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Departamento de Economía

Maestría en Economía

Inflation Expectations Under Public Information Uncertainty: The Role of Information Sets

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"Expectativas de Inflación Bajo Incertidumbre en la Calidad de la Información Pública: El Rol de los Conjuntos de Información."

Resumen

El trabajo presenta un modelo monetario de generaciones superpuestas de aprendizaje adaptativo con dos características principales: una fricción informativa y un mapeo entre varios conjuntos de información y el mecanismo de aprendizaje. La fricción informativa se introduce para capturar situaciones en las que los individuos no tienen certeza sobre el estado actual del nivel general de precios -solo observan una submuestra de todos los precios- pero necesitan formar expectativas sobre la evolución de los precios futuros. El mapeo describe la forma en que los individuos deciden su conjunto de información relevante, que luego utilizan para formar expectativas de inflación o, más precisamente, cómo forman percepciones sobre el estado actual del nivel general de precios. La predicción principal del modelo es que la información idiosincrásica sobre los precios afecta las expectativas de inflación, especialmente cuando la información pública no es totalmente creíble o es inexacta. Aplico el modelo para explicar una característica relevante de las expectativas en Argentina (2007-2014): divergencia sistemática entre los grupos de ingresos.

Palabras clave: Aprendizaje, percepciones y expectativas de inflación, fricciones informativas, información engañosa.

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Abstract

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Keywords: Learning, inflation perceptions and expectations, informational frictions, misleading information.

Códigos JEL: E31, D83, D84.

Inflation Expectations Under Public Information Uncertainty: The Role of Information Sets*

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I Introduction

How do individuals form inflation expectations under public information uncertainty? This question constitute an overriding concern when trying to understand how individuals form inflation expectations because uncertainty may affect their perceptions about the current state of the economy, which may successively affect their beliefs about future prices.

In this paper I consider a specific public information uncertainty: the general price level is a hidden state variable and individuals receive signals from public agencies which are much more informed than them. Public information may (or may not) be reliable. Additionally, individuals observe a subsample of all prices (idiosyncratic signal) which may be a biased signal of the state variable. In particular, I analyze the role of information sets (multiple public signals) and how idiosyncratic information may be used to identify which public data to use in forming inflation expectations.

The paper develops an overlapping generation (OLG) monetary model of adaptive learning (AL) with two main features: an informational friction and a mapping between information sets and the learning mechanism. The informational friction is introduced to account for the uncertainty about the general price level; the mapping represents the way in which individuals decide their set of relevant information, or more precisely, the way in which they form perceptions about the current state of the economy.

The main prediction of the model is that idiosyncratic information on prices affects inflation expectations when public information is either not fully reliable or not fully credible. I also apply the model to show that it can account for a relevant feature of expectations in Argentina (2007-2014): systematically divergence between some demographic groups.

The model contributes to the literature on AL by taking into account the uncertainty that arises from the imperfect knowledge of previous endogenous variables. Standard models in learning macroeconomic literature assume that agents learn the true parameters of the data generating process by relying on past data. Realization of endogenous variables is assumed to be fully known, or at least with some degree of noise (non-systematic errors); here, I explicitly consider the possibility of biased information (systematic errors).

Marcet and Sargent (1989a) first studies the effect of hidden state variables and private information on a learning model. However, while in their paper the price in the other market is a relevant signal of the price in one's own market, I assume here that individuals use their own consumption prices to extract information about prices in distant markets[1].

In a more general context, the paper contributes to the understanding of how inflation expectations are form. This literature is nourished by different branches of research. From the explanations who consider rational expectations without full information (e.g. Mankiw & Reis, 2002; Sims, 2003; Woodford, 2003), to the ones that consider behavioral explanations (e.g. Malmendier & Nagel, 2016; Madeira & Zafar, 2015), going through explanations that allow for "small" deviations from rational expectations (e.g. Marcet & Nicolini, 2003; Baqaee, 2019). While the paper is part of the behavioral explanations, its emphasize the role of public information uncertainty and idiosyncratic information, and is linked to the contributions of Cavallo et al. (2016, 2017).

The rest of the paper is organized as follows. Section (II) introduces an OLG monetary model of adaptive learning that explains how individuals form perceptions and expectations about aggregate inflation by using price information of their own consumption basket. The learning equilibrium is characterized in Section (III). Section (IV) expands the reference model to introduce multiple sources of information in the process of formation of perceptions Section (V) discusses the extension proposed in the context of a more general dynamic macroeconomic model and Section (VI) offers concluding remarks.

II The model

i The economy

To analyze the effect of information sets on perceptions, how these affect expectations and then influence future information sets, I propose an overlapping generation (OLG) monetary model of adaptive learning. I build upon the work of Sargent and Wallace (1987) and Marcet and Nicolini (2003). I model a closed economy with adaptive learning with two main new features: an informational friction, due to which individuals cannot observe the aggregate level of prices, and a mapping between information sets and the learning mechanism. I assume there exist two consumption goods[2] in the economy, available in separated markets. To fix ideas I name one of the markets as *Lowlands Market* (L) and the other as *Highlands Market* (H). Each market is populated by a fixed proportion of individuals.

[1] By distant markets I refer to markets in which individuals do not have access to market clearing prices.

[2] Goods may differ in their type (e.g., food and luxury goods), in their quality or in their place of sale, among others. Here, it is not important which differentiation prevails for model predictions

The government consumes in both markets by collecting seigniorage[3]. Government budget constraint in goods market i is

$$M_t^i - M_{t-1}^i = p_t^i G_t^i \tag{1}$$

with $i = \{L, H\}$. M_t^i represents the total supply of money in market i in period t, p_t^i is the price of consumption good i in terms of money and G_t^i is real government expenditure in goods market i. Equation (1) represents also the new supply of money in goods market i.

Seigniorage in each market is given by an exogenous i.i.d process $\{x_t^i\}_{t=0}^{\infty}$ that represents the growth rate of money. A government that creates money separately to finance real expenditures in different markets, like the one proposed here, can be interpreted as a central fiscal authority monetizing (unmanageable short-term exogenous) local government deficits. Hence, the evolution of monetary aggregates in each market follows

$$M_t^i = x_t^i M_{t-1}^i \tag{2}$$

Both growth rates are the only two sources of uncertainty in the model. I assume that $E(x_t^i)$ differs across markets. In any given period t, the total supply of money in each market is then

$$M_t^i = M_{t-1}^i + p_t^i G_t^i \tag{3}$$

with M_0^i given[4]. Because the creation of money is stochastic, the level of government real expenditures is also stochastic and depends on equilibrium "local" market prices.

The economy is populated by two types of agents living two periods, those born in *Lowlands* and those born in *Highlands*. I normalize the population size to one, and assume there is a fixed proportion, θ^L, of individuals born each period in the *Lowlands Market*. Young individuals know their type before making any decision. They also know that with some probability their type will change when they become old.

Let θ^L also be the probability of consuming in the *Lowlands Market* when individuals are old and $(1 - \theta^L)$ the probability of consuming in the *Highlands Market*. For simplicity, it is assumed that these probabilities are independent of young individual type. Due to this assumption, the only heterogeneity that matters to decide whether to consume today or tomorrow is young current type.

The markets are separated in the sense that once individuals, of any age and

[3]This assumption can be interpreted as a government that finances its deficit through money creation.

[4]Total supply of money in the economy is defined as $M_t \equiv \sum_i M_t^i = M_t^L + M_t^H$

cohort, are assigned to a market, they can only consume the good endowed in that market. Young individuals are endowed with $\omega^{y,i}$ and old with $\omega^{o,i}$ in each market i. Trade occurs between young and old living in the same market —is plausible by assuming $\omega^{y,i} > \omega^{o,i}$— and between young and the government. The lifetime problem of individual of type i is to maximize

$$u(c_t^{y,i}, c_{t+1}^{o}(i)) = lnc_t^{y,i} + \beta E[lnc_{t+1}^{o}(i)] \tag{4}$$

where $c_t^{y,i}$ represents young individual consumption and $c_{t+1}^{o}(i)$ represents consumption when old given initial type i. Unconditional on current type, second period consumption can be carried out either in *Lowlands Market* or in *Highlands Market*.

Money is the only asset in this economy and is used as a reserve of value. Individual born in market i faces the following constraints

$$p_t^i c_t^{y,i} + s_t^i = p_t^i \omega^{y,i} \qquad \text{1st period} \tag{5}$$

$$\left.\begin{array}{ll} \theta^L & [p_{t+1}^{L,e} c_{t+1}^{o,L} - s_t^i - p_{t+1}^{L,e}\omega^{o,L}] = 0 \\ (1-\theta^L) & [p_{t+1}^{H,e} c_{t+1}^{o,H} - s_t^i - p_{t+1}^{H,e}\omega^{o,H}] = 0 \end{array}\right\} \qquad \text{2nd period} \tag{6}$$

where s_t^i represents individual savings in terms of money and is restricted to non-negative values. $p_{t+1}^{i,e}$ represents the expected next period price of good i. The resulting Euler equation from the individual's problem pins down implicitly the nominal demand for money balances in each market and can be written as

$$\beta(p_t^i \omega^{y,i} - s_t^i) = \frac{(p_{t+1}^{L,e}\omega^{o,L} + s_t^i)(p_{t+1}^{H,e}\omega^{o,H} + s_t^i)}{\theta^L(p_{t+1}^{H,e}\omega^{o,H} + s_t^i) + (1-\theta^L)(p_{t+1}^{L,e}\omega^{o,L} + s_t^i)} \tag{7}$$

To characterize the aggregate economy, it is useful to think of the endowments as a representation of labor time[5]. Assume that there is a technology that converts one to one labor time to consumption goods. Given that labor supply is inelastic and that there is only one good in each market, individuals decide to either "eat" or sell their consumption goods. Selling consumption goods means that individuals put part of their production on the market to obtain money and consume in the future. Since the exogenous supply of labor time is greater for young people than for old people, and that all individuals of the same age are alike within a market, only a certain proportion of goods produced by young individuals are traded within a market. The non-consumed endowments are those that they provide to the market.

[5] I follow Evans et al (2001)'s interpretation to distinguish between home and market production within a market.

The size of each market is then

$$
\begin{aligned}
q_t^L &\equiv \theta^L(\omega^{y,L} - c_t^{y,L}) \\
q_t^H &\equiv (1-\theta^L)(\omega^{y,H} - c_t^{y,H})
\end{aligned} \tag{8}
$$

where q_t^L is the total amount of goods traded in *Lowlands Market* and q_t^H is the total amount of goods traded in *Highlands Market*. The respective q_t^i goods are either consumed by the old generation or is part of government expenditure[6].

q_t^i is the $p_t^i q_t^i$ is then the market value of all trades in market i and represents its aggregate expenditure. Hence, the total expenditure in the economy is the sum of the value of all traded goods in each market[7] and can be written as

$$
E_t = p_t^L q_t^L + p_t^H q_t^H \tag{9}
$$

i.1 The price index

Abstracting from the pure-exchange framework here proposed, equation (9) can be identified as the nominal value of total production in the economy. Even though I assume that endowments and the distribution of individuals across markets are fixed, the size of each market i, q_t^i, may change from period to period because optimal consumption depends on expectations regarding future prices. For this reason, any informative measure about the evolution of the general price level must take into account this fact.

It is possible to define such price index in terms of quantities in a base period. Assuming that $t = 0$ is the base period, the price index for any period t is[8]

$$
P_t = \frac{p_t^L q_0^L + p_t^H q_0^H}{p_0^L q_0^L + p_0^H q_0^H} \tag{10}
$$

(q_0^L, q_0^H) is the fixed market basket of consumer goods used to track inflation in the entire economy, i.e., in both markets. P_t measures changes in the price level of this basket in every period t and is use to track the evolution of the general price level for the average individual.

[6] Appendix B shows goods market clearing conditions in both markets.

[7] Note that money in this model not only serves as reserve of value, but also as unit of account

[8] The price index is computed according to Laspeyres formula. It is also possible to construct alternative price indexes, such as a Paasche index or a GDP Deflator. Because of model assumptions, a Paasche index and a GDP Deflator are identical in this formulation.

ii Information and perceptions

Young individuals in market i are endowed with an information set that includes the history of prices on their own market. However, they do not directly observe prices in the other market. Instead, they receive public signals about the state of the general price level that they can use to infer prices in the other market. Public signals are common knowledge and publicly observe in both markets. Individual i information set can be written as

$$\mathcal{I}^i = \{p_s^i, \bar{P}_s^z : s = 0, .., t-1; z = 1, ..., Z\}, \tag{11}$$

where Z is the total number of public signals.

The Euler equation (7) shows that optimal consumption for a young individual born in market i depends on current prices in market i and expected prices in both markets[9]. The information set $\mathcal{I}^i$ contains all necessary information to form expectations about prices in market i.

However, the individual does not observe neither current nor the history of prices in the other market. To form expectations about prices in the distant market, she first needs to form perceptions about the current state of prices and inflation. This is where public signals play their role.

For now, assume that there is only one public signal. Given its time series history, young individuals in each market are able to generate a perception about the prices in the other market by using (10); individuals replace the unobserved general price level index by its signal. Hence, the perceived price of the good that is sold in *Lowlands Market* by the young individual born in *Highlands Market* is

$$\bar{p}_{t-1}^L = \bar{P}_{t-1}\frac{E_0}{q_0^L} - p_{t-1}^H\frac{q_0^H}{q_0^L} \tag{12}$$

and the perceived price of the good that is sold in *Highlands Market* by the young individual born in *Lowlands Market* is

$$\bar{p}_{t-1}^H = \bar{P}_{t-1}\frac{E_0}{q_0^H} - p_{t-1}^L\frac{q_0^L}{q_0^H} \tag{13}$$

for a given public signal $\bar{P}_{t-1}$.

For any young individual, the perception of prices in the distant market depends on the level of prices in their own market, the common public signal and the value of the market basket of consumption goods in the base period. The only assumption needed that allows individuals to form perceptions is that individuals know the

[9]See Appendix B for an expression of equation (7) in terms of consumption.

composition of the initial market basket[10].

iii Determination of prices

The equilibrium of the economy is determined separately in each market and is given by equations (7), (12), (13), the per capita money supply in each market[11]

$$h_t^i = h_{t-1}^i + p_t^i g_t^i,$$

perceptions about the aggregate level of prices at $t-1$ ($\bar{P}_{t-1}$), and expectations about future prices in both markets.

III Learning equilibrium

i Expectations and the learning mechanism

I now assume that individuals not only have imperfect knowledge about the realization of prices in the distant market but are also learning about the actual law of motion in the economy in the spirit of Marcet and Sargent (1989b). This stylized representation helps to generate self-fulling equilibria, in the sense that the perceived law of motion affects the actual law of motion in the economy. Here, it allows us to assume that any agency in charge of releasing public statistics, that observes the evolution of prices in both markets, can exploit the asymmetric information to control the inflation rate in the economy conditional on the exogenous evolution of money creation.

Inflation expectations. Expectations of future market prices for any young individual born in *Lowlands Market* are given by

$$\begin{aligned} \widetilde{E}[p_{t+1}^L] &= \lambda_t^L p_t^L \\ \widetilde{E}[p_{t+1}^H | \bar{P}_{t-1}] &= \bar{\lambda}_t^H \bar{p}_t^H \end{aligned} \tag{14}$$

and for any young individuals born in *Highlands Market* are given by

$$\begin{aligned} \widetilde{E}[p_{t+1}^H] &= \lambda_t^H p_t^H \\ \widetilde{E}[p_{t+1}^L | \bar{P}_{t-1}] &= \bar{\lambda}_t^L \bar{p}_t^L \end{aligned} \tag{15}$$

[10]A price index constructed using a Paasche formula implies that individuals must know the total amount of goods q_t^i traded in both markets in every period t to form perceptions. Because the population in each market and the endowments are common knowledge, this implies that they must know per capita consumption in the other market. However, if the realization of the growth rate of money is not know, individuals will still have uncertainty about the aggregate price level.

[11]See Appendix B for details.

where $\widetilde{E}$ represents expectations based on a standard adaptive learning forecast function in which λ_t^i and $\bar{\lambda}_t^i$ are the expected gross inflation between period t and $t+1$ of their own and distant market, respectively. Gross inflation is estimated using historical data. $\bar{p}_t^i$ is the perception of the actual level of prices in the distant market[12]. Note that p_t^i is determined in equilibrium independently in each of the market, so only the expectations of the young inhabitants of the market matter.

Additionally, in every period t, after decisions are made and market clears, individuals use the price index given by (10) to form expectations about the general price level in $t+1$. Young individuals use their own information on prices from their market and their perceptions of current prices in the distant market to form inflation expectations.

The learning mechanism. Expected gross inflation is estimated in every period t according to a standard adaptive learning mechanism with decreasing gain parameter α_t of the form[13]

$$\lambda_t^i = \lambda_{t-1}^i + \frac{1}{\alpha_t}\left(\frac{p_{t-1}^i}{p_{t-2}^i} - \lambda_{t-1}^i\right) \tag{16}$$

I assume $\alpha_t = t$ which implies that the learning mechanism is a recursive least-squares rule and that expected gross inflation is the sample mean of past perceived rates of inflation.

Temporary equilibrium.[14] Given an expectation hypothesis by (14) and (15), a learning mechanism (16), rules for perceptions about prices in the distant market (12) and (13), and perceptions about the aggregate price level ($\bar{P}_{t-1}$), a temporary equilibrium in period t is a set of prices p_t^i for all markets i, such that, all individuals maximize their lifetime problem (4)-(6) and the money market clearing condition is satisfied[15].

ii Characterization of the equilibrium sequence

I characterize the solution of the temporary equilibrium sequence by simulation with the following parametrization. x_t^i follows a normal distribution with (μ_i, σ_i). I assume $\mu_L = 1.04$, $\mu_H = 1.01$ and $\sigma_L = \sigma_H = 0.01$. I truncate the distribution at one

[12]Since $\mathcal{I}_t^i$ includes only information up to $t-1$, we need an extra assumption as to how individuals are able to form perceptions about the current level of prices in the other market. I assume that individuals use the previous level of prices updated by the new estimation of inflation, i.e., $\bar{p}_t^H = \bar{\lambda}_t^H \bar{p}_{t-1}^H$ and $\bar{p}_t^L = \bar{\lambda}_t^L \bar{p}_{t-1}^L$

[13]Individuals also apply this rule to estimate perceived inflation of the distant market.

[14]This notion of equilibrium is in the spirit of Evans and Honkapohja (2001)

[15]A monetary equilibrium implies that the per capita money supply h_t^i is equal to the per capita money demand s_t^i. I restrict the analysis to situations in which money is valuable, hence $p_t^i \omega^{y,i} > s_t^i$.

to have a nonnegative growth in money supply. I assume $\lambda_t^i = 1$ and $\bar{\lambda}_t^i = 1.02$ for $t = \{0, 1\}$ to initialize the learning mechanism, and $\bar{p}_0^i = 1$ to be the initial perceived price. The initial stock of money in each market is $M_0^i = 0.5$ and endowments $\omega^{y,i} = 5$ and $\omega^{o,i} = 3$, respectively. I run the economy for 70 periods and I take out the first 20 to get rid off the initial noise produced due to the small number of observations use in the learning mechanism.

iii True public signal about the general price level

I first assume that agents receive a public signal $\bar{P}_{t-1}$ that is equal to the general price level. Figure (1) shows the equilibrium sequence of prices in each market and the corresponding expectations regarding inflation foe the entire economy. The actual inflation experienced by the market with an expected higher growth rate of money is higher than the inflation in the market with a lower expected growth rate of money. In other words, given that debt monetization is higher in *Lowlands Market* than in *Highlands Market*, the inflation rate in the former is on average higher. The mean market-inflation rates are 3.32% and 2.35%, respectively. The mechanism begin is very simple; under a constant population with fixed endowments, the real value of nominal balances depends on the evolution of money creation. The actual inflation tax that an individual faces is higher in the market with higher growth rate of money.

Figure (1) also shows that under a true signal of the general price level, inflation expectations for the entire economy are on average equivalent[16]. Even though individuals in different markets are exposed to different market rates of inflation, they are able to correctly infer prices in the distant market due to the informative public signal. This result is shown in Figure (2).

Figure (2) shows the expected inflation of each good by individual type. Under a true signal of the general price level, young individuals born in both markets expect the same market inflation in each market. The reason is that they extract the true information about the evolution of prices in the distant market from the public signal. As a consequence, individuals have similar perceptions about prices, which means that they use the same information set to form expectations no matter where they consume. The difference, which is non-systematic, in the expectations dynamics in Figure (1) is a consequence of the assumption of lags in the information set ($\mathcal{I}_t^i$ includes only information up to $t - 1$).

[16]Say something about variance

Figure 1: Actual and Expected Inflation by Individual Type

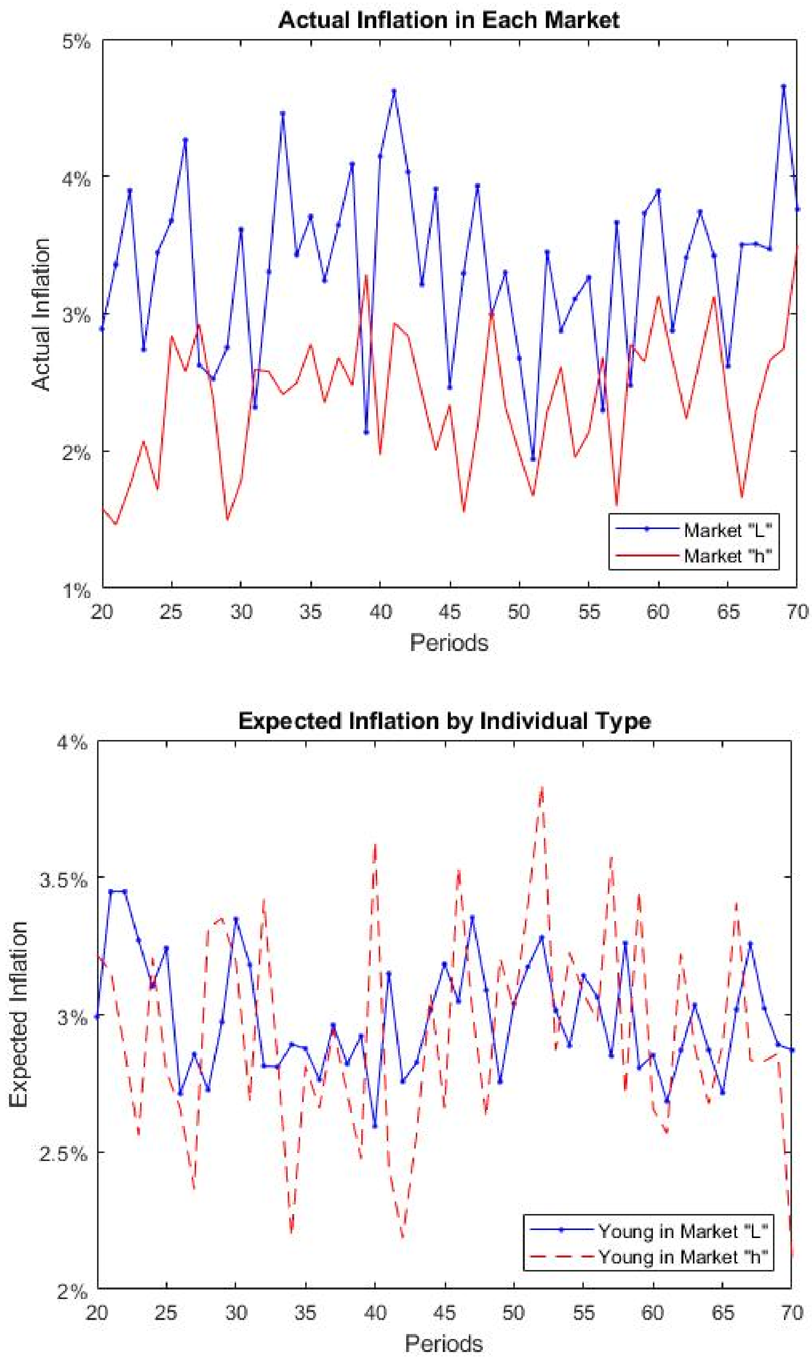
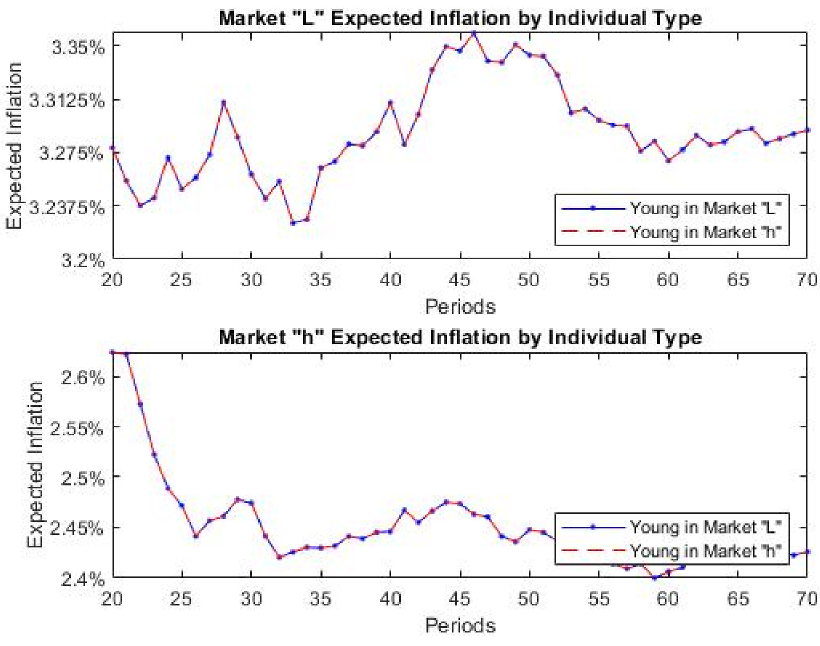


Figure 2: Expected Market-Inflation by Individual Type



iv Non-true public signal about the general price level

Now I assume that individuals receive a non-true signal about the general price level. For now, this signal is credible, i.e., they believe it and use it face value. In particular, I assume an exogenous[17] fixed signal that underestimate the actual general price level[18].

Because individuals observe the true realization of prices in their own market, any signal that is below the true general price level implies a downward perception of prices in the distant market[19]. Figure (4) shows how individuals underestimate inflation in the distant market given a credible downward bias non-true signal about the general price level.

Essentially, individuals in different markets form inflation expectations at the individual price level using the same public information but extracting different conclusions; expectations are affected by their own particular market-inflation experience.

Figure (3) shows the equilibrium sequence of prices in each market and the corresponding inflation expectations for the same parametrization and realization of shocks than in Subsection (iii). The average actual market-inflation rates are:

[17]In ii I endogenize the public signal and show why a government may have incentives to release a downward bias signal on the past realization of prices.

[18]The mean actual inflation for the simulated economy is 2.41% and I assume individuals in both markets receive a public signal of 2%.

[19]Equations (12) and (13) show the evolution of perceptions as a function of idiosyncratic prices and the public signal.

Figure 3: Actual and Expected Inflation by Individual Type

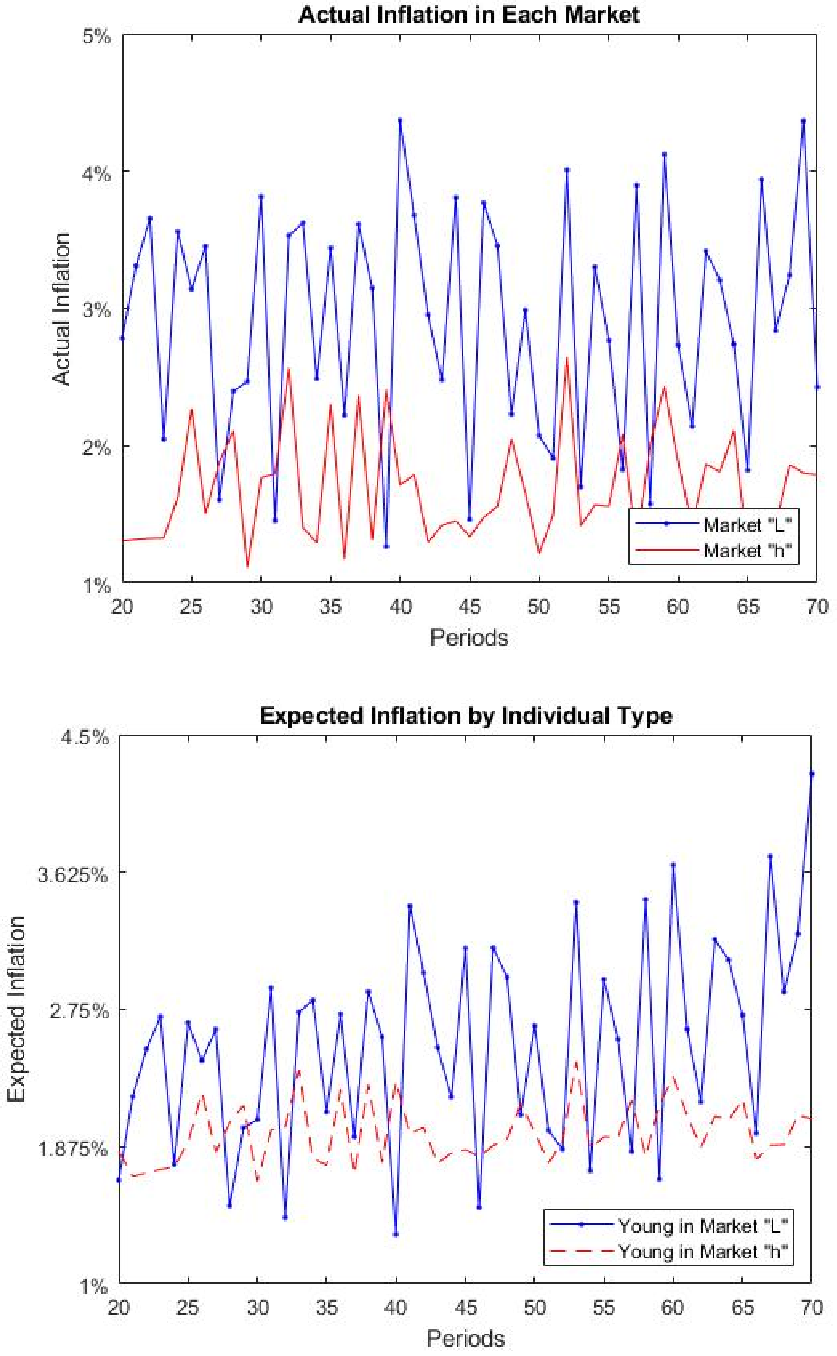
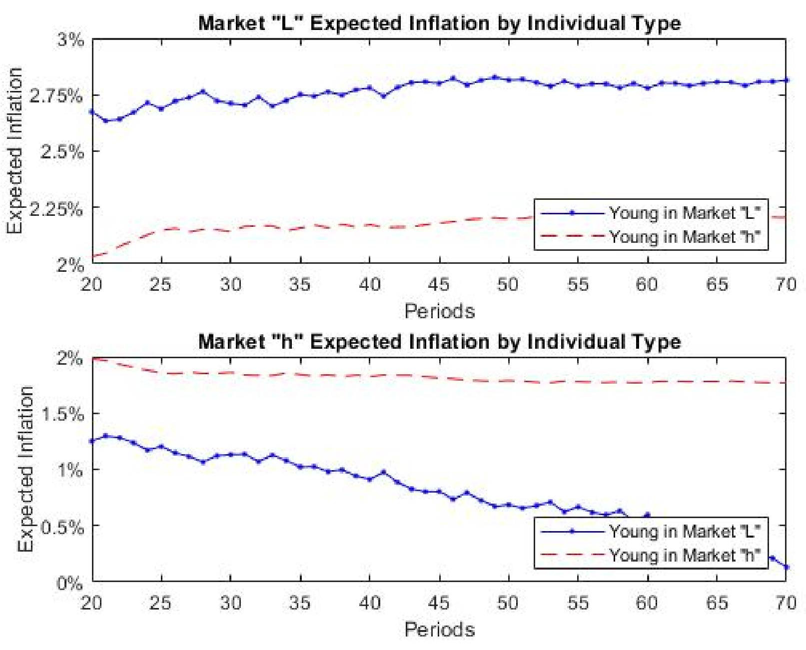


Figure 4: Expected Market-Inflation by Individual Type



2.89% and 1.69%, respectively; both below the benchmark case. This result is the consequence that prices depend positively on expectations about future prices in both markets. The underestimation of inflation in the distant market push down current prices in each market i.

Figure (3) also shows that inflation expectations about the general price level are heterogeneous in this case. Why? First, the underestimation of prices in the distant market implies an underestimation of future inflation given the learning mechanisms (16). Second, the fact that forecast errors about inflation in the distant market are higher for the young individuals born in the market with a market-inflation rate closer to the public signal, explains why under a downward bias public signal expectations diverge in line with market-inflation experience. In other words, the forecast errors made by young individuals born in the *Highlands Market* is higher than the one made by young individuals born in the *Lowlands Market*. A rapid inspection of Figure (4) allows to confirm this implication of the model. Figure (12) in Appendix C presents this prediction of the model even more clearly. The difference between the actual evolution of prices and its perception is higher for the young individual born in the *Highlands*. This is reflected in a relatively higher underestimation of current distant market-inflation, which in turn implies a relatively lower expected inflation. In summary, inflation expectations are positively correlated with market-inflation experience.

The opposite is true when individuals face a credible and non-true signal that overestimated the evolution of the general price level. Young individuals born in the market with a higher market-inflation rate have on average a lower expected inflation. See Appendix C for results.

The main prediction of the model, under a credible but non-true signal about the general price level, is that depending on the relative distance between the signal and the actual inflation, inflation expectations diverge.

IV Application: multiple public statistics

The previous section shows the behavior of the model under a unique credible signal. This section extends the benchmark model to account for situations in which individuals are uncertain about the current general level of prices but, in addition, they do not fully trust public signals. In particular, individuals face multiple inflation statistics. Hence, before making perceptions about the evolution of prices in the distant market, they need to form perceptions about the current state of the general price level. Perceptions are then, implicitly or explicitly, a decision about which information set to use.

i Perceived general price level

Theoretically, the choice of an information set is the mapping between information sets and the learning mechanism. In the context of individuals forming inflation expectations by using historical data, this choice means forming perceptions about the actual level of prices in the economy. For young individuals born in market i, perceived aggregate level of prices is

$$\bar{P}^i_{t-1} = I^i(\bar{P}^1_s, ..., \bar{P}^Z_s, p^i_s, \Gamma^i, \mu^i) \tag{17}$$

for a given set of public signals $\{\bar{P}^z_s : z = 1, ..., Z\}$ and $s = 0, ...t-1$. I^i is the mapping between multiple information sets and the learning mechanism, i.e., how individuals choose the relevant information from public signals to form perceptions about the current general price level. Γ^i represents parameters that relate idiosyncratic information on prices and public signals. μ_i represents any other variable that might affect perceived aggregate inflation.

ii Characterization of the equilibrium sequence under multiple public inflation signals: the Argentinian case

In this subsection[20], I apply the proposed extension of the model to give one possible qualitatively explanation of why lower income households expected higher inflation, than higher income households, in Argentina between 2007 and 2011. The period was characterized by uncertainty about the quality of official inflation statistics and the emergence of multiple sources of information about the state of inflation and the aggregate level of prices. Here, instead of interpreting the model as households consuming in distant markets, I reinterpret the model as households purchasing different consumption baskets that differ in the rate of price increase. Mosquera-Tarrío (2020) shows that there is a positive correlation between income-specific inflation rates and inflation expectations. This subsection shows one way to rationalize these stylized facts: it adds a mapping between information sets (multiple public signals) to the adaptive learning model previously developed.

Appendix A shows the time series of inflation expectations by income level and provides information about the different consumption experience of both groups. A deeper discussion about the period can be found in Cavallo (2013), Cavallo et al. (2016), Cavallo et al. (2017) and Mosquera-Tarrío (2020). The latter explicitly discusses how consumption experience affects inflation expectations.

The mapping: multiple inflation signals. In practice, I assume households form perceptions about past inflation according to the following rule[21]

$$\bar{\pi}^i_{t-1} = (1-\alpha^i_t)[\eta^i_t \pi^o_{t-1} + (1-\eta^i_t)\pi^u_{t-1}] + \alpha^i_t \pi^{CB_i}_{t-1} \tag{18}$$

where α^i is the weight household i assigns to their own price change information embodied in the evolution of the value of their own consumption basket, η^i is the relative weight between public signals, π^o is the official signal and π^u is the unofficial signal.

It is assumed that α^i_t follows

$$\alpha^i_t = \begin{cases} 0 & \text{if} \quad |\pi^o_{t-1} - \pi^u_{t-1}|/\bar{\pi}^i_{t-2} \leq \delta \\ 1 - \exp\left(-\frac{|\pi^o_{t-1} - \pi^u_{t-1}|}{\bar{\pi}^i_{t-2}}\right) & \text{if} \quad |\pi^o_{t-1} - \pi^u_{t-1}|/\bar{\pi}^i_{t-2} > \delta \end{cases} \tag{19}$$

where δ represents the threshold in percentage points from which households place weight to their own information of prices. Function (19) captures the idea that as

[20]The purpose of this section is to provide an applied example of the model, rather than a comprehensive explanation of the period studied.

[21]For simplicity, I assume households receive only two public signals, one that represents the official inflation and another one that represents unofficial sources.

long as the public signals remain closely related, households do not assign any weight to their own information of prices. Similarly, households assign an increasingly weight to their own information, coming from price changes in their consumption basket, as signals start to diverge.

It is assumed that η_t^i is determined by

$$(1-\eta_t^i) = \frac{|\pi_{t-1}^o - \pi_{t-1}^{CB_i}|/|\pi_{t-1}^u - \pi_{t-1}^{CB_i}|}{a + |\pi_{t-1}^o - \pi_{t-1}^{CB_i}|/|\pi_{t-1}^u - \pi_{t-1}^{CB_i}|} \tag{20}$$

where a represents the *a priori* relative confidence households assign to public signals; whenever a is greater than one, households react less strongly to deviations from their own signal with respect to the official one than to deviations from the unofficial one. Function (20) captures the correlation between the weight a household assign to a public signal and the evolution of the prices in their own consumption basket. The more similar a public signal is to the signal from the consumption basket, the more weight it has.

Note that given $\bar{P}_{t-2}^i$ and the past aggregate inflation perception implied by the above rule, young individuals born in market i perceive the aggregate level of prices $\bar{P}_{t-1}^i$.

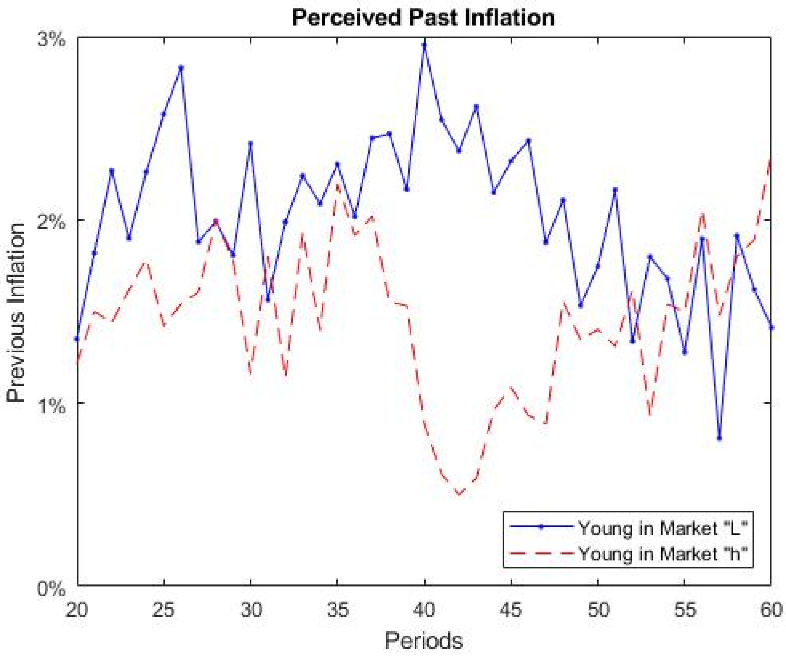
To characterize the equilibrium sequence under multiple public signals, I assume the same parameter and shock specification than in Section (III), except that now I assume the growth rate of money follows $x_t^i = c^i + \phi x_{t-1}^i + \epsilon_t^i$[22]. Additionally, I set $\delta = 0$, $a = 1$ and $\pi^o = 10\%$ and $\pi^u = 25\%$ for every period t. The perceived past aggregate inflation is graphed in Figure (5).

The fact public signals are sufficiently different and that households use their own information on prices to identify the relevant information, induce lower income households to perceive a higher current state of the unobserved aggregate price level. Because households use past information to form expectations, lower income households' expected inflation is higher than higher income households' expected inflation. Figure (6) shows the equilibrium sequence of prices in each market and the corresponding inflation expectations when households face multiple public signals and use their own information on prices to weight them. Figure (7) shows expected household-specific inflation by household type.

Figure (8) the weights that each type of household assign to their own household-specific inflation and between public signals. Note that is not the weight each type assign to their own information on prices what generates dispersion in inflation expectations (which is similar), but the fact that the rate of lower income-specific

[22]Simulations here use $\phi = 0.8$, $c^L = 0.2065$, $c^h = 0.205$ and ϵ^i is i.i.d. with zero mean.

Figure 5: Perceived Past Inflation



inflation is higher[23].

Appendix A shows the behavior of the *ad-hoc* perception formation rule in the data. The empirical analysis suggests that results are qualitatively similar[24].

V Discussion

The application proposed in Section (IV) is a particular example of a more general dynamic macroeconomic model which I discuss now.

General Framework.

Assume an economy represented by

$$y_t = f(y_{t-1}, y^e_{t+1}, \epsilon_t, \mu)$$

where y_t is a vector of all the variables of the economy in a given period t. y_{t-1} contains past realizations of the variables in y. y^e_{t+1} represents agents' expectations about future values of y. ϵ_t contains all exogenous shocks that the economy faces, while μ is a vector integrated by parameters delimiting both endogenous and exogenous variables. Function f relates current variables with past information, expectations, exogenous shocks and parameters, and it is derived from agent's choices

[23] See Appendix A.

[24] The weights between public signals are very different in the model and in the data. However, it is the fact that households assign more weight to their own information on prices, rather that their relative weight between public signals, what drives results.

Figure 6: Actual and Expected Aggregate Inflation by Market

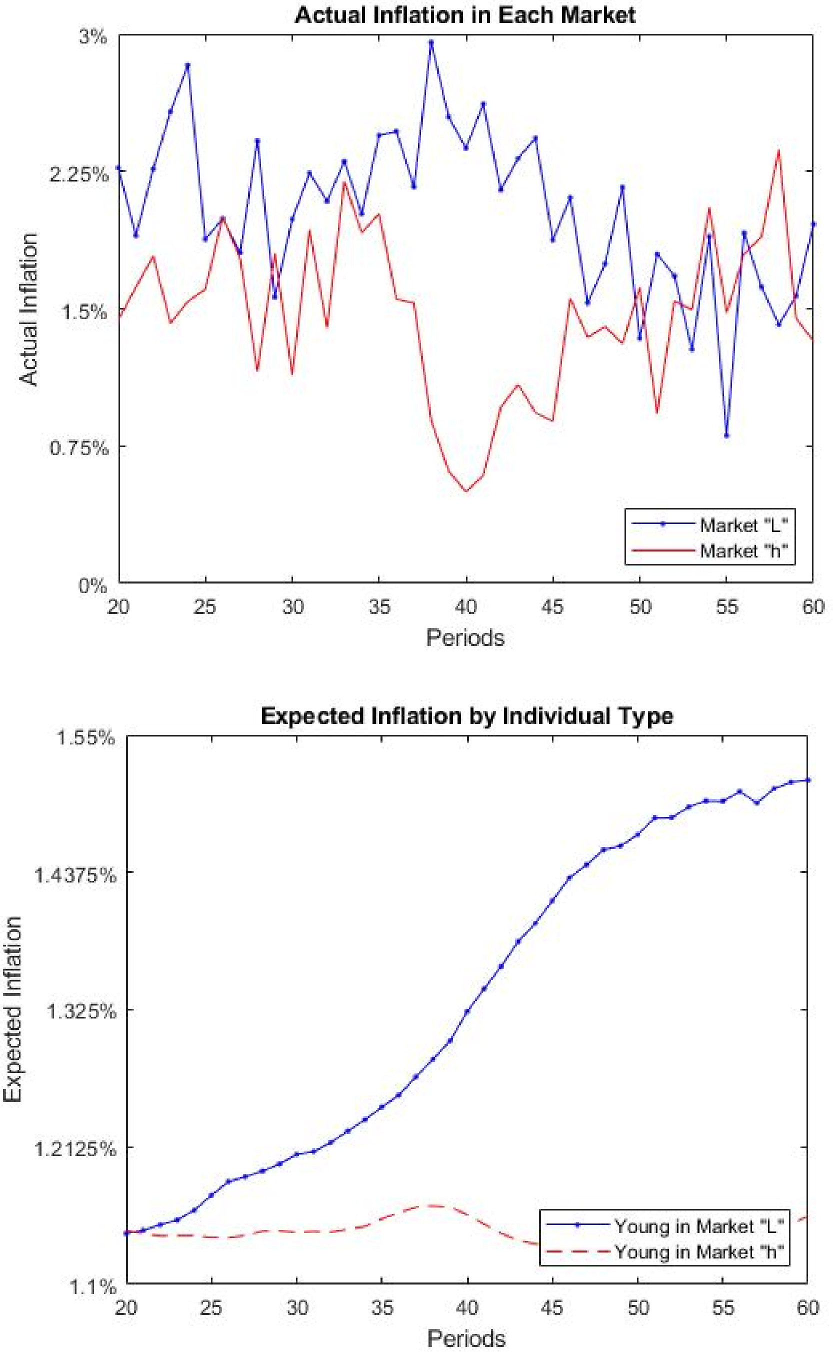
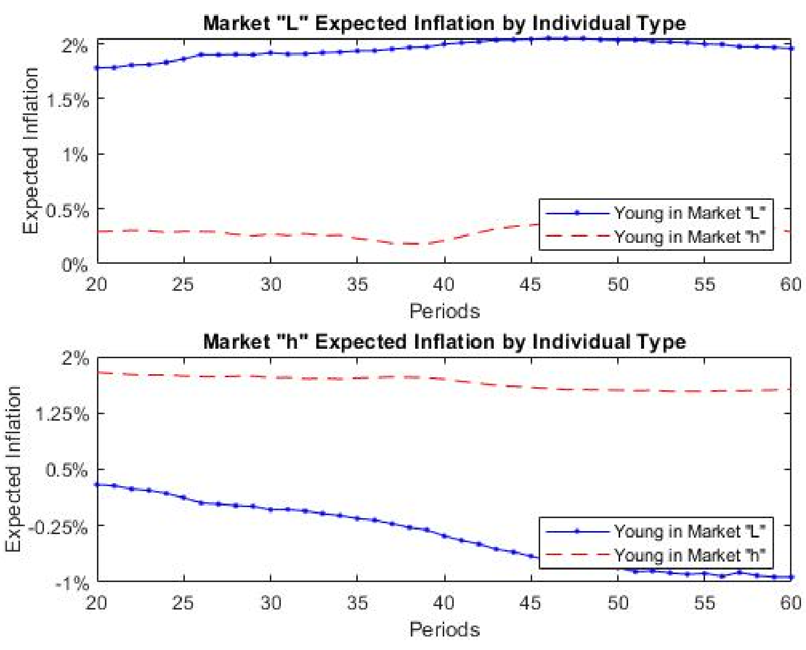


Figure 7: Expected Household-Specific Inflation by Income Type



and market equilibrium forces.

Expectations are given by

$$y^e_{t+1} = e(\beta_t(\theta), y^p_t)$$

where e is a forecast function that uses perceptions, y^p_t, of all variables in the economy and statistics, β, estimated from past data, to produce expected future values of the variables in the model[25]. Finally, a learning mechanism l is necessary to update statistics when new information arrives. θ are learning parameters that controls, for example, the differential weight between past and new information. β is updated according to

$$\beta_t(\theta) = l(\beta_{t-1}(\theta), y^p_t, \theta)$$

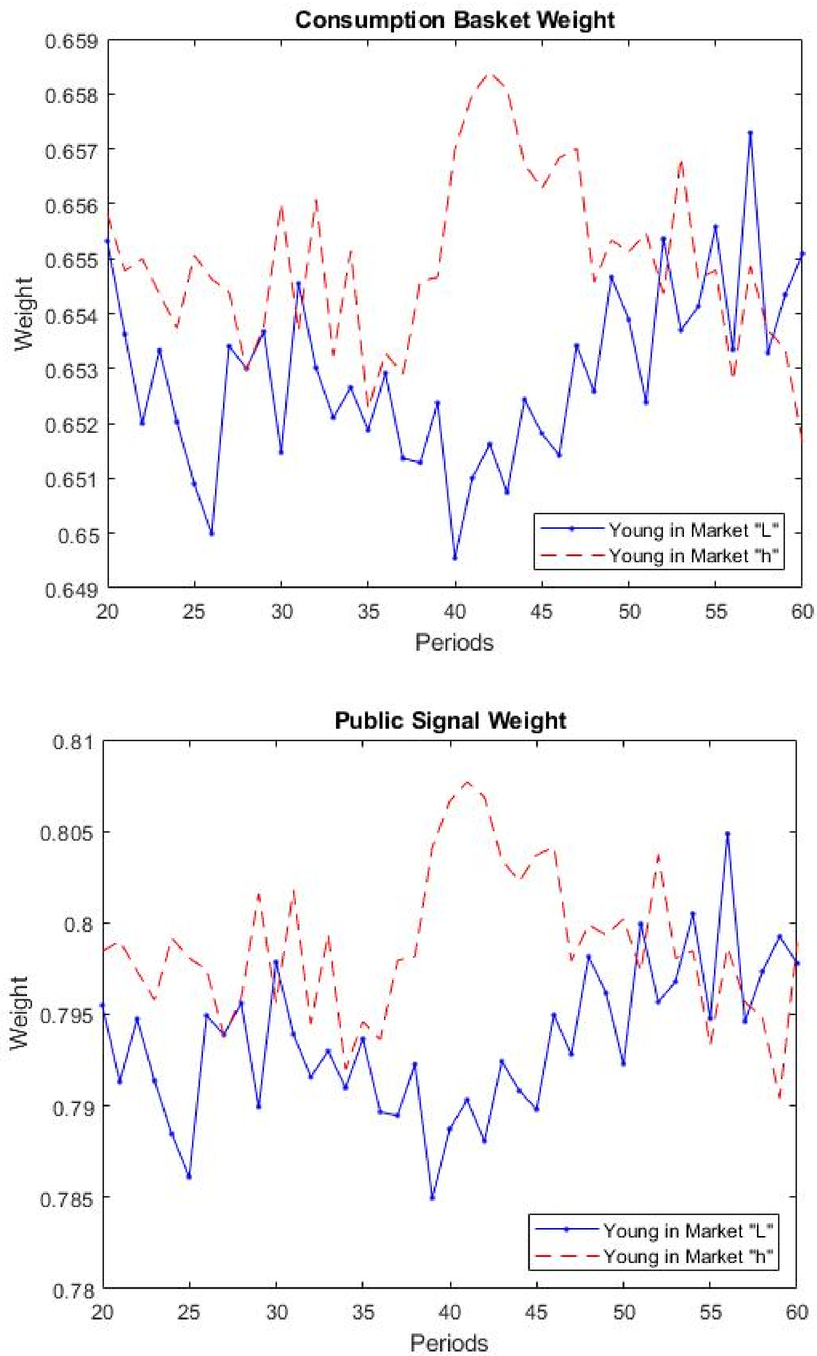
(e, l, θ) are not necessarily related to the true model (f, μ) (Marcet & Nicolini, 2003).

Assume that right before any period t agents receive ambiguous information about the realization of one or more endogenous variables. Agents face different information sets: y^i_0 for $i = 1, .., n$, where i denotes each particular information set. Any choice of an information set—how agents use different sources of past data to construct a single information set— can be summarized by the following equation

$$y^p_t = I(y^1_t, ..., y^n_t)$$

[25]In a standard AL model, $y^p_t = y_t$. Here, this equality may (or many not) hold.

Figure 8: Weights to Household-Specific Inflation and Between Public Signals



where I is a function that defines the mapping between available information sets and the chosen one. In principle, the mapping may contain behavioral rules or rational updating (i.e., Bayesian updating).

VI Conclusion

The paper develops an overlapping generation monetary model of adaptive learning with a hidden aggregate state variable: the general price level. Individuals need to form inflation expectations to decide their current and future consumption while they have access to idiosyncratic information on a subsample of prices and receive public signals about the state variable. I then analyze situations in which the quality of public information varies and whether (or not) it is credible.

The model explicitly shows the effect of the idiosyncratic information on generating heterogeneous inflation expectations in situations in which public signals are inaccurate or not fully credible.

The application to the Argentinian case shows that idiosyncratic information may help to explain heterogeneous inflation expectations across demographic groups, specially when the evolution of the general price level is uncertain.

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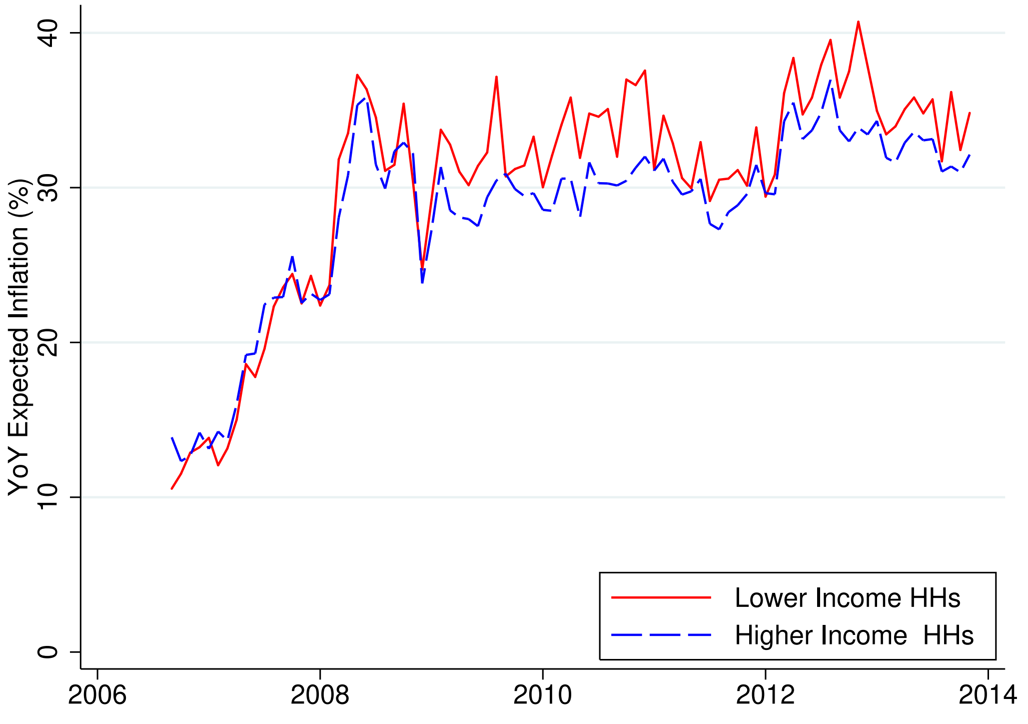
Appendix

Appendix A: Empirical Application

i Households' inflation expectations

Figure 9: Mean Annual Expected Inflation by Income, 2007-2014



Note: Expected Inflation is from the Encuesta de Expectativas de Inflación (UTDT).

ii Weight Function

This subsection shows the behavior of the *ad-hoc* inflation perception formation rule in the data (Argentina, October 2008 to March 2011). The time series of income-specific inflation rates for both lower and higher income households are from Mosquera-Tarrío (2020). See Table (2). The official inflation rate is from the CPI(Apr2008=100) and the unofficial one is from *Inflación Verdadera*[26]. Results support the proposed rule. Table (1) presents the average response of household's inflation expectations to the proposed measure of perceived past inflation by income. Results are positive and statistically significant for both groups of households.

[26]See http://www.inflacionverdadera.com/argentina/

Figure 10: Perceived Past Aggregate Inflation

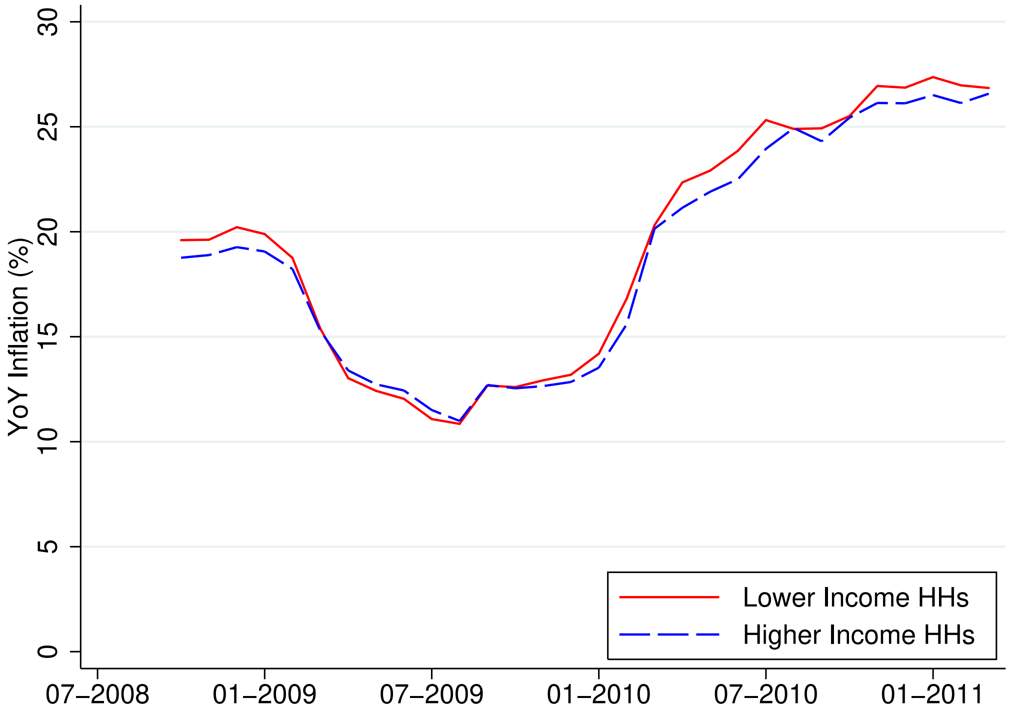
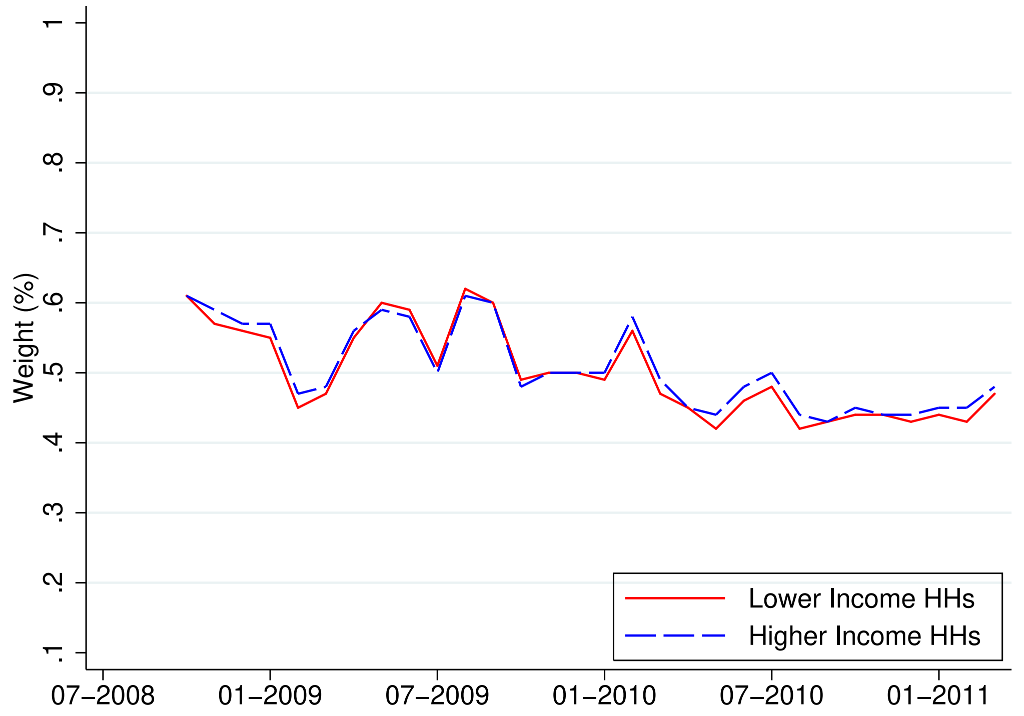


Figure 11: Weights: Income-Specific Inflation and Unofficial Public Signal

(a) Income-Specific Inflation



(b) Unofficial Public Signal

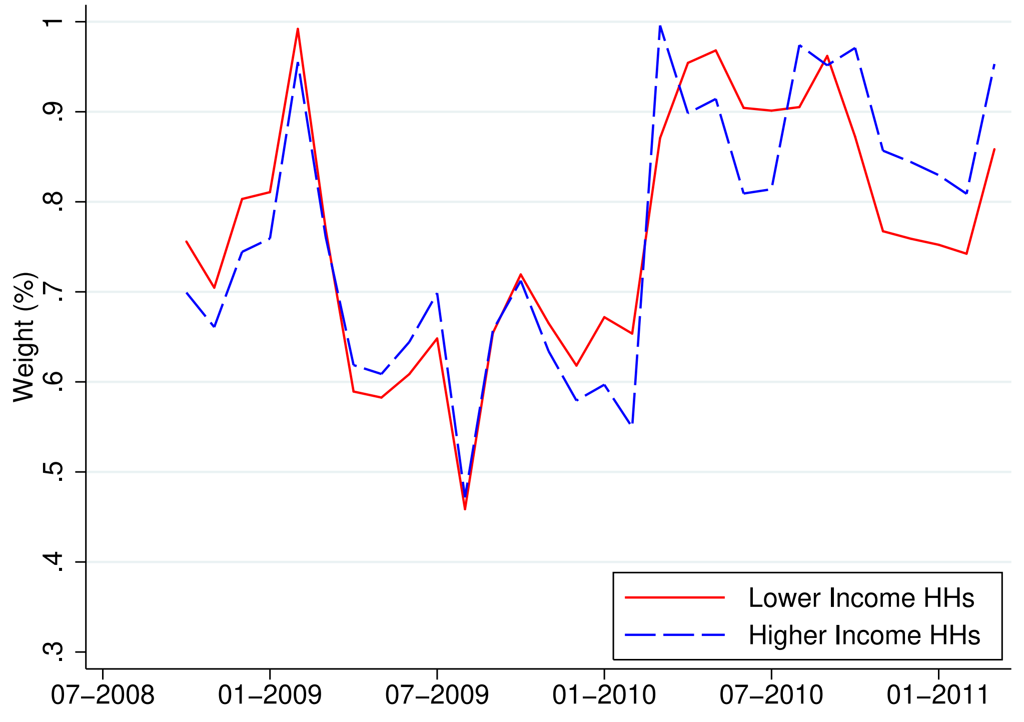


Table 1: Average Response of Household's Inflation Expectations to Perceived Past Inflation by Income

	(1) $\pi^{e,H}_{t+12\|t}$	(2) $\pi^{e,L}_{t+12\|t}$
$\bar{\pi}^{H}_{t-1}$	0.181*	
	(0.0991)	
$\bar{\pi}^{L}_{t-1}$		0.411***
		(0.139)
Dummy 2009	2.637***	3.595***
	(0.658)	(0.972)
Constant	20.73***	16.84***
	(1.977)	(3.062)
Trend	YES	YES
Observations	29	29

Standard errors in parentheses

* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

Table 2: Income-Specific Inflation Rates

Date	Lower-Income (%)	Higher-Income (%)
Oct-08	19.60	18.76
Nov-08	19.62	18.89
Dec-08	20.22	19.27
Jan-09	19.89	19.06
Feb-09	18.86	18.23
Mar-09	15.38	15.28
Apr-09	13.03	13.39
May-09	12.42	12.73
Jun-09	12.04	12.44
Jul-09	11.08	11.51
Aug-09	10.85	10.99
Sep-09	12.67	12.70
Oct-09	12.60	12.54
Nov-09	12.92	12.65
Dec-09	13.19	12.85
Jan-10	14.19	13.53
Feb-10	16.82	15.59
Mar-10	22.02	20.14
Apr-10	22.99	21.14
May-10	23.38	21.91
Jun-10	23.86	22.51
Jul-10	25.32	23.95
Aug-10	26.59	25.35
Sep-10	25.56	24.31
Oct-10	27.92	25.91
Nov-10	32.50	28.99
Dec-10	32.94	29.35
Jan-11	33.81	30.19
Feb-11	33.94	30.47
Mar-11	29.91	27.45

Source: Mosquera-Tarrío (2020)

Appendix B: Theoretical Model

iii Characterization of Individual's Behavior

Note that budget constraints imply

$$c_t^{y,i} = \omega^{y,i} - \frac{s_t^i}{p_t^i} \qquad \text{from 1st period BC}$$

$$c_{t+1}^o(i) = \omega^o + \frac{s_t^i}{p_{t+1}} \qquad \text{from 2nd period BC}$$

Hence, the individual's problem can be written as a function of prices, endowments and the choice variable s_t^i

$$\max_{\{s_t^i\}} \quad u(\cdot) = ln\left(\omega^{y,i} - \frac{s_t^i}{p_t^i}\right) + \beta E\left[ln\left(\omega^o + \frac{s_t^i}{p_{t+1}}\right)\right]$$

F.O.C.s (w.r.t. s_t^i), EE

$$\frac{1}{p_t^i\omega^{y,i} - s_t^i} = \beta E\left[\frac{1}{p_{t+1}\omega^o + s_t^i}\right]$$

or

$$\frac{1}{p_t^i\omega^{y,i} - s_t^i} = \beta\left[\frac{\theta^L}{p_{t+1}^L\omega^{o,L} + s_t^i} + \frac{1-\theta^L}{p_{t+1}^H\omega^{o,H} + s_t^i}\right]$$

EE can be also expressed as

$$\beta(p_t^i\omega^{y,i} - s_t^i) = \frac{(p_{t+1}^L\omega^{o,L} + s_t^i)(p_{t+1}^H\omega^{o,H} + s_t^i)}{\theta^L(p_{t+1}^H\omega^{o,H} + s_t^i) + (1-\theta^L)(p_{t+1}^L\omega^{o,L} + s_t^i)}$$

with $s_t^i \geq 0$. This expression implicitly define the individual of type i demand for money s_t^i.

iv Per Capita Money Supply in Each Market

$$h_t^L = h_{t-1}^L + p_t^L g_t^L$$

$$h_t^H = h_{t-1}^H + p_t^H g_t^H$$

with $h_t^L \equiv \frac{M_t^L}{\theta^L}$, $h_t^H \equiv \frac{M_t^H}{(1-\theta^L)}$, $g_t^L \equiv \frac{G_t^L}{\theta^L}$ and $g_t^H \equiv \frac{G_t^H}{(1-\theta^L)}$.

v Goods market clearing condition

The goods market clearing condition for each market i is

$$\begin{aligned} c_t^{y,i} + c_t^{o,i} + g_t^i &\leq \omega^{y,i} + \omega^{o,i} \\ c_t^{y,i} + (c_t^{o,i} - \omega^{o,i}) + g_t^i &\leq \omega^{y,i} \\ (c_t^{o,i} - \omega^{o,i}) + g_t^i &\leq (\omega^{y,i} - c_t^{y,i}) \end{aligned} \tag{21}$$

where the right-hand side represents the per capita amount of goods each young individual trades within a market. In equilibrium, the total expenditure within a market, q_t^i, is the market consumption of the old generation plus the respective government expenditure.

vi Official Consumption Price Index Release

In Section (III) I assume an exogenous public signal. In this subsection, I discuss why the government has incentives to minimize the official price index signal.

The realization of the aggregate level of prices P_t is privately observed by the government after individuals and the government make their decisions. The government has an incentive to exploit the gap in the access to information to its own interest and releases a signal $\bar{P}_t$ that differs from the true realization of aggregate prices. There are many ways through which the government can release the public signal about the aggregate level of prices. Here, I discuss one somewhat extreme case.

Assume that, given an exogenous growth rate of money, government wants to maximize real expenditure[27]. That is max $\{G_t^l + G_t^h\}$[28]. In other words, government wants to maximize

$$\frac{M_t^L - M_{t-1}^L}{p_t^L} + \frac{M_t^H - M_{t-1}^H}{p_t^H} \tag{22}$$

Recall that prices are a function of the proportion of the population in each market, the growth rates of money and expectations regarding future prices, i.e.,

$$\begin{aligned} p_t^L &= e^L(\lambda_t^L, \lambda_{t-1}^L, \bar{\lambda}_t^H, \bar{\lambda}_{t-1}^H, \theta^L, x_t^L, x_t^H) \\ p_t^H &= e^H(\bar{\lambda}_t^L, \bar{\lambda}_{t-1}^L, \lambda_t^H, \lambda_{t-1}^H, \theta^L, x_t^L, x_t^H) \end{aligned} \tag{23}$$

[27]Real expenditure is real seignorage. However here, instead of maximizing real seignorage by selecting the growth rate of money, the government use the signal about aggregate prices as the control variable.

[28]There are different ways of assuming how government derive utility from government spending. I assume there is no preference for the government in which market to consume. This assumption leads to corner solutions in all cases except the one in which the public signal induces the same value for real seignorage in both markets.

and that any young individual born in market i estimates expected inflation for market j according to

$$\bar{\lambda}_t^j = \bar{\lambda}_{t-1}^j + \frac{1}{\alpha_t}\left(\frac{\bar{p}_{t-1}^j}{\bar{p}_{t-2}^j} - \bar{\lambda}_{t-1}^j\right)$$

Note that $\bar{\lambda}_t^j$ is monotonically decreasing in $\bar{p}_{t-1}^j$ and that p_t^i is monotonically decreasing in $\bar{\lambda}_t^j$ for $i, j = \{L, H\}$ and $i \neq j$.

Government releases the common signal $\bar{P}_{t-1}$ to both markets in an effort to minimize[29] $\bar{p}_{t-1}^i$ for $i, j = \{L, H\}$. Government has one instrument for two objectives. How far the government can go in decreasing the public signal $\bar{P}_{t-1}$ is bounded by equations (12) and (13); young individuals must observe a positive price signal coming from the other market[30]. Because the maximization of real expenditure implies in general a corner solution, government has two candidates for the signal of aggregate prices: one that minimizes the perceived price of good L and one that minimizes the perceived price of good H. Formally,

$$\begin{aligned} \bar{P}_{t-1}^{(1)} &= \frac{p_{t-1}^H q_0^H}{E_0} + \epsilon_t^L \\ \bar{P}_{t-1}^{(2)} &= \frac{p_{t-1}^L q_0^L}{E_0} + \epsilon_t^H \end{aligned} \tag{24}$$

for any $\epsilon_t^L > 0$ and $\epsilon_t^H > 0$. First candidate implies $\bar{p}_{t-1}^L = \epsilon_t^L$ and the second candidate implies $\bar{p}_{t-1}^H = \epsilon_t^H$. Government chooses the candidate that maximizes equation (22)[31].

[29] Monotonically decreasing in $\bar{P}_{t-1}$

[30] Conditions $\frac{\bar{P}_t}{p_t^H} > \frac{q_0^H}{E_0}$ and $\frac{\bar{P}_t}{p_t^L} > \frac{q_0^L}{E_0}$ in H goods and L goods market, respectively, are needed to assure that young individuals observe a positive signal about the level of prices in the other market.

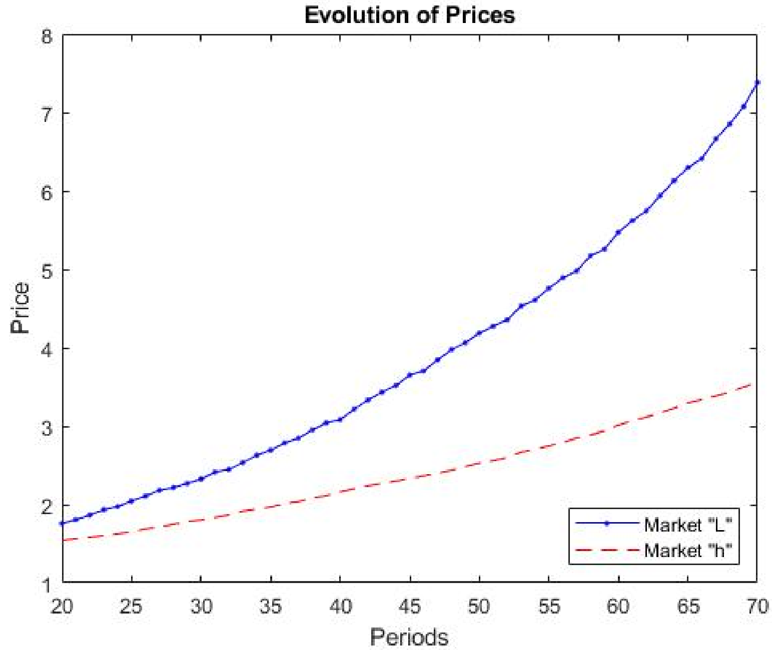
[31] In principle ϵ_t^i could be as close as possible to zero. However, one interesting candidate would be the signal that ensures that the inflation of the targeted market is equal to its REE.

Appendix C: Simulation results

Solution under a credible not true signal of the aggregate level of prices that overestimate the evolution prices .

Figures (13) and (14) show the characterization of the equilibrium sequence under a credible non-true signal that overestimate the general price level. As a result of individuals extracting information from the signal using their market-inflation experience, inflation expectations are negatively correlated with market-inflation experience. In this simulation the mean actual inflation is 3.66% and I assume individuals in both markets receive a public signal of 4%.

Figure 12: Evolution and Perception of Prices in each Market



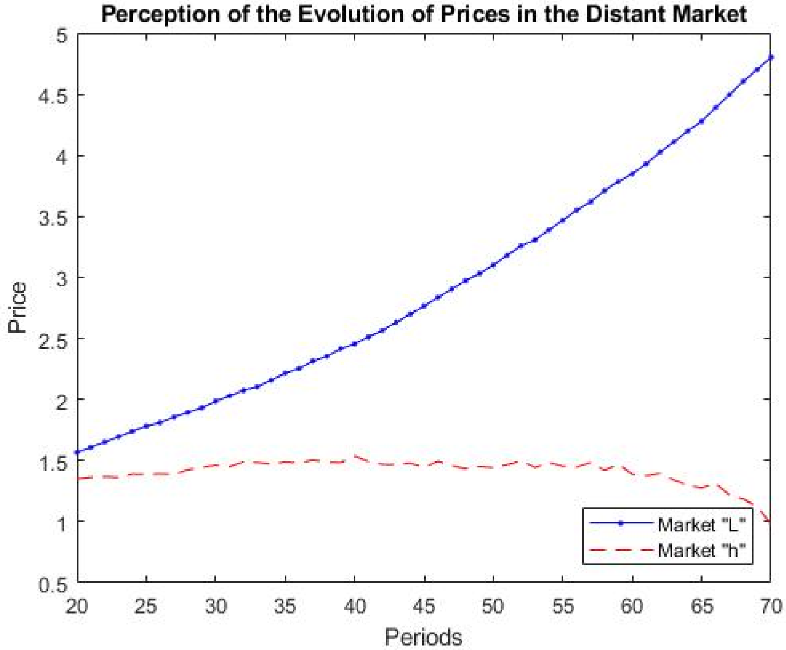
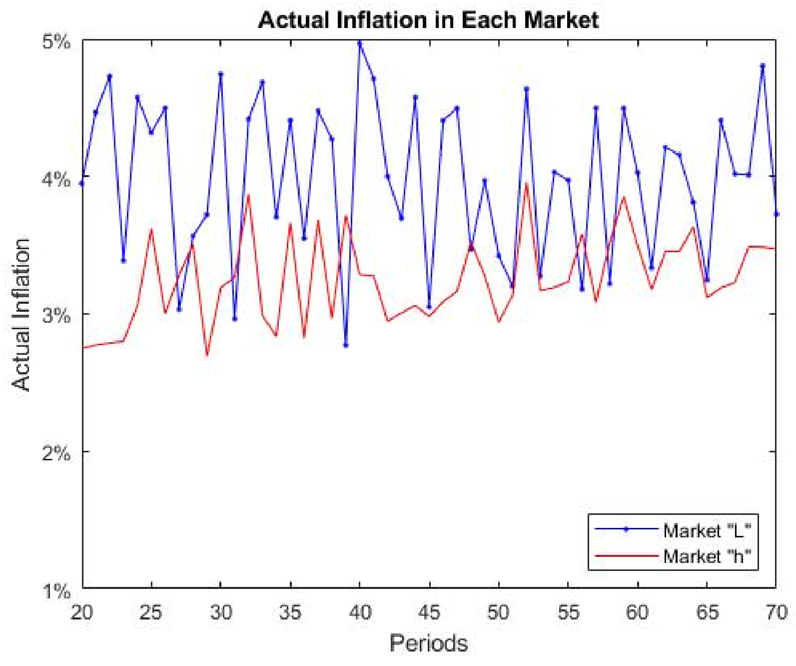


Figure 13: Actual and Expected Aggregate Inflation by Market

(a)



(b)

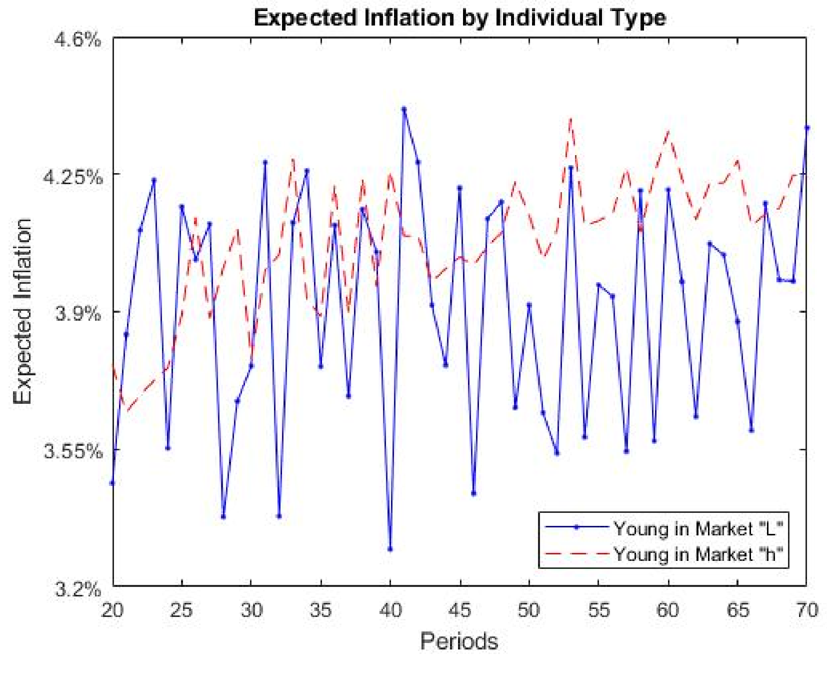


Figure 14: Expected Aggregate Inflation by Market and Type