

# Horizontal Mergers and Promotional Spending 

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## Introduction

In August 2006, the Argentine Antitrust Agency (CNDC, in Spanish) issued a document regarding the acquisition by Arcor S.A.I.C., a major producer of a broad variety of food products including candies and jams, of Benvenuto S.A.C.I., a smaller competitor of Arcor in some markets within the Argentine territory. The key issue regarding competition was the fact that both companies were major players in the jam market, which has relatively few firms competing amongst each other. The absorption of one of those companies, Benvenuto, by another, would have meant a drastic reduction in the number of firms competing in this market. Because this was happening in a context of differentiated products, this meant that the reduction of competition and the internalization of externalities surely would have implied some kind of price increase, and proliferation of brand names produced by the company emerging from the fusion.

In the long run, if the market for jams did not face significant competition from other firms or similar products, due to significant barriers to entry, this would have implied a situation close to a monopoly, as most of the market share would now be in the hands of one large firm, operating many differentiated brands of similar products. The issue of competition from other products (such as milk caramel, very popular in Argentina) and other firms (such as small conserve and jam producers, who operate in niche markets and have a small, homemade production) led to the conclusion that the fusion was to be approved under several conditions which reduced the negative impact on competition of the said acquisition.

Amongst the conditions imposed on Arcor, there was one which addressed the topic of advertising spending. The condition implied that, after the acquisition, Arcor was to maintain its spending in radio and television advertising roughly equal to the sum of both companies' spending before the merger took place. This condition restricted the possibility of further increasing the new firm's market share (and weakening competition) through an aggressive advertising campaign, and further aggravating the problems caused by the concentration of most of the market share under a single firm's control.

The CNDC does not mention a model or theoretical background to justify the policies it recommends in this document. The objective of this graduation thesis is to explore the interactions between advertising spending and mergers in a context of differentiated products. The ultimate aim is to evaluate the plausibility of the conditions imposed by the CNDC under a theoretical framework based on previous models of industrial organization, and to see how far the established theory endorses these clauses.

This graduation thesis is structured as follows: Section 1 summarizes the existing literature on advertising and promotional spending, and cites bibliography that helps us introduce the question. Section 2 briefly discusses horizontal mergers and how they are treated in the economic litreature. Section 3 develops a model which includes aspects of mergers and promotional spending, and is used to formulate predictions and evaluations based on a systematization of the
ideas exposed in the previous sections. Section 4 proceeds to inquire on the welfare consequences of the merger in this context, and section 5 concludes.


## 1 Advertising and Promotional Spending

Advertisement has been analysed in several different ways since it was first discussed in the fifties and sixties. At first, it was seen as wasteful by most economists. Nicholas Kaldor first argued that:
[...] advertising has a social function to fulfill. What requires consideration is whether it fulfills this function in a satisfactory manner, and without an unnecessary waste of resources which might have been devoted to other uses. As a means of supplying information, it may be argued that advertising is largely biased and deficient. Quite apart from the making of deliberately faked claims about products [...] the information supplied in advertisements is generally biased [...] makes no mention of alternative sources of supply; and it attempts to influence the behaviour of the consumer [...] by forcing a small amount of information through its sheer prominence to the foreground of consciousness. [Kaldor, 1950]

John Kenneth Galbraith, in his 1958 book "The Affluent Society", strongly criticizes advertisement as a social waste that induces people to consume more than they really want, and creates a desire that would otherwise never exist. This, in turn, creates more demand, and the system tends to produce more goods than would be necessary if advertisement never existed. The idea that the consumers' preferences could be altered by the companies' conscious advertising efforts clashed strongly with previous economic models that treat preferences as exogenous and given.

Later on, this idea was questioned by many authors. Lester Telser first suggested the idea of advertisement as a way of transmitting information from companies to consumers. The dynamic nature of the economy requires advertising to inform the consumers about new products, sales locations, etc.

In a static economy there would be less advertising. Information about goods and services, terms of sale, and the identity of buyers and sellers would not become obsolete. Catalogues and directories would never be changed. People would continue to use the same things in the same way. To the extent that advertising conveys pertinent information about such changes, it facilitates economic growth. [Telser, 1964]

An analysis based on conventional microeconomic theory can be revealing. We can assume that a typical firm faces a demand curve which slopes negatively with respect to price and positively against advertising spending. There is a basic trade-off in the optimal choice of advertisement, which is that, on one hand, one more unit of advertising persuades or informs more consumers about the product, shifting the firm's demand curve to the right; on the other hand, the bad news is that advertisement is expensive, and therefore we expect the firm to choose a level of advertising where the marginal cost of advertising equals the marginal benefit, for given prices and quantities.

A central result that emerges from micro theory, derived in Dorfman and Steiner (1954), is that, in a monopoly, the ratio of Promotional Spending to Total Sales Income (price times quantity) is equal to the ratio of advertisement
elasticity $\left(\eta_{\alpha}\right)$ to price elasticity $\left(\eta_{p}\right)$. Put more simply:

$$
\frac{\text { Total Advertising Spending }}{\text { Sales Income }}=\frac{\eta_{\alpha}}{\eta_{p}}
$$

This ratio suggests a positive relationship between advertising spending and market power (interpreted as low price elasticity). One possible interpretation of this is that advertising spending creates market power and thus reduces price elasticity (mathematically, that $\delta \eta_{p} / \delta \alpha<0$ ). By creating brand loyalty, it tends to distort competition, which can be seen as undesirable from a welfare point of view. However, this ratio suggests a much more interesting analysis. It states that the optimal advertising spending (as a percentage of sales income) will be a function of the price and advertisement elasticities. Therefore, it suggests a reverse causality. Where market power is high (demand elasticity is low), advertising spending will be high precisely because there is market power, and therefore the ratio of advertising/price elasticities is high (because the denominator is small). In mathematical terms, the interpretation is that $\delta \alpha$ $/ \delta \eta_{p}<0$, that increased market power (a reduction in $\eta_{p}$, for any reason) will increase advertising spending.

This finding also helps us draw some evidence to answer our question. If the merger is approved, then it is plausible to think that the new firm will face a more inelastic demand, because its consumers will now have to choose between the same number of brands, but several of these brands will be effectively controlled by the same firm. Therefore, in the aggregate, we could think of the new situation as an effective reduction of price elasticity for the new merged firm, since if prices are raised, fewer consumers will switch to brands not controlled by the firm, compared to the previous situation when it controlled a smaller number of brands. This, according to the Dorfman-Steiner condition, will cause the advertising spending to rise, due to the increase in market power (and thus, an increase in the elasticities ratio) of the new combined firm post-merger.

This is very important. The merger induces the new firm to spend more on advertising, since the new situation is farther from a competitive situation, and thus the firm can charge higher mark-ups, due to the lower price elasticity. This implies a higher benefit ensuing from capturing additional consumers through advertising, and thus leads to higher incentives to do so.

Haim Levy and Julian Simon (1989) construct a model in which the DorfmanSteiner condition is examined in a dynamic context, where advertisement determines demand in present and future periods, thus having a lagged effect which increases its effectiveness beyond the effect on the present period; however, the conclusions are essentially the same.

Gary Becker and Kevin Murphy (1993) have an interesting approach which treats advertisements as "one of the goods that enter the fixed preferences of consumers". Therefore, if the "consumption" of advertisements has some kind of complementary relation to the consumption of the advertised good (which is exactly the case, according to the authors), then advertisement affects the demand for the advertised product, not by changing the tastes of the consumer and
cheating him into things she doesn't want, but because of the complementary nature of advertisement of a given good and its consumption. This explanation helps to reject the idea that advertisement can fundamentally alter the consumers' preferences and induce them to purchase things they really didn't want in the first place, by treating advertisements as separate goods within the consumers' utility functions.

A seminal paper by Stigler (1968) also discusses the possibility of treating promotional spending as another variable under the control of firms operating in any given market. Thus, whenever facing constraints in price, firms may adjust to changes in the market through adjustments in non-price variables, such as advertising spending, so that competition still operates through other channels even though prices are fixed (for example, under a cartel). The key issue is that advertisement is seen as a variable that operates just like prices. Whenever there is a change in the conditions of the market, both price and advertising spending shift to new optimum values. This means that advertisement has a useful function within the economy, which is to allow for adjustment whenever prices are fixed. This partly helps us understand the rationale behind the conditions imposed by the CNDC. A constraint on both price and advertisement spending avoids the possibility of using adjustments in non-price variables (such as advertisement spending, which is a central determinant of demand) to achieve results which are similar to unconstrained pricing decisions, even though there is a formal constraint on prices. It is essential to view the pricing and advertising decisions as simultaneous and interconnected, and that imposing a constraint on the former can be insufficient if the firms are able to produce similar outcomes by fine-tuning their promotional spending decisions.

The view of advertisement as an activity with a useful social function was further refined by Phillip Nelson, making the distinction between advertisement in "search" goods and "experience" goods:

In consequence our results support the hypothesis that producers of experience goods advertise more than producers of search goods. [...] advertising of experience qualities increases sales through increasing the reputability of the seller, while advertising of search qualities increase sales by providing the consumer with "hard" information about the seller's products. [Nelson, 1974]

This distinction offers an explanation for most advertisements appearing to be vague and unclear. This is because, as Nelson argues, most advertisement happens in "experience goods" industries, where advertising is used as a "reputation" instrument. Nelson's argument further states the case that, although biased and misleading, advertisements do provide some information for consumers, possibly about the brand's reputation and reliability and not about the product itself.

Richard Schmalensee (1976) builds a model in a context of few sellers and differentiated products. He argues that price competition is not so common in this type of market, since the firms usually prefer avoiding "price wars" and competing through promotional spending. Thus, he models the firms as identical and taking decisions in a Cournot fashion, choosing quantities and taking the other firms' responses as given. One very interesting finding that
stems from his analysis is that entry (an increase in the number of firms) will always reduce promotional spending per firm (if the number of competing firms is large enough). This is very relevant for us, because a merger in can be interpreted as a reduction of the number of firms competing in the market. Thus, if there is a merger between two firms in this model, each firm will spend more on advertising its own products. However, the new firm will spend less than the sum of the advertising expenditures of the former two companies. Thus, from this model we can draw a preliminary hypothesis (based, however, on a very imprecise and introductory systematization of the problem we are addressing): after the merger, according to the Schmalensee's model, we would expect the new company's promotional spending to decrease, relative to the promotional spending pre-merger. However, this fails to take into account the fact that, in differentiated product industries, a merger implies a reduction in the number of firms, but not in the number of brands competing in the market. Thus, a more comprehensive analysis would take into account the interactions between different brands controlled by the same agent/firm, such as in a differentiated products Bertrand context (a good example is Grossman and Shapiro (1984), which we treat later).

One key distinction in the advertisement literature is the division between what is known as "informative advertising" and what is known as "persuasive advertising". These two sides basically have become the two leading points of view on the theory of advertisement. Informative advertising states that advertising can actually have the potential to increase market efficiency by aiding the consumers to find relevant information about the product on sale, including prices, qualities and varieties, thus strengthening competition and the flow of information. According to informative advertising theorists, advertisements can increase the size of the market. This is the case in models such as Butters (1977) and Grossman and Shapiro (1984).

Persuasive advertising, on the other hand, states that advertising is simply a "prisoner's dilemma" in which companies try to steal market shares from one another and end up in the same situation, but wastefully investing resources in something with no real value. Consumers are transferred back and forth between brands, but promotional spending is inevitable if a firm does not want to see its consumers switching to other brands. The type of advertisement found empirically clearly also depends on the type of industry under analysis, etc. The main literature to argue this case can be found in Dixit and Norman (1978) and Stigler and Becker (1978).

Greg LeBlanc (1998) investigates this question with a model based on an established-product Hotelling duopoly with the possibility to choose promotional spending for firms, and where the firms' cost functions are private information. The motivation is that:

Since informative price advertising appears to be an important feature of competition in many established markets and given that such advertising often neither creates new nor expands existing markets, the basic welfare question seems particularly compelling in these industries: is advertising a waste of resources? Do firms advertise merely to protect their market shares? Or,
alternatively, is advertising sufficiently pro-competitive that its resource cost is outweighed by an associated increase in consumer surplus?

We later develop a similar model, which omits the private cost feature, but provides insights with respect to mergers and externalities between competitors in this type of market. In LeBlanc's model, the outcome is that, whenever advertising is undertaken by firms in equilibrium, it improves social welfare, by resulting in a better match between a firm's actual cost and its price.

Another very useful model is developed by Butters (1977), where advertisement is seen as the only way in which producers of homogeneous goods can inform the public about the existence of the products they sell, and the price at which they are selling. In his model, therefore, the only source of product differentiation lies in the consumers' imperfect knowledge of the prices and existence of different products. A central result of his analysis is that the monopolistically competitive level of advertising is socially optimal, since it maximises total welfare by equating the social cost of advertising to its social benefit. This happens because the firms' incentives to advertise their location imply a social benefit through the consumers' ability to locate these businesses and purchase from them. Thus, the promotion effort that provides consumers for firms is reflected in the increased brand awareness of the consumers.

On the other hand, Dixit and Norman (1978) investigate advertising which changes consumer tastes, and find that the market equilibrium implies socially excessive levels of advertising, even when the post-advertising tastes are taken as the norm for welfare assessments, and under different market structures. Generally they find that advertising tends to create market power, which brings about the possibility of raising prices, reducing quantities and creating barriers to entry, thus tending towards a monopolistic situation and away from a competitive state.
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## 2 Horizontal Mergers

The literature on mergers has achieved a much wider consensus view than the one on advertising, with respect to implications for price variables. In particular, the literature on horizontal mergers includes mergers between firms that formerly competed in the same market, and that are now controlled by the same agent. This implies that now the focus on the decision-making process moves from considering two separate firms to considering one larger multi-product firm.

One problem that arises is usually known as the "merger paradox". It states that it is hard to construct a reasonable economic model in which two previously separate firms actually increase their combined profits after the merger (except in the case in which a monopoly is constituted).

However, one exception to the paradox is the case in which the firms compete in a differentiated goods market. The decision variables (the price each company sets) in a Bertrand model with differentiated goods are strategic complements, that is, when one firm raises its price, its competitors will find it profitable to raise their prices as well. This implies that an agent that controls two firms (or brands) will take into account the fact that raising the price of one brand will shift some consumers towards the other brand she owns. Therefore, by taking into account this positive externality between firms, in equilibrium prices are higher than the previous competitive situation, and the merger becomes profitable, both for the new larger firm, and for its competitors. One downside is that consumers face higher prices for the same products. Therefore it is reasonable to be fairly cautious when approving mergers in differentiated goods markets, as these tend to have a negative impact on aggregate welfare, especially when the number of firms is reduced.

Harold Hotelling devised a simple yet very famous model that is very relevant in this situation. In his formulation, he took into account the fact that sellers can differentiate their products to the point that a price reduction by one competitor will not necessarily take away the other competitors' customers entirely. Since consumers differ in their tastes, only the customers that were previously indifferent between two brands (or close to being so) will switch brands after a small price change. The equilibrium outcome (in competition) states that, whenever possible, firms will differentiate themselves from their competitors as much as possible, and will set prices proportional to the "transport cost" incurred by the customers. However, if the same agent controls (for the case of two firms) both brands, as is the case in a merger, then the result is radically different, and the "multi-brand monopolist" now charges "as much as possible" for the products, net of transport costs. This type of formulation of competition between firms in differentiated goods industries will be very useful in future sections, and can be extended to any number of firms.

In particular, we seek to apply this kind of horizontal merger reasoning in a setting of more than two firms, so that we can study the new interaction between brands that are now controlled by the same agent, and brands that are competing. For the case of a number of firms greater than two, the difference
with the two-firm model is that after a merger, competition still operates within the market, and an excessive price increase will imply a significant loss of market share for the firm(s) in question, if one of the remaining firms decides to keep its price lower. This will shape the incentives in such a way that the effect on prices will be much more limited than in the case of two firms, where a merger will imply a monopolistic situation ex post, and where decisions are taken in a setting where there is no competition whatsoever to keep the firms' pricing decisions bounded.

Another interesting feature of this kind of models will include the changes in advertising spending pre- and post-merger, where the union of two companies will change the incentives to advertise and induce the market to over or underprovide promotional spending, compared with a first best situation or with a competitive framework. This will enable us to deem whether the restrictions imposed by the CNDC will actually increase or decrease aggregate welfare, and give us an objective assessment of the effectiveness of the agency's ruling (for this type of setting and assumptions).
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## 3 The Model

### 3.1 Competition

At this point, we have done a fairly brief summary of different points of view on advertising and mergers. Taking into account the type of market both companies operate in, the type of competition and the type of products involved, we have found this model to be the most adequate to represent the situation and the one that allows the richest analysis. It involves aspects of both informative and persuasive advertising, as well as the possibility of analysing pre and post-merger situations, from an efficiency and welfare point of view.

It is a modification of a model first published by Grossman and Shapiro (1984), who devise a refinement of Hotelling's model, but do not assume that consumers are perfectly informed about the existence of alternative products (or any products at all). The role of advertisement in this case is to notify the customers that the products exist and have a certain price and quality (or location). Thus, firms compete in two dimensions: price and advertising spending (assuming locations are fixed). The role of advertising in this model, however, is twofold: it has a persuasive content, since it increases the firm's demand by informing consumers who were previously unaware of the product, who may now decide to consume the advertised product instead of the one they previously consumed. This reflects the vision that advertising tends to be wasteful by using resources to increase the market share of the advertiser, at the expense of its competitors. However, there is also an informative function. Consumers who do not receive any advertising do not consume the product. Thus, any advertisements that reach new consumers will increase the total size of the market and thus fulfil a social function, and increase total welfare. Therefore, both features discussed in the advertisement literature are considered.

In mathematical terms, for the case of four brands/firms, the model is as follows (the original circular model is derived in Salop (1979):

There is a "circular city", a circle with circumference normalised to length one, representing the distribution of consumers along some kind of spatial configuration, which could represent another dimension such as a preference for a certain characteristic of the goods which is continuous. There are also four firms/brands located at the same distance from each other (the location is assumed to be exogenous and given for the firms, but it could be determined by the maximal differentiation principle or some other incentive to differentiate as much as possible from its competitors).


The consumers are uniformly distributed along the circle (density is unitary), they derive a surplus $\hat{s}$ from consuming the good, and incur in a transportation cost t per unit of distance travelled to purchase the good. Their utility depends on where they consume the good. Each consumer is identified with the position on the circle that corresponds to her favourite variety/brand of the good. For a consumer whose position in the circle is $x$ :

$$
\mathrm{U}=\left\{\begin{array}{cc}
\hat{s}-p_{1}-t d\left(x, f_{1}\right) & \text { if purchase made at firm } 1 \\
\hat{s}-p_{2}-t d\left(x, f_{2}\right) & \text { if purchase made at firm } 2 \\
\hat{s}-p_{3}-t d\left(x, f_{3}\right) & \text { if purchase made at firm } 3 \\
\hat{s}-p_{4}-t d\left(x, f_{4}\right) & \text { if purchase made at firm } 4 \\
0 & \text { otherwise }
\end{array}\right.
$$

$p_{1}, p_{2}, p_{3}$ and $p_{4}$ are the prices charged by each of the four firms; $f_{1}$ through $f_{4}$ are the firms' positions in the circle. $\hat{s}$ is the consumers' common gross valuation of the good and $\mathrm{d}(\mathrm{x}, \mathrm{y})$ is the minimum distance between any two points on the edge of the circle. For any consumer that stands at position $x$, his valuation of the good (net of transport costs) will be $\hat{s}-\operatorname{td}(x, f)$. Clearly, we assume a linear transport cost. All travel happens along the edges; no consumer can cut through the circle. We also assume $\hat{s}$ to be large enough so as to guarantee that any consumer will be willing to purchase the brand farthest from her. In other words, if a consumer is only informed about the brand opposite to her, she will still be willing to consume it, since her gross valuation of the good justifies paying the transport cost, as well as the price (for a reasonable set of prices).

The difference with Hotelling's model is that a consumer can consume a product if and only if he receives an advertisement from the corresponding firm. Also, the circular nature of the space considered implies that no location is better than any other a priori.

We define $\phi_{i}(\mathrm{i}=1,2,3,4)$ as the fraction of consumers who receive an ad from firm i. $\phi_{i}$ is determined, in turn, by the advertising spending of each of the four firms. The cost of reaching a fraction $\phi_{i}$ of consumers is $\mathrm{A}\left(\phi_{i}\right)$, where $\mathrm{A}^{\prime}>0$ and A" $>0$. We also assume that the firm cannot target its advertisements so as to reach a certain part of the market; in other words, for a given $\phi_{i}$, the distribution of informed consumers for the firms will be uniform across the circumference. In this case we assume, without loss of generality, that $\mathrm{A}\left(\phi_{i}\right)=\frac{a \phi_{i}^{2}}{2}$, and
that advertising expenditure can have values between 0 (no advertising, and no customers) and $\frac{a}{2}$ (full coverage). This bears certain resemblance to Butters' model of informative advertising presented in section 1. However, in this case we assume products to be differentiated; thus, there is a more useful function for advertising which is to improve matches between consumers and product varieties.

The production technology is assumed to be subject to constant returns to scale, with a marginal production cost per unit of $c$. Because the number of firms is fixed and their location is constant and symmetric, it makes sense to look, in the beginning, for an equilibrium in which they all charge the same price $p$ and have the same level of coverage $\phi$. Because we assumed that consumers are not informed ex-ante about the existence of alternative products, there is now the possibility that the firm might sell its products to consumers as far as the other end of the circle. Thus, the firm's potential demand is actually more spread out than in Salop's perfect information model, since now the firm might retain consumers that choose its products even if there are much closer alternatives, simply because the consumer is unaware of such alternatives.

The order of the agents' actions is as follows: the firms choose their prices and advertisement spending (actually advertisement coverages) simultaneously, and consumers decide what good to consume based on the information they have. Profits are then determined for each firm.

Grossman and Shapiro partition the consumer space into 4 groups, where the $\mathrm{k}^{\text {th }}$ group corresponds to the set of consumers to whom firm k would offer the highest surplus of the n firms, were all consumers perfectly informed. However, we find it more useful to study five different cases that can happen, depending on the number of advertisements that an agent receives. We can distinguish 16 possible cases, where a consumer can be informed of the existence of any combination of the four brands (only about brand one, about brand one and two, brands two and four, etc.), or not be informed at all. To study all possible cases, we then need to characterise them by finding the indifferent consumers for all five types of firm position combinations.

The sixteen possible combinations of firms that a consumer can receive information from are:
[None]
[1,2,3 or 4$]$
[ 1 and 2,1 and 3,1 and 4,2 and 3,2 and 4,3 and 4$]$
[1, 2 and $3,1,2$ and $4,1,3$ and $4,2,3$ and 4$]$
[All Firms]
Clearly the latter cases imply a more informed decision by the consumer than the former. In the cases where the consumer only knows about the existence of


## Firm k+1

one firm, he chooses to purchase from it because he knows of no other alternative (and because we assumed his surplus $\hat{\mathrm{s}}$ to be high enough). These sixteen combinations can be summed up in five types of cases, where the demand for each of the participating firms is determined, depending on its pricing and its position. To find the total demand for any firm $k$, we characterise all possible cases for firm $k$, determine the demand in each case, and then multiply them by the probability of their occurrence. Thereby we end up with the four firms' demands in terms of their competitors' responses, as well as its own pricing and advertising decisions.

The first case, as we said before, is where the consumers are only informed about the existence of one brand. Graphically this case is represented in the top figure above.

The firm's demand in this case is 1 , since it can capture the whole market, no matter what the price it sets is.

The second case is where the consumers are informed about the existence of two adjacent firms. Graphically this can be shown as the bottom figure above.

In this case, the competition between the two firms will imply that a raise in price of one will be reflected in a lower proportion of total demand for itself, and a higher share of the demand for its competitor.To find the actual shares of demand, we calculate the indifferent consumers that determine the limits of each firm's demand on both sides.

A consumer who is at distance $d$ from the $\mathrm{k}^{\text {th }}$ firm could achieve a surplus $\hat{s}-t d-p_{k}$ by purchasing from that firm. If the consumer lies in the quarter circumference between firm k and firm $\mathrm{k}+1$, his alternative is to purchase from


Source: Grossman and Shapiro (1984)
firm $\mathrm{k}+1$, who offers a surplus of $\hat{s}-t\left(\frac{1}{4}-d_{-1}\right)-p_{k+1}$. Since between the two firms there is a continuum of consumers, there must exist a consumer for whom, were she fully informed about the existence of both firms, the utility of consuming from firm k is equal to the utility of consuming from firm $\mathrm{k}+1$. For that indifferent consumer,

$$
\hat{s}-t d_{-1}-p_{k}=\hat{s}-t\left(\frac{1}{4}-d_{-1}\right)-p_{k+1}
$$

which implies that:

$$
d_{-1}=\frac{p_{k+1}-p_{k}}{2 t}+\frac{1}{8}
$$

Consumers to the left of this point will choose to consume from firm k whenever informed about its existence; consumers to the right of this point will consume from firm $\mathrm{k}+1$.

If, instead, the consumer were to lie on the remaining three-quarters of the circle, then she would be choosing between a utility of $\hat{s}-t\left(1-d_{+1}\right)-p_{k}$ if purchasing from firm k , and a utility of $\hat{s}-t\left(d_{+1}-\frac{1}{4}\right)-p_{k+1}$ if purchasing from firm $\mathrm{k}+1$. Therefore, she will be indifferent whenever:

$$
\hat{s}-t\left(1-d_{+1}\right)-p_{k}=\hat{s}-t\left(d_{+1}-\frac{1}{4}\right)-p_{k+1}
$$

Therefore, the indifferent consumer on the "far side" will be equal to

$$
d_{+1}=\frac{p_{k}-p_{k+1}}{2 t}+\frac{5}{8}
$$



Therefore, for firm k , the demand in this case will be the distance between its two indifferent consumers, that is,

$$
\begin{aligned}
\mathrm{D}_{2, k} & =d_{-1}+\left(1-d_{+1}\right) \\
& =\frac{p_{k+1}-p_{k}}{2 t}+\frac{1}{8}+\left(1-\frac{p_{k}-p_{k+1}}{2 t}+\frac{5}{8}\right) \\
& =\frac{p_{k+1}-p_{k}}{t}+\frac{1}{2}
\end{aligned}
$$

Similarly, firm $\mathrm{k}+1$ 's demand will be:
$\mathrm{D}_{k+1}=\frac{p_{k}-p_{k+1}}{t}+\frac{1}{2}$
In this second case, we can see the familiar result that implies that, when firms set the same price, they share the market in halves, and any price increase by one of the firms not matched by the other will make it lose some market share, which depends on the transport cost $t$. The higher the transport cost, the costlier it is for a customer to choose a brand that is not her favorite, and the more inelastic demand becomes.

The third possible information case happens whenever a consumer is informed about two opposite firms; graphically, the case is presented in the figure at the top of the page.

In this case, the bottom indifferent consumer must satisfy:

$$
\hat{s}-t d_{-2}-p_{k}=\hat{s}-t\left(\frac{1}{2}-d_{-2}\right)-p_{k+2}
$$

Solving,

$$
d_{-2}=\frac{p_{k+2}-p_{k}}{2 t}+\frac{1}{4}
$$

Similarly, the other indifferent consumer satisfies the opposite condition:

$$
d_{+2}=\frac{p-p_{k+2}}{2 t}+\frac{3}{4}
$$



Therefore, demand for firm k in this case becomes:

$$
\mathrm{D}_{3, k}=\frac{p_{k+2}-p_{k}}{t}+\frac{1}{2}
$$

As in the previous case, we have that equal prices imply equal market shares, and that a higher transport cost implies lower demand elasticity.

The fourth possible case involves a situation where three firms have successfully managed to inform a customer. This situation is shown above.

We separate this case into two sub-cases. The case for firm k (which is the same as the case of firm $k+2$ ) and the case for firm $k+1$, who is surrounded by two immediateb competitors. The indifferent consumer between firm k and firm $\mathrm{k}+1$ satisfies:

$$
d_{-1}=\frac{p_{k+1}-p_{k}}{2 t}+\frac{1}{8}
$$

while the indifferent consumer between firm $k$ and firm $k+2$ satisfies:

$$
d_{+2}=\frac{p-p_{k+2}}{2 t}+\frac{3}{4}
$$

Thus, firm k's demand for the three-firm case is:

$$
\mathrm{D}_{4, k}=\frac{p_{k+1}+p_{k+2}-2 p_{k}}{2 t}+\frac{3}{8}
$$

Firm $\mathrm{k}+1$, instead, faces firm k on the left (with indifferent consumer $\mathrm{d}_{-1}$ ) and firm $\mathrm{k}+2$ on the right, for whom the indifferent consumer is:

$$
\mathrm{d}_{+1+2}=\frac{p_{k+2}-p_{k+1}}{2 t}+\frac{3}{8}
$$

Therefore, the demand faced by firm $\mathrm{k}+1$ is equal to:

$$
\mathrm{D}_{4, k+1}=\frac{p_{k}+p_{k+2}-2 p_{k+1}}{2 t}+\frac{1}{4}
$$



The interesting feature of this case is that, when firms charge the same price, they face different demands; in the example we just calculated, firm $\mathrm{k}+1$ would have a lower demand than firms k and $\mathrm{k}+2$, because it is surrounded by them and so the indifferent consumers lie closer, making the potential size of its demand smaller. As usual, elasticity of demand decreases with t.

The final case is when four firms compete. Graphically this situation implies the four firms competing, as we show in the figure at the top.

As we can see, firm k's demand is the same as firm $k+1$ 's demand in the previous case, since it does not interact with the firm that lies opposite in the circumference, because no consumer can shift from firm k to firm $\mathrm{k}+2$ because of these firms' decisions. Therefore, for firm k the demand in this case will be:

$$
\mathrm{D}_{5, k}=\frac{p_{k+1}+p_{k+3}-2 p_{k}}{2 t}+\frac{1}{4}
$$

This situation is symmetric again, since all firms face the same type of competition on both sides, and will receive one quarter of total demand if they set the same prices.

We have now calculated a set of demands that can be applied to all 16 possible cases of consumer information. To obtain each firm's demand, we need to calculate the proportion of each case that will appear once each firm decides its promotional coverage, that is, its $\phi$. We will do this for a generic firm k , and because all firms are equidistant and symmetric a priori, this will be the demand all firms will face. We define $\phi_{n}$ for $\mathrm{n}=1 \ldots 6$ as the probability of occurrence of each of the six possible cases for firm k (case six is the case where the consumer does not receive an advertisement from firm k ). Clearly the $\phi s$ will depend on the advertisement coverages of the other firms, as well as firm k's. Analytically:

$$
D_{k}\left(p_{k}, \phi_{k}\right)=N_{1, k} \phi_{1}+N_{2, k} \phi_{2}+N_{3, k} \phi_{3}+N_{4, k} \phi_{4}+N_{5, k} \phi_{5}+N_{6, k} \phi_{6}
$$

The probability of the first case, $\phi_{1}$, where only firmk is present, is equal to the probability of a consumer receiving an ad from firm $k$, but not receiv-
ing it from any of the other three firms; since the probabilities of receiving advertisements are independent, we have that

$$
\phi_{1}=\phi_{k}\left(1-\phi_{k+1}\right)\left(1-\phi_{k+2}\right)\left(1-\phi_{k+3}\right)
$$

For the second case, we need firm k and firm $\mathrm{k}+1$ or $\mathrm{k}+3$ to participate. Therefore, the total probability of case 2 is:

$$
\phi_{2}=\phi_{k} \phi_{k+1}\left(1-\phi_{k+2}\right)\left(1-\phi_{k+3}\right)+\phi_{k}\left(1-\phi_{k+1}\right)\left(1-\phi_{k+2}\right) \phi_{k+3}
$$

For the third case, firm $\mathrm{k}+2$ needs to place a successful ad. In other words:

$$
\phi_{3}=\phi_{k}\left(1-\phi_{k+1}\right) \phi_{k+2}\left(1-\phi_{k+3}\right)
$$

For case four (i), we need firms $\mathrm{k}+1$ and $\mathrm{k}+2$ to participate, or $\mathrm{k}+2$ and $k+3$. Therefore:

$$
\phi_{4}^{1}=\phi_{k} \phi_{k+1} \phi_{k+2}\left(1-\phi_{k+3}\right)+\phi_{k}\left(1-\phi_{k+1}\right) \phi_{k+2} \phi_{k+3}
$$

For case four (ii), consumers need to be informed about the existence of firms $\mathrm{k}+1$ and $\mathrm{k}+3$ :
$\phi_{4}^{2}=\phi_{k} \phi_{k+1}\left(1-\phi_{k+2}\right) \phi_{k+3}$
For case five, the consumers need to have seen ads of all four companies:

$$
\phi_{5}=\phi_{k} \phi_{k+1} \phi_{k+2} \phi_{k+3}
$$

The final case implies that the consumers have not seen firm k's ads. The probability of this is:

$$
\phi_{6}=\left(1-\phi_{k}\right)
$$

These six cases' probabilities add up to one. In other words, all cases are being contemplated. The last case is trivial, since the demand for firm $k$ is zero, and therefore it does not appear in the firm's demand function. We are now in a position to calculate firm k's total demand for given values of prices and advertisement coverages.

$$
\begin{aligned}
& \quad D_{k}\left(p_{k}, \phi_{k}\right)=N_{1, k} \phi_{1}+N_{2, k} \phi_{2}+N_{3, k} \phi_{3}+N_{4, k} \phi_{4}+N_{5, k} \phi_{5}+N_{6, k} \phi_{6} \\
& \quad=\phi_{k}\left(1-\phi_{k+1}\right)\left(1-\phi_{k+2}\right)\left(1-\phi_{k+3}\right)(1)+\phi_{k} \phi_{k+1}\left(1-\phi_{k+2}\right)\left(1-\phi_{k+3}\right)\left(\frac{p_{k+1}-p_{k}}{t}+\frac{1}{2}\right)+ \\
& + \\
& \phi_{k}\left(1-\phi_{k+1}\right)\left(1-\phi_{k+2}\right) \phi_{k+3}\left(\frac{p_{k+3}-p_{k}}{t}+\frac{1}{2}\right)+ \\
& + \\
& \phi_{k}\left(1-\phi_{k+1}\right) \phi_{k+2}\left(1-\phi_{k+3}\right)\left(\frac{p_{k+2}-p_{k}}{t}+\frac{1}{2}\right)+
\end{aligned}
$$

$$
\begin{aligned}
& +\phi_{k} \phi_{k+1} \phi_{k+2}\left(1-\phi_{k+3}\right)\left(\frac{p_{k+1}+p_{k+2}-2 p_{k}}{2 t}+\frac{3}{8}\right)+ \\
& +\phi_{k}\left(1-\phi_{k+1}\right) \phi_{k+2} \phi_{k+3}\left(\frac{p_{k+2}+p_{k+3}-2 p_{k}}{2 t}+\frac{3}{8}\right)+ \\
& +\left[\phi_{k} \phi_{k+1}\left(1-\phi_{k+2}\right) \phi_{k+3}+\phi_{k} \phi_{k+1} \phi_{k+2} \phi_{k+3}\right]\left(\frac{p_{k+1}+p_{k+3}-2 p_{k}}{2 t}+\frac{1}{4}\right)+(1- \\
& \left.\phi_{k}\right)(0)
\end{aligned}
$$

After some simplifications, we can arrive at a simpler formulation for firm k's demand:

$$
\begin{aligned}
& D_{k}\left(p_{k}, \phi_{k}\right)=\phi_{k}\left\{\phi_{k+2}\left(\frac{1}{2}-\frac{\phi_{k+1}+\phi_{k+3}}{8}\right)+\left(1-\frac{\phi_{k+1}}{2}\right)\left(1-\phi_{k+2}\right)\left(1-\frac{\phi_{k+3}}{2}\right)\right. \\
& -\frac{p_{k}}{t}\left[1-\left(1-\phi_{k+1}\right)\left(1-\phi_{k+2}\right)\left(1-\phi_{k+3}\right)\right]++\frac{\phi_{k+1} p_{k+1}}{2 t}\left[1+\left(1-\phi_{k+2}\right)\left(1-\phi_{k+3}\right)\right] \\
& +\frac{\phi_{k+2} p_{k+2}}{2 t}\left[\left(2-\phi_{k+1}-\phi_{k+3}\right]+\frac{\phi_{k+3} p_{k+3}}{2 t}\left[1+\left(1-\phi_{k+1}\right)\left(1-\phi_{k+2}\right)\right]\right\}
\end{aligned}
$$

This demand enters firm k's maximisation problem, which is slightly different from the original Salop (1979) model, since it includes variables related to promotional spending. In competition, and holding the number of firms fixed, and taking the other firms' responses as identical and given, the firm's maximisation problem is:
$\max \left\{\begin{array}{c}\left(p_{k}-c\right) \phi_{k}\left\{\phi_{k+2}\left(\frac{1}{2}-\frac{\phi_{k+1}+\phi_{k+3}}{8}\right)+\left(1-\frac{\phi_{k+1}}{2}\right)\left(1-\phi_{k+2}\right)\left(1-\frac{\phi_{k+3}}{2}\right)+\right. \\ -\frac{p_{k}}{t}\left[1-\left(1-\phi_{k+1}\right)\left(1-\phi_{k+2}\right)\left(1-\phi_{k+3}\right)\right]+\frac{\phi_{k+1} p_{k+1}}{2 t}\left[1+\left(1-\phi_{k+2}\right)\left(1-\phi_{k+3}\right)\right]+ \\ +\frac{\phi_{k+2} p_{k+2}}{2 t}\left[\left(2-\phi_{k+1}-\phi_{k+3}\right]+\frac{\phi_{k+3} p_{k+3}}{2 t}\left[1+\left(1-\phi_{k+1}\right)\left(1-\phi_{k+2}\right)\right]\right\}-\frac{a \phi_{k}^{2}}{2}\end{array}\right.$ $\left\{p_{k}, \phi_{k}\right\}$

The first order conditions are:

$$
\begin{aligned}
& {\left[p_{k}\right]: \phi_{k}\left(\frac{c-2 p_{k}}{t}\left[1-\left(1-\phi_{k+1}\right)\left(1-\phi_{k+2}\right)\left(1-\phi_{k+3}\right)\right]+\phi_{k+2}\left(\frac{1}{2}-\frac{\phi_{k+1}+\phi_{k+3}}{8}\right)+\right.} \\
+ & \left(1-\frac{\phi_{k+1}}{2}\right)\left(1-\phi_{k+2}\right)\left(1-\frac{\phi_{k+3}}{2}\right)+\frac{\phi_{k+1} p_{k+1}}{2 t}\left[1+\left(1-\phi_{k+2}\right)\left(1-\phi_{k+3}\right)\right]+ \\
+ & \frac{\phi_{k+2} p_{k+2}}{2 t}\left[\left(2-\phi_{k+1}-\phi_{k+3}\right]+\frac{\phi_{k+3} p_{k+3}}{2 t}\left[1+\left(1-\phi_{k+1}\right)\left(1-\phi_{k+2}\right)\right]\right)=0 \\
& {\left[\phi_{k}\right]:\left(p_{k}-c\right)\left\{\phi_{k+2}\left(\frac{1}{2}-\frac{\phi_{k+1}+\phi_{k+3}}{8}\right)+\left(1-\phi_{k+2}\right)\left(2-\phi_{k+1}-\phi_{k+3}+\frac{\phi_{k+1} \phi_{k+3}}{4}\right)+\right.} \\
- & \frac{p_{k}}{t}\left[1-\left(1-\phi_{k+1}\right)\left(1-\phi_{k+2}\right)\left(1-\phi_{k+3}\right)\right]+\frac{\phi_{k+1} p_{k+1}}{2 t}\left[1+\left(1-\phi_{k+2}\right)\left(1-\phi_{k+3}\right)\right]+
\end{aligned}
$$

$$
+\frac{\phi_{k+2} p_{k+2}}{2 t}\left[\left(2-\phi_{k+1}-\phi_{k+3}\right]+\frac{\phi_{k+3} p_{k+3}}{2 t}\left[1+\left(1-\phi_{k+1}\right)\left(1-\phi_{k+2}\right)\right]\right\}=a \phi_{k}
$$

In the competitive equilibrium, it is reasonable to search for a symmetric equilibrium. Therefore, we can replace $\mathrm{p}_{k+1}, \mathrm{p}_{k+2}$ and $\mathrm{p}_{k+3}$ by $\mathrm{p}^{c}$, the equilibrium competitive price fixed by all other competitors, and $\phi_{k+1}, \phi_{k+2}$ and $\phi_{k+3}$ by $\phi^{c}$, the equilibrium advertisement spending. Replacing and simplifying, we can rewrite the first FOC as:

$$
\left[p_{k}\right]: \phi^{c}\left(\frac{1}{2}-\frac{\phi^{c}}{4}\right)+\left(1-\phi^{c}\right)\left(1-\phi^{c}+\frac{\phi^{c 2}}{4}\right)+\frac{p^{c}}{t}\left[1-\left(1-\phi^{c}\right)^{3}\right]=\frac{2 p_{k}-c}{t}\left[1-\left(1-\phi^{c}\right)^{3}\right]
$$

From this equation we can rewrite the optimal price $\mathrm{p}^{*}$, for given values of competitors' prices and advertisement spending:

$$
p^{*}=\frac{c+p^{c}}{2}+\frac{t}{8}\left(\frac{4-6 \phi^{c}+4 \phi^{c^{2}}-\phi^{c^{3}}}{1-\left(1-\phi^{c}\right)^{3}}\right)
$$

This solution is equal to Salop's (1979) whenever $\phi$ equals 1. The interpretation of this is that imperfect information, by reducing demand elasticity, increases market power and thus increases equilibrium prices compared to a perfect information situation. Since the value of $\phi^{c}$ can take on values between zero and one, the term in between parentheses tends to infinity as $\phi^{c}$ tends to zero, and is equal to one when $\phi^{c}$ equals one. Therefore, as we get closer to perfect information, the mark-up tends to be equal to the original Salop model (price equals marginal cost plus $\frac{t}{4}$ ), while a situation with less information might involve considerable price increases. Therefore, we can see that information (through advertisement spending) has the crucial role of informing consumers and reducing the firms' market power, so as to avoid situations where price can exceed marginal cost considerably (depending on the magnitude of the transport cost, t). It is also noteworthy that the optimal price increases with the competitors' prices; as Bulow, Geanakoplos, and Klemperer (1985) first defined it, we can say that the pricing decisions are strategic complements. Therefore, any incentive to increase prices for one or more of the agents involved will have the equilibrium effect of raising all prices, since pricing decisions are interconnected.

Finally, it is interesting to note that the firm's own price does not depend directly on its own advertisement coverage. This is because a marginal change in $\phi$ will increase all demands proportionally within each case. Since the relative probability of occurrence of each case does not change with $\phi$, the demand the company faces is simply multiplied by a scalar. Therefore, the optimal pricing decision remains unaffected. Instead, an increase in the competitors' promotional coverage will induce the firm to reduce prices, since its demand will now fall more than proportionally in distant markets (because consumers in
distant markets are now more informed, and thus more likely to purchase other brands that offer them higher utility), and therefore it will be profitable for the firm to counteract this effect by reducing its price vis-a-vis its competitors', and regain some of the consumers it lost.

The second equation can be stated as:

$$
\phi^{*}=\frac{\left(p_{k}-c\right)}{a}\left[\left(p^{c}-p_{k}\right) \frac{1-\left(1-\phi^{c}\right)^{3}}{t}+\frac{\left.4-6 \phi^{c}+4 \phi^{c^{2}}-\phi^{c^{3}}\right)}{4}\right]
$$

In a symmetric equilibrium, $\mathrm{p}_{k}=\mathrm{p}^{c}$, so the condition becomes:

$$
a \phi^{*}=(p-c)\left(\frac{4-6 \phi^{c}+4 \phi^{c^{2}}-\phi^{c^{3}}}{4}\right)
$$

The left hand side of the last equation is the marginal cost of an extra unit of advertising coverage. The right hand side is the marginal benefit, since the probability of the new informed consumer purchasing from the firm is equal to $\frac{4-6 \phi^{c}+4 \phi^{c^{2}}-\phi^{c^{3}}}{4}$ whenever firms set the same price. If the competitors' advertising is low, then this probability will be high, since most of the potential market to be conquered by the firm will be composed of uninformed customers willing to pay any price for the firm's products. Instead, if the competitors set a high $\phi$, then the firm will only be able to gain consumers close to its location, since most consumers that lie far away from the firm will probably already be aware of other firms which offer them a higher surplus.

To obtain the symmetric equilibrium solution, we replace all $\mathrm{p}_{k}$ and $\phi_{k}$ by p and $\phi$. The equilibrium price becomes:

$$
p^{*}=c+\frac{t}{4}\left(\frac{4-6 \phi^{c}+4 \phi^{c^{2}}-\phi^{c^{3}}}{1-\left(1-\phi^{c}\right)^{3}}\right)
$$

and the equilibrium advertising coverage is:

$$
\begin{aligned}
a \phi^{*} & =(p-c)\left(\frac{4-6 \phi^{*}+4 \phi^{* 2}-\phi^{* 3}}{4}\right)=\frac{t}{16}\left(\frac{4-6 \phi^{*}+4 \phi^{* 2}-\phi^{* 3}}{1-\left(1-\phi^{c}\right)^{3}}\right)\left(4-6 \phi^{*}+4 \phi^{* 2}-\phi^{* 3}\right) \\
\frac{16 a}{t} & =\frac{16-48 \phi^{*}+68 \phi^{* 2}-56 \phi^{* 3}+28 \phi^{* 4}-8 \phi^{* 5}+\phi^{* 6}}{3 \phi^{* 2}-3 \phi^{* 3}+\phi^{* 4}}
\end{aligned}
$$

The right-hand side equation can be approximated by $\frac{1}{\phi^{* 3}}$, which has a very good fit for values of $\phi^{*}$ higher than 0,25 . Therefore, the equilibrium becomes:

$$
\phi^{*}=\sqrt[3]{\frac{t}{16 a}}
$$



Replacing in the price equation, we get that:

$$
p^{*}=c+t\left(\frac{4-\frac{t}{16 a}-3 \sqrt[3]{\frac{t}{2 a}}+\sqrt[3]{\frac{t^{2}}{4 a^{2}}}}{\frac{t}{4 a}-\frac{3}{2} \sqrt[3]{\frac{2 t^{2}}{a^{2}}}+3 \sqrt[3]{\frac{4 t}{a}}}\right)
$$

A graphical representation of the second term of the previous equation, equal to the marginal contribution $(p-c)$, is shown above, for three different values of $t$, and for values of a between 0 and 10:

From this final graph it is clear that in the competitive solution with four firms, price exceeds marginal cost by an increasing function of transport costs and marginal advertising costs. In other words, an increased transport cost will imply higher markups (due to increased market power, derived from the fact that customers are less willing to switch brands because the cost of doing so increases) and an increased marginal cost of advertising (an increase in a) will imply a higher price because of the reduction in equilibrium advertising spending, which will again imply a higher degree of market power for the firms. As we can clearly see, equilibrium mark-up tends to zero as advertising becomes less expensive, while it tends to infinity as advertsing costs increase, relative to transport costs.

### 3.2 Post-Merger

### 3.2.1 Fixed prices

Now we consider our main question: What happens when there is a merger between two firms that previously competed in the differentiated product model above? Depending from the compared welfare outcomes between the competition and post-merger situation, we will be able to give a more comprehensive answer to our main question. The case we consider is a merger between two adjacent firms, say, firms $k$ and $k+1$. Since the problem has not changed and the cases are the same, we can use them to characterise the firms' maximisation problems, now taking into account the fact that two firms are under the control of the same decision-making agent.

An attempt has been made to characterise the outcome of a situation where prices and promotional coverages are left unconstrained, and both variables are under the firms' control. However, the algebraic complexity of the problem has forced us into choosing a simpler formulation of the problem, that allows us to draw conclusions that are far more interesting that what would have been possible when analysing the full problem. The alternative we suggest is, in first place, to characterise the firms' decision as only happening in the "promotion" variable space. Therefore, we impose the constraint that prices post-merger must be equal to pre-merger prices for all four firms. This bears certain resemblance to the real-life situation we inspired ourselves from, since the CNDC actually imposed this kind of constraint on the merged brands. A second formulation will involve leaving promotional decisions fixed, and allowing the firms to decide their prices freely, while constraining all brands to maintain their pre-merger promotional coverage levels. We therefore arrive at two which study the partial effect of the merger on both decision variables, although not the effect on the simultaneous choice of both. Although this may seem insufficient to characterise the problem completely, it provides very useful insights on the workings of mergers and acquisitions in the context of differentiated goods markets.

For the merged firm, the maximisation will now involve the demand for both its brands. Therefore, the new problem will take into account the fact that one brand's decision making has externalities on the other brands. The merged firm's profit maximisation problem (choosing promotional coverages $\phi_{k}$ and $\phi_{k+1}$ ) will be:

$$
\operatorname{Max}\left\{\begin{array}{c}
\left(p_{k}-c\right) \phi_{k}\left\{\phi_{k+2}\left(\frac{1}{2}-\frac{\phi_{k+1}+\phi_{k+3}}{8}\right)+\left(1-\frac{\phi_{k+1}}{2}\right)\left(1-\phi_{k+2}\right)\left(1-\frac{\phi_{k+3}}{2}\right)+\right. \\
-\frac{p_{k}}{t}\left[1-\left(1-\phi_{k+1}\right)\left(1-\phi_{k+2}\right)\left(1-\phi_{k+3}\right)\right]+\frac{\phi_{k+1} p_{k+1}}{2 t}\left[1+\left(1-\phi_{k+2}\right)\left(1-\phi_{k+3}\right)\right]+ \\
\quad+\frac{\phi_{k+2} p_{k+2}}{2 t}\left[\left(2-\phi_{k+1}-\phi_{k+3}\right]+\frac{\phi_{k+3} p_{k+3}}{2 t}\left[1+\left(1-\phi_{k+1}\right)\left(1-\phi_{k+2}\right)\right]\right\}-\frac{a \phi_{k}^{2}}{2} \\
\quad\left(p_{k+1}-c\right) \phi_{k+1}\left\{\phi_{k+3}\left(\frac{1}{2}-\frac{\phi_{k+2}+\phi_{k}}{8}\right)+\left(1-\frac{\phi_{k+2}}{2}\right)\left(1-\phi_{k+3}\right)\left(1-\frac{\phi_{k}}{2}\right)+\right. \\
+\quad-\frac{p_{k+1}}{t}\left[1-\left(1-\phi_{k}\right)\left(1-\phi_{k+2}\right)\left(1-\phi_{k+3}\right)\right]+\frac{\phi_{k+2} p_{k+2}}{2 t}\left[1+\left(1-\phi_{k}\right)\left(1-\phi_{k+3}\right)\right]+ \\
\quad+\frac{\phi_{k+3} p_{k+3}}{2 t}\left[\left(2-\phi_{k}-\phi_{k+2}\right]+\frac{\phi_{k} p_{k}}{2 t}\left[1+\left(1-\phi_{k+2}\right)\left(1-\phi_{k+3}\right)\right]\right\}-\frac{a \phi_{k+1}^{2}}{2}
\end{array}\right.
$$

$$
\left\{\phi_{k}, \phi_{k+1}\right\}
$$

The first order conditions are:

$$
\begin{aligned}
& \quad\left[\phi_{k}\right]:\left(p_{k}-c\right)\left\{\phi_{k+2}\left(\frac{1}{2}-\frac{\phi_{k+1}+\phi_{k+3}}{8}\right)+\left(1-\phi_{k+2}\right)\left(2-\phi_{k+1}-\phi_{k+3}+\frac{\phi_{k+1} \phi_{k+3}}{4}\right)+\right. \\
& - \\
& -\frac{p_{k}}{t}\left[1-\left(1-\phi_{k+1}\right)\left(1-\phi_{k+2}\right)\left(1-\phi_{k+3}\right)\right]+\frac{\phi_{k+1} p_{k+1}}{2 t}\left[1+\left(1-\phi_{k+2}\right)\left(1-\phi_{k+3}\right)\right]+ \\
& +\frac{\phi_{k+2} p_{k+2}}{2 t}\left[\left(2-\phi_{k+1}-\phi_{k+3}\right]+\frac{\phi_{k+3} p_{k+3}}{2 t}\left[1+\left(1-\phi_{k+1}\right)\left(1-\phi_{k+2}\right)\right]\right\}+ \\
& +\phi_{k+1}\left(p_{k+1}-c\right)\left[\frac{p_{k}}{2 t}\left[1+\left(1-\phi_{k+2}\right)\left(1-\phi_{k+3}\right)\right]-\frac{p_{k+1}}{t}\left(1-\phi_{k+2}\right)\left(1-\phi_{k+3}\right)+\right. \\
& \left.-\frac{\phi_{k+2} p_{k+2}}{2 t}\left(1-\phi_{k+3}\right)-\frac{\phi_{k+3} p_{k+3}}{2 t}-\frac{\phi_{k+3}}{8}-\frac{1}{2}\left(1-\frac{\phi_{k+2}}{2}\right)\left(1-\phi_{k+3}\right)\right]=a \phi_{k} \\
& \quad\left[\phi_{k+1}\right]:\left(p_{k+1}-c\right)\left\{\phi_{k+3}\left(\frac{1}{2}-\frac{\phi_{k}+\phi_{k+2}}{8}\right)+\left(1-\phi_{k+3}\right)\left(2-\phi_{k}-\phi_{k+2}+\frac{\phi_{k} \phi_{k+2}}{4}\right)\right. \\
& -\frac{p_{k+1}}{t}\left[1-\left(1-\phi_{k}\right)\left(1-\phi_{k+2}\right)\left(1-\phi_{k+3}\right)\right]+\frac{\phi_{k+2} p_{k+2}}{2 t}\left[1+\left(1-\phi_{k}\right)\left(1-\phi_{k+3}\right)\right]+ \\
& +\frac{\phi_{k+3} p_{k+3}}{2 t}\left[\left(2-\phi_{k}-\phi_{k+2}\right]+\frac{\phi_{k} p_{k}}{2 t}\left[1+\left(1-\phi_{k+2}\right)\left(1-\phi_{k+3}\right)\right]\right\}+ \\
& + \\
& \phi_{k}\left(p_{k}-c\right)\left[\frac{p_{k+1}}{2 t}\left[1+\left(1-\phi_{k+2}\right)\left(1-\phi_{k+3}\right)\right]-\frac{p_{k}}{t}\left(1-\phi_{k+2}\right)\left(1-\phi_{k+3}\right)+\right. \\
& \left.-\frac{\phi_{k+2} p_{k+2}}{2 t}-\frac{\phi_{k+3} p_{k+3}}{2 t}\left(1-\phi_{k+2}\right)-\frac{\phi_{k+2}}{8}-\frac{1}{2}\left(1-\phi_{k+2}\right)\left(1-\frac{\phi_{k+3}}{2}\right)\right]=a \phi_{k+1}
\end{aligned}
$$

Since we are looking for an equilibrium in which the conglomerate's competitors are fixing the same prices and choose the same advertising coverage, we can replace $p_{k}, p_{k+1}, p_{k+2}$ and $p_{k+3}$ by the pre-merger price $p$, and $\phi_{k+2}$ and $\phi_{k+3}$ by $\phi$. Also, we are looking for an equilibrium in which the merged firms have a symmetric response to the non-merged firms' decision. In other words, we impose the constraint $\phi_{k}=\phi_{k+1}=\phi_{m}$, which allows us to find the optimal reaction or reaction function for the conglomerate's promotional coverage
in terms of its competitors' coverage. To do so, we simply need to use one of the FOCs, replace, and solve for $\phi_{m}$ in terms of $\phi$. This yields the following:

$$
\phi_{m}(\phi)=\frac{2-\frac{5}{2} \phi+\frac{7}{8} \phi^{2}+\frac{\phi p}{t}(1-\phi)^{2}}{\frac{a}{p-c}+\frac{2 p}{t}(1-\phi)+\left(1-\frac{\phi}{2}\right)\left(\frac{3}{2}-\phi\right)}
$$

This function is always positive and tends to get smaller as $\phi$ gets large (although there are cases for very small values of $\phi_{m}$ when the function is increasing in $\phi$ ). This is intuitive, because as competitors raise their promotional coverage, the merged firms' marginal benefit of an extra unit of advertising tends to diminish, since it is less likely that new consumers will be captured in locations where competing firms are the best option. The effect of an increase in price is ambiguous, since this variable appears as positive in both the numerator and the denominator. It is also impossible to visually sign the partial derivative with respect to transport costs and advertisement costs, since they affect the equilibrium price, which was derived in the previous section.

To find the equilibrium promotional coverages we still need to find what the competitors' reaction functions are. The demand faced by firm $\mathrm{k}+2$, one of the non-merged competing brands, is the following:

$$
\begin{aligned}
& D_{k+2}\left(p_{k+2}, \phi_{k+2}\right)=\phi_{k+2}\left(\frac{p_{k+3}+p_{k+1}-2 p_{k+2}}{2 t}+\frac{1}{4}\right)+\phi_{k+2}\left(1-\phi_{k+3}\right)\left(\frac{p_{k}-p_{k+3}}{2 t}+\frac{1}{8}\right)+ \\
& +\phi_{k+2}\left(1-\phi_{k+1}\right)\left(\frac{p_{k}-p_{k+1}}{2 t}+\frac{1}{8}\right)+\phi_{k+2}\left(1-\phi_{k+3}\right)\left(1-\phi_{k}\right)\left(\frac{p_{k+1}-p_{k}}{2 t}+\frac{1}{8}\right)+ \\
& +\phi_{k+2}\left(1-\phi_{k+1}\right)\left(1-\phi_{k}\right)\left(\frac{p_{k+3}-p_{k}}{2 t}+\frac{1}{8}\right)+ \\
& +\phi_{k+2}\left(1-\phi_{k+3}\right)\left(1-\phi_{k}\right)\left(1-\phi_{k+1}\right)\left(\frac{1}{4}+\frac{2 p_{k+2}-p_{k+3}-p_{k+1}}{2 t}\right)
\end{aligned}
$$

We can also replace the merged firms' promotional coverages by a single coverage $\phi_{m}$, since we are looking for a symmetric equilibrium. Also taking into account that we assumed prices to be equal across all firms, firm $\mathrm{k}+2$ 's demand becomes:

$$
\begin{aligned}
& D_{k+2}\left(\phi_{m}, \phi_{k+2}, \phi_{k+3}\right)= \\
& \frac{5 \phi_{k+2}}{4}+\frac{1-\phi_{k+3}}{8}+\frac{\phi_{k+2}\left(1-\phi_{k+3}\right)\left(2-\phi_{m}\right)}{8}+\frac{\phi_{k+2}\left(3-2 \phi_{k+3}\right)\left(1-\phi_{m}\right)^{2}}{8}
\end{aligned}
$$

Firm $\mathrm{k}+2$ now maximises its profits facing this demand curve, and taking the other firms' responses as given. Analytically the problem is:

$$
\begin{gathered}
\operatorname{Max} \\
\left\{\phi_{k+2}\right\}
\end{gathered}\left\{(p-c) \phi_{k+2}\left[\frac{5}{4}+\frac{1-\phi_{m}}{8}+\frac{\left(1-\phi_{k+3}\right)\left(2-\phi_{m}\right)}{8}+\frac{\left(3-2 \phi_{k+3}\right)\left(1-\phi_{m}\right)^{2}}{8}\right]-\frac{a \phi_{k+2}^{2}}{2}\right.
$$

The first order condition is:

$$
\begin{aligned}
& {\left[\phi_{k+2}\right]:} \\
& (p-c)\left[\frac{5}{4}+\frac{1-\phi_{m}}{8}+\frac{\left(1-\phi_{k+3}\right)\left(2-\phi_{m}\right)}{8}+\frac{\left(3-2 \phi_{k+3}\right)\left(1-\phi_{m}\right)^{2}}{8}\right]=a \phi_{k+2}
\end{aligned}
$$

It is simple to see that this implies:

$$
\phi_{k+2}=\frac{(p-c)}{a}\left[\frac{5}{4}+\frac{1-\phi_{m}}{8}+\frac{\left(1-\phi_{k+3}\right)\left(2-\phi_{m}\right)}{8}+\frac{\left(3-2 \phi_{k+3}\right)\left(1-\phi_{m}\right)^{2}}{8}\right]
$$

At this point, we can use the fact that, in equilibrium, the non merged competitors' prices will be the same. We can use this to find a "combined reaction function" for the conglomerate's competitors. To do so, we replace $\phi_{k+2}$ and $\phi_{k+3}$ by a single promotional coverage $\phi$. Therefore, we will find the non-merged firms' optimal promotional coverage in terms of the conglomerate's promotional coverage. In firm $\mathrm{k}+2$ 's FOC this implies that, after some simplifications:

$$
\phi\left(\phi_{m}\right)=\frac{8-8 \phi_{m}+3 \phi_{m}^{2}}{\frac{8 a}{p-c}+4-5 \phi_{m}+2 \phi_{m}^{2}}
$$

The non-merged firms' reaction function is essentially similar to the merged firms', and diminishes as the merged firms' advertising coverage increases, and also increases as the marginal contribution $p-c$ increases. This is intuitive since the marginal benefits obtained from increasing advertisement coverage depend on the size of the market that can be captured, which depends on the rivals' advertisement coverage, as well as on the marginal benefit obtained from selling the product to one additional customer, versus the cost of reaching him, which depends on the letter a.

We are now ready to characterise the equilibrium for the post-merger fixedprice case. Since the algebraic solving of the equations is virtually impossible due to the complexity of the functions, a graphical analysis can be very revealing in this case. For $t=2, a=2$ and three values of c , we have that the merged firms' reaction function is:

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As we can see, an increasing marginal cost tends to reduce the optimal advertisement coverage, since it is less profitable to sell an extra unit, and therefore it is no longer marginally convenient to inform one extra customer. This effect tends to disappear as the competitors' $\phi$ increases.

The non-merged firms' reaction function for the same parameter values is


This function is also clearly decreasing in the competitors' $\phi$, and is relatively linear.


The merged firms' reaction function, for $a=2, c=2$, and three different values of $t$, can be seen above:

The non-merged firms' reaction function is invariant to changes in marginal cost and transport costs, since these firms only include the marginal contribution, $p-c$, in their reaction function, and this value does not depend on the value of $c$, since in equilibrium price always equals marginal cost plus another term which is independent of the value of $c$. Finally, we can see the merged firms' reaction function for different values of a on the top of the next page, keeping $c=2, t=2$.
SanAndrés


The same exercise for the non-merged firms yields the bottom graph.

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As we can see from the graphical analyses presented above, there always is one single stable equilibrium for reasonable values of $t, a$ and $c$, since the reaction functions slope negatively against the other group's decision variable, and tend to be continuous and smooth. The case for $\mathrm{t}=2, \mathrm{c}=2, \mathrm{a}=2$ is presented above as an example:

### 3.2.2 Fixed Promotional Coverage

The second problem we will discuss is the case when advertisement coverages remain fixed and we allow firms to fix prices freely. In this case, we replace $\phi_{k}, \phi_{k+1}, \phi_{k+2}$ and $\phi_{k+3}$ by a single promotional coverage $\phi$ in the firms' demands, and then do the usual profit maximisation. We also replace the non merged firms' prices by a single price $p$. For the merged firms this implies that the problem becomes:

$$
\operatorname{Max}\left\{\begin{array} { r } 
{ ( p _ { k } - c ) \phi \{ \phi ( \frac { 1 } { 2 } - \frac { \phi } { 4 } ) + ( 1 - \frac { \phi } { 2 } ) ^ { 2 } ( 1 - \phi ) - \frac { p _ { k } } { t } [ 1 - ( 1 - \phi ) ^ { 3 } ] } \\
{ + \frac { \phi p _ { k + 1 } } { 2 t } [ 1 + ( 1 - \phi ) ^ { 2 } ] + \frac { \phi p } { 2 t } [ 2 - 2 \phi ] + \frac { \phi p } { 2 t } [ 1 + ( 1 - \phi ) ^ { 2 } ] \} - \frac { a \phi ^ { 2 } } { 2 } } \\
{ + ( p _ { k + 1 } - c ) \phi \{ \phi ( \frac { 1 } { 2 } - \frac { \phi } { 4 } ) + ( 1 - \frac { \phi } { 2 } ) ^ { 2 } ( 1 - \phi ) - \frac { p _ { k + 1 } } { t } [ 1 - ( 1 - \phi ) ^ { 2 } } \\
{ \{ p _ { k } , p _ { k + 1 } \} }
\end{array} \left\{\begin{array}{r}
\left.\phi \frac{\phi p}{2 t}\left[1+(1-\phi)^{2}\right]+\frac{\phi p}{2 t}[2-2 \phi]+\frac{\phi p_{k}}{2 t}\left[1+(1-\phi)^{2}\right]\right\}-\frac{a \phi^{2}}{2}
\end{array}\right.\right.
$$

The first order conditions are:

$$
\begin{aligned}
& \quad\left[p_{k}\right]: \phi\left\{\frac{c-2 p_{k}}{t}\left[1-(1-\phi)^{3}\right]+\phi\left(\frac{1}{2}-\frac{\phi}{4}\right)+\left(1-\frac{\phi}{2}\right)^{2}(1-\phi)+\right. \\
& \left.+\frac{\phi p_{k+1}}{2 t}\left[1+(1-\phi)^{2}\right]+\frac{\phi p}{2 t}[2-2 \phi]+\frac{\phi p}{2 t}\left[1+(1-\phi)^{2}\right]\right\}+\frac{\phi^{2}\left(p_{k+1}-c\right)}{2 t}\left[1+(1-\phi)^{2}\right]= \\
& 0 \\
& \quad\left[p_{k+1}\right]: \phi\left\{\frac{c-2 p_{k+1}}{t}\left[1-(1-\phi)^{3}\right]+\phi\left(\frac{1}{2}-\frac{\phi}{4}\right)+\left(1-\frac{\phi}{2}\right)^{2}(1-\phi)+\right. \\
& \left.+\frac{\phi p_{k}}{2 t}\left[1+(1-\phi)^{2}\right]+\frac{\phi p}{2 t}[2-2 \phi]+\frac{\phi p}{2 t}\left[1+(1-\phi)^{2}\right]\right\}+\frac{\phi^{2}\left(p_{k}-c\right)}{2 t}\left[1+(1-\phi)^{2}\right]= \\
& 0
\end{aligned}
$$

From the first equation we can see that:

$$
\begin{gathered}
\frac{1}{2} \phi-\frac{\phi^{2}}{4}+\left(1-\phi+\frac{\phi^{2}}{4}\right)-\phi+\phi^{2}-\frac{\phi^{3}}{4}+\frac{\phi p_{m}}{2 t}\left[1+(1-\phi)^{2}+2-2 \phi\right]+ \\
\frac{\phi p}{2 t}\left[1+(1-\phi)^{2}\right]=\frac{2 p_{m}-c}{t}\left[1-(1-\phi)^{3}\right]-\frac{\phi\left(p_{m}-c\right)}{2 t}\left[1+(1-\phi)^{2}\right]
\end{gathered}
$$

Since we are looking for a symmetric equilibrium, we can replace $p_{k}$ and $p_{k+1}$ by $p_{m}$, the equilibrium price for both brands, for a given price $p$ set by the conglomerate's competitors. Replacing in the first FOC and solving we get:

$$
p_{m}=\frac{c+p}{2}+t\left(\frac{2-3 \phi+2 \phi^{2}-\frac{\phi^{3}}{2}}{8 \phi-8 \phi^{2}+2 \phi^{3}}\right)
$$

This equation is straightforward. It states that the conglomerate's price will be an average between the competitors' prices and marginal cost, plus a term that depends on the common advertisement coverage, $\phi$. The term between parentheses tends to infinity when $\phi$ tends to zero, meaning that a lower promotional coverage will imply higher mark-ups through the weaking of competition between brands. When $\phi=1$, the term between parentheses equals one quarter. This condition differs from the pre-merger reaction function, since it implies that merged firms will charge a higher price, ceteris paribus, for a given value of $\phi$, compared to the competitive situation presented in the previous section. In other words, the internalisation of externalities implied by the merger between firm k and $\mathrm{k}+1$ will provide incentives to increase their brands' prices for given competitor prices and advertisement expenditures. To find the equilibrium values, however, we still need to find the reaction function for competitors' prices for a given price of the merged firms. To do so, we need to characterise the non-merged firms' maximisation problem. We can use the fact that we are searching for a symmetric equilibrium to replace $p_{k+1}$ and $p_{k+3}$ with $p_{m}$. For firm $\mathrm{k}+2$ the maximisation problem becomes:

$$
\max \left\{\begin{array}{l}
\left(p_{k+2}-c\right) \phi\left\{\phi\left(\frac{1}{2}-\frac{\phi}{4}\right)+(1-\phi)\left(1-\frac{\phi}{2}\right)^{2}-\frac{p_{k+2}}{t}\left[1-(1-\phi)^{3}\right]+\right. \\
\left.+\frac{\phi p_{m}}{2 t}[2-2 \phi]+\frac{\phi p_{m}}{2 t}\left[1+(1-\phi)^{2}\right]\right\}+\frac{\phi p_{k+3}}{2 t}\left[1+(1-\phi)^{2}\right]-\frac{a \phi^{2}}{2}
\end{array}\right.
$$

$\left\{p_{k+2}\right\}$
The first order condition is:

0

$$
\left[p_{k+2}\right]: \phi\left[\phi\left(\frac{1}{2}-\frac{\phi}{4}\right)+(1-\phi)\left(1-\frac{\phi}{2}\right)^{2}-\frac{2 p_{k+2}}{t}\left[1-(1-\phi)^{3}\right]+\frac{\phi p_{m}}{2 t}[2-2 \phi]+\frac{\phi p_{m}}{2 t}\left[1+(1-\phi)^{2}\right]+\right.
$$

Assuming the equilibrium will be symmetric for non-merged firms, we can replace $p_{k+2}$ and $p_{k+3}$ by a single price $p$ to find a reaction function for the price of the merged firm's competitors. Replacing and solving for $p$, we obtain:

$$
p^{*}\left(p_{m}\right)=t\left(\frac{2-3 \phi+2 \phi^{2}-\frac{\phi^{3}}{2}}{10 \phi-10 \phi^{2}+3 \phi^{3}}\right)+p_{m}\left(\frac{4-4 \phi+\phi^{2}}{10-10 \phi+3 \phi^{2}}\right)+c\left(\frac{6-6 \phi+2 \phi^{2}}{10-10 \phi+3 \phi^{2}}\right)
$$

The interpretation of this is that the conglomerate's competitors will set their prices in equilibrium taking into account the conglomerate's price $\left(p_{m}\right)$, transport costs and marginal costs of producing the good. Each of these has
a separate bearing on the final price that both firms set for a given value of promotional coverage, but whenever there is a rise in any of the three variables previously mentioned ( $\mathrm{p}_{m}$, t or c ) firms $\mathrm{k}+2$ and $\mathrm{k}+3$ raise their prices accordingly. This happens because all three of the terms between parentheses in the previous expression are positive. In particular, the first parenthesis moves between infinity (as $\phi$ tends to zero) and $\frac{1}{6}$ when $\phi=1$. The second term equals $\frac{2}{5}$ when $\phi=0$ and $\frac{1}{3}$ when $\phi=1$, and the third term equals $\frac{3}{5}$ when $\phi=0$ and $\frac{2}{3}$ when $\phi=1$. It is also interesting to note that the first and second terms are decreasing in $\phi$, while the third is strictly increasing for the possible values of $\phi$. In this case, therefore, we can also assert that prices are strategic complements, as was the case for the pre-merger situation, and that when facing a marginal increase in prices by its competitors, the non-merged firms will raise their own prices by a fraction of that marginal increase, since the term in the second set of parentheses is strictly less than one. Actually, it is interesting to see that the last two terms of the previous reaction are a weighted average of $p_{m}$ and $c$.

An interesting difference with the merged firms' reaction function is that, in the former, the last two terms are replaced by an arithmetic average of the competitors' prices and marginal cost. However, in the non-merged firms, more weight is put on marginal costs, indicating a smaller tendency to respond to competitors' price changes and more emphasis on the firms' own cost structure. Also, the term between the first set of parentheses is smaller for the non-merged firms than for the merged firms, indicating a tendency for the former to set lower prices than the merged firms, in terms of the transport cost. This happens because the merged brands reap the additional advantage of "losing customers to themselves" when implementing a price increase; therefore, in the marginal analysis that determine the brands' optimal prices, the merged brands have additional incentives to raise prices, and in equilibrium this is reflected in their reaction functions.

To compute the equilibrium in prices for the case of fixed promotional coverages, we need to solve the simultaneous equations implied in the reaction functions. Doing so yields:

$$
\begin{aligned}
& p^{*}=c+t\left(\frac{3(2-\phi)\left(2-2 \phi+\phi^{2}\right)}{2 \phi\left(16-16 \phi+5 \phi^{2}\right)}\right) \\
& p_{m}^{*}=c+t\left(\frac{\left(2-2 \phi+\phi^{2}\right)\left(7-7 \phi+2 \phi^{2}\right)}{\phi(2-\phi)\left(16-16 \phi+5 \phi^{2}\right)}\right)
\end{aligned}
$$

These equations characterise the equilibrium prices in terms of the firms' (fixed) promotional spending. Therefore, depending on the values of the transport and advertising costs (that determine optimal advertisement coverage, $\left.\phi^{*}=\sqrt[3]{\frac{t}{16 a}}\right)$, the equilibrium values of the prices set by merged and nonmerged firms for the fixed-advertisement case will fluctuate. A graphical analysis in terms of the optimal coverage shows that, in terms of $\phi^{*}$ and for $t=1$,

the equilibrium price minus marginal cost for merged and non-merged firms will be:

It is clear that merged firms charge a higher price than non-merged firms in equilibrium, since they have more incentives to do so. Also, the equilibrium price is clearly a decreasing function of the (fixed) advertisement coverage, since an increased advertisement coverage will imply more competition amongst the firms, and therefore more incentives to lower prices. The equilibrium price also depends on the transport cost, and high values of the latter will imply lower demand elasticities, with the consequent effect on prices. Average prices will also be higher than in the pre-merger equilibrium, since $p_{\text {pre-merger }}^{*}<p_{\text {post-merger }}^{*}$ implies:

$$
\begin{aligned}
& \left(\frac{4-6 \phi+4 \phi^{2}-\phi^{3}}{4-4(1-\phi)^{3}}\right)<\left(\frac{3(2-\phi)\left(2-2 \phi+\phi^{2}\right)}{2 \phi\left(16-16 \phi+5 \phi^{2}\right)}\right) \\
& 2 \phi\left(4-6 \phi+4 \phi^{2}-\phi^{3}\right)\left(16-16 \phi+5 \phi^{2}\right)<3(2-\phi)\left(2-2 \phi+\phi^{2}\right)\left(4-4(1-\phi)^{3}\right) \\
& 0<16 \phi-40 \phi^{2}+48 \phi^{3}-32 \phi^{4}+12 \phi^{5}-2 \phi^{6}
\end{aligned}
$$

Which is always true for values of $\phi$ between zero and one.
Therefore, we have that the equilibrium price of non-merged firms is greater than the equilibrium price of all firms in the competitive equilibrium. As we already knew, an undesirable consequence of mergers in differentiated product markets is the fact that the internalisation of externalities usually results in a price increase, which is reflected in all firm prices, but particularly in the merged firms' prices.

## 4 Welfare Analysis

For the welfare analysis we need to first define what our target function will be. In this particular case, it is relevant to use total net consumer surplus as an indicator of consumer welfare, since the final utility that consumers obtain is equal to this number. We simply add these up since we assume all consumers to be equal and having the same preferences. On the other hand, we include aggregate profits $\left(\pi\right.$ and $\left.\pi_{m}\right)$ as the second and third term in the welfare function, since an increase in the firms' profits will also have positive consequences for society's well-being in general. We multiply profits by a scalar $\delta$ in the welfare function, to allow us to introduce some flexibility in society's preference for the distribution of income between firms and consumers. A utilitarian welfare function would imply $\delta=1$, while a Rawlsian welfare function would put all emphasis in consumers getting as much utility as possible, implying $\delta=0$. This formulation also allows for intermediate cases. Analytically:

$$
W=\int_{0}^{1} \hat{s}-p_{x}-t d(x, f) d x+\delta \pi_{m}+\delta \pi
$$

The integral includes all consumers in the circle, and their net consumer surplus depends on the price they pay for the good, as well as the distance they have to travel to get it times the transport cost. Since the promotional coverage is random, however, it is impossible to know which consumers purchase in which firms. What we can do is calculate averages for different groups, so as to include these in the total welfare calculations. Since we assumed in section 3 that consumers have a gross surplus from consuming the good high enough to justify purchasing it from any of the four firms, even if that implies a high transport cost, we will have that all informed consumers will purchase the good. This leaves a fraction of $(1-\phi)^{4}$ consumers in the pre-merger case and $(1-\phi)^{2}\left(1-\phi_{m}\right)^{2}$ consumers in the fixed-prices post-merger case that will have a net consumer surplus of zero, because they were not informed about the existence of any of the four firms. We will not be analysing the welfare consequences of the merger in the fixed-promotional coverage setting, since we expect to find the usual result in this type of model which states that the rise in equilibrium prices has a negative effect on consumer welfare. We will thus focus on comparing the competitive case with the fixed-price, post-merger equilibrium, to see whether a merger that does not involve changes in prices can improve consumer welfare in a differentiated goods model.

Now we will find the total surplus for the consumers who purchased the good from firm k in the post-merger fixed-price case. These consumers are more concentrated close to firm k's location, where the firm's promotional effort is more effective. In fact, all customers within certain range of the firm will purchase from it if they receive a promotional message. To take advantage of this fact, we define six groups around firm $k$, within which we have that the average distant is constant. The six groups are:


The rationale for the construction of the groups is the following: Group 1 includes consumers to whom firm $k$ would offer the highest surplus of the 4 firms, were all consumers perfectly informed. The limits of this group are defined by the firms' relative prices. For example, an increase in firm k's price, ceteris paribus, would imply that consumers that lie close to $d_{+1}$ in the figure would now prefer firm $\mathrm{k}+3$ ' product instead of firm k's, since the price is now higher in firm $k$, but the distance is the same. Therefore, the size of group 1 would have to shrink until $d_{+1}$ is indifferent between consuming the good in firm k and consuming it in firm $\mathrm{k}+1$. We can say that the different $d_{n}$ are the indifferent consumers between firm k and the rest of the firms in the market. Therefore, the 4 groups' relative sizes will depend exclusively on the firms' pricing strategies, and within these groups the probability of a consumer purchasing a particular brand will be constant. Therefore, we can characterise the average transport cost as a weighted average of the transport costs of each group, each multiplied by the proportion of firm k's total customers that lie in each group. This proportion will depend on the promotional expenditures set by the four firms.

In both of the cases we are analysing, we have that prices are constant and equal across firms. Therefore, the indifferent consumers' positions will be constant all across this exercise. Consumer $d_{+1}$ 's position in the circle will be where a customer's utility derived from consuming from firm k will be equal to the utility of consuming from firm $\mathrm{k}+1$. This implies:

$$
\hat{s}-t d_{+1}-p_{k}=\hat{s}-t\left(\frac{1}{4}-d_{+1}\right)-p_{k+1}
$$

Since prices are equal, this equation simplifies to:

$$
d_{+1}=\frac{1}{8}
$$

Similarly,

$$
d_{-1}=\frac{1}{8}
$$

To calculate all other indifferent consumers, we repeat the exercise across all indifferent consumers. However, since prices are the same for all firms, the indifferent consumer always lies halfway between the two firms. Therefore, the sizes of groups 1 through $6\left(\mathrm{~N}_{n}\right.$ for $\left.\mathrm{n}=1 \ldots 6\right)$ are:

$$
\begin{aligned}
& N_{1}=N_{6}=\frac{1}{4} \\
& N_{2}=N_{3}=N_{4}=N_{5}=\frac{1}{8}
\end{aligned}
$$

Within group 1 , the average distance travelled will be $\frac{1}{16}$, since the farthest consumers lie at a distance of $\frac{1}{8}$ from brand k , and the closest are exactly at firm k's location, with a continuum of consumers in between the two. For group 2 and group 3 , the farthest consumers are at $\frac{1}{4}$, and the closest at $\frac{1}{8}$. Therefore, the average distance travelled will be $\frac{3}{16}$. Using the same logic, we can see that the average distance travelled in groups 4 and 5 is $\frac{5}{16}$ and for group 6 it is $\frac{7}{16}$. We still need to see what proportion of firm k's consumers lie in each group. The total amount of customers of firm k in group 1 will be $\mathrm{N}_{1}$ times $\phi_{m}$, since only a proportion $\phi_{m}$ of total consumers in this group receive firm k's ad. Similarly, in group 2 the total number of consumers will be $\mathrm{N}_{2}$ times $\phi_{m}(1-\phi)$, because only consumers that have not received an advertisement from firm $\mathrm{k}+3$ will be willing to purchase brand k's product. Similarly, the total number of consumers in group 3 will be $\mathrm{N}_{3}$ times $\phi_{m}\left(1-\phi_{m}\right)$. By using this logic we can arrive at the proportion of brand k's total demand that lies in each segment, which will then allow us to calculate the average distance travelled by firm k's customers.

Since the problem is symmetric for firm k and $\mathrm{k}+1$, both firms' consumers will have the same average distance travelled $\left(\bar{d}_{m}\right)$. To find this value we need to multiply the proportion of firm k's total consumers within each group $\left(\theta_{n}, n=\right.$ $1 \ldots 6, \sum_{n} \theta_{n}=1$ ) times the average distance travelled within each group. To find the total size of firm k's demand $\left(D_{k}\right)$, we add the consumers of all groups. Analytically:

$$
\begin{aligned}
& \quad \mathrm{D}_{k}=N_{1} \phi_{m}+N_{2} \phi_{m}(1-\phi)+N_{3} \phi_{m}\left(1-\phi_{m}\right)+N_{4} \phi_{m}(1-\phi)^{2}+N_{5} \phi_{m}(1- \\
& \phi)\left(1-\phi_{m}\right)+N_{6} \phi_{m}(1-\phi)^{2}\left(1-\phi_{m}\right) \\
& \quad=\phi_{m}\left[1-\phi-\frac{\phi_{m}}{2}+\frac{3}{8} \phi^{2}+\frac{5}{8} \phi_{m} \phi-\frac{1}{4} \phi_{m} \phi^{2}\right]
\end{aligned}
$$

$$
\begin{aligned}
& \theta_{1}=\frac{N_{1} \phi_{m}}{\phi_{m}\left[1-\phi-\frac{\phi_{m}}{2}+\frac{3}{8} \phi^{2}+\frac{5}{8} \phi_{m} \phi-\frac{1}{4} \phi_{m} \phi^{2}\right]}=\frac{\frac{1}{4}}{1-\phi-\frac{\phi_{m}}{2}+\frac{3}{8} \phi^{2}+\frac{5}{8} \phi_{m} \phi-\frac{1}{4} \phi_{m} \phi^{2}} \\
& \theta_{2}=\frac{N_{2} \phi_{m}(1-\phi)}{\phi_{m}\left[1-\phi-\frac{\phi_{m}}{2}+\frac{3}{8} \phi^{2}+\frac{5}{8} \phi_{m} \phi-\frac{1}{4} \phi_{m} \phi^{2}\right]}
\end{aligned}=\frac{\begin{array}{l}
\frac{\phi_{m}}{2}+\frac{3}{8} \phi^{2}+\frac{5}{8} \phi_{m} \phi-\frac{1}{4} \phi_{m} \phi^{2}
\end{array}}{1-\phi-\frac{t^{2}}{2}} .
$$

Doing this for all groups yields an expression for the average distance trav-
elled for consumers of the merged firm:

$$
\bar{d}_{m}=\frac{1}{16} \theta_{1}+\frac{3}{16} \theta_{2}+\frac{3}{16} \theta_{3}+\frac{5}{16} \theta_{4}+\frac{5}{16} \theta_{5}+\frac{7}{16} \theta_{6}
$$

This is nothing more than a weighted average of the average distances travelled by consumers of the merged firms in equilibrium, for different groups where the probabilities of the customers purchasing from firm k or $\mathrm{k}+1$ are different. Replacing the values of $\theta$ we get that:

$$
\bar{d}_{m}=\frac{1}{8}\left(\frac{\frac{1}{8}+\frac{3}{16}(1-\phi)+\frac{3}{16}\left(1-\phi_{m}\right)+\frac{5}{16}(1-\phi)^{2}+\frac{5}{16}(1-\phi)\left(1-\phi_{m}\right)+\frac{7}{8}\left(1-\phi_{m}\right)(1-\phi)^{2}}{1-\phi-\frac{\phi_{m}}{2}+\frac{3}{8} \phi^{2}+\frac{5}{8} \phi_{m} \phi-\frac{1}{4} \phi_{m} \phi^{2}}\right)
$$

After expanding and simplifying we arrive at an equation for average distance in terms of $\phi$ and $\phi_{m}$.

$$
\bar{d}_{m}=\frac{2-\frac{23}{8} \phi-\frac{11}{8} \phi_{m}+\frac{19}{16} \phi^{2}+\frac{33}{16} \phi_{m} \phi-\frac{7}{8} \phi^{2} \phi_{m}}{8-8 \phi-4 \phi_{m}+3 \phi^{2}+5 \phi_{m} \phi-2 \phi^{2} \phi_{m}}
$$

Since the group construction and demands are exactly the same for the conglomerate's competitors, only inverting the $\phi_{m}$ and $\phi$ in the original equations, it is straightforward that the average distance travelled for consumers of the non-merged firms will be:

$$
\bar{d}=\frac{2-\frac{23}{8} \phi_{m}-\frac{11}{8} \phi+\frac{19}{16} \phi_{m}^{2}+\frac{33}{16} \phi_{m} \phi-\frac{7}{8} \phi_{m}^{2} \phi}{8-8 \phi_{m}-4 \phi+3 \phi_{m}^{2}+5 \phi_{m} \phi-2 \phi_{m}^{2} \phi}
$$

Both average distance functions evaluated in $\phi_{m}=\phi=1$ imply that the average distance travelled will be $\frac{1}{16}$, which is reasonable since every firm will sell exclusively to consumers within a distance of $\frac{1}{8}$ from its location. This function also tends to decrease as the value of both firms' promotional coverage increases. This is intuitive; as more information is provided to consumers, these tend to make more informed decisions about where to purchase, and this is reflected in a lower average transport cost. This can be seen in the figure below; as the merged firm's promotional coverage increases, its customers tend to travel less on average to purchase. Also, for higher values of the non-merged firms'

promotional coverage, the average distance also falls, since consumers tend to have more information about alternative sale locations and can therefore reduce transport costs by avoiding to purchase in firms that are far from their position in the circumference.

It is also noteworthy that the average distance function is always lower than $\frac{1}{4}$. This is intuitive since a random allocation of consumers to firms would imply an average distance traveled of exactly $\frac{1}{4}$, while an efficient allocation would imply a distance of $\frac{1}{16}$. Increased promotional coverage fulfills the social function of reducing aggregate transport costs and improving the match between firms and consumers.

Another positive social function fulfilled by advertisements is the increase in the size of the market. Higher levels of promotional spending will leave fewer customers out of the market, or, in other words, reduce the fraction $(1-\phi)^{2}(1-$ $\left.\phi_{m}\right)^{2}$ of consumers that do not purchase the product because they have not received any information on the existence of any of the brands that operate in the market. Therefore, we expect the situation that implies a higher level of advertisement to improve aggregate welfare.

For the pre-merger case, since advertising spending is equal across all brands, we can replace $\phi$ and $\phi_{m}$ in the average distance equations by a single $\phi_{c}$ to calculate average distance for all consumers. Doing so, we obtain:

$$
\bar{d}_{c}=\frac{2-\frac{34}{8} \phi_{c}+\frac{52}{16} \phi_{c}^{2}-\frac{7}{8} \phi_{c}^{3}}{8-12 \phi_{c}+8 \phi_{c}^{2}-2 \phi_{c}^{3}}
$$

This function is similar to the merged firms' distance function, only that it includes one single variable, that is, the equilibrium advertisement spending of all four firms in competition. The average distance, as in the previous case, fluctuates between $\frac{1}{16}$ for the case when $\phi_{c}$ equals one, that is, perfect information, and converges to $\frac{1}{4}$ as equilibrium promotional coverage decreases.

If we call total welfare in the pre-merger case $\mathrm{W}^{1}$ and total welfare in the
fixed-prices post-merger case $W^{2}$, we can construct the following equations:

$$
\begin{aligned}
W^{1} & =\int_{0}^{1} \hat{s}-p-t d(x, f) d x+4 \delta \pi \\
& =\left[1-\left(1-\phi_{c}\right)^{4}\right]\left(\hat{s}-p-t \bar{d}_{c}\right)+\left(1-\phi_{c}\right)^{4}(0)+4 \delta \pi
\end{aligned}
$$

We can replace total profits by multiplying the equilibrium quantity, $1-$ $\left(1-\phi_{c}\right)^{4}$, by price minus marginal cost, and then subtract the total advertising cost, which will be equal to $2 a \phi_{c}^{2}$. Replacing:

$$
W^{1}=\left(1-\left(1-\phi_{c}\right)^{4}\right)\left[\hat{s}-p-t\left(\frac{2-\frac{34}{8} \phi_{c}+\frac{52}{16} \phi_{c}^{2}-\frac{7}{8} \phi_{c}^{3}}{8-12 \phi_{c}+8 \phi_{c}^{2}-2 \phi_{c}^{3}}\right)\right]+\delta\left[\left(1-\left(1-\phi_{c}\right)^{4}\right)(p-c)-2 a \phi_{c}^{2}\right]
$$

Simplifying, we can arrive at an expression for aggregate welfare for the premerger case in terms of equilibrium prices and advertisement coverage:

$$
W^{1}=\left(1-\left(1-\phi_{c}\right)^{4}\right)\left[\hat{s}-p(1-\delta)-\delta c-t\left(\frac{2-\frac{34}{8} \phi_{c}+\frac{52}{16} \phi_{c}^{2}-\frac{7}{8} \phi_{c}^{3}}{8-12 \phi_{c}+8 \phi_{c}^{2}-2 \phi_{c}^{3}}\right)\right]-2 a \delta \phi_{c}^{2}
$$

For the post-merger case, aggregate welfare is:

$$
W^{2}=\int_{0}^{1} \hat{s}-p-t d(x, f) d x+\delta\left(\pi_{k}+\pi_{k+1}+\pi_{k+2}+\pi_{k+3}\right)
$$

In this case, we have different average distances traveled, as well as different profits. Therefore, the integral will sum two terms, one corresponding to the customers of the merged brands, and the other corresponding to customers of the non-merged firms.

$$
\begin{aligned}
& W^{2}=\left(D_{k}+D_{k+1}\right)\left[\hat{s}-p-t \bar{d}_{m}\right]+\left(D_{k+2}+D_{k+3}\right)[\hat{s}-p-t \bar{d}]+\delta\left(D_{k}+D_{k+1}\right)(p- \\
& c)+\delta\left(D_{k+2}+D_{k+3}\right)(p-c)-a \delta\left(\phi_{m}\right)^{2}-a \delta(\phi)^{2} \\
& \quad=\left(D_{k}+D_{k+1}+D_{k+2}+D_{k+3}\right)(\hat{s}-p(1-\delta)-\delta c)-\left(D_{k}+D_{k+1}\right)\left[t \bar{d}_{m}\right]- \\
& \left(D_{k+2}+D_{k+3}\right)[t \bar{d}]-a \delta\left(\phi_{m}\right)^{2}-a \delta(\phi)^{2}
\end{aligned}
$$

It is useful to insert each group's equilibrium demand in terms of its advertisement spending and its competitors' in this equation. Also replacing the average distance traveled yields:

$$
\begin{gathered}
W^{2}=\left[2 \phi_{m}\left(1-\phi-\frac{\phi_{m}}{2}+\frac{3}{8} \phi^{2}+\frac{5}{8} \phi_{m} \phi-\frac{1}{4} \phi_{m} \phi^{2}\right)+\right. \\
\left.+2 \phi\left(1-\phi_{m}-\frac{\phi}{2}+\frac{3}{8} \phi_{m}^{2}+\frac{5}{8} \phi_{m} \phi-\frac{1}{4} \phi \phi_{m}^{2}\right)\right](\hat{s}-p(1-\delta)-\delta c)+
\end{gathered}
$$

$$
\begin{aligned}
& -2 t \phi_{m}\left(1-\phi-\frac{\phi_{m}}{2}+\frac{3}{8} \phi^{2}+\frac{5}{8} \phi_{m} \phi-\frac{1}{4} \phi_{m} \phi^{2}\right)\left(\frac{2-\frac{23}{8} \phi-\frac{11}{8} \phi_{m}+\frac{19}{16} \phi^{2}+\frac{33}{16} \phi_{m} \phi-\frac{7}{8} \phi^{2} \phi_{m}}{8-8 \phi-4 \phi_{m}+3 \phi^{2}+5 \phi_{m} \phi-2 \phi^{2} \phi_{m}}\right)+ \\
& -2 \phi t\left(1-\phi_{m}-\frac{\phi}{2}+\frac{3}{8} \phi_{m}^{2}+\frac{5}{8} \phi_{m} \phi-\frac{1}{4} \phi \phi_{m}^{2}\right)\left(\frac{2-\frac{23}{8} \phi_{m}-\frac{11}{8} \phi+\frac{19}{16} \phi_{m}^{2}+\frac{33}{16} \phi_{m} \phi-\frac{7}{8} \phi_{m}^{2} \phi}{8-8 \phi_{m}-4 \phi+3 \phi_{m}^{2}+5 \phi_{m} \phi-2 \phi_{m}^{2} \phi}\right)- \\
& a \delta\left(\phi_{m}\right)^{2}-a \delta(\phi)
\end{aligned}
$$

This equation simplifies to:

$$
\begin{aligned}
& \quad W^{2}=\left(\phi_{m}+\phi-\phi_{m} \phi\right)\left(2-\phi_{m}-\phi+\phi_{m} \phi\right)(\hat{s}-p(1-\delta)-\delta c)+ \\
& -\frac{t}{4}\left(2 \phi_{m}+2 \phi-\frac{23}{4} \phi \phi_{m}-\frac{11}{8} \phi_{m}^{2}-\frac{11}{8} \phi^{2}+\frac{13}{4} \phi^{2} \phi_{m}+\frac{13}{4} \phi_{m}^{2} \phi-\frac{7}{4} \phi^{2} \phi_{m}^{2}\right)-a \delta\left(\phi_{m}\right)^{2}- \\
& a \delta(\phi)^{2}
\end{aligned}
$$

Analysing both welfare equations, we can see that both have three distinct terms; the first is equal to total demand times $(\hat{s}-p(1-\delta)-\delta c)$. This represents consumer surplus net of production costs (c) but not of marketing and transport costs. This represents the utility that arises from the fact that, when a consumer purchases the product, an efficient transaction is made, in the sense that the utility derived from the good's consumption is superior to the cost of its production. In this term, we can see how an increase in advertisement coverage can improve welfare by increasing total market size. In this case, increased advertising would have a positive welfare effect, as long as this term is larger than the marginal cost of advertising. The second term is the average distance traveled by consumers in equilibrium multiplied by the transport cost, multiplied by total demand. This is where transport costs are netted from consumer surplus. In both cases, the second term is decreasing in advertisement coverage. This is the effect that involves reducing average transport costs by improving the matching of consumers and firms, which is also a positive welfare consequence of advertising. The larger the firms' promotional expenditures, the easier it will be for consumers to find a brand that suits their tastes, and the lower the transport cost they will have to pay.

The third term involves the total marketing costs involved in selling the products. The downside of advertisement is that it is expensive, and this is reflected in the welfare functions as a negative term. There is a divergence between private and public costs of advertising, as Grossman and Shapiro note, that can tend to produce equilibrium advertising coverages that are not optimal; in particular, they find that, for reasonable parameter values and a sufficiently large number of firms (four or five), advertising tends to be excessive, since the benefits of advertisement that come from reduced transport costs and increased market size are eclipsed by the "customer capture" effect that provides incentives for firms to advertise more than the social optimum, to capture other firms' customers. This duplication of advertisements tends to be wasteful, since
consumers who already have found their favourite brand derive no extra surplus from additional advertisements; yet these are costly and socially useless.

Grossman and Shapiro argue that, as more firms enter the market, the "capture effect" tends to exceed the positive consequences of advertisement, and the marginal increase in advertisement has no significant effect on the total number of consumers informed about at least one brand, and therefore advertisement is overprovided. If this were to be the case, then it is clear in our model that whenever the merger increases total advertising expenditures, it tends to reduce total welfare even further than it would already do through the price increase implied in the partial analysis of section 3.2. Therefore, if this were to be the case, then a restriction on advertising would be socially desirable, since it would prevent an increase in a good that would be overprovided relatively to the social optimum, and would therefore have a negative welfare effect. However, if the merger reduces total advertising spending, then no conditioning is necessary, since the structure of incentives will bring about a reduction of equilibrium advertisement coverage which will have a positive welfare effect. Although the quantitative response will imply the evaluation of $W^{2}-W^{1}$, a qualitative response ultimately depends on the total advertising in each case. In other words, for welfare to increase in the post-merger case compared with the pre-merger case, we need a reduction in total advertising; in other words, we need to prove that:

$$
\frac{\phi+\phi_{m}}{2}<\phi_{c}
$$



Replacing equilibrium values of $\phi$ and $\phi^{c}$ we get:

$$
\frac{8-8 \phi_{m}+3 \phi_{m}^{2}}{\frac{8 a}{p-c}+4-5 \phi_{m}+2 \phi_{m}^{2}}+\phi_{m}<\left(2 \sqrt[3]{\frac{t}{16 a}}\right)
$$

Which collapses into

$$
\phi_{m}\left(10 \sqrt[3]{\frac{t}{16 a}}-4\right)-\phi_{m}^{2}\left(2+4 \sqrt[3]{\frac{t}{16 a}}\right)<8\left(\sqrt[3]{\frac{t}{16 a}}-1\right)
$$

We can note that, after the merger, we can expect the merged firm's equilibrium advertising expenditure to decrease, since after the merger there are incentives for the firm to internalise some of the negative externalities that stem from excessive promotional activity (the "capture effect" that Grossman and Shapiro discuss). Therefore, we can expect to see that, ceteris paribus, $\phi_{m}<\phi_{c}$. However, non-merged firms will see the marginal productivity of their advertisement rise, and therefore they will increase their advertisement spending. However, the overall effect on total advertising is negative, since, replacing in the left side of the previous inequality, we have that

$$
\phi_{m}\left(10 \sqrt[3]{\frac{t}{16 a}}-4\right)-\phi_{m}^{2}\left(2+4 \sqrt[3]{\frac{t}{16 a}}\right)<\sqrt[3]{\frac{t}{16 a}}\left(10 \sqrt[3]{\frac{t}{16 a}}-4\right)-\sqrt[3]{\frac{t}{16 a}}\left(2+4 \sqrt[3]{\frac{t}{16 a}}\right)
$$

And, comparing with the right-hand side we have that:

$$
\sqrt[3]{\frac{t}{16 a}}\left(10 \sqrt[3]{\frac{t}{16 a}}-4\right)-\sqrt[3]{\frac{t}{16 a}}\left(2+4 \sqrt[3]{\frac{t}{16 a}}\right)<8\left(\sqrt[3]{\frac{t}{16 a}}-1\right)
$$

Expanding the brackets and rearranging, we get that the sufficient condition for total welfare to improve after the merger, in terms of the model's parameters, is:

$$
\begin{aligned}
& 3\left(\sqrt[3]{\frac{t}{16 a}}\right)^{2}-7\left(\sqrt[3]{\frac{t}{16 a}}\right)+4<0 \\
& \left(\sqrt[3]{\frac{t}{16 a}}-1\right)\left(3 \sqrt[3]{\frac{t}{16 a}}-4\right)<0
\end{aligned}
$$

Therefore, we have that, for sufficiently large values of $\sqrt[3]{\frac{t}{16 a}}$, so that both terms are positive $\left(\right.$ i.e. $\left.3 \sqrt[3]{\frac{t}{16 a}}>4\right)$ and for sufficiently small values of $\sqrt[3]{\frac{t}{16 a}}$, so that both terms are negative $\left(i . e \cdot \sqrt[3]{\frac{t}{16 a}}<1\right)$, the welfare consequences of the merger are negative. In other words, this condition states that, whenever a merger takes place in a differentiated goods market, if transport costs are sufficiently high vis-a-vis marketing costs, or marketing costs are sufficiently high vis-a-vis transport costs, the merger will have negative welfare consequences because it will increase total advertising spending, although presumably the merged firms' own promotional expenditure will fall as a consequence of the merger, while their competitors' will increase. Also, since this comparison has been made in a setting of partial analysis (that is, leaving prices fixed), the negative effects of the merger through changes in prices have not been taken into account.

Therefore, we can assert that there are many possible values for which the merger has negative consequences for welfare through channels associated with promotional spending. Whenever there is a merger in this type of market, there is a possibility that changes in the firm's own promotional coverage (i.e. a reduction of it) might imply negative welfare consequences for the economy as a whole, because the competitors' advertising spending will increase more than what the merged firms' spending will decrease, and whenever pre-merger advertising levels were excessive, we will have a reduction in total welfare, by any standard (this covers all cases, and includes all possible values of the social
planner's preference for distribution of utility between customers and firms, that is, for all values of $\delta$ between zero and one.


## 5 Conclusions

In the first section we have skimmed through many visions of advertising that are present in the economic literature. Retrospectively, we get the feeling that there are costs and benefits to advertising, and depending on the characteristics of the market, the cost of advertising, etc., it tends to be undersupplied or oversupplied. Depending on the function we see for advertising in a market economy, we will feel that there is a social function to fulfill for promotion by firms or not. My personal conclusion is that, whenever firms and consumers operate in a context where information is central in decision-making processes, and constant variability requires a persistent stream of new information to adapt to changing circumstances, the role of advertising as a medium for the transmission of information from firms to customers will always be likely, at least in markets where more information allows customers to make better decisions (which is certainly the case in the model we have just developed).

The model has showed us a possible way to systematise the different features we had been discussing on advertising and mergers in the first two sections; it had a cost, and provides a useful function, for firms and for society in general. Also, it reflected the fact that a firm's demand depends on its advertising, as we can see in any differentiated goods market where "standing out" through promotional expenditures is a key factor in creating a strong demand for the good. As a famous quote says, "Doing business without advertising is like turning off a plane in mid-air", which basically means that, sooner or later, companies that do not advertise enough lose market share and disappear. We also included in our model the fact that increased advertising has a "businessstealing" consequence, which means that a fraction of the firm's new demand derived from an increase in advertising expenditure will be the demand that it "steals" from its competitors, who see a reduction in their customer base.

The last section allows us to draw some interesting conclusions from the exercise we just completed. First of all, from the partial analyses of price and advertising movements after a merger, we can conclude that usually there is a movement towards inefficiency after the merger occurs, since both prices and promotional spending will tend to increase. This will be particularly relevant in the case when firms choose both variables simultaneously, because the marginal benefit of advertising depends on the profit-cost margin, and therefore a higher price necessarily will increase the incentives to spend on promotion. Also, we have seen that the negative welfare consequences of the merger arrive through the merged brands' competitors; the conglomerate itslelf reduces its equilibrium advertising (its reaction function shifts to the left), but this triggers an increase in its competitors' spending. Therefore, according to our model, the overall rationale behind the CNDC's conditioning is essentially right, but will be ineffective since all negative welfare consequences of the merger will not be responsibility of the merged firm itself.

This type of conclusion can be drawn even in a model in which advertising has a positive and useful social function; instead of simply assuming that advertising is wasteful, we allowed it to have multiple functions which have a positive bearing on total welfare; however, since the private and social benefits of advertising differ, we find that although there is a useful function in advertising, in equilibrium it tends to be excessive, and any increases in its value will have a negative net welfare effect if this is the case.

We are finally in a position to evaluate the CNDC's conditions from a theoretical point of view. In general, we find that the conditionings on advertising spending will prove to be non-operating, since the merged firms will probably reduce their advertising spending instead of increasing it. What would certainly help would be to impose constraints on the merged brands' competitors' promotional expenditures, since the negative consequences of the merger will show up (for the fixed-price assumption) through their new equilibrium advertisement values. If we were to imagine the exercise of letting prices fluctuate freely post-merger, together with advertisement expenditures, certainly we would end up with a worse situation from a welfare point of view; prices would increase in equilibrium, and advertisements would also increase, since the incentives to advertise would increase, guaranteeing an equilibrium with even higher advertising spending. All this analysis is based on the fact that increased levels of advertising are assumed to be negative for welfare. If we were in an opposite case, then total welfare calculations would have to be carried out for each case.

There is also the fact that advertising has an intertemporal dimension (as the plane analogy clearly illustrates). One possible rational behind the CNDC's conditionings which this model fails to account for is the possibility of a shift in the long-term structure of the market through the merged firms' advertising spending. For example, the new firm could increase advertising beyond its static optimal level, if dynamically this guaranteed some kind of competitive advantage in the future, as could be the case if it were to become some kind of market leader with cost advantages or strategic advantages of some kind. Also there could be the possibilty of driving firms out of the market, so as to then take advantage of a new quasi-monopoly situation with guaranteed barriers to entry through a high promotional expenditure. All these speculations lie outside the reach of our present discussion, however. As far as our model goes, we can say that the static post-merger fixed-price equilibrium will not require any promotional cover conditions for the merged firms, since these will tend to reduce advertising in equilibrium, relative to pre-merger levels.

In welfare terms, the CNDC could guarantee the welfare properties of the new equilibrium by imposing constraints on all firms on prices and advertising, so as to avoid the fact that competitors can speculate with the price constraints faced by the merged firm. The interesting case that is pending is what happens with promotional spending when prices are not constrained. This analysis is left to future research.

## 6 References

Becker, G. and Murphy, K.; A Simple Theory of Advertising as a Good or Bad. The Quarterly Journal of Economics, Vol. 108, No. 4. (Nov., 1993), pp. 941964.

Bulow, J., Geanakoplos, J. and Klemperer, P.; Multimarket oligopoly: strategic substitutes and strategic complements. Journal of Political Economy 93 (1985), pp. 488-511

Church, J. and Ware, R.; Industrial Organization: A Strategic Approach. Irwin/McGraw-Hill (1999)

Dorfman, R. and Steiner, P.; Optimal Advertising and Optimal Quality. The American Economic Review, Vol. 44, No. 5. (Dec., 1954), pp. 826-836.

Grossman, G. and Shapiro, C.; Informative Advertising with Differentiated Products. The Review of Economic Studies, Vol. 51, No. 1. (Jan., 1984), pp. 63-81.

Hotelling, H.; Stability in Competition. Economic Journal 39 (1929), pp. 41-57.

Kaldor, N.; The Economic Aspects of Advertising. The Review of Economic Studies, Vol. 18, No. 1. (1950-1951), pp. 1-27.

LeBlanc, G.; Informative Advertising Competition. The Journal of Industrial Economics, Vol. 46, No. 1. (Mar., 1998), pp. 63-77.

Levy, H. and Simon, J.; A Generalization That Makes Useful the DorfmanSteiner Theorem with Respect to Advertising. Managerial and Decision Economics, Vol. 10, No. 1. (Mar., 1989), pp. 85-87.

Nelson, P.; Advertising as Information. The Journal of Political Economy, Vol. 82, No. 4. (Jul. - Aug., 1974), pp. 729-754.

Pepall, L., Richards, D. and Norman, G.; Industrial Organization: Contemporary Theory and Practice (3 ${ }^{\text {rd }}$ Edition), South-Western College Publishing (1999)

Roberts, M. and Samuelson, L.; An Empirical Analysis of Dynamic, Nonprice Competition in an Oligopolistic Industry. The RAND Journal of Economics, Vol. 19, No. 2. (Summer, 1988), pp. 200-220.

Salop, S.; Monopolistic Competition with Outside Goods. Bell Journal of Economics 10 (1979), pp. 141-156.

Schmalensee, R.; A Model of Promotional Competition in Oligopoly. The Review of Economic Studies, Vol. 43, No. 3. (Oct., 1976), pp. 493-507.

Stigler, G.; Price and Non-Price Competition. The Journal of Political Economy, Vol. 76, No. 1. (Jan. - Feb., 1968), pp. 149-154.

Telser, L.; Advertising and Competition. The Journal of Political Economy, Vol. 72, No. 6. (Dec.1964), pp. 537-562.

Tirole, J.; The Theory of Industrial Organization. Cambridge, Mass.: MIT Press, (1988)
"Dictámen de la CNDC sobre la fusión de Arcor y La Campagnola", Comisión Nacional de Defensa de la Competencia,
http://www.mecon.gov.ar/cndc/dictamenes/dictamen_arcor.pdf, 2006.


