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"Local public works and collusion"

Martín Besfamille
(Université de Toulouse)

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Martin Besfamille[†]

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Abstract

Local public works with spillovers are often financed by grants from the Federal Government. We study a three-tier hierarchy, where the Local Government first chooses the project and the firm to undertake it and then communicates this information to the Federal Government.

If we allow collusion between the local authority and the manager of the firm in charge with constructing the project, different stakes for collusion may arise. In order to find the optimal contracts, we derive a "Collusion Proofness" property. We finally characterize the distortions imposed to attenuate the implementation costs.

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[†]GREMAQ, Université de Toulouse 1 - Sciences Sociales, Manufacture des Tabacs, 21 allée de Brienne, 31000 Toulouse, France. E-mail: Besfamil@gremaq.univ-tlse1.fr

1 Introduction

In Besfamille (1998) we analyzed the impact of local interests and multidimensional asymmetric information on the design of grants that a Federal Government sets to finance local public works with spillovers. We characterized the optimal contract offered by the Federal Government to the local one and to the manager of the firm of constructors, the unique candidate to undertake the work. The retained formalization enables us to obtain some interesting results, specially those concerning the distortions in the decisions to do or not the project, distortions *vis-à-vis* the first-best allocations. These distortions imply that both more or less local public projects are constructed. The reason for that relies on the interests that the Local Government has on the effective undertaking of the project. Henceforth, he might be tempted to misreport his information in order to obtain the funds to do it. The implementation costs associated with incentive-compatibility might be so important that the Federal Government could decide to attenuate them by imposing a non-optimal decision about the undertaking of the project.

But in our previous paper, side-contracts between the Local Government and the manager of the firm were infeasible. Here we depart from this assumption so both may collude against the Federal Government.[1] Even if the local authority does not follow a personal objective, we will see that he has many stakes to behave opportunistically against the objective of the central authority.

This is an important empirical problem. Some evidence has been discovered that inflated infrastructure costs and useless local public works are the consequence of collusion between local authorities and managers of firms of constructors. Although it is well known that this phenomenon is widespread in developing countries, some industrialized nations have also confronted problems of this kind. As was quoted by the French magazine *Le Point* in an article describing some special local projects "white elephants also exist in France".[2]

But the contractual approach of collusion has concentrated its attention only on the problem of cost-padding.[3] This is so because all authors assume that the projects are always done and the supervisors have no real interests on them. Here we are able to make the second problem emerge. In order to obtain grants to undertake a low (but non zero) valued project in his jurisdiction, a local authority may collude with the manager of the firm of constructors to declare an underestimation of its cost. By doing that, he presents to the Federal Government a project with a higher rate of return.

[1]We use the term "collusion" and "corruption" indifferently.

[2]The expression "white elephants" designates the huge although useless projects that were undertook, on behalf of international financial institutions, in developping countries during the sixties and the seventies.

[3]This literature started with the book of Rose-Ackerman (1978). The first attempt to formalize rigorously those intuitions appeared in the seminal article of Tirole (1986).

Our model uses different building blocks developed in other branches of the economic theory. First of all, our paper is related with a growing literature on incentives and fiscal federalism. Although this approach is based on informational issues (e.g. Oates (1972)), only few papers have stated them rigorously until a recent date. Among them we can signal Gilbert and Picard (1996), Cremer, Marchand and Pestieau (1996), Bucovetsky, Marchand and Pestieau (1996), Lockwood (1996) and Boadway, Horiba and Jha (1997). Even if none of these articles consider the possibility of corruption in the local jurisdiction, the structure of our model and some results are similar to them. As Gilbert and Picard (1996), we analyze a procurement problem of a public work in the local jurisdiction. We also find distortions that go in the direction of over-production like Boadway, Horiba and Jha (1997) but our results do not concern the same type of agents.

We also take as given a particular organizational framework, similar to the one adopted by the "regulatory capture" approach. As in Laffont and Tirole (1991, 1993), we analyze a three-tier hierarchy. But unlike them, we do not allow the top level of the hierarchy to communicate with the lowest. Because of informational costs, the Federal Government delegates in the Local Government the task to find the best projects for his jurisdiction and the most efficient firms to undertake them. In some sense, the local authority is an "intermediate" type of supervisor. We qualify it in this way because, as the Local Government is interested in the effective undertaking of the project chosen by him, he is neither the neutral supervisor depicted in the auditing literature nor the productive one as in the articles of delegation.[4] In fact we can say that he behaves like the agent in the model of Aghion and Tirole (1997).

We also analyze collusion in a contractual approach. Following Tirole (1986), the Local Government will be asked to report his private information in order to enable the Federal Government to implement the final allocation. In this setting, collusion is formalized as the result of side-contracting between the Local Government and the manager of the firm. This side-contract would stipulate how the Local Government should misreport his private information and the covert-transfers to set between the corrupt agents. Although our model is an "incomplete-contract" one (because of the broken communication between the top and the lowest level of the hierarchy), we are able to characterize the optimal contracts using a "Collusion Proofness" property. Therefore, unlike Kofman and Lawarrée (1996), an incentive-compatible collusion-proof allocation dominates collusion in equilibrium.

To conclude, we must stress the fact that, as was indirectly quoted above, we will not try to find the optimal organization in the presence of the threat of

[4]On one hand, the auditing literature followed Antle (1982) by assuming that the person in charge of controlling the productive agent takes only in account his retribution, independently of the action decided by the principal. On the other hand, the delegation approach as set in Baron and Besanko (1992) and Melumad, Mookherjee and Reichelstein (1992,1995) formalizes the supervisor as a productive agent.

collusion.[5]

The structure of the paper is as follows. In the next section we describe the model and its timing. Section 3 presents the benchmark: the optimal contracts when collusion is infeasible. Next we discuss about the possibility of collusion and we prove a "Collusion Proofness" property. In Section 5 we find the cost-minimizing collusion-proof contracts. In Section 6 we show the optimal contracts and then we conclude. All proofs are shown in the Appendix.

2 The model

Within a three-tier hierarchy, a Federal Government (FG) must decide, following the recommendations of the Local Government (LG), whether or not undertake an indivisible public project in the region. If it is done, its total benefits for the population are $NB = LB + SB$ where NB, LB and SB stands for the "national", "local" and "spillover" benefits. The last two are strictly positive and perfectly separable between the region and the rest of the country. Both are common knowledge.

The lowest level of this hierarchy is the manager (M) of a firm of constructors. This firm is the unique candidate to undertake the project. Its ex-post observable cost is $C = \theta - e$, where θ is an efficiency parameter (the type of the firm) and e is the effort exerted by the manager. For simplicity, we assume that $\theta \in \Theta = \{\theta_l, \theta_h\}$ and $\triangle\theta \equiv \theta_h - \theta_l > 0$. The manager faces a monetary equivalent cost of effort equal to $\Psi(e)$. The loss function $\Psi : \mathbb{R} \to \mathbb{R}_+$ satisfies the following properties

- $\forall e \leq 0,\ \Psi(e) = 0$
- $\forall e > 0,\ \Psi(e) > 0$
- $\forall e \geq 0,\ \Psi' > 0,\ \Psi'' > 0$ and $\Psi''' \geq 0$
- $\lim_{e \to \theta} \Psi(e) = +\infty$.

For accounting convention we assume that the cost C is totally reimbursed by the FG. The manager's utility is

$$U \equiv t - \delta\Psi(e)$$

where $\delta = 1$ if the project is realized and 0 otherwise. t is the net transfer received from the FG. For simplicity, the manager's reservation utility is normalized to 0. So his individual rationality constraint (MIR) is $U \geq 0$.

The LG, the middle level of this hierarchy, is an authority elected to represent the local interests. He knows the type θ. Concerning the project under analysis,

[5]There are some recent articles that analyze this issue, as Laffont and Martimort (1996, 1998) and Bardhan and Mookherjee (1998).

his unique task is to make a report to the FG. As people living in this jurisdiction enjoy LB, the local authority gains from the effective undertaking of the project. There are also monetary transfers s between the FG and the LG, which can go from one to the other, in both directions. When $s \geq 0$, the FG compensates the region. Also he can impose $s < 0$ to the LG because the region has the power to levy taxes.[6] We capture the impact of these transfers on the local welfare by formalizing it as a $\mathcal{C}^{\infty}$- function $v(s) : \mathbb{R} \rightarrow \mathbb{R}$, satisfying the following properties

- $v(0) = 0$
- $\forall s \in \mathbb{R}$, $v'(s) > 0$ and $v'(0) = 1 + \lambda$, where $\lambda > 0$ is the national shadow cost of public funds
- $\forall s \in \mathbb{R}$, $v''(s) < 0$

So the LG's utility function is[7]

$$V \equiv \delta LB + v(s)$$

Finally we assume that the FG can prohibit the LG to undertake the project by himself. Hence the individual rationality constraint (IR) for the LG is $V \geq 0$.

The Federal Government wants to undertake the project if it yields a positive social value. In order to decide if the project should be undertaken, he must rely on the LG because he is unable to distinguish between the two components of the cost C. So the fact that both levels of authority represent different populations will indeed create a conflict of interests. Hence, the FG faces a mechanism-design problem, in which the terms of the contract to offer to the LG are based on his report. The FG is an utilitarian benevolent principal, seeking to maximize a social welfare criterion of the form

$$W \equiv \delta[SB - (1+\lambda)C] - (1+\lambda)(t+s) + U + V$$

$$= \delta[NB - (1+\lambda)(C + \Psi(e))] - \lambda U - d(s)$$

where $d(s) \equiv (1+\lambda)s - v(s)$ is the deadweight loss generated by the interjurisdictions transfers. As it is minimal when $s = 0$, the FG prefers to make no transfers at all between him and the LG. Moreover, the FG dislikes to leave any extra rent to the manager of the firm because $\lambda > 0$.

The timing of the model is as follows

1. Everybody learns the probabilistic distribution of θ.

[6] This is equivalent to say that there is no limited-liability for the LG.

[7] The retained formalization for V is a short-cut that captures the most important features of the public finances of the region.

2. Nature randomly chooses θ. The manager of the firm and the LG observe this value.

3. The FG designs the public works contract or mechanism Γ to offer to the LG. This mechanism is a couple

$$\{\mathcal{M}, y(\widetilde{m})\}$$

which specifies the space of messages (or reports) to send and the final allocation

$$y = \begin{cases} (\delta = 1, C, t, s, \pi) \\ (\delta = 0, t^o, s^o) \end{cases}$$

as a vectorial function of the report $\widetilde{m} \in \mathcal{M}$. π is a penalty to impose to the LG when the terms of the undertaking of the project (accepted by the FG) are actually refused by the manager of the firm.

4. Collusion between the LG and M can take place.

5. The LG refuses or accepts the contract offered by the FG.

   (a) If he refuses, nothing happens. The LG and M get their reservation utility.

   (b) If he accepts, he must report to the FG. Then the latter decides if the project should be shutdown or undertaken.

      i. If $\delta(\widetilde{m}) = 0$, $t^o = 0$ and $\widetilde{s}^o = s^o(\widetilde{m})$ are made.

      ii. If $\delta(\widetilde{m}) = 1$, the FG gives to the LG the corresponding cost-transfer scheme (*i.e.* the couple of values $\widetilde{C} = C(\widetilde{m})$ and $\widetilde{t} = t(\widetilde{m})$) to offer to the manager of the firm. Then the LG proposes it to the latter, which can refuse or accept.

         A. If M refuses, the FG imposes the penalty π: the shutdown of the planned project and a fine $f < 0$ to the LG[8].

         B. If M accepts, the project is undertaken. Then all transfers are made.

As we can see, there is no direct communication between M and the FG. The former, if the project is planned to be done, only takes the decision to accept or refuse the cost-transfer scheme $(\widetilde{C}, \widetilde{t}\ )$ in his own self interest (perhaps after a negotiation with the LG) but without reporting to the FG. This seems to be a realistic assumption because usually there is no communication between the central government and the firm in charge of the construction of a local project. This reflects "decentralization" in a contractual sense (see Caillaud, Jullien and

[8] In this case, M gets only his reservation utility.

Picard (1996)): the FG delegates to the LG the search of the firm to undertake the local project.

More important is the consequence of this timing. In Besfamille (1998) there is a particular trade-off in the incentives of the LG to report the truth. Because he is indeed interested in the effective undertaking of the project, he might be tempted to make a report to induce it. Nevertheless, he is limited by the fact that, when a project is accepted by the FG, the cost-transfer scheme $(\widetilde{C}, \widetilde{t}\ )$ imposed to the manager of the firm depends on the report $\widetilde{m}$. And, in case of refusal by M, the LG will be penalized. The interaction of these particular issues were crucial for the description of the optimal contracts. But in this model, the LG can relax this trade-off by coordinating his announcement with M.

We adopt some methodological assumptions. The first one is usually accepted in incentive theory: the full commitment for the public works contract. The other two assumptions are more specific. Having the FG accepted to undertake a project, the LG can not change the cost-transfer scheme to offer to M. This is not so restrictive because most public works contracts have to be public. Hence, in this model, the only way to behave opportunistically is through the report. Finally, side-payments (in a monetary or non-monetary form) between the LG and M are feasible.

This paper analyzes the optimal contract that the FG offers to the LG to obtain his information. We present two useful benchmarks: the first–best allocations and the ones that would have arisen under asymmetric information but when collusion is not feasible. Then we characterize the optimal contract under asymmetric information and collusion. In order to obtain explicit solutions, sometimes we analyze a numerical example of the model. The chosen parameters and functions of this example are as follows:

1. $SB = 5$ and $\lambda = 0.3$
2. $\theta_l = 10$
3. $\Psi(e) = \begin{cases} \frac{e^2}{8} & if\ e \geq 0 \\ 0 & otherwise \end{cases}$
4. $v(s) = 1.3s - 0.1s^2$
5. $p_l = p_h = 0.5$

3 Optimal contracts when collusion is infeasible

Let's assume, in order to obtain the usual benchmarks, that side-contracts are not feasible. In Besfamille (1998) we have characterized the optimal contracts under this framework.

3.1 First-best allocations

If the FG knows θ and can also observe e, the target values are: e^*, $C_l^* = \theta_l - e^*$, $C_h^* = \theta_h - e^*$, $t^* = \Psi(e^*)$ and $s^o = s^* = 0$. Hence the first-best allocations are characterized as follows.[9]

Proposition 1

- *When $\theta = \theta_l$, the Federal Government optimally sets*
  - *if $LB < LB^*_{Inf} : \delta = 0, t^o$ and s^o*
  - *if $LB \geq LB^*_{Inf} : \delta = 1, C_l^*, t^*$ and s^**
- *When $\theta = \theta_h$, the Federal Government optimally sets*
  - *if $LB < LB^*_{Sup} : \delta = 0, t^o$ and s^o*
  - *if $LB \geq LB^*_{Sup} : \delta = 1, C_h^*, t^*$ and s^**

where $LB^*_{Inf} \equiv (1+\lambda)(C_l^* + t^*) - SB$ and $LB^*_{Sup} \equiv (1+\lambda)(C_h^* + t^*) - SB = LB^*_{Inf} + (1+\lambda)\triangle\theta$

The comparative statics of these results are straightforward. When the differential in efficiency, the lowest firm's type and the shadow cost of public funds increase, both thresholds also increase. Hence the FG funds less projects. On the contrary, when the spillover effects are important, he undertakes more local works.

3.2 Asymmetric information on θ

Although C is ex-post observable, the FG is not able to distinguish between θ and e. As he has some beliefs about θ, he faces two states of Nature $i \in \{l, h\}$, each one with a strictly positive probability $p_i \equiv \Pr(\theta = \theta_i)$. The FG could induce the LG to reveal θ truthfully by offering him a second-best incentive compatible contract. In fact, this is not necessary.

Proposition 2 *Under asymmetric information on θ, the first-best allocations are incentive compatible.*

Although there is asymmetric information on θ, the FG implements the optimal decisions δ with no extra cost by offering, for each state of Nature, the first-best allocation. This implies that at most three possible configurations of decisions about the undertaking of the project can arise:

[9] This paper maintains a conventional assumption in contract theory. When the manager or the LG are indifferent between two decisions, they do what the FG prefers.

- $[All]$: both types of firm undertake the project if $LB \geq LB^*_{Sup}$

θ_l	θ_h
$\delta_l = 1$	$\delta_h = 1$

- $[\theta_l]$: only the efficient firm undertakes the project if $LB \in [LB^*_{Inf}, LB^*_{Sup})$

θ_l	θ_h
$\delta_l = 1$	$\delta_h = 0$

- $[None]$: if $LB < LB^*_{Inf}$ the project is not undertaken

θ_l	θ_h
$\delta_l = 0$	$\delta_h = 0$

The first and the last configuration correspond to bunching. The second discriminates between both types of firm. These potential configurations are the same that would appear having the FG known θ. So asymmetric information does not impact in such a way to make new configurations arise. We gather these results in the following figure, where the subscripts indicate that the configurations are implemented through menus of first-best allocations.

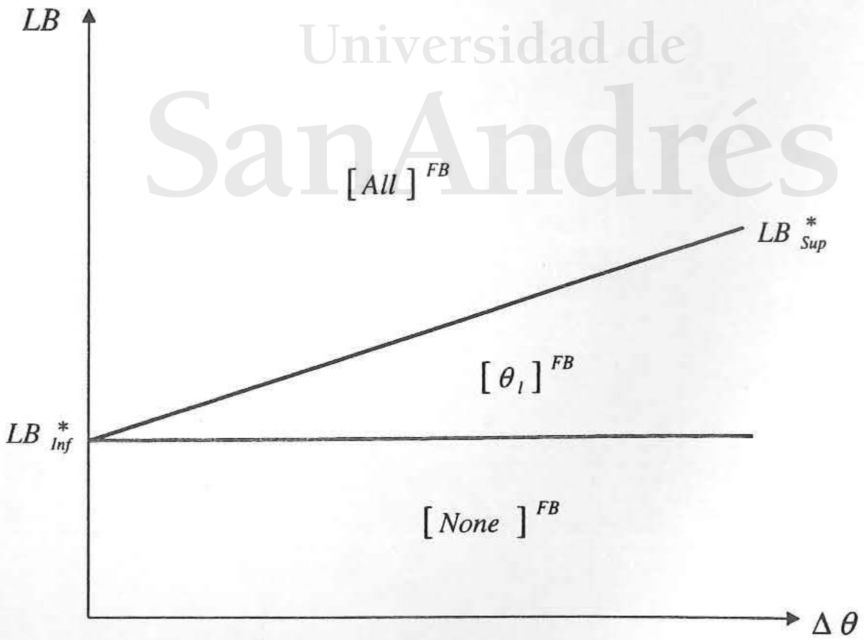

Figure 1: Optimal allocations when collusion is unfeasible

4 Collusion

4.1 The covert negotiation: timing and assumptions

Now we relax the assumption that collusion is infeasible. Henceforth side-contracts between the LG and the M are a possible threat that must be taken in account by the FG at the mechanism-design stage. We characterize the optimal contracts in this new framework.

Prior to the acceptation of the public works contract offered by the FG, the LG might want to coordinate with M the report to make. In order to formalize this, we adopt similar assumptions than the ones presented in Tirole (1986). As the LG and M negotiate under full-information and do not face transaction costs, the Coase theorem holds. For simplicity, we endow the LG with all the bargaining power: he offers to M a "take-it or leave-it" side-contract. This side contract, which is supposed to be fully enforceable, specifies the final report that the LG should make and the covert-payment b between them. If M refuses this side-contract, the LG can only play this announcement game non-cooperatively. He makes the report by himself, taking only in account the constraints imposed by the timing. Therefore, in order to be accepted, the side-contract must be Pareto-superior *vis-à-vis* this non-cooperative status quo.

4.2 A "Collusion Proofness" property

When the FG faces this threat of collusion, he must be aware of it when he designs the best contract to offer to the LG. This optimization can be hard to overcome because we have not constrained the space of messages $\mathcal{M}$ in the mechanism Γ. Fortunately we can prove the following important result

Proposition 3 *The allocations that maximize the expected welfare of the FG can be implemented by direct-revelation mechanisms. Moreover, these mechanisms are collusion-proof in the following sense: the LG does not gain by coordinating with M to deviate from truthful revelation.*

Therefore the FG can restrict himself, without any loss of generality, to offer incentive-compatible collusion-proof contracts to the LG. The design of such kind of contracts is easy to undertake: the FG must optimize his expected welfare over the set of contracts that satisfy some coalitional incentive compatibility constraints.

4.3 The stakes for collusion: existence

The optimal allocations presented in Section 3 can be implemented through direct-revelation mechanisms. But these mechanisms are not robust to collusion. To verify that, we fix an arbitrary combination of some of the parameters and functions of the model $\Lambda = \{SB, \lambda, p_i, \theta_l, \Psi, v\}$.

If the FG wants to implement $[All]$ by offering the menu of first-best allocations

$$\begin{cases} (\delta = 1, C_l^*, t^*, s^*, \pi) & if\ \theta = \theta_l \\ (\delta = 1, C_h^*, t^*, s^*, \pi) & if\ \theta = \theta_h \end{cases}$$

the LG and M have incentives to deviate from their expected behavior. When $\theta = \theta_l$, the LG can easily convince the manager that the best thing to do is to announce $\widetilde{\theta} = \theta_h$. By doing so, they could share an informational rent $\Phi(e^*) = \Psi(e^*) - \Psi(e^* - \triangle\theta) > 0$.[10] This threat of cost padding always exists (*i.e.* for any Λ, LB and $\triangle\theta$).

But this model opens the door to another potential coalitional misbehavior. When the FG wants to discriminate between the firms and implement $[\theta_l]$, he can offer the first-best contract

$$\begin{cases} (\delta = 1, C_l^*, t^*, s^*, \pi) & if\ \theta = \theta_l \\ (\delta = 0, t^o, s^o) & if\ \theta = \theta_h \end{cases}$$

When $\theta = \theta_h$, the LG might have an incentive to exaggerate the efficiency of the firm (*i.e.* to announce $\widetilde{\theta} = \theta_l$) in order to obtain the undertaking of the project. In Besfamille (1998) this was impossible. The reason is simple: as side-contracts were infeasible, he could not coordinate with M to inhibit his refusal of the cost-transfer scheme proposed by the FG. But in this model this is possible: to induce the manager of an inefficient firm to accept the cost-transfer scheme (C_l^*, t^*) designed for an efficient one, the LG should commit to compensate him for the (necessary) extra effort to attain the target cost. So this stake for collusion exists if and only if

$$LB + v(-\Phi(e^* + \triangle\theta)) > 0$$

The following lemma states that it is a real threat for the FG

Lemma 1 $\forall\Lambda, \forall LB$

$$\{\triangle\theta \in I\!R_*^+ \ /\ LB + v(-\Phi(e^* + \triangle\theta)) > 0\} \neq \varnothing$$

When the FG intends to implement $[\theta_l]$, there is always room for this stake of collusion. This last possibility concerns the subvaluation of the cost to obtain the undertaking of the project. Although it seems to be a very widespread phenomenon in public investments, it has not been well studied in the literature.

[10]Because of the initial assumptions on Ψ, it is straightforward to show that $\Phi(e) \geq 0, \Phi'(e) > 0$ and $\Phi'' \geq 0$.

5 Cost-minimizing collusion-proof contracts

In the first step to characterize the optimal contracts, we find the cost-minimizing collusion-proof contracts that implement each possible configuration. In order to design them, the FG must face the following constraints

- $IR(i)$ and $MIR(i)$: the ex-post individual rationality constraint for the LG and for M respectively.
- $CIC(i)$: the coalitional incentive constraints for the LG. They have the following shape

$$\delta_i LB + v(s_i) \geq \delta_{i'}[LB + v(s_{i'} + b_{i'})] + (1 - \delta_{i'})v(s_{i'} + b_{i'}) \quad i' \neq i$$

where $b_{i'} \geq U_{i'} - U_i$ is the covert transfer between the LG and M. As the LG has all the bargaining power, in fact $b_{i'} = U_{i'} - U_i$. We can write them clearly. The first constraint is

$$\begin{aligned} \delta_l LB + v(s_l) \geq \; & \delta_h[LB + v(s_h + U_h + \Phi(e_h) - U_l)] \\ & + (1 - \delta_h)v(s_h + U_h - U_l) \end{aligned} \qquad (CIC(l))$$

where $\Phi(e) = \Psi(e) - \Psi(e - \triangle\theta)$. This constraint enables the FG to deter the usual coalitional misbehavior: the subvaluation of the efficiency of the firm. By compensating enough the LG, up to the maximum bribe that M is willing to pay him in order to obtain the extra informational rent, the FG can induce truthful revelation.

The other constraints is

$$\begin{aligned} \delta_h LB + v(s_h) \geq \; & \delta_l[LB + v(s_l + U_l - \Phi(e_l + \triangle\theta) - U_h)] \\ & + (1 - \delta_l)v(s_l + U_l - U_h) \end{aligned} \qquad (CIC(h))$$

Now, in order to deter the subvaluation of cost, the LG must gain enough in order to not engage with M in a non-desirable project for the FG.

Thus, the FG solves the problem

$$\mathcal{P} \begin{cases} \underset{\delta_i, e_i, U_i, s_i}{Max} \sum_i p_i \{\delta_i [NB - (1+\lambda)(\theta_i - e_i + \Psi(e_i))] \\ \qquad\qquad -\lambda U_i - d(s_i)\} \\ \\ \textit{subject to} \\ \\ IR(i), MIR(i) \\ CIC(i) \end{cases}$$

It is straightforward to prove the following lemma

Lemma 2 *An optimal contract must verify* $\delta_l \geq \delta_h$.

Hence at most the three first-best configurations can arise at the optimum. Next we show the cost-minimizing collusion-proof contract that implements each configuration.

5.1 $[None]$

This allocation is collusion-proof *per se*. Hence it can be implemented through a menu of first-best allocations.

5.2 $[\theta_l]$ and $[All]$

In our previous paper, the FG does not distort neither the cost of the project nor the utility of the firm. But a priori, this is not optimal any more because both types of variables enter in the new constraints $CIC(i)$. Thus the FG can distort them in order to attenuate the overall costs of implementation. Moreover, as there are two potential misbehaviors, we show that there are also two different types of cost-distortions in equilibrium, that characterize each configuration.[11]

Proposition 4 *The cost-minimizing contract that implements* $[All]^{CP}$ *is*

$$\begin{cases} (\delta = 1, C_l^*, t^*, s_l, \pi) & if\ \theta = \theta_l \\ (\delta = 1, C_h, t_h, s_h, \pi) & if\ \theta = \theta_h \end{cases}$$

where

- $s_l > 0$ *and* $s_h < 0$
- $s_l = s_h + \Phi(e_h)$
- $C_h > C_h^*$ *and* $t_h < t^*$
- $U_h = t_h - \Psi(\theta_h - C_h) = 0$

When the FG implements $[All]^{CP}$, he must distort upwardly the cost imposed to the inefficient firm. Not surprisingly, this is the same result as the one found in Laffont and Tirole (1991, 1993). The trade-off between rent extraction and efficiency is solved by imposing a cost for the inefficient firm that is higher than the first-best one. But in this model, a fraction of this higher cost must be paid by the LG. As usual, there is no "distortion at the top" (*i.e.* the lowest cost is not distorted) but the FG offers a positive compensation to the LG to relax the coalition incentive constraints. In this case, the firm does not enjoy any rent; all the implementation cost is paid to the LG.

[11]Now the superscripts will indicate that the configurations are implemented through collusion-proof contracts.

Proposition 5 *The cost-minimizing contract that implements $[\theta_l]^{CP}$ is*

$$\begin{cases} (\delta = 1, C_l, t_l, s_l, \pi) & if\ \theta = \theta_l \\ (\delta = 0, t^o, s_h, \pi) & if\ \theta = \theta_h \end{cases}$$

where

- $C_l < C_l^*$ *and* $t_l > t^*$
- $U_l = t_l - \Psi(\theta_l - C_l) = 0$
- $s_l < 0$ *and* $s_h > 0$
- $v(s_h) = LB + v(s_l - \Phi(e_l + \triangle\theta))$

When the FG faces the threat of cost subvaluation and tries to implement $[\theta_l]^{CP}$, he must distort downwardly the cost imposed to the efficient firm. This implies that the undertaking of the project results in a higher effort imposed to the manager of an efficient firm. By doing that, the FG increases the side-transfer that the LG should pay to the manager of an inefficient firm in order to compensate him to mimic an efficient firm and to undertake the project. As in Besfamille (1998), the FG designs for the LG a cost-sharing formula when the project should be done and offers a strictly positive compensation scheme in the other case. Moreover, the utility of the manager of the firm remains unchanged.

6 The optimal contracts

Once the cost-minimizing collusion-proof contracts are found, it is straightforward to compute the FG's expected welfare under each configuration. In order to do that, we simulate the numerical example of our model for different values of $\triangle\theta$ and LB.[12] Finally we look under which conditions on $\triangle\theta$ and LB the FG implements each configuration.

[12] All numerical simulations have been done with Mathematica 2.0. The complete list of results are available upon request to the author.

Proposition 6 *Under the threat of collusion, the optimal configurations are set as in the following graphic.*

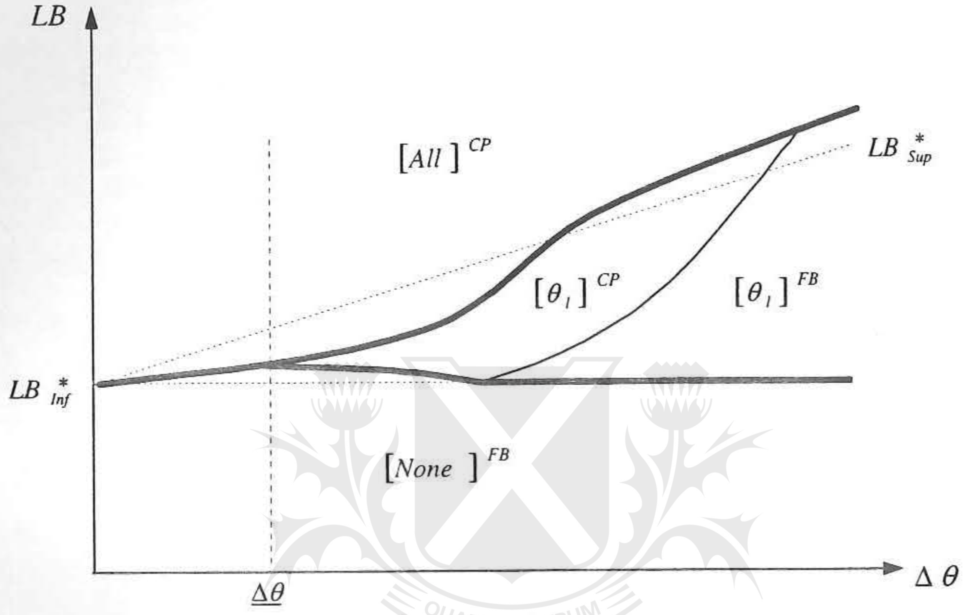

Figure 2: Optimal allocations under the threat of collusion

The graphic shows that for every configuration there exists a non-empty parametric region where it is optimal to implement it. Collusion is not so constraining so as to completely vanish one configuration.

Collusion entails upward and downward distortions on the optimal decisions about the effective realization of the project. The first case occurs when $[All]^{CP}$ is implemented instead of $[\theta_l]^{CP}$. The second occurs in two cases: when $[None]^{FB}$ is implemented instead of $[\theta_l]^{CP}$ and when $[\theta_l]$ is implemented instead of $[All]$.

These distortions can be very important. For $\triangle\theta \leq \underline{\triangle\theta}$, the configuration $[\theta_l]$ is no longer optimal. There, when the FG wants to discriminate between firms that do not differ in efficiency too much, the stake for collusion is too constraining. Therefore, the distortions in the cost of the project and in the transfers to the LG needed to attenuate the implementation costs associated to $[\theta_l]^{CP}$ are so important that the FG shifts towards more drastic distortions in δ. This result was not present in our previous paper. The numerical computations state that $\underline{\triangle\theta} \geq 0,6$. That means that the differential in efficiency between two types of firms that make the discrimination too costly is more than 6%, which is a non-negligeable figure.

7 Conclusion

We summarize the main results of this paper. We were able to show how in our three-tier (national administrative) hierarchy the problems of collusion in the undertaking of local public works are more complicated than the usual ones presented by the three-tier hierarchy approach of corruption.

Our model endogenizes the decision about the undertaking of the local project. When it should be done independently of its cost, then the usual stake for collusion implying cost-padding arises. But when the project should be undertaken only if it cost is low, the local authority may be tempted to overstate the efficiency of the firm to obtain the funds to construct it.

Although our model is an incomplete-contract one, we prove a "Collusion Proofness" property which enables us to easily characterize the optimal contracts. As in our previous paper, we found distortions concerning the decision about the undertaking of the project. Here, these distortions are more important in two aspects. Not only they appear whereas in Besfamille (1998) there were no room for them but also they imply that, for a non negligeable region of parameters, one configuration completely vanishes. The Federal Government shifts towards a more drastic way to decide about the funding of local projects.

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8 Appendix

8.1 Proof of a "Collusion Proofness" Principle

The proof of this important result, made in different steps, is similar to the one presented in Laffont and Tirole (1991, 1993). First we characterize the equilibrium allocations of the covert game of collusion. Then we find the allocation that maximizes the expected welfare of the FG. Finally we show that this allocation can be implemented through an incentive-compatible collusion-proof contract.

8.1.1 The equilibrium allocations

Any mechanism offered by the FG leads to a side-contract between the LG and the manager of the firm and to some equilibrium allocation. We index the final incomes and utilities by a hat: $\{\widehat{t}_i, \widehat{s}_i, \widehat{U}_i, \widehat{V}_i\}_{i\in\{l,h\}}$. The actual transfers from the FG to the LG and to M are denoted by s_i and t_i. Letting b_i denote the side-transfer between the LG and M, we have:

$$\begin{aligned}\widehat{s}_i &= s_i - b_i\\ \widehat{t}_i &= t_i + b_i\\ \widehat{U}_i &= \widehat{t}_i - \delta_i\Psi(e_i)\\ \widehat{V}_i &= \delta_i LB + v(\widehat{s}_i)\end{aligned}$$

Then, in any state of Nature, the equilibrium welfare of the FG is

$$\begin{aligned}\widehat{W}_i &\equiv \delta_i[SB - (1+\lambda)C_i] - (1+\lambda)(t_i + s_i) + \widehat{U}_i + \widehat{V}_i\\ &= \delta_i[NB - (1+\lambda)(C_i + \Psi(e_i))] - \lambda\widehat{U}_i - d(\widehat{s}_i)\end{aligned}$$

We characterize the equilibrium allocations for a particular case; the other are strictly equivalent. Assume that the FG wants to implement $\delta_i = 1\ \forall i \in \{l, h\}$ (*i.e.* he wants to implement $[All]$). The necessary conditions for an allocation to be an equilibrium are the following:

- State l : $\widehat{U}_l \geq 0$, $\widehat{V}_l \geq 0$ and one of the following possibilities
  - Case l_1

$$\begin{cases}\widehat{U}_h + \Phi(e_h) \geq \widehat{U}_l\\ v(\widehat{s}_l) \geq v(\widehat{s}_h + \widehat{U}_h + \Phi(e_h) - \widehat{U}_l)\end{cases}$$

    M wants to be in state h but LG not, even if M gives to the LG all the potential benefits from the deviation.

  - Case l_2

$$\begin{cases}\widehat{U}_h + \Phi(e_h) \leq \widehat{U}_l\\ v(\widehat{s}_h) \geq v(\widehat{s}_l)\\ v(\widehat{s}_l) \geq v(\widehat{s}_h + \widehat{U}_h + \Phi(e_h) - \widehat{U}_l)\end{cases}$$

Although the LG wants to deviate, it is too expensive for him to obtain the acceptation of M.

- State h : $\widehat{U}_h \geq 0$ and $\widehat{V}_h \geq 0$ and one of the following possibilities, whose intuition are equivalent to the previous ones

  - Case h_1

$$\begin{cases} \widehat{U}_l - \Phi(e_l + \triangle\theta) \geq \widehat{U}_h \\ v(\widehat{s}_h) \geq v(\widehat{s}_l + \widehat{U}_l - \Phi(e_l + \triangle\theta) - \widehat{U}_h) \end{cases}$$

  - Case h_2

$$\begin{cases} \widehat{U}_l - \Phi(e_l + \triangle\theta) \leq \widehat{U}_h \\ v(\widehat{s}_l) \geq v(\widehat{s}_h) \\ v(\widehat{s}_h) \geq v(\widehat{s}_l + \widehat{U}_l - \Phi(e_l + \triangle\theta) - \widehat{U}_h) \end{cases}$$

The conditions that characterize an equilibrium allocation result from a combination of one of the possible cases, each for a different state of Nature. Hence the FG faces four possible cases denoted $(l_1, h_1), (l_1, h_2), (l_2, h_1)$ and (l_2, h_2).

8.1.2 The allocation that maximize $\mathbb{E}\widehat{W}$ in each case

As the aim of the FG is to find the optimal equilibrium allocation, first he looks, in each one of the possible cases mentioned above, for the allocation that maximizes his expected welfare $\mathbb{E}\widehat{W}$.

Case (l_1, h_1) : The FG solves the program

$$\begin{cases} \underset{\substack{e_l, \widehat{U}_l, \widehat{s}_l \\ e_h, \widehat{U}_h, \widehat{s}_h}}{Max} \ \mathbb{E}\widehat{W} \\ \\ subject\ to \\ \\ \widehat{U}_l \geq 0 & (1) \\ \widehat{U}_h \geq 0 & (2) \\ LB + v(\widehat{s}_l) \geq 0 & (3) \\ LB + v(\widehat{s}_h) \geq 0 & (4) \\ \widehat{U}_h + \Phi(e_h) \geq \widehat{U}_l & (5) \\ \widehat{U}_l - \Phi(e_l + \triangle\theta) \geq \widehat{U}_h & (6) \\ v(\widehat{s}_l) \geq v(\widehat{s}_h + \widehat{U}_h + \Phi(e_h) - \widehat{U}_l) & (7) \\ v(\widehat{s}_h) \geq v(\widehat{s}_l + \widehat{U}_l - \Phi(e_l + \triangle\theta) - \widehat{U}_h) & (8) \end{cases}$$

If (7) and (8) hold, then $\widehat{s}_l = \widehat{s}_h = 0$. So (5) and (6) hold with equality, implying $e_l + \triangle\theta = e_h$ and therefore $e_l < e_h$. Moreover, $\widehat{U}_l = \Phi(e_l + \triangle\theta) > 0$ and $\widehat{U}_h = 0$. The Lagrangian of the reduced problem is:

$$\begin{aligned} \mathcal{L} = \; & -p_l[(1+\lambda)\left(\theta_l - e_l + \Psi\left(e_l\right)\right) + \lambda\Phi(e_l + \triangle\theta)] \\ & -p_h\left(1+\lambda\right)\left(\theta_h - e_h + \Psi\left(e_h\right)\right) + \gamma[e_l + \triangle\theta - e_h] \end{aligned}$$

where γ is the multiplier associated with the equality constraint. The first-order conditions yield to

$$\Psi'\left(e_l\right) = \frac{1+\lambda}{p_l} - \frac{p_h+\lambda}{p_l}\Psi'\left(e_h\right)$$

As $e_l < e_h$, it is straightforward to see that $e_l < e^* < e_h$. *Vis-à-vis* the first-best, this allocation generates an extra cost equal to

$$\begin{aligned} \mathcal{C}_1 = \; & p_l[(1+\lambda)(H(e_l) - H(e^*)) + \lambda\Phi(e_l + \triangle\theta)] \\ & + p_h(1+\lambda)(H(e_h) - H(e^*)) \end{aligned}$$

where $H(e) \equiv \Psi\left(e\right) - e$.

Case (l_1, h_2) The FG solves the program

$$\left\{ \begin{array}{ll} \underset{\substack{e_l,\widehat{U}_l,\widehat{s}_l \\ e_h,\widehat{U}_h,\widehat{s}_h}}{Max} \; \mathbb{E}\widehat{W} & \\ & \\ subject\ to & \\ & \\ \widehat{U}_l \geq 0 & (1) \\ \widehat{U}_h \geq 0 & (2) \\ LB + v(\widehat{s}_l) \geq 0 & (3) \\ LB + v(\widehat{s}_h) \geq 0 & (4) \\ \widehat{U}_h + \Phi(e_h) \geq \widehat{U}_l & (5) \\ \widehat{U}_l - \Phi(e_l + \triangle\theta) \leq \widehat{U}_h & (6) \\ v(\widehat{s}_l) \geq v(\widehat{s}_h + \widehat{U}_h + \Phi(e_h) - \widehat{U}_l) & (7) \\ v(\widehat{s}_h) \geq v(\widehat{s}_l + \widehat{U}_l - \Phi(e_l + \triangle\theta) - \widehat{U}_h) & (8) \end{array} \right.$$

If (4), (5) and (7) hold, (3) also holds. If (7) and (8) are verified, $\Phi(e_l + \triangle\theta) \geq \Phi(e_h)$ because v is strictly monotonic. Therefore, this last inequality combined with (5) yields to (6). The Kuhn-Tucker conditions of the reduced problem are

$$\begin{cases}
-p_l(1+\lambda)(\Psi'(e_l)-1)+\gamma_6 v'(\widehat{s}_l+b)\Phi'(e_l+\triangle\theta)=0 & (i1)\\
-p_h(1+\lambda)(\Psi'(e_h)-1)+\gamma_4\Phi'(e_h)-\gamma_5 v'(\widehat{s}_h+a)\Phi'(e_h)=0 & (i2)\\
-\lambda p_l+\gamma_1-\gamma_4+\gamma_5 v'(\widehat{s}_h+a)-\gamma_6 v'(\widehat{s}_l+b)=0 & (i3)\\
-\lambda p_h+\gamma_2+\gamma_4-\gamma_5 v'(\widehat{s}_h+a)+\gamma_6 v'(\widehat{s}_l+b)=0 & (i4)\\
-p_l d'(\widehat{s}_l)+\gamma_5 v'(\widehat{s}_l)-\gamma_6 v'(\widehat{s}_l+b)=0 & (i5)\\
-p_h d'(\widehat{s}_h)+\gamma_3 v'(\widehat{s}_h)-\gamma_5 v'(\widehat{s}_h+a)+\gamma_6 v'(\widehat{s}_h)=0 & (i6)\\
\gamma_1\widehat{U}_l=0 \qquad \gamma_1\geq 0 & (ii1)\\
\gamma_2\widehat{U}_h=0 \qquad \gamma_2\geq 0 & (ii2)\\
\gamma_3[LB+v(\widehat{s}_h)]=0 \qquad \gamma_3\geq 0 & (ii3)\\
\gamma_4[\widehat{U}_h+\Phi(e_h)-U_l]=0 \qquad \gamma_4\geq 0 & (ii4)\\
\gamma_5[v(\widehat{s}_l)-v(\widehat{s}_h+a)]=0 \qquad \gamma_5\geq 0 & (ii5)\\
\gamma_6[v(\widehat{s}_h)-v(\widehat{s}_l+b)]=0 \qquad \gamma_6\geq 0 & (ii6)
\end{cases}$$

where $\gamma_j \;/\; j\in\{1,2,3,4,5,6\}$ are the multipliers associated with the inequality constraints, $a\equiv\widehat{U}_h+\Phi(e_h)-U_l\geq 0$ and $b\equiv U_l-\Phi(e_l+\triangle\theta)-\widehat{U}_h\leq 0$.

1. The first immediate result is, from $(i3)$ and $(i4)$ $\gamma_1+\gamma_2=\lambda$.

2. Then we show that $\gamma_6=0$. Assume $\gamma_6>0$. Hence, from $(ii6)$, $\widehat{s}_h=\widehat{s}_l+b$ and from $(i1)$, $e_l>e^*$. But if $(ii5)$ holds, it yields to $e_h\leq e_l+\triangle\theta$.

   (a) If $e_h=e_l+\triangle\theta, e_h>e^*$. But from $(i2)$, this should imply that $\gamma_4-\gamma_5 v'(\widehat{s}_h+a)>0$ and therefore $\gamma_4>0$. If so, $a=0$ and $U_l>0$, which then implies that $\gamma_1=0$. But $(i3)$ becomes

   $$0>-\lambda p_l-\gamma_6 v'(\widehat{s}_l+b)=\gamma_4-\gamma_5 v'(\widehat{s}_h)>0$$

   which is a contradiction.

   (b) If $e_h<e_l+\triangle\theta$, $a+b<0$ and so $(ii5)$ is slack, implying $\gamma_5=0$. If this was the case, $(i2)$ yields to

   $$e_h\geq e^*$$

   $(i5)$ yields to $\widehat{s}_l<0$ and $(i6)$ yields to $\widehat{s}_h\geq 0$. Hence, as $b\leq 0$

   $$\widehat{s}_l<0\leq\widehat{s}_h=\widehat{s}_l+b\leq\widehat{s}_l$$

   which is another contradiction.

   So $\gamma_6=0$ and $e_l=e^*$. Moreover, $(i5)$ becomes

   $$p_l d'(\widehat{s}_l)=\gamma_5 v'(\widehat{s}_l) \qquad (1)$$

   so, as $\gamma_5\geq 0,\widehat{s}_l\geq 0$.

3. Next we prove that $\widehat{U}_h + \Phi(e_h) > U_l$. Assume that $\widehat{U}_h + \Phi(e_h) = U_l$ so $a = 0$. As $\Phi(e_h) > 0$, $U_l > 0$ and so $\gamma_1 = 0$. In that case, $(i3)$ yields to $\gamma_5 > 0$ and $\widehat{s}_h = \widehat{s}_l$. This equality, combined with $(i6)$, yields to

$$\gamma_3 v'(\widehat{s}_h) = d'(\widehat{s}_h)$$

which implies that $\widehat{s}_h \geq 0$ and therefore $\gamma_3 = 0$ because $(ii3)$ is slack. If so, $\widehat{s}_h = 0$. But then, $\widehat{s}_l = 0$ which must imply that $\gamma_5 = 0$ in order to satisfy (1). But this is a contradiction.

So $a \equiv \widehat{U}_h + \Phi(e_h) - U_l > 0$ and therefore $\gamma_4 = 0$. If so, $(i4)$ becomes

$$\gamma_2 = \lambda p_h + \gamma_5 v'(\widehat{s}_h + a) > 0$$

so $\widehat{U}_h = 0$.

4. Next we prove that $\widehat{s}_l > 0$. Assume $\widehat{s}_l = 0$. Hence $\gamma_5 = 0$ and $(i6)$ becomes

$$\gamma_3 v'(\widehat{s}_h) = d'(\widehat{s}_h)$$

which yields to $\widehat{s}_h \geq 0$. But from $(ii5)$ and the fact that $a > 0$

$$0 = \widehat{s}_l \geq \widehat{s}_h + a > 0$$

which is obviously a contradiction.

Hence $\widehat{s}_l > 0$ so, from (1), $\gamma_5 > 0$ and therefore $\widehat{s}_l = \widehat{s}_h + a$ and $e_h < e^*$ from $(i2)$.

5. Then we show that $\widehat{U}_l = 0$. Assume that $\widehat{U}_l > 0$ so $\gamma_1 = 0$.

   - From $(i3)$,
   $$\gamma_5 v'(\widehat{s}_h + a) = \lambda p_l$$
   so
   $$\Psi'(e_h) = 1 - \frac{p_l}{p_h}\frac{\lambda}{1+\lambda}\Phi'(e_h) \tag{2}$$
   As the "Maximum Principle" holds, (2) characterizes a function $e_h(\triangle\theta)$.
   - As $\widehat{s}_l = \widehat{s}_h + a$ and v is strictly monotonic, $\gamma_5 v'(\widehat{s}_l) = \lambda p_l$. So $(i5)$ yields to
   $$v'(\widehat{s}_l) = 1$$
   and either $v(\widehat{s}_h) = -LB$ or, from $(i6)$, $v'(\widehat{s}_h) = 1 + \frac{\lambda}{p_h}$ (in both cases, $\widehat{s}_h < 0$). These equalities imply that $\widehat{s}_l$ and $\widehat{s}_h$ do not depend on $\triangle\theta$.

Although we were not able to prove it in general, our numerical example shows that the function defined as

$$F(\triangle\theta) \equiv \widehat{s}_h - \widehat{s}_l + \Phi(e_h(\triangle\theta))$$

verifies

(a) $\lim_{\triangle\theta \to 0^+} F(\triangle\theta) \equiv \widehat{s}_h - \widehat{s}_l < 0.$

(b) $\forall \triangle\theta > 0,\ F(\triangle\theta) < 0.$

Hence, there exist no $\triangle\theta$ such that the value of $\widehat{U}_l$ characterized by the Kuhn-Tucker conditions is strictly positive, which is a contradiction. So $\widehat{U}_l = 0$ and $a = \Phi(e_h)$.

6. Finally, either $v(\widehat{s}_h) = -LB$ or $v(\widehat{s}_h) > -LB$. In the last case, $\gamma_3 = 0$ so, from $(i6)$, $\widehat{s}_h < 0$. Hence $\widehat{s}_h < 0$ always.

This allocation generates an extra cost

$$\mathcal{C}_2 = \ p_l d(\widehat{s}_l) + p_h[(1+\lambda)(H(e_h) - H(e^*)) + d(\widehat{s}_l)]$$

Case (l_2, h_1) The FG solves the program

$$\begin{cases} \underset{\substack{e_l, \widehat{U}_l, \widehat{s}_l \\ e_h, \widehat{U}_h, \widehat{s}_h}}{Max} \ \mathbb{E}\widehat{W} & \\ \textit{subject to} & \\ \widehat{U}_l \geq 0 & (1) \\ \widehat{U}_h \geq 0 & (2) \\ LB + v(\widehat{s}_l) \geq 0 & (3) \\ LB + v(\widehat{s}_h) \geq 0 & (4) \\ \widehat{U}_h + \Phi(e_h) \leq \widehat{U}_l & (5) \\ \widehat{U}_l - \Phi(e_l + \triangle\theta) \geq \widehat{U}_h & (6) \\ v(\widehat{s}_h) \geq v(\widehat{s}_l) \geq v(\widehat{s}_h + \widehat{U}_h + \Phi(e_h) - \widehat{U}_l) & (7) \\ v(\widehat{s}_h) \geq v(\widehat{s}_l + \widehat{U}_l - \Phi(e_l + \triangle\theta) - \widehat{U}_h) & (8) \end{cases}$$

If (2) and (5) hold, then (1) also holds. If (6) and (8) hold, (7) also holds. If (3) and (8) hold, then (4) is satisfied. So the Kuhn-Tucker conditions of the reduced problem are

$$\begin{cases} -p_l(1+\lambda)(\Psi'(e_l)-1)+[\gamma_6 v'(\widehat{s}_l+b)-\gamma_4]\Phi'(e_l+\triangle\theta)=0 & (i1)\\ -p_h(1+\lambda)(\Psi'(e_h)-1)-[\gamma_3+\gamma_5 v'(\widehat{s}_h+a)]\Phi'(e_h)=0 & (i2)\\ -\lambda p_l+\gamma_3+\gamma_4+\gamma_5 v'(\widehat{s}_h+a)-\gamma_6 v'(\widehat{s}_l+b)=0 & (i3)\\ -\lambda p_h+\gamma_1-\gamma_3-\gamma_4-\gamma_5 v'(\widehat{s}_h+a)+\gamma_6 v'(\widehat{s}_l+b)=0 & (i4)\\ -p_l d'(\widehat{s}_l)+\gamma_2 v'(\widehat{s}_l)+\gamma_5 v'(\widehat{s}_l)-\gamma_6 v'(\widehat{s}_l+b)=0 & (i5)\\ -p_h d'(\widehat{s}_h)-\gamma_5 v'(\widehat{s}_h+a)+\gamma_6 v'(\widehat{s}_h)=0 & (i6)\\ \gamma_1\widehat{U}_h=0 \qquad \gamma_1\geq 0 & (ii1)\\ \gamma_2[LB+v(\widehat{s}_h)]=0 \qquad \gamma_2\geq 0 & (ii2)\\ \gamma_3[\widehat{U}_l-\widehat{U}_h-\Phi(e_h)]=0 \qquad \gamma_3\geq 0 & (ii3)\\ \gamma_4[\widehat{U}_l-\widehat{U}_h-\Phi(e_l+\triangle\theta)]=0 \qquad \gamma_4\geq 0 & (ii4)\\ \gamma_5[v(\widehat{s}_l)-v(\widehat{s}_h+a)]=0 \qquad \gamma_5\geq 0 & (ii5)\\ \gamma_6[v(\widehat{s}_h)-v(\widehat{s}_l+b)]=0 \qquad \gamma_6\geq 0 & (ii6) \end{cases}$$

where $a \leq 0$ and $b \geq 0$. From now on, we will assume that $(ii2)$ is slack. Then we have to verify this statement.

1. The first result is, from $(i3)$ and $(i4)$, $\gamma_1 = \lambda > 0$. Hence $\widehat{U}_h = 0$.

2. Next we show that at the maximum $\widehat{U}_l = \Phi(e_l + \triangle\theta)$. If we assume that $\widehat{U}_l > \Phi(e_l + \triangle\theta)$, we must have $\gamma_4 = 0$. We show that this statement yields to a global contradiction. We can have either $\gamma_6 = 0$ or $\gamma_6 > 0$.

   - If $\gamma_6 = 0$, from $(i1)$, $e_l = e^*$ and from $(i4)$

   $$\gamma_3 + \gamma_5 v'(\widehat{s}_h + a) = \lambda p_l > 0 \tag{3}$$

   which implies that $e_h < e^*$. So $\Phi(e_h) < \Phi(e^*) < \Phi(e_l + \triangle\theta)$. Hence $\widehat{U}_l > \Phi(e_h)$ so $a < 0$ and $\gamma_3 = 0$. If so, (3) imply that $\gamma_5 > 0$ so

   $$\widehat{s}_l = \widehat{s}_h + a \tag{4}$$

   which yields to $\widehat{s}_l < \widehat{s}_h$ because $a < 0$. Plugging (3) in $(i6)$ yields to $\widehat{s}_h < 0$ so, from (4), $\widehat{s}_l < 0$. But also plugging the mentioned expression in $(i5)$ yields to $\widehat{s}_l \geq 0$ which is a contradiction.

   - If $\gamma_6 > 0$, $\widehat{s}_h = \widehat{s}_l + b$, $e_l > e^*$ and from $(i3)$

   $$0 < \lambda p_l + \gamma_6 v'(\widehat{s}_l + b) = \gamma_3 + \gamma_5 v'(\widehat{s}_h + a)$$

   so $e_h < e^*$. Hence $\widehat{U}_l > \Phi(e_h)$ and $\gamma_3 = 0$. If so, $\gamma_5 > 0$ and therefore (3) holds again. But combined with the value of $\widehat{s}_h$, it follows that $\Phi(e_h) = \Phi(e_l + \triangle\theta)$ which is a contradiction.

So $\widehat{U}_l = \Phi(e_l + \triangle\theta)$ and therefore $b = 0$. As $(ii5)$ and $(ii6)$ hold, $v(\widehat{s}_h) \geq v(\widehat{s}_l) \geq v(\widehat{s}_h + a)$.

3. Next we show that $\widehat{s}_h = \widehat{s}_l$. Assume $\widehat{s}_h > \widehat{s}_l$ so $\gamma_6 = 0$. $(i5)$ yields to $\widehat{s}_l \geq 0$ while $(i6)$ to $\widehat{s}_h \leq 0$, so by the assumption, $\widehat{s}_l < 0$ which is a contradiction. So as $\widehat{s}_h = \widehat{s}_l$, $(i5)$ becomes

$$\gamma_6 - \gamma_5 = \frac{-p_l d'(\widehat{s}_l)}{v'(\widehat{s}_l)} \tag{5}$$

and, as $a \leq 0$, $(i6)$ becomes

$$p_h d'(\widehat{s}_h) = \gamma_6 v'(\widehat{s}_h) - \gamma_5 v'(\widehat{s}_h + a) \leq (\gamma_6 - \gamma_5) v'(\widehat{s}_h)$$

Combining the equality between the transfers to the LG and (5), we obtain $\widehat{s}_h = \widehat{s}_l \leq 0$.

4. Next we show that, at the maximum, $\Phi(e_l + \triangle\theta) > \Phi(e_h)$. Assume they are equal. Hence $a = 0$ and $e_l < e_l + \triangle\theta = e_h \leq e^*$ so

$$e_l < e^* \tag{6}$$

$(i6)$ yields to

$$\gamma_6 - \gamma_5 = \frac{p_l d'(\widehat{s}_h)}{v'(\widehat{s}_h)}$$

Combining this last expression with (5) yields to $\widehat{s}_h = \widehat{s}_l = 0$ and hence $\gamma_5 = \gamma_6$. Therefore $(i3)$ becomes $\gamma_3 = \lambda p_l - \gamma_4$. We thus obtain the following system

$$\begin{cases} \Psi'(e_l) = 1 + \frac{1}{p_l(1+\lambda)}[\gamma_6(1+\lambda) - \gamma_4]\Phi'(e_h) \\ \Psi'(e_h) = 1 - \frac{1}{p_h(1+\lambda)}[\gamma_3 + \gamma_6(1+\lambda)]\Phi'(e_h) \\ \gamma_3 = \lambda p_l - \gamma_4 \end{cases}$$

From the first equation, we can obtain

$$\gamma_6 \Phi'(e_h) = p_l[\Psi'(e_l) - 1] + \frac{\gamma_4}{(1+\lambda)}\Phi'(e_h)$$

If we plug it in the second equation, we have

$$\Psi'(e_h) = \frac{1+\lambda}{p_h + \lambda} - \frac{p_l}{p_h + \lambda}\Psi'(e_l)$$

But from (6), $\Psi'(e_l) < 1$ so $\Psi'(e_h) > 1$ which is a contradiction. Hence $\Phi(e_l + \triangle\theta) > \Phi(e_h)$.

5. As a consequence of the previous result, $a < 0$ so $\gamma_5 = 0$ and $(ii3)$ is slack so also $\gamma_3 = 0$. But then $\gamma_3 = \gamma_5 = 0$ so $e_h = e^*$ and from $(i3)$, $\gamma_4 - \gamma_6 v'(\widehat{s}_l) = \lambda p_l > 0$ so $e_l < e^*$.

6. Finally, from $(i6)$,
$$\gamma_6 = p_h(\frac{1+\lambda}{v'(\widehat{s}_h)} - 1)$$
which yields to $\gamma_6 = 0$ as the only compatible solution. So $\widehat{s}_l = \widehat{s}_h = 0$ and $\gamma_4 = \lambda p_l$.

This allocation generates a cost of implementation equals to

$$\mathcal{C}_3 = \quad p_l[(1+\lambda)(H(e_l) - H(e^*)) + \lambda\Phi(e_l + \triangle\theta))]$$

Case (l_2, h_2) The FG solves the program

$$\begin{cases} \underset{\substack{e_l,\widehat{U}_l,\widehat{s}_l \\ e_h,\widehat{U}_h,\widehat{s}_h}}{Max} \ \mathbb{E}\widehat{W} \\ \\ subject\ to \\ \\ \widehat{U}_l \geq 0 & (1) \\ \widehat{U}_h \geq 0 & (2) \\ LB + v(\widehat{s}_l) \geq 0 & (3) \\ LB + v(\widehat{s}_h) \geq 0 & (4) \\ \widehat{U}_h + \Phi(e_h) \leq \widehat{U}_l & (5) \\ \widehat{U}_l - \Phi(e_l + \triangle\theta) \leq \widehat{U}_h & (6) \\ v(\widehat{s}_h) \geq v(\widehat{s}_l) \geq v(\widehat{s}_h + \widehat{U}_h + \Phi(e_h) - \widehat{U}_l) & (7) \\ v(\widehat{s}_l) \geq v(\widehat{s}_h) \geq v(\widehat{s}_l + \widehat{U}_l - \Phi(e_l + \triangle\theta) - \widehat{U}_h) & (8) \end{cases}$$

If (2) and (5) hold, then (1) is satisfied. If (7) and (8) hold, then $\widehat{s}_l = \widehat{s}_h = 0$. So if (4) holds, then (3) also holds. So (5) and (6) hold with equality, implying that $e_l + \triangle\theta = e_h$ and therefore $e_l < e_h$. Moreover, $\widehat{U}_l = \Phi(e_l + \triangle\theta) > 0$ and $\widehat{U}_h = 0$. This case is formally identical with the first case so $\mathcal{C}_4 = \mathcal{C}_1$.

8.1.3 The optimal allocation

As we have just seen, the FG has four different ways to implement the configuration $[All]$. In order to find the one that minimize the extra cost, we compare them. As for each value of $\triangle\theta$ there exist a solution for the different programs, we can show

1. $\lim\limits_{\triangle\theta\to 0^+} \mathcal{C}_1 = \lim\limits_{\triangle\theta\to 0^+} \mathcal{C}_2 = \lim\limits_{\triangle\theta\to 0^+} \mathcal{C}_3 = 0$

2. Next we compute the derivatives of the different costs.

- $\frac{d\mathcal{C}_1}{d\triangle\theta} = p_l[(1+\lambda)(\Psi'(e_l)-1) + \lambda\Phi'(e_l+\triangle\theta)]\frac{de_l}{d\triangle\theta} + \lambda p_l \Psi'(e_l+\triangle\theta) + p_h(1+\lambda)(\Psi'(e_h)-1)\frac{de_h}{d\triangle\theta}$.
  As $e_h = e_l + \triangle\theta$, $\frac{de_h}{d\triangle\theta} = \frac{de_l}{d\triangle\theta} + 1$ so

$$\begin{aligned} \frac{d\mathcal{C}_1}{d\triangle\theta} = & \; [p_l\Psi'(e_l) - (1+\lambda) + (p_h+\lambda)\Psi'(e_h)]\frac{de_l}{d\triangle\theta} \\ & +\lambda(\Psi'(e_h) - p_h) + p_h(\Psi'(e_h)-1) \end{aligned}$$

  From the first-order conditions, the term in brackets vanishes. So finally

$$\frac{d\mathcal{C}_1}{d\triangle\theta} = \frac{d\mathcal{C}_4}{d\triangle\theta} = (p_h+\lambda)(\Psi'(e_h)-1) + \lambda p_l > \lambda p_l$$

  because as $e_h > e^*$, $\Psi'(e_h) > 1$.
- $\frac{d\mathcal{C}_2}{d\triangle\theta} = p_l d'(\widehat{s}_l)\frac{d\widehat{s}_l}{d\triangle\theta} + p_h[(1+\lambda)(\Psi'(e_h)-1)\frac{de_h}{d\triangle\theta} + d'(\widehat{s}_h)\frac{d\widehat{s}_h}{d\triangle\theta}]$.
  As $\widehat{s}_l = \widehat{s}_h + \Phi(e_h)$, $\frac{d\widehat{s}_l}{d\triangle\theta} = \frac{d\widehat{s}_h}{d\triangle\theta} + \Phi'(e_h)\frac{de_h}{d\triangle\theta} + \Psi'(e_h - \triangle\theta)$ so

$$\begin{aligned} \frac{d\mathcal{C}_2}{d\triangle\theta} = & \; [p_l(1+\lambda-v'(\widehat{s}_l)) + p_h(1+\lambda-v'(\widehat{s}_h))]\frac{d\widehat{s}_h}{d\triangle\theta} \\ & [p_l\Phi'(e_h)(1+\lambda-v'(\widehat{s}_l)) + p_h(1+\lambda)(\Psi'(e_h)-1)]\frac{de_h}{d\triangle\theta} \\ & +p_l d'(\widehat{s}_l)(\Psi'(e_h-\triangle\theta) \end{aligned}$$

  From the first-order conditions, we know that $\Psi'(e_h-\triangle\theta) < 1$ and $v'(\widehat{s}_l) \geq 1$. Hence we obtain

$$\frac{d\mathcal{C}_2}{d\triangle\theta} = p_l d'(\widehat{s}_l)\Psi'(e_h-\triangle\theta) < \lambda p_l$$

  because $v'(\widehat{s}_l) \geq 1$ and $\Psi'(e_h-\triangle\theta) < 1$.
- $\frac{d\mathcal{C}_3}{d\triangle\theta} = p_l[(1+\lambda)(\Psi'(e_l)-1) + \lambda\Phi'(e_l+\triangle\theta)]\frac{de_l}{d\triangle\theta} + \lambda p_l\Psi'(e_l+\triangle\theta)$ which, by using again the first-order conditions and the fact that $e_l + \triangle\theta > e^*$, yields to

$$\frac{d\mathcal{C}_3}{d\triangle\theta} = \lambda p_l \Psi'(e_l+\triangle\theta) > \lambda p_l$$

The final result is immediate. The FG implements $[All]$ by offering a mechanism that yields to the second case of equilibrium allocation. It is straightforward to verify that this allocation can be implemented by the direct-revelation mechanism characterized in Proposition 5, where there were no equilibrium bribes. This contract is collusion-proof because the LG has no incentive to misreport the type of the firm ■

8.2 Proof of Lemma 1

We fix a combination of parameters and functions $\Lambda = (SB, \lambda, p_i, \theta_l, \Psi, v)$. We take any given $LB > 0$. We try to find if there exist values of $\triangle\theta$ such that

$$LB + v(-\Phi(e^* + \triangle\theta)) \geq 0 \tag{7}$$

We analyze the shape of the function $G_{LB}(\triangle\theta) \equiv LB + v(-\Phi(e^* + \triangle\theta))$.

- $\lim\limits_{\triangle\theta \to 0^+} G_{LB} = LB > 0$
- $\frac{dG_{LB}}{d\triangle\theta} = -v'(-\Phi(e^* + \triangle\theta))\Psi'(e^* + \triangle\theta) < -(1+\lambda) < 0$
- $\frac{d^2 G_{LB}}{d\triangle\theta^2} = v''(-\Phi(e^* + \triangle\theta))\Psi'(e^* + \triangle\theta) - v'(-\Phi(e^* + \triangle\theta)\Psi''(e^* + \triangle\theta) < 0$

So there exists a unique value $\overline{\triangle\theta}_{LB} > 0$ such that $F_{LB}(\overline{\triangle\theta}_{LB}) = 0$. Hence we have found an open non-empty interval $(0, \overline{\triangle\theta}_{LB}]$ where $\forall \triangle\theta \leq \overline{\triangle\theta}_{LB}$, $F_{LB}(\triangle\theta) \geq 0$ ∎

8.3 Proof of Lemma 2

Assume that an optimal contract yields to $\delta_l < \delta_h$ (*i.e.* $\delta_l = 0$ and $\delta_h = 1$). From the coalition incentive constraints we can state that $v(s_l) > 0$ and $U_h > 0$. Moreover, in order to attenuate the distortions in the transfers, $e_h \leq e^*$. Hence

$$\mathbb{E}W_{[\delta_l=0,\delta_h=1]} = -p_l d(s_l) + p_h\{NB - (1+\lambda)\left(C_h + \Psi\left(e_h\right)\right) - \lambda U_h - d(s_h)\}$$

where, at least, $v(s_l) \geq LB + v(s_h + U_h + \Phi(e_h))$. But if this configuration is implemented, it means that $\mathbb{E}W_{[\delta_l=0,\delta_h=1]} \geq 0$ or equivalently that $NB - (1+\lambda)\left(C_h + \Psi\left(e_h\right)\right) - \lambda U_h - d(s_h) > 0$. But as $NB - (1+\lambda)\left(C_h + \Psi\left(e_h\right)\right) - \lambda U_h - d(s_h) < NB - (1+\lambda)\left(C_l^* + \Psi\left(e^*\right)\right)$, it is worth to undertake the project in the state l. So this configuration is always dominated by a configuration undertaking the project in both states of Nature and letting $U_l = 0$ and $e_l = e^*$, which is a contradiction ∎

8.4 Cost-minimizing collusion-proof contracts

8.4.1 [*All*]

The cost-minimizing collusion-proof contract that implements [*All*] solves the following problem

$$
\mathcal{P}_1 \left\{
\begin{array}{ll}
\underset{\substack{e_l,U_l,s_l \\ e_h,U_h,s_h}}{Max} \; p_l\{NB-(1+\lambda)\left(\theta_l-e_l+\Psi\left(e_l\right)\right)-\lambda U_l-d(s_l)\}+ & \\
\qquad\qquad p_h\{NB-(1+\lambda)\left(\theta_h-e_h+\Psi\left(e_h\right)\right)-\lambda U_h-d(s_h)\} & \\
 & \\
subject\ to & \\
 & \\
U_l \geq 0 & MIR(l) \\
U_h \geq 0 & MIR(h) \\
LB+v(s_l) \geq 0 & IR(l) \\
LB+v(s_h) \geq 0 & IR(h) \\
v(\widehat{s}_l) \geq v(s_h+U_h+\Phi(e_h)-U_l) & CIC(l) \\
v(s_h) \geq v(s_l+U_l-\Phi(e_l+\triangle\theta)-U_h) & CIC(h)
\end{array}
\right.
$$

For the moment, we do not take in account $IR(h)$ because we claim that it is slack at the maximum. We have to check later this statement. Hence, the Kuhn-Tucker conditions of $\mathcal{P}_1$ are

$$
\left\{
\begin{array}{ll}
-\lambda p_l+\gamma_1+\gamma_4 v'(s_h+a)-\gamma_5 v'(s_l+b)=0 & (i1) \\
-\lambda p_h+\gamma_2-\gamma_4 v'(s_h+a)+\gamma_5 v'(s_l+b)=0 & (i2) \\
-p_l(1+\lambda)(\Psi'(e_l)-1)+\gamma_5 v'(s_l+b)\Phi'(e_l+\triangle\theta)=0 & (i3) \\
-p_h(1+\lambda)(\Psi'(e_h)-1)-\gamma_4 v'(s_h+a)\Phi'(e_h)=0 & (i4) \\
-p_l d'(s_l)+(\gamma_3+\gamma_4)v'(s_l)-\gamma_5 v'(s_l+b)=0 & (i5) \\
-p_h d'(s_h)+\gamma_5 v'(s_h)-\gamma_4 v'(s_h+a)=0 & (i6) \\
\gamma_1 U_l=0 \qquad \gamma_1\geq 0 & (ii1) \\
\gamma_2 U_h=0 \qquad \gamma_2\geq 0 & (ii2) \\
\gamma_3[LB+v(s_l)]=0 \qquad \gamma_3\geq 0 & (ii3) \\
\gamma_4[v(s_l)-v(s_h+a)]=0 \qquad \gamma_4\geq 0 & (ii4) \\
\gamma_5[v(s_h)-v(s_l+b)]=0 \qquad \gamma_5\geq 0 & (ii5)
\end{array}
\right.
$$

where $\gamma_j \ / \ j\in\{1,2,3,4,5\}$ are the multipliers associated to the inequality constraints, $a\equiv U_h+\Phi(e_h)-U_l$ and $b\equiv U_l-\Phi(e_l+\triangle\theta)-U_h$.

1. By simple observation we can see that adding $(i1)$ and $(i2)$ yields to

$$\gamma_1+\gamma_2=\lambda>0 \tag{8}$$

2. Moreover, if $(i3)$ and $(i4)$ hold, then $e_h\leq e^*\leq e_l$ because $\gamma_j\geq 0$ and the derivatives of the functions v and Φ are strictly positive. Hence

$$\Phi(e_h)<\Phi(e_l+\triangle\theta) \tag{9}$$

3. Next we have a useful lemma, whose proof is immediate from (9).

Lemma 3

- *If $CIC(l)$ binds, $CIC(h)$ is slack*
- *If $CIC(h)$ binds, $CIC(l)$ is slack*

4. Now we prove that $\gamma_2 > 0$ at the maximum. Assume that $\gamma_2 = 0$. By (8), $\gamma_1 = \lambda$ so $U_l = 0$. Hence $a > 0$ and $b < 0$. As $(ii4)$ must hold, then

$$s_l > s_h \tag{10}$$

Therefore $(ii3)$ is slack so $\gamma_3 = 0$.[13] Plugging the value of γ_1 in $(i1)$ yields to

$$\gamma_5 = \frac{\lambda(1-p_l)}{v'(s_l+b)} + \gamma_4 \frac{v'(s_h+a)}{v'(s_l+b)} > 0$$

so

$$v(s_h) = v(s_l + b) \tag{11}$$

Next we subtract $(i5)$ from $(i1)$ which gives

$$\lambda + p_l[1 - v'(s_l)] = \gamma_4[v'(s_l) - v'(s_h + a)] \leq 0$$

because $(ii4)$ must hold and $v'' < 0$. As $\lambda > 0$, $v'(s_l) > 1$. Moreover by subtracting $(i6)$ from $(i2)$ and noting that (11) implies that $v'(s_h) = v'(s_l + b)$ (as the function v is monotonic) we obtain

$$d'(s_h) = \lambda$$

implying that $v'(s_h) = 1$. So we have $v'(s_h) < v'(s_l)$ which is a contradiction with (10). Hence $\gamma_2 > 0$ and $U_h = 0$.

5. We claim that at the maximum $e_l = e^*$. Assume that $e_l > e^*$ so $\Psi'(e_l) > 1$. In order to verify $(i3)$, $\gamma_5 > 0$. Hence $v(s_h) = v(s_l + b)$ and, by Lemma 7, $(ii4)$ is slack and $\gamma_4 = 0$. So $(i1)$ becomes

$$\gamma_1 = \gamma_5 v'(s_l + b) + p_l \lambda > 0$$

implying that $U_l = 0$ and therefore $a > 0$. As $(ii4)$ holds strictly

$$s_l > s_h \tag{12}$$

Moreover $(i6)$ gives

$$p_h d'(s_h) = \gamma_5 v'(s_h) > 0$$

[13] Remember that we have assumed that $LB + v(s_h) > 0$.

which implies that $s_h > 0$. Hence $s_l > s_h > 0$. Then $(i5)$ is

$$\gamma_3 v'(s_l) = \gamma_5 v'(s_l + b) + p_l d'(s_l)$$

If $\gamma_3 = 0$, $-p_l d'(s_l) = \gamma_5 v'(s_l + b) > 0$. This is only satisfied for $s_l < 0$ which is a contradiction. If $\gamma_3 > 0$, $LB + v(s_l) = 0$ which implies that $v(s_l) = -LB < 0$, again a contradiction. So $e_l = e^*$ and $\gamma_5 = 0$.

6. Then $(i5)$ becomes $p_l d'(s_l) = (\gamma_3 + \gamma_4)v'(s_l) \geq 0$. So $s_l \geq 0$ and therefore $LB + v(s_l) > 0$ which implies that $\gamma_3 = 0$.

7. Next we claim that $\gamma_4 > 0$. Assume that $\gamma_4 = 0$. $(i1)$ gives $\gamma_1 = \lambda p_l > 0$ so $U_l = 0$ and $a > 0$. $(i5)$ yields to

$$s_l = 0 \tag{13}$$

Moreover, $(i6)$ becomes $p_h d'(s_h) = 0$ and therefore

$$s_h = 0 \tag{14}$$

Together, (13) and (14) contradict $(ii4)$. Hence $\gamma_4 > 0$, $v(s_l) = v(s_h + a)$ and then, by Lemma 3, $(ii5)$ is slack.

8. As $\gamma_4 > 0$, we obtain

$$\Psi'(e_h) = 1 - \frac{1}{p_h}\frac{1}{1+\lambda}\gamma_4 v'(s_h + a)\Phi'(e_h) < 1$$

so $e_h < e^*$.

9. As $v(s_l) = v(s_h + a)$ and v is monotonic, $v'(s_l) = v'(s_h + a)$. Hence $(i5)$ becomes

$$p_l d'(s_l) = \gamma_4 v'(s_h + a) = \gamma_4 v'(s_l) > 0$$

implying that $s_l > 0$. Plugging in $(i1)$ gives

$$\gamma_1 = p_l(\lambda - d'(s_l)) \geq 0$$

and therefore $v'(s_l) \geq 1$.

10. $(i6)$ becomes

$$-p_h d'(s_h) = \gamma_4 v'(s_h + a) > 0$$

so $1 + \lambda < v'(s_h)$ implying that $s_h < 0$.

11. Finally, we have to study if $(ii1)$ binds. Assume that at the maximum $U_l > 0$. So, in order to verify $(ii1)$, $\gamma_1 = 0$. From $(i1)$ we have

$$\gamma_4 v'(s_h + a) = \lambda p_l$$

As $v'(s_h + a) = v'(s_l)$, also $\gamma_4 v'(s_l) = \lambda p_l$. Hence we obtain the following results:

- $(i4)$ becomes

$$\Psi'(e_h) = 1 - \frac{p_l}{p_h}\frac{\lambda}{1+\lambda}\Phi'(e_h) \tag{15}$$

- $(i5)$ becomes

$$-p_l d'(s_l) + \lambda p_l = 0$$

which yields to

$$v'(s_l) = 1 \tag{16}$$

so $s_l > 0$.

- $(i6)$ becomes

$$-p_h d'(s_h) - \lambda p_l = 0$$

which yields to

$$v'(s_h) = 1 + \frac{\lambda}{p_h} \tag{17}$$

so $s_h < 0$.

Our numerical example shows that there exist no value of $\triangle\theta$ such that the value of U_l characterized by the Kuhn-Tucker conditions is strictly positive. Hence $(ii1)$ binds at the maximum and therefore $s_l = s_h + \Phi(e_h)$.

8.4.2 $[\theta_l]$

We characterize the cost-minimizing collusion-proof contract that implements $[\theta_l]$ when, by assumption, the stake for collusion is effective. This is true for values such that

$$LB + v(-\Phi(e^* + \triangle\theta)) > 0 \tag{18}$$

The FG must solve the following problem

$$\mathcal{P}_2 \begin{cases} \underset{\substack{e_l,U_l,s_l \\ s_h}}{Max}\ p_l\{NB - (1+\lambda)(\theta_l - e_l + \Psi(e_l)) - \lambda U_l - d(s_l)\} + \\ \qquad -p_h d(s_h) \\ \\ subject\ to \\ \\ U_l \geq 0 & MIR(l) \\ LB + v(s_l) \geq 0 & IR(l) \\ v(s_h) \geq 0 & IR(h) \\ LB + v(s_l) \geq v(s_h - U_l) & CIC(l) \\ v(s_h) \geq LB + v(s_l + U_l - \Phi(e_l + \triangle\theta)) & CIC(h) \end{cases}$$

We forget for the moment $CIC(l)$. Then we check if the solution verifies it. The Kuhn-Tucker conditions of $\mathcal{P}_2$ are

$$\begin{cases} -\lambda p_l + \gamma_1 - \gamma_5 v'(s_l + b) = 0 & (i1) \\ -p_l(1+\lambda)(\Psi'(e_l) - 1) + \gamma_4 v'(s_l + b)\Phi'(e_l + \triangle\theta) = 0 & (i2) \\ -p_l d'(s_l) + \gamma_2 v'(s_l) - \gamma_4 v'(s_l + b) = 0 & (i3) \\ -p_h d'(s_h) + (\gamma_3 + \gamma_4) v'(s_h) = 0 & (i4) \\ \gamma_1 U_l = 0 \qquad \gamma_1 \geq 0 & (ii1) \\ \gamma_2[LB + v(s_l)] = 0 \qquad \gamma_2 \geq 0 & (ii2) \\ \gamma_3 v(s_h) = 0 \qquad \gamma_3 \geq 0 & (ii3) \\ \gamma_4[v(s_h) - LB - v(s_l + b)] = 0 \qquad \gamma_4 \geq 0 & (ii4) \end{cases}$$

where $b \equiv U_l - \Phi(e_l + \triangle\theta)$.

1. By simple observation of $(i1)$, $\gamma_1 = \lambda p_l + \gamma_4 v'(s_l + b) > 0$. Hence $U_l = 0$ and $b < 0$.

2. From $(i2)$,
$$\Psi'(e_l) = 1 + \gamma_4 \frac{1}{p_l} \frac{v'(s_l + b)}{1 + \lambda} \Phi'(e_l + \triangle\theta) \geq 1$$
which implies that $e_l \geq e^*$.

3. Next we claim that $v(s_h) = LB + v(s_l + b)$. Assume that $v(s_h) > LB + v(s_l + b)$ so $\gamma_4 = 0$. This has the following consequences:

   - from $(i2)$, $e_l = e^*$
   - $(i3)$ becomes
   $$\gamma_2 v'(s_l) = p_l d'(s_l) \geq 0 \tag{19}$$
   In order to satisfy it, $s_l \geq 0$. Hence $(ii2)$ is slack and then $\gamma_2 = 0$. Therefore, to verify (19), $s_l = 0$.
   - $(i4)$ becomes
   $$\gamma_3 v'(s_h) = p_h(1 + \lambda - v'(s_h)) \tag{20}$$

   So, from the initial assumption about the stake for collusion and the statement at the beginning of this point, we have that
   $$v(s_h) > LB + v(-\Phi(e^* + \triangle\theta)) > 0$$
   which implies that $v(s_h) > 0$ and so $\gamma_3 = 0$. But in that case, the only way to satisfy (20) is by $s_h = 0$, which is a contradiction. Hence $v(s_h) = LB + v(s_l + b)$.

4. As $b < 0$ and $(ii3)$ must hold,

$$LB + v(s_l) > LB + v(s_l + b) = v(s_h) \geq 0$$

so $(ii2)$ is slack and $\gamma_2 = 0$. Moreover, $CIC(l)$ is effectively slack.

5. Next we claim that $\gamma_4 > 0$. Assume that it is equal to 0, which has the following consequences

   - from $(i2)$, $e_l = e^*$
   - $(i3)$ becomes
   $$p_l d'(s_l) = 0$$
   which implies that $s_l = 0$
   - $(i4)$ becomes
   $$\gamma_3 v'(s_h) = p_h(1 + \lambda - v'(s_h)) \tag{21}$$

   But we have already proved that $v(s_h) = LB + v(s_l + b)$ so $v(s_h) = LB + v(-\Phi(e^* + \triangle\theta)) > 0$ from the initial assumption. So $s_h > 0$ and therefore $\gamma_3 = 0$ if $(ii4)$ has to be verified. But then, the only way to verify (21) is $s_h = 0$, which is a contradiction. Hence $\gamma_4 > 0$ and so $e_l > e^*$.

6. $(i3)$ becomes
$$-p_l d'(s_l) = \gamma_4 v'(s_l + b) > 0$$
so $s_l < 0$.

7. Next we claim that $s_h > 0$. Assume that $s_h = 0$ so $v'(s_h) = 1 + \lambda$. $(i4)$ becomes
$$(\gamma_3 + \gamma_4)(1 + \lambda) = 0$$
which is a contradiction because we have already proved that $\gamma_4 > 0$. Hence $s_h > 0$ and then $\gamma_3 = 0$ ■