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"Insuring Social Security"

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Insuring Social Security

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Abstract

The current debate about social security reform has sprung a renewed interest in studying the way risks are shared through different social security programs. There is a widespread consensus that a fully funded, or investment based, system would result in a larger exposure to risk by retirees. One way to reduce the risk to retirees is by a government insurance that shifts some of the financial market performance risk to future taxpayers. We study the conditions under which such a guarantee would be voted and sustained as a Markovian equilibrium in an economy with income inequality within generations. If there is a separte choice of insurance characteristics and social security's portfolio, there will be incentives to distort the choice of the first in order to strategically affect the latter. This results in an inefficient level of insurance.

1 Introduction

Introduction

The secular increase in the ratio of retirees to employees has pushed up the cost of maintaining pay-as-you-go programs in industrial countries. In the US the cost of providing promised benefits is expected to rise from about 12 percent of covered payroll earnings now to more than 17 percent by 2030. This has lead to a debate on the need to reform social security, either by cutting benefits, increasing contributions or improving the rate of return of contributions, now estimated at around 2 percent a year in the US.

Although the US social security system is essentially a pay-as-you-go program in which each years tax receipts are used to pay the benefits of concurrent retirees, there is also a trust fund

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that is invested in government bonds. The rate of return of these bonds affects the amount that is available to pay annual benefits and therefore affects the overall rate of return that participants receive on the taxes that they pay. A way of increasing the rate of return of social security would be to have some prefunding of benefits. This requires that a part of contributions be invested in capital accumulation.

In thinking the practical aspects of social security reform, the prefunding solution requires the transition generation to "pay twice", since they have to pay taxes to cover benefits of existing retirees and to provide savings for their own retirement. Although it has been argued that these transitional costs are not very large, making the transition politically feasible, this does not imply that such a policy is desirable. There are three issues that have to be considered to determine whether a switch to a fully-funded program is desirable: administrative costs, distributional effects, and exposure to market risks.

If part or all of future social security benefits are converted from an explicit defined-benefit plan to some form of defined-contribution system based on individual investment accounts, future retirees will experience the risk of fluctuations in asset prices. It is not optimal to have a generation to bear all this risk, and Shiller (1999) considers the interaction between intergenerational, intragenerational, and international risk sharing, and how the government should design social security to promote risk sharing.The extent of retirees exposure to asset market risk can be reduced by using a mixed system with both defined-contribution and defined-benefit parts. Another way of reducing this risk exposure is to provide a contingent pay-as-you-go benefit that varies inversely with the performance of social security portfolios. Such a proposal for a government guarantee on minimum benefits has been advocated by Feldstein, Ranguelova, and Sandwick (2000).

We will abstract from considerations about the transition from a pay-as-you-go to a fully-funded system, and will take as given the existence of a defined-benefit program. We want to study whether the presence of income heterogeneity might lead to conflicts of interest in the design of a government guarantee on minimum benefits. If the choice of insurance characteristics is done after the choice of tax rates, and if the identity of both decision makers is different, then the latter one has an incentive to distort the choice of tax rates to influence the insurance characteristics chosen by the former. This creates an inefficiency resulting in a higher level of taxes voted in equilibrium.

To have a tractable model, we make some simplifying assumptions. The most controversial would be that although we want to study government guarantees in a fully-funded social security system, we will have no public program of capital accumulation. The rationale for this is the well know result that the implementation of a fully-funded social security program has no effects on capital accumulation, as savers react reducing their private savings leaving the overall level of savings constant. Of course this is no longer true if social security is redistributive, or if there are government guarantees to the return on social security savings, but not on private savings. But in this last case, it is straight forward to see that savers will find it optimal to reduce as much as possible their exposure to risk in their private savings, and have the social security portfolio to be a risky one (even with a full exposure to market risk). To avoid this corner solution in social security porfolio choice, we will simply assume that social security provides a contingent pay-as-you-go benefit that insures a minimum return on private savings. In this first draft we will assume that there is a unique risky asset that can be used for savings. Later we will refine our analysis introducing a riskless asset.

We find general conditions to have a median voter result in both the choice of tax rates, and the choice of insurance characteristics. We concentrate on the case of a redistributive social security program, as we think this is a defining characteristics of these programs. An important result that we get is that the contingent pay-as-you-go system is sustained in a political vote without the threat of the collapse of the system if any generation fails to maintain it. Traditionally two explanations have been advanced for the persistence of social security systems in the world. Hansson and Stuart (1989) and Tabellini (1991) are among the works that look at some form of altruism between generations as the driving force behind the political support of social security. Later, Cooley and Soares (1999), and Rangel (2000) among others, see the threat of system collapse as the mechanism that helps supporting a pay-as-you-go system as an equilibrium in a game between generations. In our model it is the strategic incentive to increase tax rates and affect the choice of insurance characteristics that results in a positive tax rate chosen in equilibrium. In the absence of income heterogeneity, there is no gain to distort tax rates, and the only equilibrium is one with no social security.

The next section presents a model of the economy, describing the determinants of tax, and insurance characteristics choices, and savings decisions. Section 3 looks at the steady state equilibrium with a positive tax rate. Section 4 reports on the results of numerical simulations, section 5 concludes and a mathematical appendix follows.

2 Distorted Portfolio Choice

We consider an economy with a continuum of households in each generation and population growth rate $\nu - 1 > 0$. Households are two period lived and indexed by their labor productivity, n, which is positive and bounded. $\phi(n)$ denotes the time invariant density of n. Young households in period t inelastically supply labor at a fixed wage w per efficiency unit. Their labor income, nw, is then taxed at rate τ_t. Households are also endowed with another source of income, e, that is not taxed and is independent of labor productivity. (We introduce this untaxed form of income as a simple device to allow for progressive taxation.) Disposable income, $e + nw(1 - \tau_t)$, is either consumed, $c_{1,t}^n$, or saved, a_t^n. Consumption of old households, $c_{2,t+1}^n$, consists of the random gross return on their savings, $a_t^n R_{t+1}$ (R_{t+1} denotes the random gross rate of return on private savings between period t and $t+1$), plus a state contingent benefit, b_{t+1}^n. A young household of type n who is born in period t chooses savings such that,

$$\max_{a_t^n} \; u(e + nw(1-\tau_t) - a_t^n) + \beta E_t \left[u(a_t^n R_{t+1} + b_{t+1}^n) \right] \tag{1}$$

taking as given the tax rate τ and benefits b. The households' felicity function $u(\cdot)$ satisfies the Inada conditions; β denotes the subjective time discount factor.

Social Security balances its budget period by period, but not necessarily state by state. It can provide such aggregate insurance by accessing an international reinsurance market (to which individual households are assumed to have no access) at actuarially fair terms. In particular, the government's insurance contract with the outside financiers swaps the deterministic tax collections from the young against state contingent (depending on the ralization of R) payments to the old:

$$\int_{R_t} \int_n b_t^n \, f(R_t) \, dn dR_t = \nu \int_n \tau_t \, nw \, dn \; , \; \forall t \tag{2}$$

with $f(\cdot)$ denoting the p.d.f. of R_t. Note that this insurance scheme partially resembles a state contingent pay-as-you-go social security system in that tax collections from the young are immediately distributed among the old. The difference from a pay-as-you-go system is that the presence of outside insurers removes the state by state identity between contributions and benefits.

Decisions are taken sequentially. First all generations alive vote on the tax rate. Then the characteristics of aggregate insurance are chosen. Finally, given the tax rate, and expected benefit payments, the savings decisions are taken. We solve for the equilibrium by backward induction. The optimal savings decision of a household is characterized by the Euler equation

$$u'(c_{1,t}^n) \geq \beta E_t \left[u'(c_{2,t+1}^n) R_{t+1} \right] \tag{3}$$

(with equality if the restriction on short-selling is not binding.)

2.1 The choice of insurance characteristics

We will consider a general environment in which Social Security chooses a unique insurance portfolio for all young savers in the economy. The choice of this portfolio is the result of a decision process that results in its characteristic being those of a median saver. We present now two examples of institutional arrangements for Social Security that have this result.

Social Security could be centralized provided through a trust fund working under public management. The trust fund manager could provide different insurance portfolios to suit the needs of different groups of consumers. But transaction costs and asymmetric information will most likely deter her from providing tailor-made insurance to individual characteristics. We will consider the case that a single portfolio is used to insure everybody's savings. Then, if the fund manager selection is through a political process, we will expect her to choose the portfolio preferred by a majority of young savers (under the assumption that the old are not interested in this choice since it will only be having effects on the allocation of resources in the future, when they are no longer around). We show in the appendix that if social security is redistributive, i.e. if b_n is the same for all n, then if savers are risk averse, a median voter result follows and the insurance characteristic of social security are those preferred by him.

Under a decentralized institutional arrangement for social security, private fund managers compete to attract savers' contributions. In general, a variety of funds will be offered to attract different types of savers. As before, transaction costs and problems of asymmetric information will make it impossible to provide tailor-made insurance. But in general competition will result in a variety of funds offered, as funds' managers try to differentiate their products in order to gain some market power. Nevertheless we can still find an outcome with a single insurance portfolio if regulations distort the choice of trust funds' managers. An example of this, would be the case of two fund managers competing to get workers contributions and subject to the restriction that they can offer a unique contract to all their customers. Then they will try to satisfy the needs of a majority of savers. If fund's fees are proportional to the value of the funds they manage, and given that there is income heterogeneity, they will put more weight to the insurance needs of the wealthiest. As a result the chosen portfolio will be the preferred one by the median wealth saver, the young saver given by the value of n^w such that the cumulative wealth of those with $n < n^w$ is the same as the cumulative wealth of those with $n > n^w$.

We restrict the state contingent payments to the old to be of the form $b_t^n \equiv b(n) \max[I_t - \theta_t R_t, 0]$. To the extent that $b(\cdot)$ varies with n, old age benefits increase with individual contributions. Moreover, benefits are paid only if the return on savings is lower than the threshold return $\bar{R}_t \equiv I_t/\theta_t$. Below this threshold benefits are decreasing in R_t (for $\theta_t > 0$). The characteristics of the insurance scheme, I_t and θ_t, are chosen period by period as described above. The redistributive characteristics of the benefit formula, $b(n)$, are assumed to be given and are thus exogenous to our model. The idea of a government guarantee on minimum benefits has been advocated among others by Feldstein, Ranguelova, and Sandwick (2000). The idea has shown up in other countries with fully-funded social security programs. In Argentina there was a recent debate to replace a non-contingent pay-as-you-go benefit provided to all retirees for a contingent benefit that would provide a minimum retirement level.

Let $\bar{n} \equiv \int_n n \; dn$ and normalize $\int_n b(n) \; dn$ to $\bar{n}$. Social Security's budget constraint then reads

$$\int_{R_t} \max[I_t - \theta_t R_t, 0] \; f(R_t) \; dR_t = \tau_t \; w \; \nu, \; \forall t. \tag{4}$$

In period t, before the young choose their individual savings, decisions about the insurance scheme to be applicable in period $t+1$ are made. For that decision, young savers form expectations on the tax rate in the coming period and thus on the expected value of benefit payments. (Below, we will discuss how tax rates are set and whether state contingent intergenerational benefits are sustainable.). Social Security trust fund managers choose the insurance characteristics in order to maximize (1) for the median saver, subject to the above budget constraint and taking into consideration how the choice of insurance characteristics affect agent n^w optimal savings. Formally, fund managers solve

$$\max_{I_{t+1},\theta_{t+1}} \quad u(e + n^w w(1-\tau_t) - a_t^w) + \beta E_t \left[u(a_t^w R_{t+1} + b(n^w)\Omega_{t+1})\right] \tag{5}$$

$$\text{s.t.} \quad (4),$$

$$\tau_t, \tau_{t+1}^e \text{ given},$$

$$\Omega_{t+1} \equiv \max[I_{t+1} - \theta_{t+1}R_{t+1}, 0].$$

Denoting the multiplier on (4) by $-\lambda\beta$ we find the following first order conditions with respect to θ_{t+1} and I_{t+1}:

$$\int_{-\infty}^{I_{t+1}/\theta_{t+1}} u'(c_{2,t+1}^w)\ b(n^w)\ R_{t+1}\ f(R_{t+1})\ dR_{t+1} = \lambda \int_{-\infty}^{I_{t+1}/\theta_{t+1}} R_{t+1}\ f(R_{t+1})\ dR_{t+1},$$

$$\int_{-\infty}^{I_{t+1}/\theta_{t+1}} u'(c_{2,t+1}^w)\ b(n^w)\ f(R_{t+1})\ dR_{t+1} = \lambda \int_{-\infty}^{I_{t+1}/\theta_{t+1}} f(R_{t+1})\ dR_{t+1}.$$

Eliminating the multiplier we see that fund managers choose the insurance characteristics such as to equalize agent n^w's marginal rate of substitution with the marginal rate of transformation:

$$\frac{\int_{-\infty}^{I_{t+1}/\theta_{t+1}} u'(c_{2,t+1}^w)\ b(n^w)\ R_{t+1}\ f(R_{t+1})\ dR_{t+1}}{\int_{-\infty}^{I_{t+1}/\theta_{t+1}} u'(c_{2,t+1}^w)\ b(n^w)\ f(R_{t+1})\ dR_{t+1}} = \frac{\int_{-\infty}^{I_{t+1}/\theta_{t+1}} R_{t+1}\ f(R_{t+1})\ dR_{t+1}}{\int_{-\infty}^{I_{t+1}/\theta_{t+1}} f(R_{t+1})\ dR_{t+1}}. \tag{6}$$

As shown in the Appendix the previous conditions imply that n^w's second period consumption is constant over the range of returns to savings for which insurance is paid. The insurance characteristics thus satisfy

$$\theta_{t+1} = \frac{a_t^w}{b(n^w)}, \tag{7}$$

$$\int_{-\infty}^{I_{t+1}b(n^w)/a_t^w} [I_{t+1} - a_t^w/b(n^w)R_{t+1}]\ f(R_{t+1})\ dR_{t+1} = \tau_{t+1}^e\ w\ \nu. \tag{8}$$

Equations (7) and (8) (implicitly) define functions $\theta_{t+1}(a_t^w, b(n^w))$ and $I_{t+1}(a_t^w, b(n^w); \tau_{t+1}^e w\nu)$; they thus implicitly define $\Omega_{t+1}^w \equiv \max[I_{t+1}(a_t^w, b(n^w); \tau_{t+1}^e w\nu) - \theta_{t+1}(a_t^w, b(n^w))\ R_{t+1}, 0]$.

2.2 The choice of tax rates

In period t, before the young choose their individual savings and before fund managers choose the insurance characteristics, a decision on the tax rate to be levied on labor income in period t is taken by majority vote. Taxes are restricted to be weakly positive. Clearly, old voters favor as high a tax rate as possible. For young voters, increasing the tax rate has a negative effect on their available income. This affects savings decision, in particular it affects the median saver choice, and thus the common hedging ratio θ. Thus it might be possible that the median voter deciding on the tax rate might find it profitable to choose a positive tax rate if the benefit of a better hedge to her risk exposure outweighs the cost in terms of foregone income.

In the appendix we show that if social security is redistributive and if preferences are characterized by non-increasing absolute risk aversion and non-decreasing relative risk aversion, preferred tax rates are decreasing with income for $n < n^w$. Under the same conditions on preferences, young savers with $n \leq n^w$ would prefer a negative tax rate. Thus we have that under fairly general conditions a median voter result follows for the choice of tax rate. Denote by v the young household with productivity n^v who represents this median voter,

$$1 + \nu \int_0^{n^v} \phi(n)\, dn = \nu \int_{n^v}^{\infty} \phi(n)\, dn.$$

The median voter chooses τ_t in order to maximize (1) for $n = n^v$ subject to (7), (8) and subject to given (expectations about) τ_{t+1}. Formally, v solves

$$\begin{aligned} \max_{\tau_t} \quad & u(e + n^v w(1-\tau_t) - a_t^v) + \beta E_t \left[u(a_t^v R_{t+1} + b(n^v)\Omega_{t+1}^w) \right] \\ \text{s.t.} \quad & \tau_{t+1} \text{ given}, \\ & \tau_t \geq 0. \end{aligned} \tag{9}$$

Denoting the (non negative) multiplier on the non-negativity constraint for τ_t by μ we find the following first order conditions with respect to τ_t:

$$u'(c_{1,t}^v) n^v w = \beta E_t \left[u'(c_{2,t+1}^v)\, b(n^v)\, \frac{\partial \Omega_{t+1}^w}{\partial a_t^w} \right] \frac{\partial a_t^w}{\partial \tau_t} + \mu, \quad \mu \tau_t = 0. \tag{10}$$

If young households were homogeneous the median voter and fund managers would have the same objectives and would face the same constraints, thus $\Omega_{t+1}^w = \Omega_{t+1}^v$. By an envelope condition, (10) would thus reduce to $\tau_t = 0$. With heterogeneous young households, however, and with $n^w \neq n^v$ the median voter's first order condition with respect to the current tax rate is distorted by the incentive to strategically affect n^w's savings. The median voter may gain from distorting n^w's savings because a change in a_t^w translates into a change of θ_{t+1} and I_{t+1} and thus Ω_{t+1}^w. The median voter might therefore choose to implement a positive tax rate if the direct cost is compensated by the expected utility gain due to the induced change in insurance characteristics.

3 Intergenerational Equilibrium

Up to this point we have discussed the static choice of τ_t and θ_{t+1}, I_{t+1} forming expectations on the level of resources available for insurance as given. In a dynamic equilibrium, today's expectations of future tax rates coincide with the actual choices taken in the future. We thus have to verify the existence of equilibria in which expected and actual tax rate choices are identical. We will concentrate on steady state equilibrium.

The literature on the sustainability of social security generally rationalizes the existence of positive intergenerational transfers by a trigger strategy argument. Examples are Cooley and Soares (1995), Rangel (1999), Boldrin and Rustichini (1999). A fundamental ingredient of such trigger strategy equilibria is a "reputational" state variable that links current voting decisions to the choices of future generations. Absent this reputational intergenerational link most models of social security would predict each generation to implement as low a tax rate as possible. In our model, conventional trigger strategies would also work. We restrict ourselves to Markovian equilibria, were we mean an equilibrium in which the decisions of agents depend only on the current state of the economy and not on its history.

It is clear from the outset that repeated choices of zero taxes constitute an intergenerational equilibrium. Subject to the expectation that $\tau_{t+1} = 0$ and thus $b^n_{t+1} = 0$ there is no benefit from strategically affecting n^v's saving decision and the median voter in period t will thus choose $\tau_t = 0$ herself. More interestingly, however, an intergenerational Markovian equilibrium with strictly positive tax rates is implementable in our economy.

Let's take a look at the FOC for tax choice of the median saver for the case that future tax rates are zero.

$$-u'(c_1)n^w w + \beta E\left[u'(c_2)b_n(E(R|1) - R)1\frac{d\theta}{d\tau}\right] = -u'(c_1)n^w w < 0$$

were we used the fact that 1 is an empty set if future tax rates are zero. If follows that there is a neighborhood around $\tau^e = 0$ for which the choice by median voter is to have zero taxes today. As the future tax rate increases further, the present tax rate chosen by median voter also increases. A necessary condition for an equilibrium with positive tax rates is to have $\frac{d\tau}{d\tau^e} > 1$ over some range of future tax rates. A sufficient condition for the existence of an equilibrium is to have that for some $\tau^e \leq 1$ there is full taxation of income today, i.e. $\tau = 1$. Obviously this will not be the case if all sources of income are taxable, i.e. if $e = 0$.

From the analysis of the existence of a median voter result we know that the more uneven the distribution of income, the higher the tax rate chosen. Also, we can see under what conditions an increase in e leads to an increase in the chosen τ for a given expected future tax rate. This analysis is done in the appendix and we find that in particular this is the case when preferences are characterized by constant relative risk aversion. We use this results to study the determinants of equilibrium in numerical simulations.

4 Simulations

We want to verify the existence of an equilibrium with positive tax rates. From the previous analysis we know that in an economy where Social Security is redistributive, there is income inequality, and taxation is progressive, such an equilibrium can arise if preferences are of the CRRA type. Thus the parameters that will determine the equilibrium tax rate are the coefficient of relative risk aversion, γ, the relative taxable income between the median tax voter and the median saver, $\frac{n^v}{n^w}$, the ratio between taxable income and non-taxable income, $\frac{w}{e}$. We will use both a lognormal distribution with parameters μ and σ, and a uniform distribution for R. We report results for one of these simulations.

5 Conclusions

We show that the introduction of a government guarantee on minimum benefits can result in an excessive level of insurance voted in equilibrium. If the choice of tax rates precedes the choice of insurance characteristics, the vote on the first one will reflect the desire to strategically affect the second choice. This result is obtained when social security is redistributive and when there is heterogeneity in labor productivity. Preference must reflect non-increasing absolute risk aversion and non-decreasing relative risk aversion, and the coefficient of relative risk aversion must be larger than one.

Since we are abstracting from reputational considerations to sustain social security, in the absence of income heterogeneity the only equilibrium is one with zero tax rates. Thus we have found a new possible explanation for the persistence of social security programs. A positive tax rate to pay benefits of concurrent retirees is chosen because it strategically affects the savings of the decisive median saver and thus the hedging characteristics of social security insurance. The same logic extends to a simpler scenario of a closed economy with capital accumulation and a riskless technology. When voting for tax rates, the median voter knows that it affects capital accumulation and therefore the rate of return on its savings and the wage of future generations. If the effect on the interest rates outweighs the effect on future wages and on current income, then she might find it profitable to vote for a positive tax rate (Gonzalez-Eiras and Niepelt 2002).

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7 Appendix

7.1 Median saver's consumption

Let's rewrite the condition characterizing the choice of insurance characteristics that we derived maximizing the expected utility of the median saver.

$$\int_{-\infty}^{\frac{I_{t+1}}{\theta_{t+1}}} u'(c^w_{2,t+1})\, R_{t+1}\, f(R_{t+1})\, dR_{t+1} = \frac{\int_{-\infty}^{\frac{I_{t+1}}{\theta_{t+1}}} R_{t+1}\, f(R_{t+1})\, dR_{t+1} \int_{-\infty}^{\frac{I_{t+1}}{\theta_{t+1}}} u'(c^w_{2,t+1})\, f(R_{t+1})\, dR_{t+1}}{\int_{-\infty}^{\frac{I_{t+1}}{\theta_{t+1}}} f(R_{t+1})\, dR_{t+1}} \tag{11}$$

dividing both terms by $\int_{-\infty}^{\frac{I_{t+1}}{\theta_{t+1}}} f(R_{t+1})\, dR_{t+1}$ we can rewrite the integrals as conditional expectations.

$$E[u'(c^w_{2,t+1})\, R_{t+1}|R \leq \frac{I_{t+1}}{\theta_{t+1}}] = E[R_{t+1}|R \leq \frac{I_{t+1}}{\theta_{t+1}}]E[u'(c^w_{2,t+1})|R \leq \frac{I_{t+1}}{\theta_{t+1}}] \tag{12}$$

The only way for (12) to be satisfied is to have second period consumption smoothed for low realizations of the shock, i.e. for $R \leq \frac{I_{t+1}}{\theta_{t+1}}$. To see this we can rewrite the left hand side of (12) as the conditional covariance between marginal utility of second period consumption and the rate of return R, plus the product of the conditional expectations of these random variables. But this last product is exactly what we have in the right hand side of (12). Thus the conditional covariance between $u'(c^w_{2,t+1})$ and R has to be zero, and given that $c^w_{2,t+1}$ is linear in R and $u(.)$ is concave, the only possibility for this is to have $u'(c^w_{2,t+1})$ constant over the range of rates of return $R \leq \frac{I_{t+1}}{\theta_{t+1}}$.

It is straightforward to find that the hedging ratio will be given by (7) and through the budget constraint, the expected level of insurance I_{t+1} is given by (8).

7.2 Conditions for a median voter over insurance characteristics

Young savers will vote on the hedging coefficient θ and on the level of insurance I, both related by the budget constraint of Social Security. Thus we can think that the choice is one dimensional and look for conditions to have single-peakedness in preferences over this choice. The problem that voters face is

$$\max_{\theta, a^n_t}\ u(e + nw(1-\tau_t) - a^n_t) + \beta E_t\left[u(a^n_t R_{t+1} + b^n_{t+1})\right]$$

Thus the problem is that of a saver that chooses at the same time his level of savings, and the hedging of Social Security. Therefore the FOC will call for perfect hedging of low realizations of the shock and the choice of θ by saver n is given by

$$\theta^n = \frac{a(n)}{b_n}$$

For single-peakedness, it must be the case that $\frac{d\theta^n}{dn} \geq 0$, i.e.,

$$\frac{da(n)}{dn} - \frac{a(n)}{b_n} db_n dn \geq 0$$

For the special case of a redistributive Social Security, i.e. b_n constant across n, this reduces to the condition that savings increase with income. From FOC for individual saving decision we get that

$$\frac{da}{dn} = \frac{u''(c_1)w(1-\tau)}{u''(c_1) + \beta E\left[u''(c_2)R(E(R|1)1 + R\bar{1})\right]} > 0$$

thus for a redistributive Social Security condition, for general concave preferences we get a median voter result for the choice of insurance characteristics.

7.3 Conditions for a median voter over tax rates

Let $v(\tau, \theta, n) = max_{a_n} u(c_1) + \beta E[u(c_2)]$ be the indirect utility function of a young voter with productivity n. To prove that preferences over tax rates are single peaked we have to proceed in two steps. First let's note that for the median saver, the one with $n = n^w$, it must be the case that $\frac{dv}{d\theta} = 0$ because that was one of the FOC characterizing the choice of insurance characteristics. Therefore we have to consider if the Spence-Mirrlees condition is satisfied separately for young agents with $n < n^w$ and $n > n^w$. We know that voter's preferences over feasible tax shedules, $v(\tau, \theta(\tau), n)$ are single crossing in (τ, n) if voter's marginal rates of substitution between θ and τ are increasing in n. This marginal rate of substitution, which we call $\Lambda(n)$ is given by,

$$\Lambda(n) = \frac{-v_\tau}{v_\theta}$$

Using the envelope theorem we can calculate this derivatives obtaining,

$$\frac{dv}{d\tau} = -u'(c_1)nw$$

$$\frac{dv}{d\theta} = \beta E[u'(c_2)b_n(E(R|INS) - R)INS]$$

were first and second period consumption should be the optimal choices for consumer n and where INS is an indicator for the states of nature for which insurance payments are positive. We thus obtain the following expression for the marginal rate of substitution,

$$\Lambda(n) = \frac{u'(c_1)nw}{\beta E[u'(c_2)b_n(E(R|INS) - R)INS]}$$

Using the FOC for the consumer's consumption savings decision we can rewrite this as,

$$\Lambda(n) = \frac{n}{b_n} \frac{E[u'(c_2)R]w}{E[u'(c_2)(E(R|INS) - R)INS]}$$

We now have to determine under what conditions this marginal rate of substitution will be increasing in n for $n < n^w$. Let's write an expression for $\frac{1}{\Lambda(n)} \frac{d\Lambda(n)}{dn}$.

$$\frac{1}{\Lambda(n)}\frac{d\Lambda(n)}{dn} = \frac{b_n}{n}\frac{d\frac{n}{b_n}}{dn} + \frac{E[u''(c_2)R\frac{dc_2}{dn}]}{E[u'(c_2)R]} - \frac{E[u''(c_2)((E(R|INS)-R)INS)\frac{dc_2}{dn}]}{E[u'(c_2)((E(R|INS)-R)INS)]}$$

We can rewrite the second derivative of the utility function using the definition of the coefficient of relative risk aversion, $R_R(c) = \frac{u''(c)c}{u'(c)}$.

$$\frac{1}{\Lambda(n)}\frac{d\Lambda(n)}{dn} = \frac{b_n}{n}\frac{d\frac{n}{b_n}}{dn} + \frac{E[u'(c_2)R\frac{R_R(c_2)}{c_2}\frac{dc_2}{dn}]}{E[u'(c_2)R]} - \frac{E[u''(c_2)((E(R|INS)-R)INS)\frac{R_R(c_2)}{c_2}\frac{dc_2}{dn}]}{E[u'(c_2)((E(R|INS)-R)INS)]}$$

Replacing in the numerators of the last two terms the expectation of two random variables as the covariance between them plus the product of their expectations we get the simple expression,

$$\frac{1}{\Lambda(n)}\frac{d\Lambda(n)}{dn} = \frac{b_n}{n}\frac{d\frac{n}{b_n}}{dn} + cov(\frac{R_R(c_2)}{c_2}\frac{dc_2}{dn}); u'(c_2)\{\frac{(E(R|INS)-R)INS}{E[u'(c_2)((E(R|INS)-R)INS)]} - \frac{R}{E[u'(c_2)R]}\})$$

The first term must be non-negative, therefore we have to see under what conditions the covariance term is positive. To do this we have to see the behavior of both random variables as R increases. We have to distinguish the intervals for which the insurance payments are positive and zero. Doing this we get that the second term, $u'(c_2)\{\frac{(E(R|INS)-R)INS}{E[u'(c_2)((E(R|INS)-R)INS)]} - \frac{R}{E[u'(c_2)R]}\}$ is increasing in R when the insurance is positive, i.e. when $R < \frac{I}{\theta}$. For $R > \frac{I}{\theta}$ this term will be negative, and increasing in R iff the coefficient of relative risk aversion is larger than 1.

For the covariance to be positive we need that $\frac{R_R(c_2)}{c_2}\frac{dc_2}{dn}$ be increasing in R. For $R > \frac{I}{\theta}$ we have that

$$\frac{1}{c_2}\frac{dc_2}{dn} = \frac{1}{a}\frac{da}{dn} > 0$$

thus for the case of $R_R > 1$ it must be the case that preferences are characterized by non-decreasing relative risk aversion. Finally, for $R < \frac{I}{\theta}$ we have

$$\frac{dc_2}{dn} = (\frac{da}{dn} - \theta\frac{db_n}{dn})R + I\frac{db_n}{dn}$$

for this term to be positive and increasing in R we need that $\frac{da}{dn} - \theta\frac{db_n}{dn} > 0$. This is a strong assumption, and together with preferences characterized by non-increasing absolute risk aversion guarantee that the covariance term be non-negative. In summary the conditions required are non-decreasing relative risk aversion, non-increasing absolute risk aversion, $R_R > 1$, and

$$\frac{da}{dn} - \theta\frac{db_n}{dn} > 0$$

Recalling the condition for a median voter result in the choice of insurance characteristics, we see that those conditions implied that for $n < n^w$ $\frac{a(n)}{b_n} < \theta$. This requirement is thus stronger than the condition for a median voter result in choice of insurance characteristics. If we consider the particular case of a redistributive Social Security, i.e. the case of b_n constant across the population, this last requirement reduces to

$$\frac{da}{dn} > 0$$

Although it seems as we can directly apply the result from the median voter result on insurance characteristics, it is not the same condition. The change in savings with income before was under the assumption that the hedging coefficient changed with income in a way that consumption was always smoothed for low values of R. Now we have to consider that θ is unchanged as we increase income. The result we get from saving's FOC is

$$\frac{da}{dn} = \frac{u''(c_1)w(1-\tau)}{u''(c_1) + \beta E\left[u''(c_2)R^2\right]} > 0$$

Thus with redistributive Social Security we get a median voter result for tax choice if preferences are characterize by non-decreasing relative risk aversion, non-increasing absolute risk aversion, and $R_R > 1$.

7.4 Conditions for chosen tax rates to be increasing in non-taxable income

The analysis parallels that of the previous subsection. We look at the same Spence-Mirrlees condition, but now we want to see under what conditions an increase in the parameter e reduces the marginal rate of substitution between taxes and the hedging coefficient. A reduction will imply that for higher non-taxable income the median voter is going to favor a higher tax rate.

$$\Lambda(e) = \frac{nw}{b_n}\frac{E[u'(c_2)R]}{E[u'(c_2)(E(R|INS) - R)INS]}$$

$$\frac{1}{\Lambda(e)}\frac{d\Lambda(e)}{de} = \frac{E[u''(c_2)R\frac{dc_2}{de}]}{E[u'(c_2)R]} - \frac{E[u''(c_2)((E(R|INS) - R)INS)\frac{dc_2}{de}]}{E[u'(c_2)((E(R|INS) - R)INS)]}$$

$$\frac{1}{\Lambda(e)}\frac{d\Lambda(e)}{de} = \frac{E[u'(c_2)R\frac{R_R(c_2)}{c_2}\frac{dc_2}{de}]}{E[u'(c_2)R]} - \frac{E[u''(c_2)((E(R|INS) - R)INS)\frac{R_R(c_2)}{c_2}\frac{dc_2}{de}]}{E[u'(c_2)((E(R|INS) - R)INS)]}$$

$$\frac{1}{\Lambda(e)}\frac{d\Lambda(e)}{de} = cov(\frac{R_R(c_2)}{c_2}\frac{dc_2}{de}); u'(c_2)\{\frac{(E(R|INS) - R)INS}{E[u'(c_2)((E(R|INS) - R)INS)]} - \frac{R}{E[u'(c_2)R]}\})$$

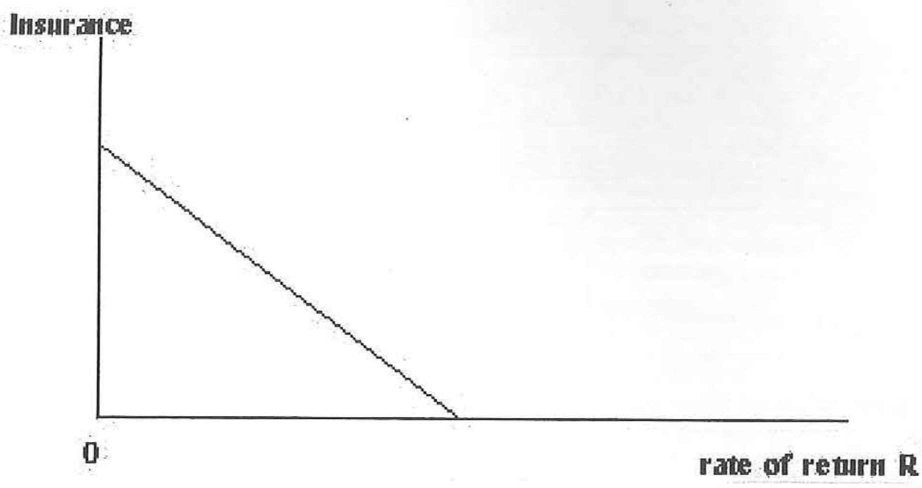
For the covariance to be negative we need that $\frac{R_R(c_2)}{c_2}\frac{dc_2}{de}$ be non-increasing in R. For $R > \frac{I}{\theta}$ we have that

$$\frac{1}{c_2}\frac{dc_2}{de} = \frac{1}{a}\frac{da}{de} > 0$$

thus for the case of $R_R > 1$ it must be the case that preferences are characterized by non-increasing relative risk aversion. Finally, for $R < \frac{I}{\theta}$ we have

$$\frac{dc_2}{de} = (\frac{da}{de} - b_n\frac{d\theta}{de})R + b_n\frac{dI}{de}$$

for this term to be decreasing in R we need that $\frac{da}{de} - b_n\frac{d\theta}{de} < 0$.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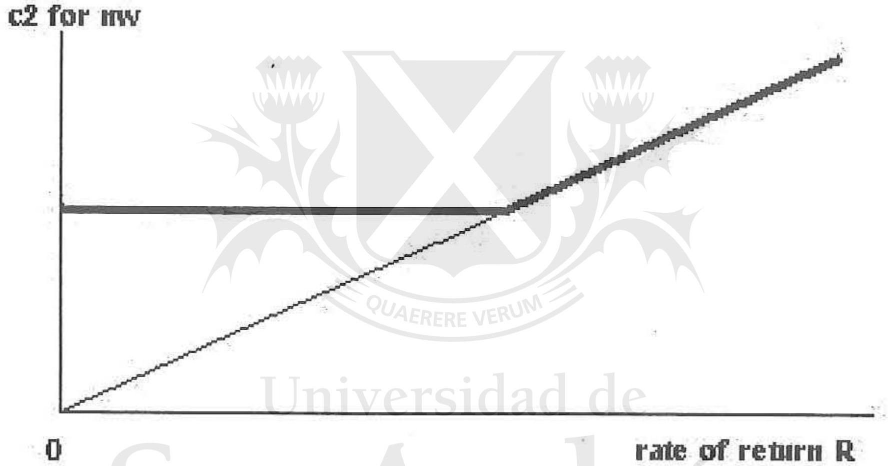
rate of return R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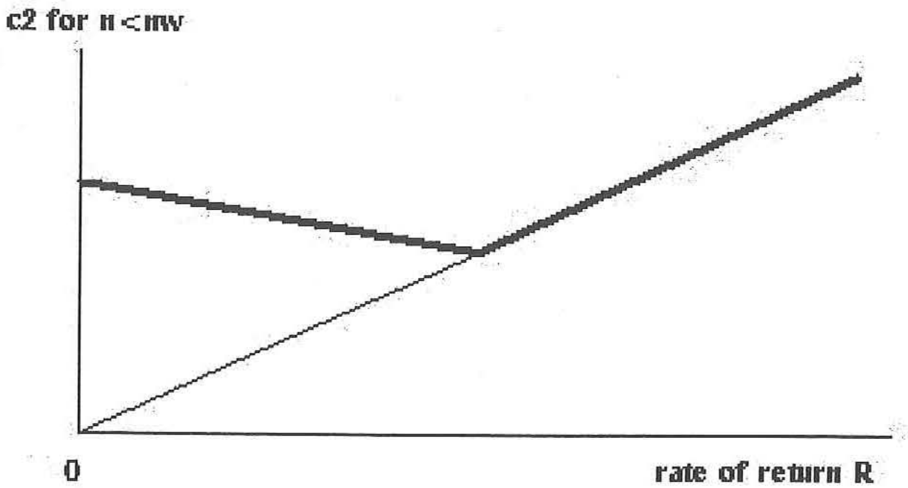
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rate of return R