho_L}X_I^{\rho_X} \;-\; \widetilde{w}L$$

The expression indicates the aggregate (future) income of a (now) young capitalist, given that the level of bequests is b, a fraction μ has received skills $\bar{h}$, so that the average level of skills is $\mu\bar{h}$, and all workers L receive a wage

$\widetilde{w}$. *Clearly, all choice variables must be non negative. A solution would satisfy first order conditions given by:*

$$X_I : \qquad -f' + \rho_X \frac{p_I y^I}{X_I} \;\leq\; 0 \quad (\mathbf{i})$$

$$\mu : \qquad -f' Le \;+\; \frac{\varphi'}{\varphi}\bar{h}(e) p_I y^I \;\leq\; 0 \quad (\mathbf{ii})$$

$$L : \qquad -f'\mu e \;+\; \rho_L \frac{p_I y^I}{L} \;-\; \widetilde{w} \;\leq\; 0 \quad (\mathbf{iii})$$

$$e : \qquad -f'\mu L \;+\; \frac{\varphi'}{\varphi}\bar{h}'(e)\mu p_I y^I \;\leq\; 0 \quad (\mathbf{iv})$$

For $\mu = 0$ *we obtain from* (**iii**) *that* $\widetilde{w} = \rho_L \frac{p_I y^I}{L}$ *(at an interior solution for* L*).*

On the other hand, (**ii**) *is equivalent to:*

$$-f' + \frac{\varphi'(e)}{\varphi(e)} \frac{h(e)}{e} \frac{p_I y^I}{L} \leq 0$$

and replacing by the expression for $\widetilde{w}$ *we have:*

$$-f' + \frac{\varphi'(e)}{\varphi(e)} \frac{\bar{h}(e)}{e} \frac{\widetilde{w}}{\rho_L} \leq 0$$

Taking the limit of this expression for $e \to 0$*, for* $\mu = 0$ *we have that:*

$$\frac{\widetilde{w}}{\rho_L} \frac{\varphi'(0)}{\varphi(0)} \lim_{e \to 0} \frac{\bar{h}(e)}{e} \leq f'$$

which recalling the L'Hopital rule ($\lim_{e\to 0} \frac{\bar{h}(e)}{e} = h'(0)$*) yields the desired result.*

On the other hand, from $r_I = (s\varphi(0) p_I \widetilde{w}^{-\rho_L})^{1/\rho_X}$*, with* s *a constant, we have the equivalent expression sought.* □

7.3 Early industrialization for the domestic market

The discussion in the previous exercise assumed that the price p_I was determined in the international market. But, if there is no developed world market (or the country is the "first comer"), industrial goods cannot be purchased abroad, but must be produced internally when the demand arises. Production diversifies in parallel with consumption. Assuming that the no-education condition holds, the economy would now be described by the following equations:

- Demand for industrial goods:

$$p_I y^I \;=\; (1-\beta)\alpha_{I3}(i^K - \widetilde{i}_2)$$

  where the notation is as before, and i^K is the income of the landlord/capitalists.

- Income of agents K:

$$i^K = f(X_A)\; r_I(p_I, \widetilde{w}, \bar{h} = 0) X_I$$

  where, as before, $X_{A,}X_I$ are the capital stocks in each sector, r_I the rate of return of capital in manufacturing.

- Supply of good I:

$$y^I = y^I(p_{I,}\widetilde{w}, h)$$

- Allocation of capital:

$$f'(X_A) = r_I(p_I, \widetilde{w}, 0)$$

- Allocation of bequests:

$$b = X_A + X_I$$

- Employment in manufacturing:

$$L = X_I l(p_{I,}\widetilde{w}, h)$$

  where l is the labor/capital ratio.

- Dynamics of bequests:

$$b^o = \beta(i^K - \widetilde{i}_2) + \widetilde{b}_2^o$$

The system can be completed by specifying the choices on education, which could be determined, as before, by a government that optimizes on behalf of group K.