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Regional Environmental Agreements on Renewable Resources

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(Preliminary Version)

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This paper addresses the issue of conflicts between countries who share a renewable natural resource using a two-level framework. Contrary to the usual modeling of countries as representative agents who sign an international treaty to protect the resource that they share (level I), this research considers the existence of firms and consumers within each country (level II). It discusses the influence of both domestic characteristics (consumers' preferences and firms' costs) and the presence of some national environmental policies on the resulting regional environmental agreement. The level II outcome is analyzed as the result of a dynamic game in which each government (taking other countries' pollution as given) decides a domestic environmental regulation. While domestically there is a regulatory authority able to dictate norms, the same does not happen at the international level. Hence, regional environmental agreements are viewed as the result of some form of bargaining among countries. Another insight, of a more technical kind, is the incorporation of a numerical simulation (for a linear-quadratic example) to depict the dynamics of the model. In particular, its main result is an estimation of the path of pollution regulations that countries should agree upon.

I. Introduction

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Environmental spillovers have been widely analyzed in the economic literature. They appear mainly as typical examples of failure of the market mechanism to achieve efficiency, the origin of that failure being the presence of real externalities. Several kinds of theoretical solutions to those problems have been devised. The most classical is perhaps the imposition of taxes and subsidies to correct prices for the distortion in such a way that private decisions result in a social optimum (Pigou, 1920). Another well-known possibility is to identify the problem of externalities as the absence of markets and associated property rights (Coase, 1960). In this view, developing a market for the externality, and letting the parties bargain with each other is optimal, irrespective of the allocation of the property rights between victim and perpetrator (when property rights are "well-defined" and there are no "transaction's costs").

Controversies exist between one line of thought and the other. Pigovian taxes are criticized mainly because they are informationally very demanding (since the government is supposed to know all private information in order to decide the precise tax to correct the externality). The Coasian approach is reproved mainly because it does not consider that pecuniary externalities can persist or even be created while solving real externalities.

Beyond those discrepancies about which solution has to be applied, international environmental problems represent a situation in where neither of the above "solutions" is easily applicable. This happens because, when pollution generated in one country degrades the environment of other countries or an international common property resource, there is no supra-national government that can directly intervene. The most common problem of this kind, *unidirectional externalities,* occurs between polluting and suffering countries (e.g., pollution or withdrawal of water along rivers). *Regional reciprocal externalities* arise when a region contains both the polluter and the victim (e.g., pollution of common lakes or seas, acid rain). Finally, there are also *global environmental problems,* which affect most of the countries of the world in one way or another (e.g., conservation of biodiversity, global climate, the deep sea or the Antarctica)[1].

This research deals basically with cases of renewable natural resources passing or forming the boundaries of countries in a region. It addresses the issue of conflicts between countries which share a renewable natural resource using a two-level framework. Contrary to the usual modeling of countries as representative agents who sign an international treaty to protect the resource that they share (level I), this paper considers the existence of firms and consumers within each country (level II). It discusses the influence of domestic characteristics (consumers' preferences and firms' costs) and national environmental policies on the resulting regional environmental

[1]This is basically the classification of international environmental problems adopted by Mäler (1990).

agreement. The level II outcome is analyzed as the result of a dynamic game in which each government (taking other countries' pollution as given) decides a domestic environmental regulation.

While domestically there is a regulatory authority able to dictate norms, the same does not happen at the international level. Hence, regional environmental agreements are viewed as the result of negotiations among countries. Another insight, of a more technical kind, is the incorporation of a numerical simulation (for a linear-quadratic example) to depict the dynamics of the model. In particular, its main result is an estimation of the path of pollution regulations that countries should agree upon.

The paper is organized in the following way. Part II contains a review of the literature on environmental cooperation between countries studied in the framework of dynamic games. Part III presents a model to interpret different hypothetical situations which can derive in international environmental agreements. Part IV deals with a numerical exercise to illustrate the model. Finally, part V summarizes the main results.

II. Review of the literature

In recent years, various authors have employed the theory of repeated non-cooperative games to analyze economic behavior in cases of oligopolistic exploitation of common-property resources. The basis of that argument is the tragedy of the commons (Hardin, 1968) in a "prisoner's dilemma" game, which emphasizes the impossibility of cooperation between the parties because independent exploitative actions are a dominant strategy for all participants. If various governments agreed to maintain a given level of stock of a resource, then each government would have strong incentives to reduce its own stock, free-riding on their neighbors. However, by a repeated game argument (the Folk theorem), countries can reach efficient outcomes through mutual agreement. The idea is to design a treaty which makes credible to each country that future cooperation from the others will occur only if that country complies. Each country has then a choice: to incur the loss from future cooperation in exchange for the short run benefit of cheating, or to comply with the agreement and enjoy the benefits of continuing cooperation.

One dissatisfying aspect of repeated games is that they are based on the assumption that the environment of the game does not change over time. However, in many economic applications there is a state variable which evolves along time, changing the way in which the game being is played (for example, the capital stock of the economy, the stock of reputation or "goodwill", the stock of natural resources). To incorporate those features, it is therefore necessary to deal with state-dependent dynamic games.

Strictly theoretical papers on state-dependent dynamic games are numerous[2]. In general, they focus on the study of "Markov" or "state-space" strategies for which the state variable summarizes the past play. The equilibrium concept linked to those games is called Markov Perfect Equilibrium, and is the profile of Markov strategies that gives a Nash equilibrium in every subgame. The most important papers are due to Sundaram (1989), Dutta (1995b), and Dutta and Sundaram (1993a, 1993b). In addition, Dutta (1995a) presents a Folk Theorem for these types of dynamic games[3].

There are also several papers that deal with more applied research involving dynamic game theory and environmental problems (a good review of them is Clemhout and Wan, 1990). The case usually more analyzed in that literature is fisheries, of which Levhari and Mirman (1980) is the most important reference. It depicts the situation of two countries simultaneously fishing in the same sea as a dynamic game in which the number of fish available is the state variable and its evolution depends on how much fish is caught in each fishing period. The authors show that, for particular utility and reproduction functions, the difference along time between the optimal evolution of the resource and the one arising from a Nash equilibrium. Another line of publication on the same topic has its origins in Europe, basically through papers like Hämäläinen, Haurie and Kaitala (1985), which deals with the idea of using threats to sustain cooperation in fishery games. Kaitala's (1985) comprehensive survey of game theoretic models of fishery management also constitutes a key reference for applications to fisheries.

Other problems involving natural resources studied in a dynamic game framework are air pollution (mainly acid rain) and global warming effects. For the former, Tahvonen, Kaitala, and Pohjola (1993) compare the sustainability and cost-effectiveness of the agreement for reductions of sulfur emissions between Finland and the former Soviet Union with what would be the alternatives under purely non-cooperative and cooperative outcomes. Pallage (1995) performs a simulation for air pollution between two countries. For global warming, Martin, Patrick and Tolwinski (1993) analyze, within an asymmetric game of transboundary pollution, what are the effects of using a global carbon tax as a scheme of agreement where the players cooperate to set the level of that tax. For a similar problem, Beltratti (1995) discusses a linear-quadratic model where two countries share an environmental resource, emphasizing the positive correlation between the ratio of Markov equilibrium and efficient resource stocks, and the rate of time preference.

The main difference between this research and the existing literature is that

[2]For a general review of dynamic games, see Fudenberg and Tirole (1991) or Basar and Olsder (1994).

[3]Other important references are Benhabib and Radner (1987), who consider the possibility of delays in the detection of deviations from cooperative strategies to exploit a common productive asset, and Reinganum and Stokey (1985), who analyze the importance of the periods of commitment in that kind of games.

countries involved in the negotiation are not modeled as representative agents but as governments whose aim is to maximize the gains from trade in their own country (defined as the sum of consumers' and producers' surpluses). Hence, in addition to international negotiations, there is the possibility that each government decides to impose national policies to influence consumers and firms' behavior towards the resource. Instead of viewing conflicts among countries in exactly the same way as a two-person situation, then, this approach adds another dimension and new possibilities of considering links between domestic regulations and international treaties. Another new insight is the incorporation of a numerical simulation that depicts the way in which the model works, even if functions are such that there are no closed solutions for the dynamic problems. The methodology employed to perform the simulation is a variation of the method used in the real business cycle literature to solve social planning and recursive competitive equilibrium problems.

III. The model

The simplest modeling framework to employ in describing the situations involved in environmental conflicts is to consider one common natural resource (a), one money commodity (m), and one non-money commodity (c) in each country. Then, a few countries (indexed by i) share a resource that is on (or crosses) their common political border. Each country has several types of consumers (indexed by h) and firms (indexed by f). There is a continuum of consumers and a continuum of firms of each type, but the number of types is finite[4].

Consumers like to consume both commodities and value the stock of the resource. For example, if part of the boundary of the two countries is constituted by a lake, then consumers may want it to be clean, and a^i is the quality of the water that they see on the side of the lake that corresponds to country i. More precisely, each type of consumer has the following quasi-linear utility function:

$$U_h^i = v_h^i(c_h^i, a^i) + m_h^i$$

and a budget constraint of the form:

[4]No trade among countries is assumed, in order to keep the model simple. There is an abundant literature on the link of trade and environment. Environmental regulation is supposed to play a role both in determining the composition of trade and the pattern of investments. However, empirical research has not been able to confirm that opinion (see, for example; Tobey, 1990; Grossman and Krueger, 1991; or Jaffe et al, 1995).

$$p^i \cdot c^i_h + m^i_h = 0$$

where c^i_h is the quantity consumed of the non-money commodity by consumers of type h, m^i_h is the quantity consumed of the money commodity by the same kind of households, and p^i is the price of the non-money commodity, all in country i. The functions v^i_h are all increasing in consumption and resource quality, continuously differentiable and strictly concave. The main advantage of using this type of utility function comes from the fact that it makes aggregation across individuals easier, and it is equivalent (once the constraint is substituted into the objective function) to consider consumers who maximize their surplus.

On the other hand, each type of firm in country i chooses its level of production so as to maximize its profit (or producer surplus). It also generates some pollution which affects the quality of the natural resource, and its objective function can therefore be written as:

$$\Pi^i_f = p^i \cdot c^i_f - TC^i_f(c^i_f, x^i_f)$$

where c^i_f is the quantity of the non-money commodity produced by firms of type f in country i, and x^i_f is the amount of pollution generated by those firms. The functions TC^i_f are all continuously differentiable, strictly convex, and increasing in output.

Each country has a government whose aim is to maximize the gains from trade of its nation, defined as the sum of all domestic consumers' and firms' surpluses[5]. This is:

$$g^i = \sum_h s^i_h \cdot \left[v^i_h(c^i_h, a^i) - p^i \cdot c^i_h\right] + \sum_f r^i_f \cdot \left[p^i \cdot c^i_f - TC^i_f(c^i_f, x^i_f)\right].$$

where s^i_h is the mass of consumers of type h and r^i_f is the mass of producers of type f.

Finally, the resource involved is renewable (e.g., water). Its stock exhibits a natural growth (e.g., water naturally cleansing through biodegradation of organic pollutants and precipitation of solids). That natural rate of amelioration is partially offset by the harm caused by polluters (in this case, firms). In general,

$$a^i_{t+1} = Z^i(a^i_t, \sum_f r^1_f \cdot x^1_{f,t}, \ldots, \sum_f r^I_f \cdot x^I_{f,t}) \qquad i = 1,\ldots,I$$

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[5]This function assumes that the government is "benevolent", in the sense that its goal is not a selfish or politically-oriented objective but rather to maximize the welfare of the inhabitants of its country. Chapter 3 of my disseration contains variations on this point, and it also incorporates informational problems which arise in these kinds of situations.

where a^i_t and a^i_{t+1} are the states of the resource at country i in period t and t+1 respectively, and the other sums represent the aggregate pollution level in each country. This way of modeling pollution takes into account the fact that the damage to the resource depends not only on how much do firms pollute but also on where those pollutants are emitted, because harm may vary negatively with distance[6].

The issues concerning equilibrium and efficiency for this model can be analyzed in two different levels: domestic and international. For the former, two main scenarios can be considered. First, the there is an equilibrium without government intervention, which results from the free interaction among firms and consumers within each country. This is inefficient due to the presence of a unidirectional externality from producers to consumers (i.e. producers pollute a lake or river and domestic consumers are harmed). Second, there is a partially efficient equilibrium in which governments adopt some domestic policy measures: each country instruments a domestic environmental policy taking the pollution of other countries as given. Full efficiency, however, requires further policy, because each government does not consider the harm that the national firms impose on other countries. Solving that problem is the role played by an international agreement (level I).

As seen, there is more in international environmental agreements than bargaining as if governments were simple individuals. The additional complexity appears because there is an interaction between each country's characteristics, its domestic environmental policy, and what is agreed on at the international level.

A. General case: a reciprocal externality among countries

This section of the paper deals with regional reciprocal externalities. Those conflicts occur when a region is both the polluter and the victim, as in the case of pollution of common lakes (i.e., the Great Lakes), seas (i.e., Mediterranean, Baltic, or North Sea) or acid rain.

1) Domestic situation (level II)

i. Absence of any environmental regulation

When there is not any environmental regulation, and agents are price takers in the market of the non-money commodity, each consumer in each country maximizes his

[6]Note that it could also be assumed (as is usual in the literature) that the resource is common in a more complete sense: pollution in the neighbor country affects the consumer in the same way as the pollution in his own country. That simplification allows the use of a single stock variable, but it is less realistic in terms of what actually happens in regional environmental problems.

intertemporal utility subject to his budget constraint following this rule:

$$p^i = \frac{\partial v_h^i(c_h^i, a^i)}{\partial c_h^i}$$

Besides, each firm chooses production and pollution levels in order to maximize its intertemporal profits, such that

$$p^i = \frac{\partial TC_f^i(c_f^i, x_f^i)}{\partial c_f^i} \qquad \text{and} \qquad \frac{\partial TC_f^i(c_f^i, x_f^i)}{\partial x_f^i} = 0$$

Therefore, firms and consumers interact in a Walrasian fashion in the market for the non-money commodity[7] (equating their marginal costs and marginal utilities to the price), while *pollution is pursued by the firm until the marginal cost from its emissions is equal to zero*.

A particularity of these conditions is that, even when they care about it, consumers cannot control the state of the resource, so their problem is in fact static. In the same way, firms exercise control over the level of pollution that they generate, but the state of the resource has no influence on their payoff, so their problem is also static. So in each country consumers and firms demand and supply the non-money commodity, firms choose pollution levels and, as a result of these independent decisions and its natural growth, the resource evolves through time according to the function Z^i.

Clearly, the unregulated equilibrium outcome within each country is inefficient, since neither the national nor the international damages to the resource are taking into account. The result of this type of interaction can be interpreted as an "occupational equilibrium" (Makowski and Ostroy, 1995) in which firms choose their optimal "occupation" (i.e, level of pollution), while consumers have no relevant occupational choice.

ii. Governments apply some domestic environmental regulation

The unregulated equilibrium outcome within each country being inefficient, the attainment of efficiency requires some corrective measures. So governments which seek to maximize the gains from trade in each country (g^i) can establish some domestic environmental regulation. The problem is then fully dynamic, because both the level of pollution and the quality of the resource are part of the intertemporal objective function

[7]To assume a Walrasian context for the market of the good is a reasonable hypothesis here, since there is a continuum of firms and consumers who cannot influence the price of the non-money commodity.

of the government. Hence, each country i has a dynamic programming intertemporal problem of the following type:

$$V^i(a^1,\ldots,a^I) = \max_{\substack{(c_h^i, c_f^i, x_f^i, p^i)_{t=0}^{\infty} \\ \forall f,h}} \left\{ \left[\sum_h s_h^i \cdot v_h^i(c_h^i, a^i) - \sum_h s_h^i \cdot p^i \cdot c_h^i + \sum_f r_f^i \cdot p^i \cdot c_f^i \right.\right.$$

subject to $\quad a'^i = Z^i(a^i, \sum_f r_{f,t}^1 \cdot x_{f,t}^1, \ldots, \sum_f r_{f,t}^I \cdot x_{f,t}^I)$

where the prime corresponds to period t+1 and the discount factor (β) is assumed to be the same for every country. V^i is the value function of each country, which reflects the maximized intertemporal value for its objective function. Note that the value function V^i also depends on the state of the resource at the neighboring countries (even if the government of country i does not control them), because each country, when looking at the state of the resource on the other countries, can deduce what their pollution will be, and this has an influence on its intertemporal gains from trade through its effect on domestic consumers' utility.

To solve this problem, the government must find a way to induce the citizens of its country to fulfill the following first order conditions (FOC):

for consumption (c_h^i); $\quad s_h^i \cdot \dfrac{\partial v_h^i(c_h^i, a^i)}{\partial c_h^i} = s_h^i \cdot p^i \quad$, for all h

for production (c_f^i); $\quad r_f^i \cdot p^i = r_f^i \cdot \dfrac{\partial TC_f^i(c_f^i, x_f^i)}{\partial c_f^i} \quad$, for all f

for price (p_i); $\quad \sum_h s_h^i \cdot c_h^i = \sum_f r_f^i \cdot c_f^i$

These conditions correspond to the market of the non-money commodity. Note that the FOC for p^i represents a market-clearing condition (production is equal to consumption), since p^i is in fact a Lagrange multiplier (the shadow-price of the resource constraint). However, in a competitive market like this, the government does not need to intervene at all, since the results shown are identical to the ones obtained through a Walrasian equilibrium.

When the government wishes to implement the domestically optimal level of pollution, conversely, it has to design a certain mechanism to be imposed to the other economic agents. One of those mechanisms consists of establishing *emission standards* for each firm[8]. In that case, the FOC for pollution is:

$$r_f^i \cdot \frac{\partial TC_f^i(c_f^i, x_f^i)}{\partial x_f^i} = \beta \cdot \frac{\partial V^i}{\partial a''} \cdot \frac{\partial Z^i}{\partial r_f^i x_f^i} \cdot r_f^i \quad \text{, for all f}$$

while the envelope condition is:

$$\frac{\partial V^i}{\partial a^i} = \sum_h \left[s_h^i \cdot \frac{\partial v_h^i(c_h^i, a^i)}{\partial a^i} \right] + \beta \cdot \frac{\partial V^i}{\partial a''} \cdot \frac{\partial Z^i}{\partial a^i}$$

Combining the envelope and first-order conditions for pollution yields:

$$\frac{\dfrac{\partial TC_f^i}{\partial x_f^i}}{\dfrac{\partial Z^i}{\partial r_f^i x_f^i}} = \beta \cdot \frac{\dfrac{\partial TC_f^i}{\partial x''^i_f}}{\dfrac{\partial Z^i}{\partial r_f^i x''^i_f}} \cdot \frac{\partial Z^i}{\partial a''} + \beta \cdot \sum_h \left(s_h^i \cdot \frac{\partial v_h^i}{\partial a''}\right) \text{, for all f}$$

The combination of the FOC for pollution and the envelope condition can be interpreted as the *equality between the marginal abatement cost of domestic emissions today in each country i and the loss (in terms of utility for national consumers) that pollution will have through the deterioration of the resource.*

An alternative corrective mechanism would be setting Pigovian taxes (or *emission charges*, t_f^i)[9]. This practice is very common in several countries. For example, emissions' charges for air pollution are used in Canada (British Columbia), France (for some installations), Japan, and Sweden. Taxes are also used for waste water effluents in several European countries as Germany or Belgium, and even in Canada (OECD, 1994). In this model, taxes should be designed in such a way that each firm is induced to behave optimally when it maximizes:

$$p^i \cdot c_f^i - TC_f^i(c_f^i, x_f^i) - t_f^i \cdot x_f^i$$

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[8]Note that in this model, each single government can not set ambient limits (i.e., to allow a limited amount of pollutants to accumulate on its border) because that would imply choosing the level of pollution of firms which are located in other countries.

[9] In terms of efficiency, this is equivalent to assigning an amount of allowed emissions to the generator of the externality when there is complete information. With asymmetrically held information, taxes and quotas are not substitutes (see Weitzman, 1974).

and therefore decides according to the FOC

$$p^i = \frac{\partial TC_f^i(c_f^i, x_f^i)}{\partial c_f^i} \qquad \text{and} \qquad \frac{\partial TC_f^i(c_f^i, x_f^i)}{\partial x_f^i} = t_f^i$$

Then, governments should set the tax equal to the domestic marginal loss caused by pollution, and this implies that

$$t_f^i = \left[\beta \cdot \frac{\dfrac{\partial TC_f^i}{\partial x'^i_f}}{\dfrac{\partial Z^i}{\partial r_f^i x'^i_f}} \cdot \frac{\partial Z^i}{\partial a'^i} + \beta \cdot \sum_h \left(s_h^i \cdot \frac{\partial v_h^i}{\partial a'^i} \right) \right] \cdot \frac{\partial Z^i}{\partial r_f^i x_f^i}$$

Another alternative is to limit emissions through fixed quotas for each type of firm, or to decide a quota on the total level of the externality ($\sum_f r_f^i \cdot x_f^i$) and distribute tradable pollution permits among firms. Then, the government could decide the path of the aggregate pollution, and firms could be allowed to trade them among themselves. In fact, the problem is the same as the one above, with the only difference that, instead of the sum of the cost for each type of firm, the government can use a global cost function (an approximation of the minimal cost envelope of the entire range of cost functions for the economy) defined as:

$$TC^i(C^i, X^i) = \min_{\{c_h^i, x_f^i\}_{t=0}^{\infty} \forall f} \left\{ \left[\sum_f r_f^i \cdot TC_f^i(c_f^i, x_f^i) \right] / \sum_f r_f^i \cdot c_f^i = C^i, \;\; \sum_f r_f^i \cdot x_f^i = X^i \right\}$$

so that its problem becomes:

$$V^i(a^1, \ldots, a^I) = \max_{\substack{\{c_h^i, C^i, X^i, p^i\}_{t=0}^{\infty} \\ \forall h}} \left\{ \left[\sum_h s_h^i \cdot v_h^i(c_h^i, a^i) - \sum_h s_h^i \cdot p^i \cdot c_h^i + p^i \cdot C^i - TC^i(C^i, X^i) \right] \right.$$

subject to $\quad a'^i = Z^i(a^i, X^1, \ldots, X^I) \qquad$ for all i.

Then, the FOC for X^i is:

$$\frac{\partial TC^i(C^i,X^i)}{\partial X^i}=\beta\cdot\frac{\partial V^i}{\partial a^{ii}}\cdot\frac{\partial Z^i}{\partial X^i}$$

and the envelope condition is exactly the same as before. The combined condition that the aggregate pollution for country i should satisfy is similar to the one above:

$$\frac{\dfrac{\partial TC^i}{\partial X^i}}{\dfrac{\partial Z^i}{\partial X^i}}=\beta\cdot\frac{\dfrac{\partial TC^i}{\partial X^{ii}}}{\dfrac{\partial Z^i}{\partial X^{ii}}}\cdot\frac{\partial Z^i}{\partial a^{ii}}+\beta\cdot\sum_h\left(s^i_h\cdot\frac{\partial v^i_h}{\partial a^{ii}}\right)$$

This quasi-market mechanism of setting quotas is not a merely theoretical convenience but a policy used in environmental regulation, particularly in the United States. Examples involve emission trading under the Clean Air Act (see Hahn, 1989)[10], and regulation on water pollution at the states' level[11].

Regardless of the instrument chosen to regulate, the steady state for the resource at this level of analysis should be higher than in the unregulated situation. This happens because the country considers the positive effect of the resource quality on its consumers' utility. But the government's allowed emissions evolve trough time as the resource change. Beginning at the state of the resource without any policy, the governments aim to follow a path of emissions that goes from the steady state of the completely unregulated equilibrium to what they consider the optimum quality of the

[10]The Clean Air Act originally specified that no new emission sources would be allowed in "non-attainment areas" (regions which do not meet specified ambient standards). However, concerns that this prohibition would impact strongly on economic growth in those regions, led the EPA (Environmental Protection Agency) to institute the so-called "offset rule". This rule consists of allowing new sources to locate in those areas provided that they offset their new emissions by reducing pollution from existing plants (owned by this firm or others firms).

[11]The US regulation on water consists mainly of discharge standards based on available technology. In fact, the Clean Water Act converted the earlier system of discharge permits into a mix between effluent technology-based limitations (which differ according to the kind of pollutant and if the source is new or not) and standards imposed through individual "National Pollutant Discharge Permits". In 1981, the state of Wisconsin implemented a program to control pollution in the Fox River, mainly directed at waste generated by pulp and paper plants (which are some of the most significant point sources) and for municipal waste treatment plants. In the state of Colorado, local authorities have issued restrictive limits for pollution from all sources because their discharges were endangering drinking water supplies in the Dillon Reservoir. Then Colorado allowed point sources to increase their discharges if they acquired allowances from non-point sources. Similar experiments were conducted for the Tar-Pimlico watershed in North Carolina (Dudek, Stewart, and Wiener, 1994).

resource[12]. This implies that, when the new environmental regulation is instituted, there is a "jump" down in pollution. Then, as the resource begins to improve, policy measures for pollution become less tight (see figure 1)[13].

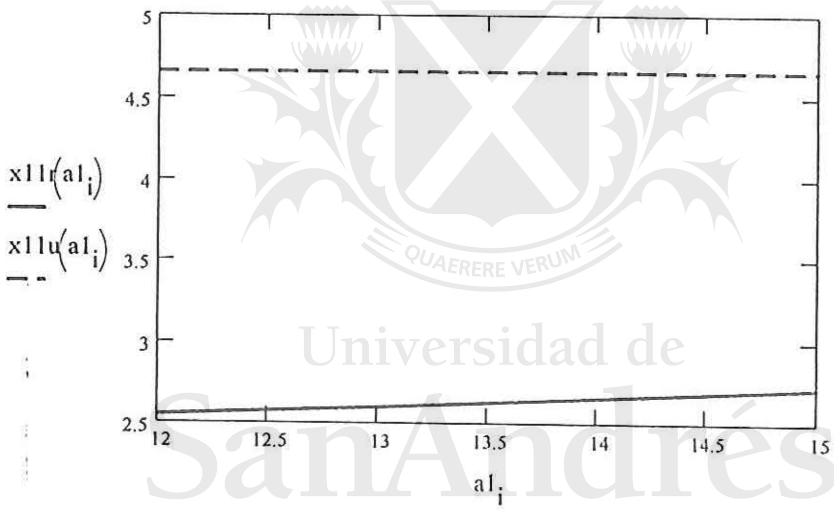
To follow that policy function implies that the corresponding national intertemporal gains from trade (or value function V^i) are maximized. Those gains increase according to the evolution of the resource until a steady state is attained. In addition, the discount factors have an effect on the *shape* of the value function and the policy functions. The level II solution just described corresponds to a Markov Perfect equilibrium, which is not optimum from the point of view of the resource.

Figure 1: Domestic pollution policy versus no regulation

$i := 12..1 \quad a1_i := 1\cdot$

$x11r(a1) := 1.85715 + 0.05715a1$

$x11u(a1) := 4.6475$



2) International situation (level I)

[12]This is what Chari and Kehoe (1990) call "the commitment version" in the sense that the government is assumed to have commitment technologies to bind its actions and those of future governments. Hence, the government sets a sequence of maximum allowed emissions once and for all at the beginning of time and then consumers and firms choose allocations in each subsequent time period.

[13]All figures in this section are based on the policies required for firms of type 1 in country 1, as derived from the numerical simulation of Part IV.

Even if domestic regulations were successful and perfectly applied in every country, they would not by themselves lead to efficiency. Each country would tend to internalize the harm that its firms impose on its consumers, but not to take into account the fact that they also affect other countries' consumers. Hence, efficiency is not fully reached unless the sum of the gains from trade functions for every country is maximized. That could be possible if there were a supra-national authority that solved the following problem:

$$V(a^1,\dots,a^I) = \max_{\substack{\{c_h^i, c_f^i, x_f^i, p^i\}_{t=0}^{\infty} \\ \forall f,h,i}} \left\{ \sum_i \left[\sum_h s_h^i \cdot v_h^i(c_h^i, a^i) - \sum_h s_h^i \cdot p^i \cdot c_h^i + \sum_f r_f^i \cdot p^i \cdot c_f^i \right.\right.$$

subject to one constraint for the resource at each border:

$$a'^i = Z^i(a^i, \sum_f r_f^1 \cdot x_f^1, \dots, \sum_f r_f^I \cdot x_f^I) \qquad \text{, for all i}$$

This is again a dynamic programming problem because, even more than the national governments, the supra-national authority would be conscious that the resource can be depleted along time if it is not well managed.

The solution to this problem yields the same FOCs that were obtained in level II for the non-money commodity consumed and produced in each country, but different ones for the emission decisions' variables:

$$\text{for } x^i_f;\ r_f^i \cdot \frac{\partial TC_f^i(c_f^i, x_f^i)}{\partial x_f^i} = \beta \cdot \frac{\partial V}{\partial a'^1} \cdot \frac{\partial Z^1}{\partial r_f^i x_f^i} \cdot r_f^i + \dots + \beta \cdot \frac{\partial V}{\partial a'^I} \cdot \frac{\partial Z^I}{\partial r_f^i x_f^i} \cdot r_f^i \qquad \text{for all f in all i}$$

Once the r^i_f are simplified, these conditions clearly state that *the marginal abatement cost from domestic emissions has to be equal to the marginal loss that they generate for consumers in all the countries affected.* Finally, the envelope conditions can be expressed as:

$$\frac{\partial V}{\partial a^i} = \sum_h \left[s_h^i \cdot \frac{\partial v_h^i(c_h^i, a^i)}{\partial a^i} \right] + \beta \cdot \frac{\partial V}{\partial a'^i} \cdot \frac{\partial Z^i}{\partial a^i} \qquad \text{, for all } a^i$$

Regulation by a supranational authority typically implies higher taxes or lower quotas than the ones decided domestically, since they would also take into account the harm that domestic firms cause to consumers from other countries. To reach efficiency as established by a supranational planner, each country should follow a policy function for pollution (derived from the solution to the dynamic programming problem described above) which depends on the state of the resource at each border.

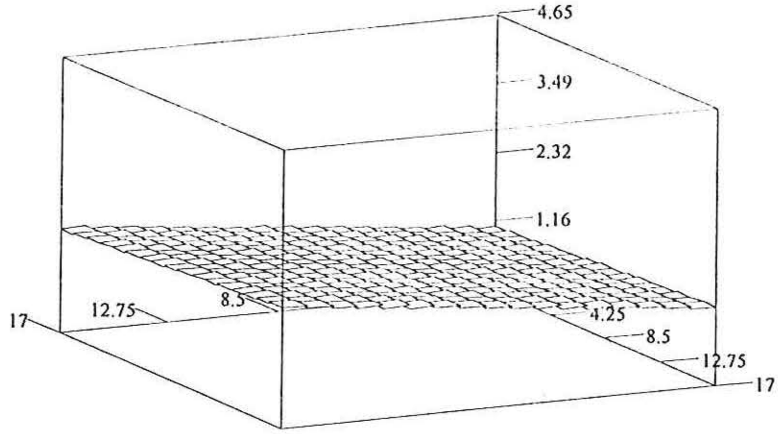
Figure 2 shows the policy for pollution at level I as a function of *any* a_1 and a_2. Figure 3 shows that same situation when a_1 and a_2 follow their optimal paths. Compared to the situation at level II (either i. or ii.), there is a "jump" down in the path of pollution and then a new increasing trajectory until the optimal steady state for the resource is reached[14].

Figure 2: Efficient pollution policy (Level I)

$i := 0..1 \quad j := 0..1 \quad a1_i := 1\cdot \quad a2_j := 1\cdot$

$f(a1, a2) := 1.06205 + 0.04224\,a1 + 0.01658\,a2 \quad M_{i,j} := f(a1_i, a2_j)$

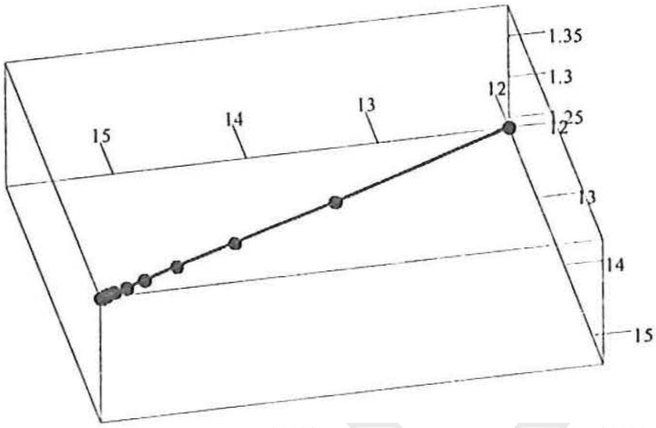
[14]In fact, air pollution agreements usually prescribe a decrease in emissions linked to a time frame. For example, the Sulphur Protocol of the Geneva Convention on Long-Range Transboundary Air Pollution (1979) reduce emissions 50 % in a schedule with intermediate limits at the years 2000, 2005, and 2010.



M

Figure 3: Pollution efficient policy followed by all countries

M := READPRN(f3) a1 := $M^{<0>}$ a2 := $M^{<1>}$ x11 := $M^{<2>}$



a1 , a2 , x11

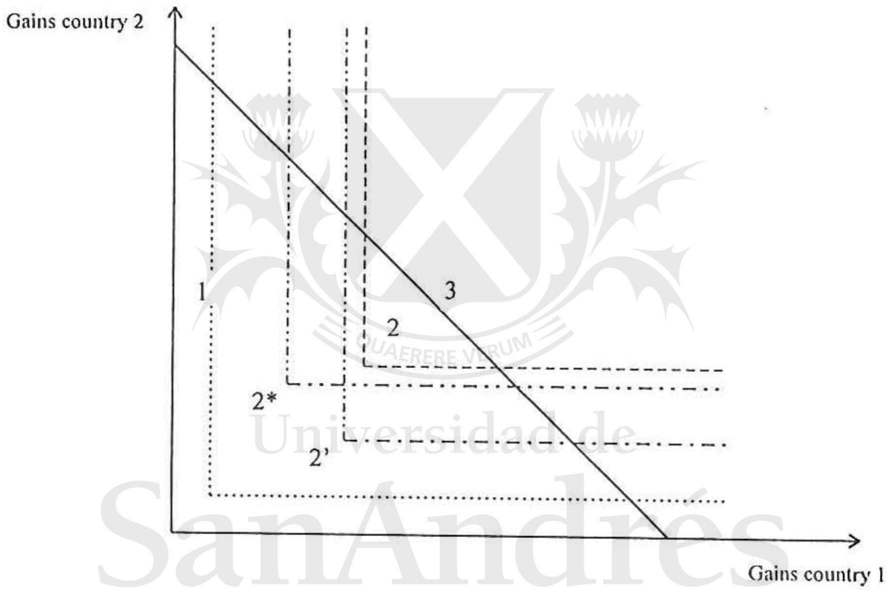
If each country followed that optimal policy, its intertemporal gains from trade would also depend on the level of the resource. Hence, values for the welfare of both countries would change as the resource evolves to the steady state. If the discount factor is equal to one, however, the gains from trade do not depend on the initial quality of the resource, because the horizon being infinite. Therefore, if all periods are equally valued, most of the overall time is spent at the steady state, so it is not important which is the point of departure.

The discount factor being close to 1 allows efficiency to be depicted by a single frontier instead of a moving one. Moreover, efficiency (as decided by a supra-national authority) is represented by a unique point of the utility possibility frontier, because of the use of a quasi-linear utility function for consumers. Therefore, to reach other points on that frontier, a set of transfers is needed. Those transfers can take the form of direct financial transfers or favors in areas in which the countries have common interests. For example, it is possible that the US had agreed to build a desalinization plant on the Colorado River only to maintain good relations with Mexico (1973), and that the same spirit had leaded to the Columbia River Treaty between the United States and Canada (1961).

The problem of international agreements is that there is not such thing as a supranational authority, so countries have to make some sort of arrangement among themselves if they want to improve upon the situation at level II. However, cooperation exists, because there are numerous international agreements on environmental problems which reflect negotiations among governments[15]. On those agreements, cooperation takes different forms. Countries can agree on a kind of Pigovian tax to apply internally (for example, a global carbon tax to decrease air pollution). They can also set different kinds of quantitative limits (e.g., for the Great Lakes, Canada and the US agree on maximum levels of pollution that the Lakes should have; in the Rhine chloride agreement, countries agree on concentrations measured at some points; and in air pollution agreements in Europe, they set goals about decreases in aggregate emissions).

Moreover, governments can begin negotiations in different circumstances, and their outcome may be strongly determined by that initial state of affairs (figure 4).

Figure 4: Gains from trade for both countries at the three levels (symmetric case)



One possibility is that countries bargain in a situation where there exists no previous internal environmental policy (number 1 in figure 4). Another is that they are initially in a situation where both have already implemented optimal national environmental policies (number 2 in the figure 4). The third and fourth cases arise when one of the countries has no previous environmental policy and the other one has

[15]In 1992, there were already a total of 885 legal international instruments which had provisions on environmental matters (Weiss, Szasz, and Magraw, 1992).

(depicted in the figure as 2' and 2^*)[16].

Anyway, it is clear from the observation of figure 4 that the space of negotiation is different if countries begin at 1 than at 2, and so is their outcome. The idea is that even when there is no supranational authority, the departing situation determines what are the countries' individually rational payoffs. These are associated to the different solutions which can be attained as equilibria of the repeated negotiation among countries, as long as their respective discount factors are large enough (Dutta, 1995a). In the case of state-dependent dynamic games as this one, those solutions imply that governments follow certain previously agreed policy functions while the treaty is respected by every country, but switch to a different pollution path when a deviation is detected.

B. Particular case: a unilateral externality among countries

Until now, the model has dealt with a case of a reciprocal externality among countries. This means that firms pollute the resource on the boundary, pollution is transported from one border to the others, and consumers of all countries are affected by it. However, even if the same pollution is generated on every side of the border, consumers in each country suffer more from emissions originated in their own countries than from the ones which come from abroad since the effect of pollution is decreasing with respect to distance. This fact is implicitly taken into account by the laws of motion for the natural resource involved (through Z^i), and by having a state variable for every one of the resource-sharing countries.

However, another possibility occurs when the harm among countries is unidirectional, i.e., when some countries pollute and others are victims. In that case, the consumers and the resource in the polluting countries are affected only by firms of the same country, but in other nations consumers are in the same situation as in part A.

In terms of the results of the model, the problem without any kind of environmental regulation does not change because neither the firms' nor the consumers' optimization changes when environmental policy is implemented. At level II, the difference is that there are two kinds of countries: victims and polluters. Victims have the same law of motion of the resource than before ($a'^i = Z^i(a^i, \sum_f r_f^1 \cdot x_f^1, \dots, \sum_f r_f^I \cdot x_f^I)$), but for polluters the quality of the resource depends only on their firms' emissions ($a'^i = Z^i(a^i, \sum_f r_f^i \cdot x_f^i)$). The first-order conditions, however, are the same because

[16]Note that the assumption behind figure 4 is that the increase in welfare resulting from greater consumer but lower producer surpluses in the country which implements a domestic environmental policy is higher than the resulting increase in the neighbor's consumers utility due to that policy.

countries are myopic about the effect of their pollution on their neighbors.

The main change in the optimality conditions appears at level I. Although the objective function remains the same, the various constraints of the problem are now different. Moreover, the FOCs for pollution also differ. For countries that are victims, the FOC is:

$$r_f^i \cdot \frac{\partial TC_f^i(c_f^i, x_f^i)}{\partial x_f^i} = \beta \cdot \frac{\partial V}{\partial a'^1} \cdot \frac{\partial Z^1}{\partial r_f^i x_f^i} \cdot r_f^i + \ldots + \beta \cdot \frac{\partial V}{\partial a'^I} \cdot \frac{\partial Z^I}{\partial r_f^i x_f^i} \cdot r_f^i$$

whereas for the polluters, it holds that:

$$r_f^i \cdot \frac{\partial TC_f^i(c_f^i, x_f^i)}{\partial x_f^i} = \beta \cdot \frac{\partial V}{\partial a'^i} \cdot \frac{\partial Z^i}{\partial r_f^i x_f^i} \cdot r_f^i$$

The problem is easier to solve for this case than for the one with reciprocal externalities, because the envelope conditions (which are still the same) can be used to solve for the optimal polluters emissions and the resulting formula can be plugged into the victims' FOC.

IV. Numerical application

This section deals with a numerical exercise designed to illustrate the model, performed using GAUSS. Its main characteristic is that it allows to simulate the policy functions resulting from the two levels' problems (even if they have no closed solutions), instead of dealing solely with the steady states. The present section states the methodology employed and the results obtained. The actual computer program written to perform the simulation is reproduced in Appendix A.

A. Information necessary for the simulation

The example assumes some specific forms for the utility and cost functions, for the law of motion of the resource, and some arbitrary values for the parameters. This gives rise to a linear-quadratic problem, making the simulation easier. A key hypothesis to simplify the exposition is to assume only two types of consumers, two types of firms, and two countries involved.

The utility is different for each type of consumer but it does not depend on where they live. The function for each type of individual is:

$$v_h^i = A_h \cdot \ln(c_h^i) + B_h \cdot \ln(a^i)$$

where the first and second term refer respectively to the preferences that each consumer has toward consuming the non-money commodity and his taste for seeing (for example) the water clean in a lake. A_h and B_h are both constants greater than zero, and A_h is greater than B_h.

Firms are assumed to have a cost function of the following form (which again varies among types but not among countries)[17]:

$$TC_f^i = \frac{1}{2} \cdot D_f \cdot {y_f^i}^2 + \frac{1}{2} \cdot (E_f \cdot y_f^i - x_f^i)^2$$

where the constants D_h, E_h are also greater than zero. The quadratic term referring to pollution implies that the marginal cost from pollution is negative only if $E_f^i \cdot y_f^i > x_f^i$. Thus, less pollution increases costs as long as that inequality holds.

The resource evolves according to a certain "radioactive law" (Neher, 1990). That law is equivalent to assuming an exponential decay of the pollutant as a way to characterize the natural cleansing of the resource (for example, water bodies). Hence, firms in both countries also pollute the resource but this also regenerates itself. The equation of movement of the resource in each border (derived in Appendix B) takes the following form:

$$a'^i = \overline{a^i} \cdot \delta + a^i \cdot (1-\delta) - \psi^{li} \cdot \sum_f r_f^l \cdot x_f^l - \psi^{li} \cdot \sum_f r_f^l \cdot x_f^l$$

where $\overline{a^i}$ is the state of the resource at the border of the country I when there is absolutely no pollution, δ is its rate of natural cleaning, and the ψs determine how pollutants affect the resource at each border (given that they are transported from one country to the other). Note that the heterogeneous harm of the different kind of emissions is already incorporated in x, being the ψs only indicators of transport. Since it does not matter which firm is polluting in a particular country (what is taken into account are total emissions), the so-called "transport matrix" can be illustrated by a table of the following form:

Emit/Recep	**Country 1**	**Country 2**
Country 1	$\psi^{11} = .3$	$\psi^{12} = .2$
Country 2	$\psi^{21} = .2$	$\psi^{22} = .3$

[17]The notation here is different than the one used in the theoretical part: for production, y is used instead of c.

Note that the elements in the diagonal of the matrix are greater, reflecting the larger effect of local emission. The value of the parameters has been arbitrarily assumed but it is possible to find estimations for this kind of matrices in the environmental literature (e.g., for acid rain in Europe: Tahvonen, Kaitala and Pohjola, 1993 or Mäler, 1990).

In addition to the parameters of the transport matrix, the other values needed to calibrate the model are related to the utility functions, the cost functions and the masses of each type of consumers and firms. For the utility functions, it is assumed that type 1 consumers are more environmentally oriented than type 2 consumers, so they prefer to have a smaller consumption but a cleaner water (i.e., $A_1 < A_2$ and $B_1 > B_2$).

Consumers	**A**	**B**
Type 1	$A_1 = 90$	$B_1 = 50$
Type 2	$A_2 = 110$	$B_2 = 10$

That same criterion is used for the cost functions, where firms of type 1 are assumed to have better cleaning technologies than firms of type 2, even if they are equally efficient in other aspects of production (hence, $D_1 = D_2$ and $E_1 < E_2$).

Firms	**D**	**E**
Type 1	$D_1 = 10$	$E_1 = 1.5$
Type 2	$D_2 = 10$	$E_2 = 2.5$

Since consumers and firms of the same type are equal among countries, the difference between countries has to rely on the masses of each type of consumers and firms that are at each location. In this case, those masses are the following:

Consumers	**Country 1**	**Country 2**
Type 1	$s^1_1 = .7$	$s^2_1 = .3$
Type 2	$s^1_2 = .3$	$s^2_2 = .7$

Firms	**Country 1**	**Country 2**
Type 1	$r^1_1 = .7$	$r^2_1 = .3$
Type 2	$r^1_2 = .3$	$r^2_2 = .7$

Note that country 1 is inhabited by a larger percentage of environmentally oriented consumers and cleaner firms than country 2. Finally, the discount factor is assumed to be very close to 1, the rate of decay of pollutants (δ) is assumed to be 0.4 and the pristine state of the resource $(\bar{a})$ is defined as equal to 20.

B. Methodology employed at each level

1) Steps for the simulation of level I

The pollution policy function simulated for this level corresponds to the problem of a supra-national planner. The methodology employed in this subsection follows the one used by the real business cycle literature (Hansen and Prescott, 1994) for social planning cases. Its only difference is that the problem of the planner here has behind consumers and producers, it is not a representative agent. Several steps are necessary to solve the corresponding linear quadratic dynamic programming problem:

♦ *Define the variables and the function to maximize*

There are 2 state variables (a_1 and a_2) and 14 decision variables (4 consumptions, 4 productions, 4 pollutions, and 2 implicit prices or Lagrange multipliers). The objective function is the sum of the gains from trade in both countries (g) as stated in part III. section A.2).

♦ *Define a matrix (C) with the intertemporal constraints*

In this case, C is a 3x20 matrix. The number of rows corresponds to one row for constant terms and two other rows for the equations of movement of each of the resources. The number of columns is determined by a column for constants, 2 for the state variables in period t, 14 columns for the decision variables, and 3 more columns for the constant and the state variables in period t+1.

♦ *Define all the steady states for variables' from the FOC*

Using the FOC, each type of consumer in each country consumes the non-money commodity, which for the functions utilized in this example corresponds to:

$$c_1^1 = \frac{A_1}{p^1} \qquad c_2^1 = \frac{A_2}{p^1} \qquad c_1^2 = \frac{A_1}{p^2} \qquad c_2^2 = \frac{A_2}{p^2}$$

Additionally, the FOCs for production of each type of firm in each country with the assumed functions are:

$$p^1 = y_1^1 \cdot [D_1 + (E_1)^2] - E_1 \cdot x_1^1 \qquad p^1 = y_2^1 \cdot [D_2 + (E_2)^2] - E_2 \cdot x_2^1$$
$$p^2 = y_1^2 \cdot [D_1 + (E_1)^2] - E_1 \cdot x_1^2 \qquad p^2 = y_2^2 \cdot [D_2 + (E_2)^2] - E_2 \cdot x_2^2$$

The first-order conditions for pollution are:

$$x_1^1 = E_1 \cdot y_1^1 - \psi^{11} \cdot \beta \cdot \frac{\partial V}{\partial a'^1} - \psi^{12} \cdot \beta \cdot \frac{\partial V}{\partial a'^2} \qquad x_2^1 = E_2 \cdot y_2^1 - \psi^{11} \cdot \beta \cdot \frac{\partial V}{\partial a'^1} - \psi^{12} \cdot \beta \cdot \frac{\partial V}{\partial a'^2}$$

$$x_1^2 = E_1 \cdot y_1^2 - \psi^{21} \cdot \beta \cdot \frac{\partial V}{\partial a'^1} - \psi^{22} \cdot \beta \cdot \frac{\partial V}{\partial a'^2} \qquad x_2^2 = E_2 \cdot y_2^2 - \psi^{21} \cdot \beta \cdot \frac{\partial V}{\partial a'^1} - \psi^{22} \cdot \beta \cdot \frac{\partial V}{\partial a'^2}$$

while the envelope conditions can be expressed as:

$$\frac{\partial V}{\partial a^1} = \left(\frac{s_1^1 \cdot B_1}{a^1} + \frac{s_2^1 \cdot B_2}{a^1} \right) + \beta \cdot \frac{\partial V}{\partial a'^1} \cdot (1-\delta) \;\; ; \quad \frac{\partial V}{\partial a^2} = \left(\frac{s_1^2 \cdot B_1}{a^2} + \frac{s_2^2 \cdot B_2}{a^2} \right) + \beta \cdot \frac{\partial V}{\partial a'^2} \cdot (1-\delta)$$

Plugging the envelope conditions into each pollution FOC, and then into the FOC for production, it has to be true that, in the steady state:

$$p^1 = y_1^1 \cdot D_1 + E_1 \cdot \frac{\beta}{1-\beta \cdot (1-\delta)} \cdot \left[\psi^{11} \cdot \left(\frac{s_1^1 \cdot B_1 + s_2^1 \cdot B_2}{a^1} \right) + \psi^{12} \cdot \left(\frac{s_1^2 \cdot B_1 + s_2^2 \cdot B_2}{a^2} \right) \right]$$

$$p^1 = y_2^1 \cdot D_2 + E_2 \cdot \frac{\beta}{1-\beta \cdot (1-\delta)} \cdot \left[\psi^{11} \cdot \left(\frac{s_1^1 \cdot B_1 + s_2^1 \cdot B_2}{a^1} \right) + \psi^{12} \cdot \left(\frac{s_1^2 \cdot B_1 + s_2^2 \cdot B_2}{a^2} \right) \right]$$

$$p^2 = y_1^2 \cdot D_1 + E_1 \cdot \frac{\beta}{1-\beta \cdot (1-\delta)} \cdot \left[\psi^{21} \cdot \left(\frac{s_1^1 \cdot B_1 + s_2^1 \cdot B_2}{a^1} \right) + \psi^{22} \cdot \left(\frac{s_1^2 \cdot B_1 + s_2^2 \cdot B_2}{a^2} \right) \right]$$

$$p^2 = y_2^2 \cdot D_2 + E_2 \cdot \frac{\beta}{1-\beta \cdot (1-\delta)} \cdot \left[\psi^{21} \cdot \left(\frac{s_1^1 \cdot B_1 + s_2^1 \cdot B_2}{a^1} \right) + \psi^{22} \cdot \left(\frac{s_1^2 \cdot B_1 + s_2^2 \cdot B_2}{a^2} \right) \right]$$

where the formulas between brackets express the harm that national *and* foreign consumers suffer because of domestic pollution. Production for the non-money commodity can be expressed as:

$$y_1^1 = \frac{p^1 - E_1 \cdot K_1}{D_1} \qquad y_2^1 = \frac{p^1 - E_2 \cdot K_1}{D_2} \qquad y_1^2 = \frac{p^2 - E_1 \cdot K_2}{D_1} \qquad y_2^2 = \frac{p^2 - E_2 \cdot K_2}{D_2}$$

where the brackets and their immediate precedent multiplicative term of the previous expression are called K_1 and K_2 for countries 1 and 2 respectively. Then, production by each type of firm in each country not only depends positively on prices but also on another term which contains the state of the resource. Hence, the decisions with respect to pollution do not depend only on the level of production, but they are also related to the harm that production causes to all consumers in the region:

$$x_1^1 = E_1 \cdot y_1^1 - K_1 \qquad x_2^1 = E_2 \cdot y_2^1 - K_1 \qquad x_1^2 = E_1 \cdot y_1^2 - K_2 \qquad x_2^2 = E_2 \cdot y_2^2 - K_2$$

The implicit prices in the steady state imply full employment of the resources from the point of view of the planner. Those Lagrange multipliers are the ones that allow the fulfillment of the market-clearing constraint in both countries. For country 1, they come from solving the equation:

$$(p^1)^2 \cdot \left(\frac{r_1^1}{D_1} + \frac{r_2^1}{D_2}\right) - p^1 \cdot \left[r_1^1 \cdot \frac{E_1 \cdot K_1}{D_1} - r_2^1 \cdot \frac{E_2 \cdot K_1}{D_2}\right] - \left(s_1^1 \cdot \frac{A_1}{p^1} + s_2^1 \cdot \frac{A_2}{p^1}\right) = 0$$

and a similar condition holds for country 2. However, since both equations contain K_1 or K_2 (which are functions of the resource), they also depend on a^1 and a^2.

In order to determine the steady states for prices and resource levels, two more equations are needed. These are the laws of movement of motion of the resource at each border, evaluated at the steady state. They imply:

$$a^1 = \overline{a^1} - \frac{1}{\delta} \cdot \left[\psi^{11} \cdot (r_1^1 \cdot x_1^1 + r_2^1 \cdot x_2^1) - \psi^{21} \cdot (r_1^2 \cdot x_1^2 + r_2^2 \cdot x_2^2)\right]$$

for the resource in country 1, and a similar condition for country 2. Then, knowing the steady states for a^1, a^2, p^1, and p^2, the steady states for all the other variables can be easily calculated using the corresponding formulas[18].

♦ *Quadratize the objective function around the steady state*

For the case of a two variable objective function, the quadratic approximation around the steady state implies using the following Taylor's series expansion:

$$\begin{aligned} f(x,y) \cong\ & f(\bar{x},\bar{y}) + f_x(\bar{x},\bar{y}) \cdot (x-\bar{x}) + f_y(\bar{x},\bar{y}) \cdot (y-\bar{y}) \\ & + \frac{1}{2} \cdot \left[f_{xx}(\bar{x},\bar{y}) \cdot (x-\bar{x})^2 + 2 \cdot f_{xy}(\bar{x},\bar{y}) \cdot (x-\bar{x}) \cdot (y-\bar{y}) + f_{yy}(\bar{x},\bar{y}) \cdot (y-\bar{y})^2\right] \end{aligned}$$

In the computer, this procedure uses approximated derivatives for all the variables of the problem so as to convert the objective function into a quadratic form $w' \cdot q \cdot w$, where w is a 17x1 vector of the state and decision variables in the problem (plus a constant) and q is a 17x17 matrix which contains the coefficients which approximate the objective function.

---

[18] The resulting steady states are obtained using a non-linear simultaneous equation solving procedure in GAUSS.

♦ *Solve for the value function of the Bellman's equation*

After the quadratization of the objective function, the overall problem is now a linear-quadratic one, and can be written as:

$$V(a^1,a^2) = \max\{w'\cdot q\cdot w + \beta\cdot V(a'^1,a'^2)\}$$

$$\text{s.t. } z'\cdot C\cdot x$$

where z is the vector of the state variables in t+1 and x is a 20x1 vector containing both w and z. Hence, the problem can be rewritten in a matrix form as:

$$V(a^1,a^2) = \max\{x'\cdot R\cdot x\}$$

$$\text{s.t. } z'\cdot C\cdot x$$

where R is a 20x20 matrix of the form: $\begin{matrix} q & 0 \\ 0 & \beta\cdot V \end{matrix}\Bigg]$.

Given an arbitrary guess for V, the problem can be solved using a fixed-point argument of the form T(V)=V. More precisely, C is used to reduce 3 rows and 3 columns from R, and then the FOCs of the problem are used to reduce 14 more rows and columns[19]. The remaining matrix has only 3 rows and 3 columns, and gives an expression for V after several iterations.

♦ *Calculate the decision variables as functions of the state variables*

Once the value function is obtained, the policy functions can be derived from because the rows of the R matrix which correspond to the coefficients of the FOC of the Bellman's equation. In this problem, with 14 decision variables, the set of all rows is equivalent to a system of FOC from which the policy functions arise for all decision variables. For pollution, those functions indicate the optimum amount of emissions for each state of the resource. Therefore, they also give the path that a supra-national authority would dictate and that countries should follow in an agreement..

2) Steps for the simulation of level II

[19]For the reduce procedure, the order of the variables is important because it consist basically in replacing one FOC into the other as a way to solve the problem. Then, Lagrange multipliers have to be placed before consumption, production and pollution because those are all a function of prices.

i. Absence of any environmental regulation

The equilibrium without any kind of environmental policy does not require much calculation, since the problems of consumers and producers are static. Each type of consumer in each country decides its demand for the non-money commodity so as to equate price to marginal utility). Each type of firm chooses the supply of the non-money commodity so as to equate price to marginal cost. Hence, the corresponding functions are:

$$c_1^1 = \frac{A_1}{p^1} \qquad c_2^1 = \frac{A_2}{p^1} \qquad c_1^2 = \frac{A_1}{p^2} \qquad c_2^2 = \frac{A_2}{p^2}$$
$$y_1^1 = \frac{p^1}{D_1} \qquad y_2^1 = \frac{p^1}{D_2} \qquad y_1^2 = \frac{p^2}{D_1} \qquad y_2^2 = \frac{p^2}{D_2}$$

and emissions are proportional to production:

$$x_1^1 = E_1 \cdot y_1^1 \qquad x_2^1 = E_2 \cdot y_2^1 \qquad x_1^2 = E_1 \cdot y_1^2 \qquad x_2^2 = E_2 \cdot y_2^2$$

In equilibrium, consumption has to be equal to production; so that:

$$s_1^1 \cdot c_1^1 + s_2^1 \cdot c_2^1 = r_1^1 \cdot y_1^1 + r_2^1 \cdot y_2^1 \qquad s_1^2 \cdot c_1^2 + s_2^2 \cdot c_2^2 = r_1^2 \cdot y_1^2 + r_2^2 \cdot y_2^2$$

Prices in each country depend on tastes and production costs, and therefore:

$$p^1 = \sqrt{\frac{s_1^1 \cdot A_1 + s_2^1 \cdot A_2}{\frac{r_1^1}{D_1} + \frac{r_2^1}{D_2}}} \qquad p^2 = \sqrt{\frac{s_1^2 \cdot A_1 + s_2^2 \cdot A_2}{\frac{r_1^2}{D_1} + \frac{r_2^2}{D_2}}}$$

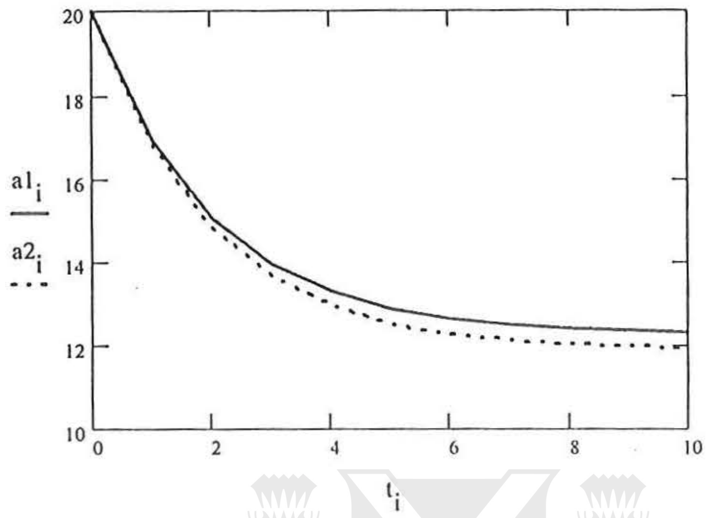
As a result of these choices, the stock of the resource in each border changes according to the following equations:

$$a'^1 = \overline{a^1} \cdot \delta + a^1 \cdot (1-\delta) - \psi^{11} \cdot (r_1^1 \cdot x_1^1 + r_2^1 \cdot x_2^1) - \psi^{21} \cdot (r_1^2 \cdot x_1^2 + r_2^2 \cdot x_2^2)$$
$$a'^2 = \overline{a^2} \cdot \delta + a^2 \cdot (1-\delta) - \psi^{12} \cdot (r_1^1 \cdot x_1^1 + r_2^1 \cdot x_2^1) - \psi^{22} \cdot (r_1^2 \cdot x_1^2 + r_2^2 \cdot x_2^2)$$

The movement of the resource over time which results from a completely unregulated equilibrium in both countries can be seen in figure 4. The steady states for all the variables can be calculated directly by first substituting the parameters in the prices, and then these in the demands and supplies. Finally, after plugging production into the FOC for emissions, the steady state for the resource results from its equation of motion.

Figure 4: Movement of the resource along time without any regulation

M := READPRN(f5) $a1 := M^{<1>}$ $a2 := M^{<2>}$ $i := 0..10$ $t_i := 1\cdot$



ii. Governments apply some domestic environmental regulation

To analyze the case of both countries doing domestic environmental policy requires the solution of a more complicated problem. The government of each country decides on its domestic pollution taking the other country's actions as given, so in the problem of country 1, emissions by country 2 are not controls and viceversa. The steps to follow in this case are a variation of the method employed by Hansen and Prescott (1994).

♦ *Define the variables and the functions to maximize*

The number of decision variables is different from the one seen in section B.1), because each country's problem is half the dimension of the supra-national planner. However, the number of state variables is the same, because the intertemporal gains from trade in each country change when the other one takes less care of the resource.

♦ *Define matrices with the intertemporal constraints*

In this case, two matrices C have to be defined, one for each country. The number of rows corresponds to one row for a constant term and two rows for the equation of motion of each of the resources. The number of columns is determined by a column for a constant, 2 columns for the state variables in period t, 11 columns for the decision variables and 3 more columns for the constant and the state variables in period t+1.

♦ *Define the steady states for all variables from the FOC*

As the objective functions are different, so are the FOC for pollution. The terms K_1 and K_2 are smaller because at level 2 only the harm from domestic firms to domestic consumers is internalized, instead of the whole externality. Except for this, the way to solve for the steady states is the same than in level I.

$$p^1 = y_1^1 \cdot D_1 + E_1 \cdot \frac{\beta}{1-\beta\cdot(1-\delta)} \cdot \left[\psi^{11} \cdot \left(\frac{s_1^1 \cdot B_1 + s_2^1 \cdot B_2}{a^1}\right)\right]$$

$$p^1 = y_2^1 \cdot D_2 + E_2 \cdot \frac{\beta}{1-\beta\cdot(1-\delta)} \cdot \left[\psi^{11} \cdot \left(\frac{s_1^1 \cdot B_1 + s_2^1 \cdot B_2}{a^1}\right)\right]$$

$$p^2 = y_1^2 \cdot D_1 + E_1 \cdot \frac{\beta}{1-\beta\cdot(1-\delta)} \cdot \left[\psi^{22} \cdot \left(\frac{s_1^2 \cdot B_1 + s_2^2 \cdot B_2}{a^2}\right)\right]$$

$$p^2 = y_2^2 \cdot D_2 + E_2 \cdot \frac{\beta}{1-\beta\cdot(1-\delta)} \cdot \left[\psi^{22} \cdot \left(\frac{s_1^2 \cdot B_1 + s_2^2 \cdot B_2}{a^2}\right)\right]$$

♦ *Quadratize the objective function around the steady state*

The procedure is the same one used for level I, except that the number of variables is smaller.

♦ *Solve for the value function of the Bellman's equation*

The way to solve for V^1 is a bit different because each country takes the pollution of the other as given, so both dynamic programming problems have to be solved simultaneously. The first step to solve those two problems consists in replacing the 5 FOC for countries' own productions, consumptions and price, only then comes the simultaneous part. The problem is that in countries' 1 problem there is no FOC to reduce pollution from country 2 because the former countries does not decide foreign emissions, and viceversa. So, while "reducing" the value function, the FOC for pollution of country 1 (which depends on pollution in country 2) is considered together with the FOC for country 2's pollution (which depends on country 1' s emissions). The idea is that in equilibrium, countries have some expectations about the policy function of the other country. More precisely, the pairs of FOC of the two problems are used to express emissions as functions only of a^1 and a^2. After that, the corresponding results for emissions in country 2 are plugged into country 1's problem and the reverse happens with country 2. Then, pollution FOC are reduced from the two problems, and the same procedure than in level I is used to get the value functions of each country as functions of the state variables.

C. Summary of the results of the simulation

The results of the simulation can be summarized by the steady states, the policy functions for pollution, and the resulting gains from trade and prices.

♦ *The steady states of all the variables*

Table 1 shows that the values of the steady states for all the variables in the case of some domestic policy (II.ii.) are between those of the unregulated equilibrium (II.i.) and those of the overall optimum (I). For example, the state of the resource is better if there is international cooperation than in the other two situations, basically because the pollution allowed is much lower. In addition, the country which has the greater proportion of environmentally oriented consumers and cleaner firms also has the higher resource at the steady state in all circumstances.

Table 1: Steady states for all the variables in the simulation

		Level II Case i.	Case ii.	Level I
Consumption	*country 1/type 1*	2.90474	2.74919	2.70516
	country 1/type 2	3.55023	3.36012	3.30631
	country 2/type 1	2.79078	2.68414	2.58788
	country 2/type 2	3.41096	3.28061	3.16297
Production	*country 1/type 1*	3.09839	2.98934	2.95908
	country 1/type 2	3.09839	2.79978	2.71382
	country 2/type 1	3.22490	3.18165	3.14549
	country 2/type 2	3.22490	3.06739	2.92399
Pollution	*country 1/type 1*	4.64758	2.58837	1.98599
	country 1/type 2	7.74597	5.10380	4.33192
	country 2/type 1	4.83735	3.62992	2.50322
	country 2/type 2	8.06226	6.52593	5.09496
Resource	*country 1*	12.26978	14.66418	15.82395
	country 2	11.89036	14.08565	15.41704

There is a clear (but small) trade-off between production, consumption and a cleaner environment, since the former is lower at level I for both countries and for every type of firm and consumer. Within each country, consumers of type 1 (who care more about the resource) consume less than the other type. Firms produce the same in absence of any regulation, because they are equally efficient and their only difference is the extent to which their technology pollutes. However, once the government begins to regulate them, the firm that is "more dirty" produces less than the one that uses a cleaner technology, reflecting the achievement of the regulation's goal. This fact is reinforced when both countries sign an agreement to lower emissions (level I).

◆ *The policy functions for pollution*

Since the particular interest of this paper is to clarify the difference among different environmental regulatory situations, the focus has to be more on the policy functions for pollution than on those for consumption and production. In an unregulated equilibrium, firms always pollute the same amount, which in the case of this exercise is:

For country 1, firms of type 1: $x_1^1 = 4.64758$
For country 1, firms of type 2: $x_2^1 = 7.74597$
For country 2, firms of type 1: $x_1^2 = 4.83735$
For country 2, firms of type 2: $x_2^2 = 8.06226$

When the government of each country regulates, it will do so according to the level of the resource. Therefore, so it will establish taxes, quotas or permits to induce paths of emission, which in this case are the following:

For country 1, firms of type 1: $x_1^1 = 1.85715 + 0.05078 \cdot a^1 - 0.00095 \cdot a^2$
For country 1, firms of type 2: $x_2^1 = 4.15149 + 0.06612 \cdot a^1 - 0.00123 \cdot a^2$
For country 2, firms of type 1: $x_1^2 = 3.20993 - 0.00087 \cdot a^1 + 0.03073 \cdot a^2$
For country 2, firms of type 2: $x_2^2 = 5.97985 - 0.00112 \cdot a^1 + 0.03993 \cdot a^2$

These policy functions are the result of solving a system of 4 equations, because the direct result that the simulation yields has the policy function for each each type of firm in each country depending on the other one (for example, $x_1^1 = d(a^1, a^2, x_1^2, x_2^2)$, $x_1^2 = d_*(a^1, a^2, x_1^1, x_2^1)$). Each government is less strict in its environmental regulation the better the state of the resource perceived in that country. In addition, if the neighbors do not take care of the resource, then regulations have to be more stringent. This is because the government that makes firms pollute less expects the other country to allow more pollution in the following periods.

If all externalities are internalized through an international agreement (like the one a supranational planner would propose), governments have to adopt a stricter regulation to match the optimal emissions which follow the policy functions derived for level I:

For country 1, firms of type 1: $x_1^1 = 1.06205 + 0.04224 \cdot a^1 + 0.01658 \cdot a^2$
For country 1, firms of type 2: $x_2^1 = 3.14834 + 0.05411 \cdot a^1 + 0.02123 \cdot a^2$
For country 2, firms of type 1: $x_1^2 = 1.69438 + 0.02660 \cdot a^1 + 0.02516 \cdot a^2$
For country 2, firms of type 2: $x_2^2 = 4.06860 + 0.03376 \cdot a^1 + 0.03192 \cdot a^2$

In this case, pollution levels depend positively on the two levels of the resource, because coordination makes each country treat the other's resource in the same way than its own.

♦ *Resulting gains and (implicit) prices*

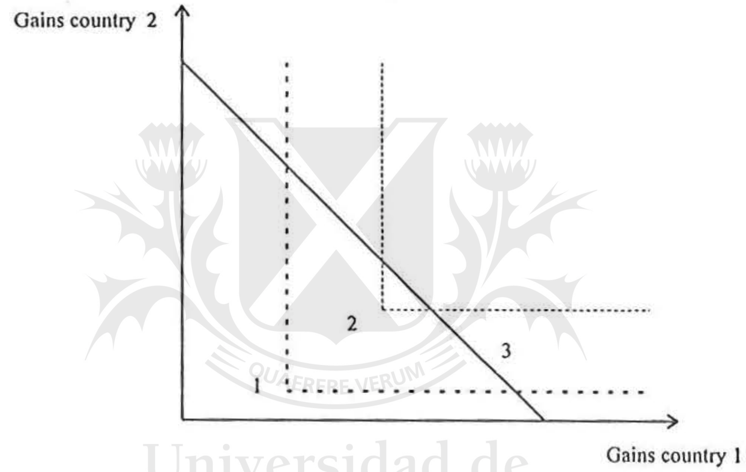
Since the assumption used has been to assume a discount factor very close to 1, it is easy to compare the gains from trade (and the surpluses for the firms and consumers in each country) at the steady state. In both countries, consumers and firms of type 1 (more worried about the environment) benefit from all sorts of environmental regulation. In general, consumers and firms of type 2 are worse off with regulations due to the impact that those have on prices, production and consumption (Table 2).

Table 2: Gains and Surpluses at each Level

	Level II		*Level I*
	Case i.	*Case ii.*	
Country 1	156.26237	160.92172	162.39401
Consumer 1	131.32786	135.28807	137.64071
Consumer 2	54.44291	50.17167	49.15671
Firm 1	126.15536	131.12020	132.34926
Firm 2	49.72644	47.13490	44.02091
Country 2	124.65512	127.56227	127.30102
Consumer 1	42.00000	51.38417	51.65943
Consumer 2	62.00000	50.66548	50.45635
Firm 1	38.00000	55.41454	57.46849
Firm 2	58.00000	55.15346	56.48723
Both countries	280.91749	288.48399	289.69503

Not only does the level I result in a higher resource but also in a higher overall surplus, because it reflects an efficient situation. The differences are not too large due to the values of the parameters chosen, but the fact that the problem is not symmetric creates a particularity in the space of payoffs: country country 2 is better off doing only domestic policy (at the same time that country 1 also does that) than going to a stage of international cooperation. Hence, any such agreement must be accompanied by money transfers from country 1 to country 2, in order to induce the latter to accept the change in its domestic environmental policy (figure 6).

Figure 6: Gains from trade of both countries at the three levels (asymmetric case, country 1 more "environment-oriented")



The value functions for each problem reflect the fact that the intertemporal gains from trade depend on the state of the resource. Hence, in the space of payoffs, the attainable point will shift trough time until the steady state is reached. For example, for level I, the value function is:

$$V = 8441.85161 + (-0.11578)\cdot(a^1)^2 + (-0.07110)\cdot(a^2)^2$$
$$+ 2\cdot(4.77092)\cdot a^1 + 2\cdot(2.83446)\cdot a^2 + 2\cdot(0.00120)\cdot a^1\cdot a^2$$

Finally, prices also change according to each level in a consistent way, in the sense that they are higher at level I (p^1=33.26976 and p^2=34.77747), lower at level II.ii. (p^1=32.73688 and p^2=33.53032), and even lower at level II.i. (p^1=30.98387 and p^2=32.24903). Prices are lower in the country in which consumers are more environmentally oriented and firms are cleaner.

V. Summary and Conclusions

This paper provides a deeper analysis of international negotiations than other models in the literature of dynamic games applied to environmental economics, since it considers that governments may already be engaged in domestic environmental regulation. Moreover, the inclusion of this government policy goes a step further, because countries are modeled as spaces that also contain consumers and firms.

The model delineates some possible circumstances in which an international environmental negotiation may begin. It may be that all countries involved are doing domestic environmental policy, and so they tighten their regulation in order to take into account the harm that they impose on each other. It may also happen that only some countries are previously engaged in some environmental policy, or that none of them have actual policies in effect. The occurrence of each of these cases limits in one way or another the full range of possibilities of agreements among countries. The framework utilized is helpful to conceptualize the need for transfers in some circumstances (for example when one of the countries is populated by consumers and firms that are more environmentally oriented than the other). Then, both domestic policies and domestic agents' characteristics influence the possible outcomes from cooperation.

The numerical simulation for a renewable resource in a linear-quadratic framework allows for the derivation of policy trajectories for pollution which result in different equilibria. The aggregate pollution allowed to each country and the intertemporal surpluses generated in different circumstances depend on the state of the resource.

Further research can enrich this model by incorporating two additional features: a more complete framework which includes information problems and political pressure (both at the national and the international levels), and the possible calibration of the model using parameters derived from an empirical study of a concrete case.

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Appendix A: Computer Program for the numerical simulation

```
new;
library nlsys,pgraph,user;
output file=c:\gauss\disser1.out reset;
format 9,5;

@PART 1) INFORMATION NECESSARY FOR THE SIMULATION@

@ Setting the dimensions of the problem @
        np=2; @number of countries sharing the resource@
        nc=2; @number of types of consumers in each country@
        nf=2; @number of types of firms in each country@
@ The following functional forms are used:
         vih=[Ah*log cih+Bh*log ai] and TCif=(Df/2)*cif^2-(1/2)*(Ef*cif-xif)^2@
@ Parameter Values :@
  beta=.99;
  pva=zeros(nc,1);pva[1:nc,1]=seqa(90,20,nc);  @A for all consumers@
  pvb=zeros(nc,1);pvb[1:nc,1]=seqa(50,-40,nc); @B for each type of consumers@
  pvd=zeros(nf,1);pvd[1:nf,1]=1./seqa(10,0,nf); @1/D for each type of firms@
  pve=zeros(nf,1);pve[1:nf,1]=seqa(1.5,1,nf);  @E for each type of firms@
  massc=zeros(nc,nc);
  massc[1,1]=.7;massc[2,2]=.7;        @% people of each type in each country@
  massc[1,2]=.3;massc[2,1]=.3;
  massf=zeros(nf,nf);
  massf[1,1]=.7;massf[2,2]=.7;        @% firms of each type in each country@
  massf[1,2]=.3;massf[2,1]=.3;
  abar=20;                            @pristine state of the resource@
  delta=.4;                           @natural decay of pollutants@
  psi=zeros(np,np); @transport matrix@
  psi[1,1]=.3;psi[2,2]=.3;          @effect of own country pollution@
  psi[1,2]=.2;psi[2,1]=.2;          @effect of other country pollution@
format 9,2; "PARAMETER VALUES USED IN THE SIMULATION";
"As";pva;"Bs";pvb;"1/Ds";pvd;"Es";pve;
"Mass consumers (types/country)";massc;
"Mass firms (types/country)";massf;
"Transport matrix between countries (emitter/receptor)";psi;
"beta";beta;"abar";abar;"delta";delta;


@PART 2) LEVEL I: SUPRA-NATIONAL AUTHORITY@

@ Step 1. The model @
ns=3;  @Number of state variables (1,a1,a2)@
nd=14; @Number of decision variables (c11,c12,c21,c22,y11,y12,y21,y22,x11,x12,x21,x22,p1,p2)@

@ Step 2: Procedure that defines the return function:
 w[1,1]=a1; w[2,1]=a2; w[3,1]=p1; w[4,1]=p2; w[5,1]=c11; w[6,1]=c12;
 w[7,1]=c21; w[8,1]=c22; w[9,1]=y11; w[10,1]=y12; w[11,1]=y21; w[12,1]=y22;
 w[13,1]=x11; w[14,1]=x12; w[15,1]=x21; w[16,1]=x22@
```

```
proc retf(w);
local c1,c2,c3,c31,c4,c41,r1,r2,r3,r4,r;
 c1=ln(w[5,1]|w[6,1]); r1=massc[.,1].*(pva.*c1+pvb*ln(w[1,1])-w[3,1]*(w[5,1]|w[6,1]));
 c2=ln(w[7,1]|w[8,1]);  r2=massc[.,2].*(pva.*c2+pvb*ln(w[2,1])-w[4,1]*(w[7,1]|w[8,1]));
 c3=(w[9,1]|w[10,1]); c31=(w[13,1]|w[14,1]);
 r3=massf[.,1].*(w[3,1]*c3-(.5/pvd).*(c3^2)-(.5)*((pve.*c3-c31)^2));
 c4=(w[11,1]|w[12,1]); c41=(w[15,1]|w[16,1]);
 r4=massf[.,2].*(w[4,1]*c4-(.5/pvd).*(c4^2)-(.5)*((pve.*c4-c41)^2));
 r=r1[1,1]+r1[2,1]+r2[1,1]+r2[2,1]+r3[1,1]+r3[2,1]+r4[1,1]+r4[2,1];
retp(r);
endp;

@ Step 3: Compute steady state values of w which will be used to form
a quadratic approximation of the return function.@
proc reso(m);
local b,K1,K2,p1,p2,y1,y2,x1,x2,xx,xxa,xxa1,xxa2,xx1,xx2,eqns; b=beta/(1-beta*(1-delta));
 K1=b*(((psi[1,1]*massc[.,1]'*pvb)/m[1,1])+((psi[1,2]*massc[.,2]'*pvb)/m[2,1]));
 K2=b*(((psi[2,1]*massc[.,1]'*pvb)/m[1,1])+((psi[2,2]*massc[.,2]'*pvb)/m[2,1]));
 y1=(m[3,1]*pvd)-(pvd.*pve*K1); @production both types of firms country 1@
 y2=(m[4,1]*pvd)-(pvd.*pve*K2); @production both types of firms country 2@
 x1=(pve.*y1)-K1; x2=(pve.*y2)-K2;
 xx=x1~x2;xxa=massf.*xx; xxa1=xxa*psi[.,1]; xxa2=xxa*psi[.,2];
 xx1=xxa1[1,.]+xxa1[2,.]; xx2=xxa2[1,.]+xxa2[2,.];
 eqns=zeros(4,1);
 eqns[1,1]=m[1,1]*delta-abar*delta+xx1;eqns[2,1]=m[2,1]*delta-abar*delta+xx2;
 eqns[3,1]=(m[3,1]^2)*massf[.,1]'*pvd-m[3,1]*(massf[.,1]'*(pve.*pvd*k1))-massc[.,1]'*pva;
 eqns[4,1]=(m[4,1]^2)*massf[.,2]'*pvd-m[4,1]*(massf[.,2]'*(pve.*pvd*k2))-massc[.,2]'*pva;
 retp(eqns);
 endp;
{wp,ttf,ttj,ttrc}=nlsys(&reso,10|10|40|40);
if ttrc>1; "Return code is";;ttrc;;endif;

ws=zeros(ns+nd-1,1); ws[1,1]=wp[1,1]; @a1@ ws[2,1]=wp[2,1];  @a2@
b=beta/(1-beta*(1-delta));
K1=b*(((psi[1,1]*massc[.,1]'*pvb)/ws[1,1])+((psi[1,2]*massc[.,2]'*pvb)/ws[2,1]));
K2=b*(((psi[2,1]*massc[.,1]'*pvb)/ws[1,1])+((psi[2,2]*massc[.,2]'*pvb)/ws[2,1]));
p1I=wp[3,1];p2I=wp[4,1];c1I=pva./p1I;c2I=pva./p2I;
y1I=(p1I*pvd)-(pve*K1.*pvd);y2I=(p2I*pvd)-(pve*K2.*pvd);
x1I=(pve.*y1I)-K1;x2I=(pve.*y2I)-K2;
ws[5,1]=c1I[1,1];              @Consumption ss@
ws[6,1]=c1I[2,1];ws[7,1]=c2I[1,1];ws[8,1]=c2I[2,1];
ws[9,1]=y1I[1,1];              @Production ss@
ws[10,1]=y1I[2,1];ws[11,1]=y2I[1,1];ws[12,1]=y2I[2,1];
ws[13,1]=x1I[1,1];             @Pollution ss@
ws[14,1]=x1I[2,1];ws[15,1]=x2I[1,1];ws[16,1]=x2I[2,1];
ws[3,1]=p1I;ws[4,1]=p2I;      @Implicit prices@
ssI=ws;
Print "STEADY STATES AT LEVEL I (by calculation)";ssI;
```

```
@ Step 4: Define a matrix containing the laws of motion for the state variables@
cI=zeros(ns,2*ns+nd);
cI[1,1]=1;cI[2,1]=abar*delta;cI[3,1]=abar*delta;
cI[2,2]=(1-delta);cI[3,3]=(1-delta);
cI[2,14]=-psi[1,1]*massf[1,1]; cI[2,15]=-psi[1,1]*massf[1,2];
cI[2,16]=-psi[2,1]*massf[2,1]; cI[2,17]=-psi[2,1]*massf[2,2];
cI[3,14]=-psi[1,2]*massf[1,1]; cI[3,15]=-psi[1,2]*massf[1,2];
cI[3,16]=-psi[2,2]*massf[2,1]; cI[3,17]=-psi[2,2]*massf[2,2];format 9,3;"matrix c at level I" cI;

@ Step 5: Compute quadratic approximation of return function. @
qI=quad(ws,0.0001,&retf);"qI" qI;

@ Step 6:  Solve dynamic program.  @
test=10;n=ns+nd;v=eye(ns)*(-.0001); format /rd 8,5;
iter=0;
do until test lt 1E-8 or iter>1000;
    tv=qI~zeros(n,ns)|zeros(ns,n)~(v*beta);nv=rows(tv);
   @Reduce out laws of motion for state variables (l',a')@
    i=1;
    do until i>ns;
        tv=reduce(tv,cI[ns-i+1,1:nv-i]);
        i=i+1;
    endo;
    if iter<1;format 9,3;print "tv after reduce resources constraints" tv;endif;
  @Reduce out first order conditions@
   dsave=tv[ns+1:n,1:n];
    i=1;
    do until i>nd;
        tv=reduce(tv,-tv[n-i+1,1:n-i]./tv[n-i+1,n-i+1]);
    if iter<1;format 9,3;print "tv after reduce each of FOC" tv;endif;
        i=i+1;
    endo;
test=abs(tv-v);test[1,1]=0;v=tv;  @will be 3x3@
iter=iter+1;
endo;
format 9,3;"Dynamic program required ";; iter;; "iterations."; ?;format 9,5;"v for level I" v;

@ Step 7: Compute decision rules@
  decis=-dsave[.,1:ns]/dsave[.,ns+1:n];"decis for level I" decis;    @matrix will be 14x17@

@ Step 8: Compute steady states from decision rules and compare with original
steady states (to get ss for a have to use the law of motion of the resource)@
proc ma(a);
 local mam1,mam2,mam,mama,mama1,mama2,pa1,pa2,eqnsa;
 mam1=decis[11:12,.]*(1|a[1,1]|a[2,1]); mam2=decis[13:14,.]*(1|a[1,1]|a[2,1]);
 mam=mam1~mam2; mama=massf.*mam; mama1=mama*psi[.,1]; mama2=mama*psi[.,2];
 pa1=mama1[1,.]+mama1[2,.]; pa2=mama2[1,.]+mama2[2,.];
 eqnsa=zeros(2,1); eqnsa[1,1]=a[1,1]*delta-abar*delta+pa1; eqnsa[2,1]=a[2,1]*delta-abar*delta+pa2;
retp(eqnsa);
endp;
```

```
{ap,ttf,ttj,ttrc}=nlsys(&ma,19|19);
if ttrc>1; "Return code is";;ttrc;;endif;

ssst=1|ap;ssm=decis*ssst;ssm=ssst|ssm;sss=ssm[2:17,.];
format 9,5;"STEADY STATES AT LEVEL I (by approximation) ";;sss;

VI=(ssst'*v*ssst); "Value at level I";;VI;
GFTqI=ssm'*qI*ssm; "Gains from trade with q";GFTqI;
@Calculate the overall gains from trade from the return function@
 h1=ln(sss[5,1]|sss[6,1]);hh1=massc[.,1].*(pva.*h1+pvb*ln(sss[1,1])-sss[3,1]*(sss[5,1]|sss[6,1]));
 h2=ln(sss[7,1]|sss[8,1]);hh2=massc[.,2].*(pva.*h2+pvb*ln(sss[2,1])-sss[4,1]*(sss[7,1]|sss[8,1]));
 h3=(sss[9,1]|sss[10,1]); h33=(sss[13,1]|sss[14,1]);
 hh3=massf[.,1].*(sss[3,1].*h3-(.5/pvd).*(h3^2)-(.5)*((pve.*h3-h33)^2));
 h4=(sss[11,1]|sss[12,1]);h44=(sss[15,1]|sss[16,1]);
 hh4=massf[.,2].*(sss[4,1]*h4-(.5/pvd).*(h4^2)-(.5)*((pve.*h4-h44)^2));
gI1a=hh1[1,1]+hh1[2,1]+hh3[1,1]+hh3[2,1];gI2a=hh2[1,1]+hh2[2,1]+hh4[1,1]+hh4[2,1];
gIa=hh1[1,1]+hh1[2,1]+hh2[1,1]+hh2[2,1]+hh3[1,1]+hh3[2,1]+hh4[1,1]+hh4[2,1];
"Gains from trade level I for both countries and the sum"; gI1a;gI2a;gIa;
"CS and PS at level I";
(pva.*h1+pvb*ln(sss[1,1])-sss[3,1]*(sss[5,1]|sss[6,1]));
(pva.*h2+pvb*ln(sss[2,1])-sss[4,1]*(sss[7,1]|sss[8,1]));
(sss[3,1].*h3-(.5/pvd).*(h3^2)-(.5)*((pve.*h3-h33)^2));
(sss[4,1].*h4-(.5/pvd).*(h4^2)-(.5)*((pve.*h4-h44)^2));


@PART 3) LEVEL II (UNREGULATED EQUILIBRIUM)@

@Calculate all variables steady states in each country@
  p1IIU=((massc[.,1]'*pva)/(massf[.,1]'*pvd))^(1/2);
  p2IIU=((massc[.,2]'*pva)/(massf[.,2]'*pvd))^(1/2);
  y1IIU=p1IIU*pvd; @production both types of firms country 1@
  y2IIU=p2IIU*pvd; @production both types of firms country 2@
  x1IIU=pve.*y1IIU; x2IIU=pve.*y2IIU;c1IIU=pva/p1IIU; c2IIU=pva/p2IIU;
  abaro=seqa(abar,0,np);  @the water completely clear@
  xxIIU=abaro; xxfIIuIIU=x1IIU~x2IIU;
  xxaIIU=massf.*xxfIIuIIU; xxa1IIU=xxaIIU*psIIu[.,1];xx1IIU=xxa1IIU[1,.]+xxa1IIU[2,.];
  xxa2IIU=xxaIIU*psIIu[.,2]; xx2IIU=xxa2IIU[1,.]+xxa2IIU[2,.]; xxrIIU=xx1IIU|xx2IIU;
  asIIU=abaro-(xxrIIU/delta);
  ssIIU=asIIU|p1IIU|p2IIU|c1IIU|c2IIU|y1IIU|y2IIU|x1IIU|x2IIU;

format 9,5;
Print "STEADY STATES AT LEVEL IIU";ssIIU;

ga1IIU=(pva.*ln(c1IIU)+pvb*ln(asIIU[1,1])-p1IIU*c1IIU).*massc[.,1]
          +(p1IIU*c1IIU-(1/pvd)*(.5).*(y1IIU^2)-(.5)*((pve.*y1IIU-x1IIU)^2)).*massf[.,1];
gIIU1=ga1IIU[1,1]+ga1IIU[2,1];
ga2IIU=(pva.*ln(c2IIU)+pvb*ln(asIIU[2,1])-p2IIU*c2IIU).*massc[.,2]
          +(p2IIU*c2IIU-(1/pvd)*(.5).*(y2IIU^2)-(.5)*((pve.*y2IIU-x2IIU)^2)).*massf[.,2];
gIIU2=ga2IIU[1,1]+ga2IIU[2,1];gIIU=gIIU1+gIIU2;
"Gains from trade for country 1 at level IIU";gIIU1;"Gains from trade for country 2 at level IIU";gIIU2;
```

```
"Sum of gains from trade at level IIU";gIIU;
"CS and PS at level IIU"
(pva.*ln(c1IIU)+pvb*ln(asIIU[1,1])-p1IIU*c1IIU);(pva.*ln(c2IIU)+pvb*ln(asIIU[2,1])-p2IIU*c2IIU);
(p1IIU.*c1IIU-(1/pvd)*(.5).*(y1IIU^2)-(.5)*((pve.*y1IIU-x1IIU)^2));
(p2IIU.*c2IIU-(1/pvd)*(.5).*(y2IIU^2)-(.5)*((pve.*y2IIU-x2IIU)^2));


@PART 4) LEVEL II: ONLY DOMESTIC ENVIRONMENTAL POLICY@

@Step 1: Define the number of variables@
nss=3; @(1,a are the states,then the decision of the other country xj)@
ndd=7; @c for each type,y for each type,x for each type,p1@

@ Step 2: Procedure that defines the return functions@
 t1[1,1]=a1; t1[2,1]=a2; t1[3,1]=x21; t1[4,1]=x22; t1[5,1]=x11; t1[6,1]=x12;
  t1[7,1]=p1; t1[8,1]=c11; t1[9,1]=c12; t1[10,1]=y11; t1[11,1]=y12;
 t2[1,1]=a1; t2[2,1]=a2; t2[3,1]=x11; t2[4,1]=x12; t2[5,1]=x21; t2[6,1]=x22;
  t2[7,1]=p2; t2[8,1]=c21; t2[9,1]=c22; t2[10,1]=y21; t2[11,1]=y22

  proc retf1(t1);
  local c1,c3,c31,r1,r3,rr;
  c1=ln(t1[8,1]|t1[9,1]);
  r1=massc[.,1].*(pva.*c1+pvb*ln(t1[1,1])-t1[7,1]*(t1[8,1]|t1[9,1]));
  c3=(t1[10,1]|t1[11,1]); c31=(t1[5,1]|t1[6,1]);
  r3=massf[.,1].*(t1[7,1]*c3-(.5/pvd).*(c3^2)-(.5)*((pve.*c3-c31)^2));
  rr=r1[1,1]+r1[2,1]+r3[1,1]+r3[2,1];
 retp(rr);
endp;

  proc retf2(t2);
  local c2,c4,c41,r2,r4,rrr;
  c2=ln(t2[8,1]|t2[9,1]);
  r2=massc[.,2].*(pva.*c2+pvb*ln(t2[2,1])-t2[7,1]*(t2[8,1]|t2[9,1]));
  c4=(t2[10,1]|t2[11,1]); c41=(t2[5,1]|t2[6,1]);
  r4=massf[.,2].*(t2[7,1]*c4-(.5/pvd).*(c4^2)-(.5)*((pve.*c4-c41)^2));
  rrr=r2[1,1]+r2[2,1]+r4[1,1]+r4[2,1];
 retp(rrr);
endp;

@ Step 3: Compute steady state values of y which will be used to form
    a quadratic approximation of the return functions. @
t1s=zeros(nss+ndd+1,1); t2s=zeros(nss+ndd+1,1);

proc resol1(m1);
local b,K1,K2,p1,p2,y1,y2,x1,x2,xx,xxa,xxa1,xxa2,xx1,xx2,l,eqns1;
b=beta/(1-beta*(1-delta));
K1=b*((psi[1,1]*massc[.,1]'*pvb)/m1[1,1]);K2=b*((psi[2,2]*massc[.,2]'*pvb)/m1[2,1]);
y1=(m1[3,1]*pvd)-(pvd.*pve*K1); @production both types of firms country 1@
y2=(m1[4,1]*pvd)-(pvd.*pve*K2); @production both types of firms country 2@
x1=(pve.*y1)-K1; x2=(pve.*y2)-K2;
```

```
xx=x1~x2;xxa=massf.*xx; xxa1=xxa*psi[.,1]; xxa2=xxa*psi[.,2];
xx1=xxa1[1,.]+xxa1[2,.]; xx2=xxa2[1,.]+xxa2[2,.];
eqns1=zeros(4,1);
eqns1[1,1]=m1[1,1]*delta-abar*delta+xx1;eqns1[2,1]=m1[2,1]*delta-abar*delta+xx2;
eqns1[3,1]=(m1[3,1]^2)*massf[.,1]'*pvd-m1[3,1]*(massf[.,1]'*(pve.*pvd*k1))-massc[.,1]'*pva;
eqns1[4,1]=(m1[4,1]^2)*massf[.,2]'*pvd-m1[4,1]*(massf[.,2]'*(pve.*pvd*k2))-massc[.,2]'*pva;
retp(eqns1);
endp;
{ts,ttf,ttj,ttrc}=nlsys(&resol1,15|15|30|30);
if ttrc>1; "Return code is";;ttrc;;endif;

t1s[1,1]=ts[1,1]; t2s[1,1]=t1s[1,1]; @a1@
t1s[2,1]=ts[2,1]; t2s[2,1]=ts[2,1]; @a2@
b=beta/(1-beta*(1-delta));
K1s=b*((psi[1,1]*massc[.,1]'*pvb)/ts[1,1]);K2s=b*((psi[2,2]*massc[.,2]'*pvb)/ts[2,1]);
t1s[7,1]=ts[3,1]; @p1@ t2s[7,1]=ts[4,1]; @p2@
c1s=pva./t1s[7,1]; c2s=pva./t2s[7,1];
y1s=(t1s[7,1]*pvd)-(pve*K1s.*pvd); y2s=(t2s[7,1]*pvd)-(pve*K2s.*pvd);
x1s=(pve.*y1s)-K1s; x2s=(pve.*y2s)-K2s;
t1s[8,1]=c1s[1,1];t1s[9,1]=c1s[2,1];t2s[8,1]=c2s[1,1];t2s[9,1]=c2s[2,1];@consumption ss@
t1s[10,1]=y1s[1,1];t1s[11,1]=y1s[2,1];t2s[10,1]=y2s[1,1];t2s[11,1]=y2s[2,1];@production ss@
t1s[5,1]=x1s[1,1];t2s[3,1]=x1s[1,1];t1s[6,1]=x1s[2,1];t2s[4,1]=x1s[2,1];@pollution ss@
t1s[3,1]=x2s[1,1];t2s[5,1]=x2s[1,1];t1s[4,1]=x2s[2,1];t2s[6,1]=x2s[2,1];
ssIIR1=t1s;ssIIR2=t2s;
format 9,5;
"STEADY STATES AT LEVEL II FOR COUNTRY 1 (by calculations)";ssIIR1;
"STEADY STATES AT LEVEL II FOR COUNTRY 2 (by calculations)";ssIIR2;

@Step 4: Define the constraint matrices@
cc1=zeros(nss,2*nss+ndd+2);     @add 1 for xj@ cc2=zeros(nss,2*nss+ndd+2);
cc1[1,1]=1;cc1[2,1]=abar*delta;cc1[2,2]=(1-delta);

cc1[2,4]=-psi[2,1]*massf[2,1];cc1[2,5]=-psi[2,1]*massf[2,2];
cc1[2,6]=-psi[1,1]*massf[1,1];cc1[2,7]=-psi[1,1]*massf[1,2];
cc1[3,1]=abar*delta;cc1[3,3]=(1-delta);
cc1[3,4]=-psi[2,2]*massf[2,1];cc1[3,5]=-psi[2,2]*massf[2,2];
cc1[3,6]=-psi[1,2]*massf[1,1];cc1[3,7]=-psi[1,2]*massf[1,2];

cc2[1,1]=1;cc2[2,1]=abar*delta;cc2[2,2]=(1-delta);
cc2[2,6]=-psi[2,1]*massf[2,1];cc2[2,7]=-psi[2,1]*massf[2,2];
cc2[2,4]=-psi[1,1]*massf[1,1];cc2[2,5]=-psi[1,1]*massf[1,2];
cc2[3,1]=abar*delta;cc2[3,3]=(1-delta);
cc2[3,6]=-psi[2,2]*massf[2,1];cc2[3,7]=-psi[2,2]*massf[2,2];
cc2[3,4]=-psi[1,2]*massf[1,1];cc2[3,5]=-psi[1,2]*massf[1,2];
format 9,3;
"Constraint for country 1 at level II";;cc1;
"Constraint for country 2 at level II";;cc2;

@ Step 5: Compute quadratic approximation of return function. @
q1=quad(t1s,0.000001,&retf1);q2=quad(t2s,0.000001,&retf2);format 9,3;"q1" q1;"q2" q2;
```

```
@ Step 6: Solve dynamic program. @
test1=10;test2=10;
nn=nss+ndd;
v1=eye(nss)*(-.0001); format /rd 8,5;v2=eye(nss)*(-.0001); format /rd 8,5;
iter1=0;
do until test1 lt 1E-10 or test2 lt 1E-10 or iter1>1000;
tv1=q1~zeros(nn+2,nss)|zeros(nss,nn+2)~(v1*beta);tv2=q2~zeros(nn+2,nss)|zeros(nss,nn+2)~(v2*beta);
nvv1=rows(tv1); nvv2=rows(tv2);
if iter1<1;format 9,3;"original full tv1";tv1;endif;if iter1<1;format 9,3;"original full tv2";tv2;endif;

@Reduce out laws of motion for state variable.@
   i=1;
   do until i>nss;
      tv1=reduce(tv1,cc1[nss-i+1,1:nvv1-i]); tv2=reduce(tv2,cc2[nss-i+1,1:nvv2-i]);
      i=i+1;
   endo;
if iter1<1;format 9,3;"partial tv1 after reduce law of motions of a";tv1;endif;
if iter1<1;format 9,3;"partial tv2 after reduce law of motions of a";tv2;endif;

@Reduce out first order conditions for consumption and production@
   dsave1=tv1[nss+3:nn+2,1:nn+2];dsave2=tv2[nss+3:nn+2,1:nn+2];
   i=1;
   do until i>5;
      d1=-tv1[nn-i+3,1:nn-i+2]./tv1[nn-i+3,nn-i+3]; tv1=reduce(tv1,d1);
      i=i+1;
   endo;
   i=1;
   do until i>5;
      d2=-tv2[nn-i+3,1:nn-i+2]./tv2[nn-i+3,nn-i+3]; tv2=reduce(tv2,d2);
      i=i+1;
   endo;
if iter1<1;format 9,3;"partial tv1 after reduce FOC of c and y";tv1;endif;
if iter1<1;format 9,3;"partial tv2 after reduce FOC of c and y";tv2;endif;

@Solve x11,x12,x21,x22 as fn of a1 and a2 in order to be able to reduce both
variables for which one of the countries has no FOC because no control on them@
sis=zeros(4,7);sis[1,.]=tv1[6,.];sis[2,.]=tv1[7,.];sis[3,.]=tv2[6,.];sis[4,.]=tv2[7,.];
@rearranging because diff order x var@
auxi=zeros(2,2);auxi[1:2,1:2]=sis[1:2,4:5];sis[1:2,4:5]=sis[1:2,6:7];sis[1:2,6:7]=auxi[1:2,1:2];
if iter<1;format 9,5;print sis;endif;
siss=-sis[.,1:3]/sis[.,4:7];msave=siss[.,.];
if iter1<1;format 9,5;"solving x1 and x2 as function of a (iter1)";;siss;endif;
tv1[4,4]=-1;tv1[4,5:7]=zeros(1,3);tv1[5,4:7]=zeros(1,4);
tv1[5,5]=-1;tv2[4,4]=-1;tv2[4,5:7]=zeros(1,3);tv2[5,4:7]=zeros(1,4);
tv2[5,5]=-1;tv1[4:5,1:3]=siss[3:4,.];tv2[4:5,1:3]=siss[1:2,.];
if iter1<1;format 9,3;"partial tv1 before reduce x1 and x2 cont";tv1;endif;
if iter1<1;format 9,3;"partial tv2 before reduce x1 and x2 cont";tv2;endif;

@Reduce the controlled x's as usual FOC@
```

```
  i=1;
  do until i>2;
   d11=-tv1[ndd-i+1,1:ndd-i]./tv1[ndd-i+1,ndd-i+1]; tv1=reduce(tv1,d11);
   i=i+1;
  endo;
  i=1;
  do until i>2;
    d21=-tv2[ndd-i+1,1:ndd-i]./tv2[ndd-i+1,ndd-i+1];
    tv2=reduce(tv2,d21);
     i=i+1;
  endo;
if iter1<1;format 9,3;"partial tv1 before reduce x1 and x2 uncont";tv1;endif;
if iter1<1;format 9,3;"partial tv2 before reduce x1 and x2 uncont";tv2;endif;

@Reduce the uncontrolled x's @
   i=1;
   do until i>2;
      d12=-tv1[ndd-2-i+1,1:ndd-2-i]./tv1[ndd-2-i+1,ndd-2-i+1];
      d22=-tv2[ndd-2-i+1,1:ndd-2-i]./tv2[ndd-2-i+1,ndd-2-i+1];
      tv1=reduce(tv1,d12); tv2=reduce(tv2,d22);
      i=i+1;
   endo;
if iter1<1;format 9,3;"partial tv1 before reduce x1 and x2 cont";tv1;endif;
if iter1<1;format 9,3;"partial tv2 before reduce x1 and x2 cont";tv2;endif;

test1=abs(tv1-v1); test1[1,1]=0;test2=abs(tv2-v2); test2[1,1]=0;
v1=tv1;v2=tv2;
iter1=iter1+1;
endo; format 9,1;
"Dynamic program required ";; iter1;; "iterations."; ?;
format 9,5;
"v1";tv1;"v2";tv2;

@ Step 7. Compute decision rules. @
"dsave1" dsave1; "dsave2" dsave2;
decis1=-dsave1[.,1:nss+2]/dsave1[.,nss+3:nn+2];decis2=-dsave2[.,1:nss+2]/dsave2[.,nss+3:nn+2];
format 9,5;"decis1" decis1; "decis2" decis2;
@Auxiliary calculations for pollution policy functions to check@
car=zeros(4,7);car[1:2,1:3]=decis1[1:2,1:3];car[1:2,6:7]=decis1[1:2,4:5];car[3:4,1:5]=decis2[1:2,.];
aux=car[.,1:3]/(eye(4)-car[.,4:7]);
print "Policy fn of pollution (at the end)"; aux;
print "Policy fn of pollution (intermediate)";siss;

@ Step 8. Compute steady states from decision rules and compare
 with original steady states (use law of motion for a)@
proc ger(fp);
local mam1,mam2,mam,mama,mama1,mama2,pa1,pa2,eqnsf;
mam1=decis1[1:2,.]*(1|fp[1,1]|fp[2,1]|fp[5,1]|fp[6,1]);
mam2=decis2[1:2,.]*(1|fp[1,1]|fp[2,1]|fp[3,1]|fp[4,1]);
mam=mam1~mam2;mama=massf.*mam;mama1=mama*psi[.,1];mama2=mama*psi[.,2];
```

```
pa1=mama1[1,.]+mama1[2,.];pa2=mama2[1,.]+mama2[2,.];
eqnsf=zeros(6,1);
eqnsf[1,1]=fp[1,1]*delta-abar*delta+pa1;eqnsf[2,1]=fp[2,1]*delta-abar*delta+pa2;
eqnsf[3,1]=fp[3,1]-decis1[1,.]*(1|fp[1,1]|fp[2,1]|fp[5,1]|fp[6,1]);
eqnsf[4,1]=fp[4,1]-decis1[2,.]*(1|fp[1,1]|fp[2,1]|fp[5,1]|fp[6,1]);
eqnsf[5,1]=fp[5,1]-decis2[1,.]*(1|fp[1,1]|fp[2,1]|fp[3,1]|fp[4,1]);
eqnsf[6,1]=fp[6,1]-decis2[2,.]*(1|fp[1,1]|fp[2,1]|fp[3,1]|fp[4,1]);
retp(eqnsf);
endp;

{fpp,ttf,ttj,ttrc}=nlsys(&ger,19|19|2|2|2|2);
if ttrc>1; "Return code is";;ttrc;;endif;

ssst1=1|fpp[1:2,1]|fpp[5:6,1];ss11=decis1*ssst1;ss1=ssst1[2:5,.]|ss11;
format 9,5;"STEADY STATE AT LEVEL II FOR COUNTRY 1";;ss1;
ssst2=1|fpp[1:2,1]|fpp[3:4,1];ss22=decis2*ssst2;ss2=ssst2[2:5,.]|ss22;
format 9,5;"STEADY STATE AT LEVEL II FOR COUNTRY 2";;ss2;

VIIR1a=(1|ss1[1:2,.])'*v1*(1|ss1[1:2,.]);"Value at level II for country 1";VIIR1a;
VIIR2a=(1|ss2[1:2,.])'*v2*(1|ss2[1:2,.]); "Value at level II for country 2";VIIR2a;
GFTqIIR1a=(1|ss1)'*q1*(1|ss1);"Gains from trade with q1";GFTqIIR1a;
GFTqIIR2a=(1|ss2)'*q2*(1|ss2);"Gains from trade with q2";GFTqIIR2a;
c1=ln(t1s[8,1]|t1s[9,1]);
r1=massc[.,1].*(pva.*c1+pvb*ln(t1s[1,1])-t1s[7,1]*(t1s[8,1]|t1s[9,1]));
c3=(t1s[10,1]|t1s[11,1]); c31=(t1s[5,1]|t1s[6,1]);
r3=massf[.,1].*(t1s[7,1]*c3-(.5/pvd).*(c3^2)-(.5)*((pve.*c3-c31)^2));
rr=r1[1,1]+r1[2,1]+r3[1,1]+r3[2,1];
c2=ln(t2s[8,1]|t2s[9,1]);
r2=massc[.,2].*(pva.*c2+pvb*ln(t2s[2,1])-t2s[7,1]*(t2s[8,1]|t2s[9,1]));
c4=(t2s[10,1]|t2s[11,1]); c41=(t2s[5,1]|t2s[6,1]);
r4=massf[.,2].*(t2s[7,1]*c4-(.5/pvd).*(c4^2)-(.5)*((pve.*c4-c41)^2));
rrr=r2[1,1]+r2[2,1]+r4[1,1]+r4[2,1];
gIIR1a=rr;"Gains from trade at level II country 1";gIIR1a;
gIIR2a=rrr;"Gains from trade at level II country 2";gIIR2a;
"CS and PS at level II"
(pva.*c1+pvb*ln(t1s[1,1])-t1s[7,1]*(t1s[8,1]|t1s[9,1]));
(pva.*c2+pvb*ln(t2s[2,1])-t2s[7,1]*(t2s[8,1]|t2s[9,1]));
(t1s[7,1].*c3-(.5/pvd).*(c3^2)-(.5)*((pve.*c3-c31)^2));
(t2s[7,1].*c4-(.5/pvd).*(c4^2)-(.5)*((pve.*c4-c41)^2));
end;
```