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The Bigger the Stickier: Asymmetric Adjustment to Negative Demand Shocks

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"A mayor tamaño mayor rigidez: respuestas asimétricas ante shocks de demanda negativos"

Resumen:
En un duopolio en donde las empresas compiten de manera simultánea, el presente trabajo estudia la respuesta de las firmas ante un shock de demanda no anticipado. La respuesta también es simultánea, consiste en una sola acción entre ajustar y no hacerlo y no tiene costo alguno. Al igual que en trabajos previos las respuestas de las firmas son asimétricas en dos dimensiones. Por un lado, las firmas siempre reaccionan ante un shock de demanda positivo, mientras que no siempre lo hacen cuando el shock es negativo. Por otro lado, cuando el shock de demanda es pequeño solamente una sola firma ajusta las decisiones previas, mientras que la competidora decide no hacerlo. Al permitir que los costos marginales difieran aparece una tercera asimetría: existen shocks de demanda negativos de magnitud media ante los cuales solamente la firma de altos costos reacciona. Este resultado sugiere que la firma más grande tiene menos incentivos a modificar sus planes luego de un shock. Los resultados son robustos a otras especificaciones de la función de demanda y no dependen de que el shock sea no anticipado. Además, el análisis presentado en este trabajo sugiere que es posible que un shock de demanda reduzca la concentración de mercado, pero ese escenario es poco probable.

Palabras clave: ajuste asimétrico, duopolio, rigidez de precios, shocks de demanda.

"The Bigger the Stickier: Asymmetric Adjustment to Negative Demand Shocks"

Abstract:
This article studies the response to unanticipated demand shocks in a simultaneous competition duopoly model, where adjustment is simultaneous as well it has no cost and it is characterized by a single choice to adjust or not previous plans. In line with former analyses, responses are asymmetric in two dimensions: firms always react to positive demand shocks while they may not react to negative ones and, when demand shocks are negative and small-sized, only a single firm adjusts its previous decisions. Since in the baseline model firms are identical, it is undetermined which firm will adjust its decisions. Allowing for different marginal costs generates a third type of asymmetry: There are medium-sized negative shocks such that only the firm with higher marginal costs adjusts its price or quantities. This result suggest that the bigger firm is less willing to modify its plans after the shock. This result is robust to other demand specifications and does not depend on the shock being unanticipated. Furthermore, the analysis suggest that it is possible for a negative shock to reduce market concentration, but this situation is not likely.

Keywords: Asymmetric adjustment, duopoly, price stickiness, demand shock.

Códigos JEL: L1 L2 D4

1 Introduction

Rigidities are a key issue in economic analysis. In particular, price rigidities are an important feature in different topics going from the analysis of specific markets to the explanation of real effects of monetary shocks, including many other phenomena such as wage stickiness and unemployment. Several different explanations are available. First, it may not be free to modify a price that was set in a previous period and this fact is modeled as menu costs. Second, there may exists forces that limit the price movement of a good and in turn this limitation may also have effects on other related markets. Finally, price stickiness can arise as a consequence of strategic interactions between agents. The work presented here is concerned with this latter explanation.

Empirical work has documented differences in sign, magnitude and speed of price adjustment in a great variety of markets. Surveys of this literature can be seen in Frey and Manera (2007) and Meyer and von Cramon-Taubadel (2004). Peltzman (2000) analyzes many markets and finds differences in the speed of adjustment to positive or negative cost shocks. Many studies address how gasoline prices respond to oil shocks, see Borenstein, Cameron, and Gilbert (1997), Bacon (1991), Bachmeier and Griffin (2003) and Galeotti, Lanza, and Manera (2003) to mention a few. The fact that small price increases occur more frequently than small price decreases while large price variations do not exhibit that pattern has come into light with works like Ray, Chen, Bergen, and Levy (2006) and Chen, Levy, Ray, and Bergen (2008).

Menu costs have been studied by economist under many circumstances. They were introduced by Mankiw (1985) and their macroeconomics implications can be reviewed in Ball and Mankiw (1994) and Kuran (1983). Imperfect competition is a key issue in menu cost models: see Barro (1972), Sheshinski and Weiss (1977) for an introduction to these models and Alvarez, Beraja, Gonzalez-Rozada, and Neumeyer (2018) for a recent empirical study using Argentine history of high inflation. Strategic interactions and menu costs have been studied together in Hansen, Møllgaard, Overgaard, and Sørensen (1996) and in Bennett and La Manna (2001). Menu costs are not the only explanation for price rigidities and asymmetric responses to shocks. Part of the literature has focused on search costs to explain why prices react differently to a positive or negative cost shock. Chen et al. (2008), Tappata (2009) and Cabral and Fishman (2012) are only some references of this topic.

Interestingly, adjustment asymmetries can arise from strategic interaction even if menu costs are absent and consumers do not search. That is the case described by Hansen et al. (1996), Hansen, Møllgaard, Overgaard, and Sørensen (2001) considering a simultaneous competition model. They analyze the also simultaneous response to an unanticipated demand shock and show an additional asymmetry: in some cases only one firm adjust its price while its competitor leaves it unchanged. Damania and Yang (1998) can explain different price adjustment in a repeated duopoly game with collusion. Also in a context of collusion Obradovits (2014) shows that under some circumstances positive cost shocks are transmitted instantaneously while negative cost shocks require more time. Kovenock and Widdows (1998) find asymmetric responses to demand shocks in an oligopoly with price leadership: prices are rigid for a negative demand shock and the source of this rigidity is given only by strategic interactions since price setting has no cost. This article consists of an extension of the model presented in Hansen et al. (1996) and Hansen et al. (2001) allowing for different marginal costs.

By considering a duopoly where two firms react to an unanticipated shift of the demand curve, this article shows that the response to a demand shock can be asymmetric in three dimensions. First, firms react adjusting the price to any positive demand shock regardless of its size, while the same is not true when the shock is negative. A second type of asymmetry arises when the demand shifts inwards: in some cases a single firms adjusts its price while the competitor sticks to the previous one. This is true when the shock is negative and *small-sized* and in this situation the identity of the firm which sticks to the previous price is undetermined. Finally if the shock is negative and *medium-sized* it is still true that only one firm modifies the price as a reaction to the shock and its competitor does not, but the identity can be determined: the biggest firm sticks to the previous price and the smallest one adjusts. Since there are no menu cost these asymmetries arise purely as a strategic interaction between players. The first two types of asymmetries were already present in the baseline model and the main contribution of this paper consists on presenting and studying the third type.

The main result is present in both types of simultaneous competition considered, Bertrand and Cournot. It is shown that this result does not depend on the demand specification and it does not depend on the shock being unanticipated either. The asymmetric response to negative demand seems to affect the market concentration. After the introduction presented here, the article is divided in three more sections. Section 2 develops the extension of the model and describes its main results, section 3 discuss those results and, finally, section 4 concludes.

2 The Model

Let γ be given which is positive when goods are substitutes and negative when goods are complements, the inverse demand system is described in (1).

$$\begin{cases} p_1 = a - q_1 - \gamma q_2 \\ p_2 = a - q_2 - \gamma q_1 \end{cases} \quad \text{where} \quad |\gamma| \leq 1 \tag{1}$$

There are two firms producing with constant marginal cost. Assume w.l.o.g. $0 \leq c_1 \leq c_2$. The timing of the game is described as follows:

- **Stage 1**: Firms choose their prices (or quantities) simultaneously.
- Demand shifts. Specifically $a' = a + \Delta a$, where $\Delta a \neq 0$. This change is completely unanticipated and once it happens it is known by both players.
- **Stage 2**: There is a simultaneous decision to adjust (or not) to this shock by changing the price (or quantities). This adjustment has no cost.
- Profits are realized and the game ends.

The following two subsections develop standard Bertrand and Cournot models to analyze both forms of simultaneous competition. Since the demand change is unanticipated, we can analyze the two stages independently. It turns out that the results seem to be qualitatively the same if the shock is anticipated, as we will discuss in the next section.

2.1 Bertrand Competition

Firms engage in price competition. Inverting (1) the demand system is described by (2). As mentioned above, if $\gamma > 0$ goods are substitutes and if $\gamma < 0$ goods are complements.

$$\begin{cases} q_1 = \frac{1}{1-\gamma^2} \cdot [a(1-\gamma) - p_1 + \gamma p_2] \\ q_2 = \frac{1}{1-\gamma^2} \cdot [a(1-\gamma) - p_2 + \gamma p_1] \end{cases} \quad \text{where} \quad |\gamma| < 1 \tag{2}$$

Prices (less marginal cost), quantities and profits of first stage Bertrand competition are shown in tables (1) and (2).

Table 1: Prices and quantities in Bertrand competition

	Prices	Quantities
Firm 1	$(p_1^* - c_1) = \frac{1-\gamma}{2-\gamma} \cdot (a - c_1 + d)$	$q_1^* = \frac{1}{(2-\gamma)(1+\gamma)} \cdot (a - c_1 + d)$
Firm 2	$(p_2^* - c_2) = \frac{1-\gamma}{2-\gamma} \cdot (a - c_2 - d)$	$q_2^* = \frac{2}{(2-\gamma)(1+\gamma)} \cdot (a - c_2 - d)$

Table 2: Profits in Bertrand competition

	Profits
Firm 1	$\Pi_1^{B*} = \frac{1-\gamma}{(1+\gamma)(2-\gamma)^2} \cdot (a - c_1 + d)^2$
Firm 2	$\Pi_2^{B*} = \frac{1-\gamma}{(1+\gamma)(2-\gamma)^2} \cdot (a - c_2 - d)^2$

The parameter d is defined in equation (3). Given $c_2 > c_1$, note that $sg(d) = sg(\gamma)$ which implies d will be positive when goods are substitutes and negative when goods are complements. Assume $a - c_1 + d > 0$ and $a - c_2 - d > 0$ so quantities (and profits) are positive.

$$d \equiv \frac{\gamma}{(2+\gamma)(1-\gamma)} \cdot (c_2 - c_1) \tag{3}$$

After the demand change, firms decide simultaneously to adjust (action A) or not (action D) their own price. The adjustment has no cost. Let

$$B \equiv \frac{1-\gamma}{(1+\gamma)(2-\gamma)^2} > 0$$

If both firms decide to adjust their price, they engage in a new round of Bertrand competition and, according to the shock, their profits change in the following way:

$$\Delta\Pi_1^B(A,A) = B\cdot\Delta a\cdot[\Delta a + 2(a-c_1+d)]$$
$$\Delta\Pi_2^B(A,A) = B\cdot\Delta a\cdot[\Delta a + 2(a-c_2-d)]$$

If none of them decides to adjust their price, changes in profits are given by:

$$\Delta\Pi_1^B(D,D) = B\cdot(2-\gamma)\cdot\Delta a\cdot(a-c_1+d)$$
$$\Delta\Pi_2^B(D,D) = B\cdot(2-\gamma)\cdot\Delta a\cdot(a-c_2-d)$$

If (only) firm 1 decides not to adjust, it sticks to p_1^* chosen previously and its competitor, plays its best response taking into account that demand has shifted.

$$\Delta\Pi_1^B(D,A) = B\cdot(4-\gamma^2)\cdot\frac{\Delta a}{2}\cdot(a-c_1+d)$$
$$\Delta\Pi_2^B(D,A) = B\cdot(2-\gamma)^2\cdot\Delta a\cdot\left(\frac{\Delta a}{4}+\frac{a-c_2-d}{2-\gamma}\right)$$

The last case is when (only) firm 2 does not adjust its price and change in profits are given by:

$$\Delta\Pi_1^B(A,D) = B\cdot(2-\gamma)^2\cdot\Delta a\cdot\left(\frac{\Delta a}{4}+\frac{a-c_1+d}{2-\gamma}\right)$$
$$\Delta\Pi_2^B(A,D) = B\cdot(4-\gamma^2)\cdot\frac{\Delta a}{2}\cdot(a-c_2-d)$$

Note that in all cases previous profits were subtracted. Table (3) shows the normal form of the simultaneous adjustment game firms play during the second stage. For different shocks, proposition (1) characterizes the pure strategy Nash equilibrium of the simultaneous adjustment game.

Table 3: Simultaneous adjustment game

		Firm 2	
		A	D
Firm 1	A	$\Delta\Pi_1^i(A,A)$, $\Delta\Pi_2^i(A,A)$	$\Delta\Pi_1^i(A,D)$, $\Delta\Pi_2^i(A,D)$
	D	$\Delta\Pi_1^i(D,A)$, $\Delta\Pi_2^i(D,A)$	$\Delta\Pi_1^i(D,D)$, $\Delta\Pi_2^i(D,D)$

Note: $i = B, C$ for Bertrand and Cournot competition.

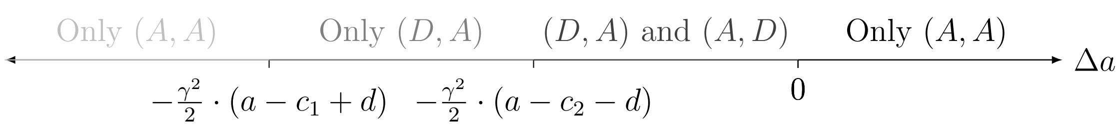
Proposition 1. *Given $\Delta a \neq 0$,*

- *(D, D) is never an equilibrium.*
- *If $\Delta a > 0$, (A, A) is the only equilibrium.*
- *If $\Delta a < 0$, three situations can arise:*
  1. *When $-\frac{\gamma^2}{2} \cdot (a - c_2 - d) < \Delta a < 0$, both (A, D) and (D, A) are equilibrium.*
  2. *When $-\frac{\gamma^2}{2} \cdot (a - c_1 + d) < \Delta a < -\frac{\gamma^2}{2} \cdot (a - c_2 - d)$, (D, A) is the only equilibrium.*
  3. *When $\Delta a < -\frac{\gamma^2}{2} \cdot (a - c_1 + d)$, (A, A) is the only equilibrium.*

Proof. (See appendix) □

Proposition (1) describes that there is a downward price rigidity that arises from strategic interactions between firms after a demand shift. On the other hand, prices are not rigid upwards. This fact was already present in Hansen et al. (1996, 2001). The novelty is that there are sizes of a negative demand shift that produce a single asymmetric equilibrium where only the firm with higher costs adjust its price. Figure (1) illustrates the situation for different shock sizes.
In short, when competition is in prices an asymmetric equilibrium where the firm with a smaller marginal cost does not adjust its price downwards is more likely to be observed. The result presented here suggest that bigger firms endure bigger (negative) shocks without reducing their prices. This fact appears even if price adjustment is free and only as a consequence of strategic interactions between competitors in the adjustment decision game.

Figure 1: Equilibria for different shock sizes



2.2 Cournot Competition

Now firms choose quantities and demand system is given by (1). Prices, quantities and profits are described in tables (4) and (5).

Table 4: Prices and quantities in Cournot competition

	Prices	Quantities
Firm 1	$(p_1^* - c_1) = \frac{1}{(2+\gamma)} \cdot (a - c_1 + \delta)$	$q_1^* = \frac{1}{(2+\gamma)} \cdot (a - c_1 + \delta)$
Firm 2	$(p_2^* - c_2) = \frac{1}{(2+\gamma)} \cdot (a - c_2 - \delta)$	$q_2^* = \frac{1}{(2+\gamma)} \cdot (a - c_2 - \delta)$

Where

$$\delta \equiv \frac{\gamma}{2-\gamma} \cdot (c_2 - c_1)$$

Note that $p_1^* - c_1 = q_1^*$ and $p_2^* - c_2 = q_2^*$. Then profits are given by:

Table 5: Profits in Cournot competition

	Profits
Firm 1	$\Pi_1^{C*} = \frac{1}{(2+\gamma)^2} \cdot (a - c_1 + \delta)^2$
Firm 2	$\Pi_2^{C*} = \frac{1}{(2+\gamma)^2} \cdot (a - c_2 - \delta)^2$

Let

$$C \equiv \frac{1}{(2+\gamma)^2} > 0$$

If both firms decide to adjust in the simultaneous adjustment game, profits are modified by:

$$\Delta\Pi_1^C(A, A) = C \cdot \Delta a \cdot [\Delta a + 2(a - c_1 + \delta)]$$
$$\Delta\Pi_2^C(A, A) = C \cdot \Delta a \cdot [\Delta a + 2(a - c_2 - \delta)]$$

If none of them decides to adjust their quantities, profits are modified by:

$$\Delta\Pi_1^C(D, D) = C \cdot (2+\gamma) \cdot \Delta a \cdot (a - c_1 + \delta)$$
$$\Delta\Pi_2^C(D, D) = C \cdot (2+\gamma) \cdot \Delta a \cdot (a - c_2 - \delta)$$

If only firm 2 adjust its quantities.

$$\Delta\Pi_1^C(D,A) = C \cdot (4-\gamma^2) \cdot \frac{\Delta a}{2} \cdot (a - c_1 + \delta)$$
$$\Delta\Pi_2^C(D,A) = C \cdot (2+\gamma) \cdot \Delta a \cdot \left[\frac{\Delta a(2+\gamma)}{4} + (a - c_2 - \delta)\right]$$

If only firm 1 adjust its quantities.

$$\Delta\Pi_1^C(A,D) = C \cdot (2+\gamma) \cdot \Delta a \cdot \left[\frac{\Delta a(2+\gamma)}{4} + (a - c_1 + \delta)\right]$$
$$\Delta\Pi_2^C(A,D) = C \cdot (4-\gamma^2) \cdot \frac{\Delta a}{2} \cdot (a - c_2 - \delta)$$

For different demand shocks proposition (2) characterizes the pure strategy Nash equilibrium of the simultaneous adjustment game which, in turn, is described in table (3). Figure (2) illustrate the result.

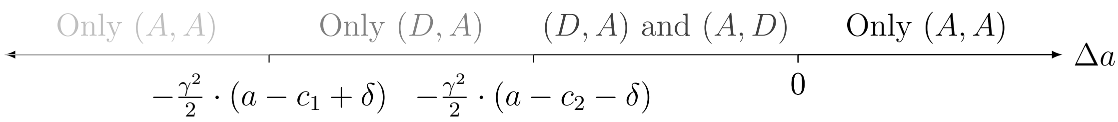
Proposition 2. *Given* $\Delta a \neq 0$*,*

- (D,D) *is never an equilibrium.*
- *If* $\Delta a > 0$*,* (A,A) *is the only equilibrium.*
- *If* $\Delta a < 0$*, three situations can arise:*
  1. *When* $-\frac{\gamma^2}{2} \cdot (a - c_2 - \delta) < \Delta a < 0$*, both* (A,D) *and* (D,A) *are equilibrium.*
  2. *When* $-\frac{\gamma^2}{2} \cdot (a - c_1 + \delta) < \Delta a < -\frac{\gamma^2}{2} \cdot (a - c_2 - \delta)$*,* (D,A) *is the only equilibrium.*
  3. *When* $\Delta a < -\frac{\gamma^2}{2} \cdot (a - c_1 + \delta)$*,* (A,A) *is the only equilibrium.*

Proof. (See appendix) □

Again, if firms are identical the result is equivalent to the one presented in Hansen et al. (2001). If demand shifts outwards the only equilibrium is that both firms adjust their quantities to this new situation. For an inward demand shift three different situations can arise. First, if the shift is (relatively) small there are two asymmetric equilibria where only a single competitor adjust its quantities. Second,

Figure 2: Equilibria for different shock sizes



an unique asymmetric equilibrium where the higher cost firm adjust its quantities appears for (relatively) greater shocks. Finally, if the shock is negative enough the only equilibrium is that both firms adjust their quantities.

This model suggest that out of asymmetric equilibria, the one where the most efficient firm does not adjust its quantities is more likely to be observed. In this framework a smaller marginal cost implies a greater share of the market, so the bigger can face greater (negative) shocks without modifying its original choice.

Next subsection contains a description of the results seen in propositions (1) and (2).

2.3 Main Results

Focusing on price competition, we can easily rationalize the main result established in proposition (1). If the demand curve shifts inwards and provided that the competitor will adjust its price, a firm deciding whether to cut down the price must take into consideration that the eventual reduction of the price will affect all the quantities sold. Firms with relatively smaller marginal costs will eventually have engaged (relatively) more units and, in consequence, will be less willing to cut down their price.

Suppose the cost difference is given and, additionally, $\gamma \neq 0$ and $|\gamma| < 1$. It turns out that $d > \delta$ and a comparison between the price adjustment game and the quantities game can be made. Recall propositions and figures (1) and (2), keeping only the absolute values of the expressions:

$$\frac{\gamma^2}{2} \cdot (a - c_2 - d) < \frac{\gamma^2}{2} \cdot (a - c_2 - \delta) \tag{4}$$

$$\frac{\gamma^2}{2} \cdot (a - c_1 + d) > \frac{\gamma^2}{2} \cdot (a - c_1 + \delta) \tag{5}$$

The range of negative shocks that generates a single asymmetric equilibrium is broader in the price adjustment game than in the quantities adjustment game. This is true because of two reasons present in inequalities (4) and (5) respectively: On one hand, in the price adjustment game the range where multiple equilibria are present is reduced with respect to the quantities adjustment game. On the other hand, also in the price adjustment game a greater negative demand shift is required for (A, A) to be an equilibrium.

As seen before, the asymmetric situation where the firm with lower marginal costs does not adjust its previous decision is more likely emerge as a result of a demand shock. Furthermore, inequalities (4) and (5) imply that this asymmetric situation should be seen more frequently when firms compete in price. This result only arises if competitors have different marginal costs.

Finally, if menu costs were to be paid in order to modify the prices or quantities previously chosen, the adjustment game changes as one would expect. With menu costs the situation where neither firm adjust its price, namely (D, D), is a Nash equilibrium if the demand shock is small enough independently of its sign. The asymmetries described in the model presented here remain, but prices and quantities become more rigid since explicit adjustment costs have been added.

3 Discussion

In the following three subsections we analyze to what extent the results presented in this model can be generalized and what are its implications.

In section 3.1 we see that the general result does not depend on the shock being completely unanticipated. If players have perfect foresight of a negative demand shock it is still true that the biggest firm is less willing to adjust its previous plans. Furthermore, if marginal costs are equal in the subgame perfect Nash equilibrium the firm that will not adjust will send more units in the first stage.

In section 3.2 we repeat the exercise with a different demand function. If demand is isoelastic the results are very similar. For a negative demand shock the results are qualitatively the same. In turn, it is no longer true that both firms will choose to adjust to any positive demand shock. In the future a fourth type of asymmetric response could be studied under similar specifications.

Finally in section 3.3 we present an example that shows that it is possible for a negative demand shock to reduce market concentration. Although this situation is possible, it is not likely: in most cases a negative demand shock increases market concentration measured using the Herfindahl index.

3.1 Anticipating the Shock

The following example suggest that if firms anticipate the negative demand shock, the results are qualitatively the same. Additionally, firms with the same marginal costs may have different productions plans whether they plan to adjust or not to the demand shock.

Assume the two firms produce an homogeneous good with constant and equal marginal costs (normalized to zero) $c_1 = c_2 = 0$. The timing goes as follows: In the first stage of the game firms face the inverse demand function $p = 1 - q_1 - q_2$ and engage in Cournot competition. After that, demand curve shifts inwards $\Delta a = -1/4$ and this change is perfectly anticipated by both players. In the second stage of the game firms decide simultaneously to adjust (action A) or not (action D) their quantities. There is no discounting between stages.

As shown in the appendix this game has two subgame perfect Nash equilibria. Let $\bar{q}_i$ be the quantity of firm $i = 1, 2$ in the first stage of the game.

$$\begin{aligned} SPNE_1 &= \{(\bar{q}_1 = 7/20, D), (\bar{q}_2 = 13/40, A)\} \\ SPNE_2 &= \{(\bar{q}_1 = 13/40, A), (\bar{q}_2 = 7/20, D)\} \end{aligned} \tag{6}$$

The bigger firm is the one that decides not to adjust its quantities according to the shock, but in this case the size of the firm is endogenous: The firm that does not adjust its quantities chooses produce more in the Cournot stage of the game. Note that $7/20 > 1/3$ the latter being the quantities of a standard Cournot duopoly.

It is easy to show that if the shock were positive ($\Delta a = 1/4$) there is a single subgame perfect Nash equilibrium where both firms adjust after the shock. Furthermore we can extend the results of this subsection allowing the shock to be $\Delta a = -1/4$ with probability p and $\Delta a = 1/4$ with probability $1 - p$.

The example presented here suggest that the main result of the paper does not depend on the shock being completely unanticipated. If, in contrast, it was perfectly anticipated we see that in any subgame perfect Nash equilibrium the firm that is bigger in the first stage does not adjust its quantities in the second stage of the game. Although marginal costs are symmetric, an asymmetry between firms emerges as a consequence of the anticipated negative demand shock. Further extensions may repeat this analysis with heterogeneous goods and price competition.

3.2 Robustness to Other Demand Specifications

As the following extension suggest, the main results does not seem to depend on the parametric expression of the demand function either.
Consider a Cournot duopoly. Each firm produces with constant marginal costs $0 < c_1 \leq c_2$ and demand function is isoelastic with inverse demand function given by $p(q_1, q_2) = \frac{1}{q_1+q_2}$. For simplicity assume $c_1 + c_2 = 1$. As usual the timing of the game consists on two stages:

- **Stage 1**: Players engage in a Cournot duopoly.
- There is an unanticipated demand shock, such that $p'(q_1, q_2) = \frac{\alpha}{q_1+q_2}$ for some $\alpha > 0$. Note that if $0 < \alpha < 1$ the shock is negative and if $\alpha > 1$ it is positive.
- **Stage 2**: Players simultaneously decided whether to adjust (action A) or not (action D) their previous decisions.

Proposition 3. *Let $\alpha \neq 1$ be given and focus on the adjustment subgame.*

- *(D, D) is never an equilibrium.*
- *If $\alpha < 1$, three situations can arise. For a constant $0 < k \leq 1$:*
  1. *When α is close enough to 1, both (A, D) and (D, A) are Nash equilibria. $\sqrt{\alpha} > \frac{k}{2c_1}$ which corresponds to a small shock.*
  2. *For intermediate values of α, (D, A) is the only Nash equilibrium. $\frac{k}{2c_2} \leq \sqrt{\alpha} \leq \frac{k}{2c_1}$ which corresponds to a medium sized shock.*

3. *When α is far enough from 1, (A, A) is the only Nash equilibrium. $\sqrt{\alpha} < \frac{k}{2c_2}$ which corresponds to a large shock.*

Proof. (See appendix) □

Proposition (3) suggest that the results of this paper are independent of the linear demand system considered. An interesting extension may explore if the degree of stickiness is associated with the elasticity of demand. A good starting point could be repeating this exercise with $p = \frac{1}{(q_1+q_2)^r}$ with $r < 2$ since there are two firms.
This example indicates that the general result does not seem to depend on the linear form of the demand function. With the isoelastic demand proposed here we cannot conclude that when demand shifts upwards the only equilibrium is (A, A) because that also depends on the size of the positive demand shock. In this context, elasticity of demand may be important to explain rigidities to positive demand shocks.

3.3 Effects on Market Concentration

In this subsection we will explore the implications of a negative demand shock on the market concentration using the framework presented here. Recall Herfindahl index and let σ^2 be the variance of the market shares s_i. One can show that if there are N firms in the market:

$$H = \frac{1}{N} + N\sigma^2 \tag{7}$$

Note that if all firms have the same market share $\sigma^2 = 0$ and the index takes its minimum value which is $H_{MIN} = 1/N$.
For the remainder of this subsection we will assume that firms play a Cournot duopoly game and that the good is homogeneous.
If marginal costs are symmetric the analysis is straightforward. Any negative demand shock that lands the market in a situation where a firm does not adjust its previous plans, namely (A, D) or (D, A), will increase the Herfindahl index. The variance of the market shares will go from $\sigma^2 = 0$ to $\sigma^2 > 0$ because markets shares where both equal to each other in the initial situation and they end up being different after the asymmetric response.

If marginal costs are asymmetric $c_1 < c_2$ and we are in the equilibrium where the bigger firm does not adjust its previous plans (D, A), the index increases as well. The firm that had the highest market share will see its market share increased even more because its quantities remain unchanged while total units fall.
The most interesting case arises if marginal costs are asymmetric and we are in the other equilibrium (A, D). We will briefly discuss this case with an example. Consider a shock of $\Delta a = -0.25$ and $a = 1$, that is demand goes from $p = 1 - q_1 - q_2$ to $p' = 0.75 - q_1 - q_2$. Normalize $c_1 = 0$ and consider $c_2 = 0.05$. Before the shock the Herfindahl index is $H \simeq 0.5029$ and it ends up increasing $H' \simeq 0.5102$. Now we repeat the same exercise with $c_2 = 0.1$. Before the shock $H \simeq 0.5125$. After the shock it turns out that the new value of the Herfindahl index is $H' \simeq 0.5012$. In the example we can see that if the difference between marginal costs is large enough, a negative demand shock that generates an asymmetric response where the biggest firm adjusts its quantities can reduce the Herfindahl index. This conclusion was not obvious because there are two effects present. On the one hand, the index goes down because the firm that had the more market share in the first stage will produce less. On the other hand the index goes up because there is less total output. It turns out that this result also depends on the size of the shock. Future extension may continue on this direction and provide a better characterization of the effects on concentration indexes.

4 Conclusion

This article studies to what extent the response to unanticipated demand shocks can be asymmetric in a duopoly. The response to the shock is assumed to be a single decision between two actions which is itself simultaneous and free. The response can be asymmetric in three dimensions. First, firms adjust to any positive demand shock regardless of its size, but they do not adjust to any negative demand shock. Second, if the shock is negative and *small-sized* only one firm will modify its previous decisions, but the identity of the firm is undetermined. Third, if the shock is negative and *medium-sized* only the smallest firm modifies its previous plans while the biggest firm leaves them unchanged. The first two asymmetries have already been studied by the literature and the main contribution of this paper is centered around the third one: Bigger firms are less willing to adjust to a demand shock, even when this adjustment is free. There exists a range of negative demand shocks where a single equilibrium arises in which the biggest firm does not change its previous plans while the smallest one does. In addition to this result, we show that the size of this range is larger when firms engage in Bertrand competition than when they engage in Cournot competition.

The main result does not depend on the shock being unanticipated and it does not depend on the demand function being linear either. If we allow players to have perfect foresight of a negative demand shock, the asymmetry arises even if marginal costs are equal purely as a result of the strategic interactions between both players in the subgame prefect Nash equilibrium. If the demand function is isoelastic, the main result is qualitatively the same. In this case it is no longer true that both firms will adjust to a positive demand shock regardless of its size. Future extensions may explore the relation between the shape of the demand function and this new source of asymmetry.

While it is possible for a negative demand shock to reduce market concentration measured using the Herfindahl index, it does not seem to be likely. In most cases a negative demand shock will increase the value of the index. In order to the index to reduce its value two conditions need to hold. On the one hand marginal costs must be different enough. On the other hand, the shock must be negative and it must also induce a situation where the biggest firm adjusts its previous plans, which is in turn the less likely scenario. These two conditions are not independent of the size of the shock and this fact may be studied in the future.

The discussions of section 3 can be extended to provide an exhaustive characterization of the response to demand shock and the implications of the strategic interactions between the players. Empirically it would be interesting to identify demand shocks and analyze the evolution of the Herfindahl index of that market. We believe this could be relevant to see to what extent the conclusions of this model arise in practice.

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Appendix

Proof of proposition (1). Suppose $\Delta a \neq 0$:

- For (D, D) to be an equilibrium it should be true that:

  $$\Delta\Pi_1^B(D, D) \geq \Delta\Pi_1^B(A, D)$$

  Which never holds independently of the direction of the demand shift. Then, (D, D) cannot be an equilibrium.

- For $\Delta a > 0$ first it is shown (A, A) is an equilibrium. To be an equilibrium the following inequalities must be true:

  $$\Delta\Pi_1^B(A, A) \geq \Delta\Pi_1^B(D, A) \quad \text{and} \quad \Delta\Pi_2^B(A, A) \geq \Delta\Pi_2^B(A, D)$$

  Which imply:

  $$\Delta a \geq -\frac{\gamma^2}{2} \cdot (a - c_1 + d) \quad \text{and} \quad \Delta a \geq -\frac{\gamma^2}{2} \cdot (a - c_1 - d)$$

  Both satisfied with strict inequality because it was assumed quantities were positive. Now, since inequality is strict and since (D, D) is not an equilibrium, it follows (A, A) is the only equilibrium for a positive demand shift.

- Given $\Delta a < 0$, conditions under (A, D) and (D, A) are equilibria are shown first.

  (A, D) is an equilibrium if the following inequalities are true:

  $$\Delta\Pi_1^B(A, D) \geq \Delta\Pi_1^B(D, D) \quad \text{and} \quad \Delta\Pi_2^B(A, D) \geq \Delta\Pi_2^B(A, A)$$

  As seen in the beginning of this proof, the first inequality always holds. The second inequality implies:

  $$\Delta a \geq -\frac{\gamma^2}{2} \cdot (a - c_2 - d)$$

(D, A) is an equilibrium if the following inequalities are true:

$$\Delta\Pi_1^B(D, A) \geq \Delta\Pi_1^B(A, A) \quad \text{and} \quad \Delta\Pi_2^B(D, A) \geq \Delta\Pi_2^B(D, D)$$

Here the second inequality always holds and the first one implies:

$$\Delta a \geq -\frac{\gamma^2}{2} \cdot (a - c_1 + d)$$

Since $(a - c_1 + d) > (a - c_2 - d)$ holds for the range of γ considered, if the shock is negative enough (D, A) will be an equilibrium while (A, D) will not.

For a negative demand shock (A, A) is an equilibrium if:

$$\Delta\Pi_1^B(A, A) \geq \Delta\Pi_1^B(D, A) \quad \text{and} \quad \Delta\Pi_2^B(A, A) \geq \Delta\Pi_2^B(A, D)$$

Implying

$$\Delta a \leq -\frac{\gamma^2}{2} \cdot (a - c_1 + d) \quad \text{and} \quad \Delta a \leq -\frac{\gamma^2}{2} \cdot (a - c_2 - d)$$

If the first inequality is strict (A, A) is the only equilibrium under a negative demand shock and this completes the proof.

$\square$

Proof of proposition (2). Suppose $\Delta a \neq 0$:

- Since $\Delta\Pi_1^C(A, D) > \Delta\Pi_1^C(D, D)$ always holds when there is a demand shift, (D, D) is not an equilibrium.

- When there is a positive demand shift, (A, A) is an equilibrium because the following two inequations are satisfied with strict inequality since it was assumed quantities are positive.

$$\Pi_1^C(A, A) \geq \Pi_1^C(D, A) \quad \text{and} \quad \Pi_2^C(A, A) \geq \Pi_2^C(A, D)$$

Given that (D, D) is not an equilibrium, it follows (A, A) is the only equilibrium.

- For a negative demand shift (A, D) is an equilibrium if $\Delta\Pi_1^C(A, D) \geq \Delta\Pi_1^C(D, D)$ and $\Delta\Pi_2^C(A, D) \geq \Delta\Pi_2^C(A, A)$. Note that the first inequality always holds and the second one implies:

$$\Delta a \geq -\frac{\gamma^2}{2} \cdot (a - c_2 - \delta)$$

Similarly (D, A) is an equilibrium if:

$$\Delta a \geq -\frac{\gamma^2}{2} \cdot (a - c_1 + \delta)$$

In the range of γ considered $a - c_1 + \delta > a - c_2 - \delta$ and, then, if Δa is negative enough (D, A) (but not so much) is the only equilibrium.

Finally, as in proposition (1), (A, A) is an equilibrium if the shock is negative and big enough. In this case:

$$\Delta a \leq -\frac{\gamma^2}{2} \cdot (a - c_1 + \delta)$$

This completes the proof.

□

Finding the subgame perfect Nash equilibria described in (6). First we list the profits of the adjustment subgame. $\bar{q}_i$ are the quantities of firm i in the first stage and q_i^* are the quantities of firm i after the adjustment. We use a similar notation for profits.

- Case (A, A): $\Pi_1^*(A, A) = \Pi_2^*(A, A) = 1/16$ with $q_1^* = q_2^* = 1/4$ and $p^* = 1/4$.
- Case (D, D): $\Pi_1^*(D, D) = \bar{\Pi}_1 - \bar{q}_1/4$ and $\Pi_2^*(D, D) = \bar{\Pi}_2 - \bar{q}_2/4$ with $q_i^* = \bar{q}_i$ and $p = 3/4 - \bar{q}_1 - \bar{q}_2$.
- Case (D, A): $\Pi_1^*(D, A) = (3/8 - \bar{q}_1/2)\bar{q}_1$ and $\Pi_2^*(D, A) = (3/8 - \bar{q}_2/2)^2$
- Case (A, D): Analogue to the previous case.

For (D, A) to be an equilibrium of the adjustment game it must be the following. In brackets we have the actual values the profits take in the subgame perfect Nash equilibrium.

$$\Pi_1^A(D,A) \geq \Pi_1^A(A,A) = \frac{1}{16} \qquad \left[\frac{7}{100} > \frac{1}{16}\right]$$

$$\Pi_2^A(D,A) \geq \Pi_2^A(D,D) \qquad \left[\frac{1}{25} > \frac{39}{1600}\right]$$

If this is the case, in the first stage firm 1 solves:

$$\max_{\bar{q}_1} \quad \bar{\Pi}_1 + \Pi_1^*(D,A) = (1 - \bar{q}_1 - \bar{q}_2)\bar{q}_1 + \left(\frac{3}{8} - \frac{\bar{q}_1}{2}\right)\bar{q}_1$$

$$BR_1(\bar{q}_2) = \frac{11}{24} - \frac{\bar{q}_2}{3}$$

In the first stage firm 2 solves:

$$\max_{\bar{q}_2} \quad \bar{\Pi}_2 + \Pi_2^*(D,A) = (1 - \bar{q}_1 - \bar{q}_2)\bar{q}_2 + \left(\frac{3}{8} - \frac{\bar{q}_2}{2}\right)^2$$

$$BR_2(\bar{q}_1) = \frac{1}{2} - \frac{\bar{q}_1}{2}$$

And we get $\bar{q}_1 = 7/20$ and $\bar{q}_2 = 13/40$.
There are two more steps to fully characterize the SPNE of this game.

- Rule out (A, A) case.
  - For (A, A) to be a Nash equilibrium in the adjustment subgame we should have the following for firm 1 (with a similar inequality valid for firm 2 too).

$$\frac{1}{16} = \Pi_1^*(A,A) \geq \Pi_1^*(D,A) \quad \left[\frac{1}{16} \ngeq \frac{5}{72}\right]$$

  - It is easy to see $\bar{q}_1 = \bar{q}_2 = \bar{p} = 1/3$ which implies $\Pi_1^*(D,A) = 5/72$.

- Rule out (D,D) case.
  - For (D,D) to be a Nash equilibrium in the adjustment subgame we should have the following for firm 1 (and also for the other player).

$$\Pi_1^*(D,D) \geq \Pi_1^*(A,D) \quad \left[\frac{7}{144} \ngeq \frac{361}{3136}\right]$$

  - Focus on the left hand side. Firm 1 solves in the first stage.

$$\max_{\bar{q}_1} \quad \bar{\Pi}_1 + \Pi_1^*(D,D) = 2(1-\bar{q}_1-\bar{q}_2)\bar{q}_1 - \frac{\bar{q}_1}{4}$$

$$BR_1(\bar{q}_2) = \frac{7}{16} - \frac{\bar{q}_2}{2}$$

  - Since the game is symmetric $\bar{q}_1 = \bar{q}_2 = 7/24$ which implies $\Pi_1^*(D,D) = 7/44$.
  - Focus on the right hand side. Firm 1 solves in the first stage.

$$\max_{\bar{q}_1} \quad \bar{\Pi}_1 + \Pi_1^*(A,D) = (1-\bar{q}_1-\bar{q}_2) + \left(\frac{3}{8} - \frac{\bar{q}_1}{2}\right)^2$$

$$BR_1(\bar{q}_2) = \frac{5}{12} - \frac{2}{3}\bar{q}_2$$

  - The game is **not** symmetric. But we have solved the problem of the firm 2 before.

$$BR_2(\bar{q}_1) = \frac{11}{24} - \frac{\bar{q}_1}{3}$$

  - Solving the system $\bar{q}_1 = 1/7$ and $\bar{q}_2 = 23/56$ which implies $\Pi_1^*(A,D) = 361/3136$ (the best adjustment is $q_1^* = 19/112$).

So one SPNE is $\{(\bar{q}_1 = 7/20, D), (\bar{q}_2 = 13/40, A)\}$ and we can repeat the process to find the other one.

$\square$

Proof. Proof of proposition (3)

- Focus on firm 2. For (D, D) to be an equilibrium:

$$\Delta\Pi_2(D, D) \geq \Delta\Pi_2(D, A)$$

  Considering that $c_1 + c_2 = 1$ the inequality never holds for any $\alpha \neq 1$. Furthermore, if $\alpha = 1$ both changes in profits are equal to zero because there was no demand shock.

$$\Delta\Pi_2(D, D) = (\alpha - 1)c_1$$

$$\Delta\Pi_2(D, A) = (\sqrt{\alpha} - c_2)^2 - (1 - c_2)c_1$$

- Find the values of $\alpha < 1$ such that (D, A) and (A, D) are Nash equilibria.

  – Case (A, D).
    Provided that $\Delta\Pi 1(A, D) \geq \Delta\Pi_1(D, D)$ always holds, we need to check:

$$c_1(\sqrt{\alpha} - 1) = \Delta\Pi_2(A, D) \geq \Delta\Pi_2(A, A) = (\alpha - 1)c_1(1 - c_2)$$

    Which implies

$$\sqrt{\alpha} \geq \frac{1 - \sqrt{1 - 4c_1c_2}}{2c_1} = \frac{k}{2c_1}$$

  – Case (D, A).
    Provided that $\Delta\Pi_2(D, A) \geq \Delta\Pi_2(D, D)$ always holds, we need to check:

$$c_2(\sqrt{\alpha} - 1) = \Delta\Pi_1(D, A) \geq \Delta\Pi_1(A, A) = (\alpha - 1)c_2(1 - c_1)$$

    It follows that:

$$\sqrt{\alpha} \geq \frac{1 - \sqrt{1 - 4c_1c_2}}{2c_2} = \frac{k}{2c_2}$$

Since $c_1 \leq c_2 \implies \frac{k}{2c_1} \geq \frac{k}{2c_2}$ we get the desired range of values. The third case where (A, A) was omitted from the proof because it follows from previous cases.

$\square$