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The Composition Effect of the Current Account in Small Open Economies

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Abstract

A common problem in international finance consists of the indeterminacy of the equilibrium asset portfolio in small open economy models. This paper develops a simple approach to compute this portfolio under the assumption of incomplete financial markets. The procedure involves the limiting allocation of a class of two-country world economies where the relative size of one of them tends to zero. Such approach allows to identify the effect of portfolio decisions on the dynamics of the net foreign asset position of a small open economy in a structural fashion. As an illustration, an approximated closed-form solution is obtained for a highly stylized model that is isomorphic to the class of Dynamic Stochastic General Equilibrium (DSGE) models typically used in the literature.

JEL Classification: F32.

Keywords: net foreign assets, endogenous portfolios, small open economies, DSGE models.

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1 Introduction

In macroeconomics, it is commonly assumed that financial markets for insurable risks are complete. This assumption states that there exist as many state-contingent assets as possible states of nature. For economies that satisfy such assumption, Modigliani and Miller (1958) conclude that the equilibrium portfolio decisions of agents are irrelevant in the determination of the remaining equilibrium variables.

Nonetheless, such result turns out to be at odds with empirical evidence. For example, in the international macroeconomics literature Bénétrix (2009) examines the large shifts in a country's net asset position due to the re-valuation of its foreign assets and/or foreign liabilities. Equivalently, the author examines, the impact of capital gains on the value of foreign asset and liability positions (also referred to as the *valuation channel* of external adjustment). Furthermore, Lane and Shambaugh (2010) also report that the quantitative significance of the valuation effect has grown in recent years in line with the rapid growth in the scale of cross-border financial holdings.

From a theoretical standpoint, the relaxation of the *complete financial markets* assumption in models based on the Real Business-Cycle framework by King et al. (1988) has led to the indeterminacy of equilibrium asset portfolios because it assumes certain equivalence. In this regard, Devereux and Sutherland (2011) provide a solution method for economies with two equally-sized countries. In this paper, I extend the aforementioned framework to solve for the case of a small open economy. The key result is that in order to pin down the portfolio of interest a portfolio for the representative agent of the the world must also be introduced, which also forces the foreign variables to exhibit a minimum extent of structure. The main advantage of my approach relies on its capacity of characterize a small open economy as the component of a general equilibrium setting in which there is other component that behaves as a big closed economy in the spirit of Obstfeld and Rogoff (1995) and in this regard a small open economy model may be seen as more than a partial equilibrium model. In addition, it provides an alternative to the traditional solution of models that relies on continuous-time frameworks and ranges from Grinols and Turnovsky (1994) to Bhamra et al. (2014), only to cite two examples.

In order to illustrate my approach, instead of providing a general setting I provide a stylized model which is isomorphic to the class of models analyzed by Clarida et al. (1999). In this equilibrium, by construction, the relative variability of the uncorrelated endowment shocks affects the asset portfolio of the small open economy.

The rest of this paper is organized as follows. Section 2 presents a non-exhaustive review of the related literature. Section 3 describes in detail the setup and the assumptions therein. Section 4 defines and characterizes the equilibrium to be analyzed. Section 5 elaborates on the way the equilibrium is approximated. Section 6 explicitly solves the model and analyzes its main properties. Section 7 concludes.

2 Related literature

The portfolio solution method originally proposed by Devereux and Sutherland (2011) focuses on the steady-state value of such portfolio by partially relying on approximation techniques. However, unlike the perturbation approach by Judd (1996) where the deterministic steady-state is defined as the equilibrium position of the system in absence of shocks (certain equivalence), the relevant notion of steady-state portfolio is related to the risky steady-state approach developed by Juillard (2011) where the risky steady-state is defined as the point where, in absence of shocks in the current period, agents decide to stay while expecting shocks in the future and knowing the probability distribution (the risky steady-state is affected by future uncertainty).[1] Also, from a computational standpoint, the iterative algorithm by Juillard (2011) requires a second-order approximation of the entire dynamical system and, therefore, the risky steady-state is simultaneously determined with the other variables of interest. Such algorithm differs from the three-step method by Devereux and Sutherland (2011), although they deliver equivalent result when applied to portfolio choice problem.

Such two-country framework has also been extended in several directions. For example, Okawa and van Wincoop (2012) employ a N-country framework to develop a theory of bilateral asset holdings that takes a gravity form and conclude that very strong assumptions are needed to be made in order to derive at such a theory whereas reasonable extensions of such framework no longer generate a gravity form. Bergin and Pyun (2016) generalize the aforementioned method to a N country setting with $N + 1$ assets and non-zero covariance structure on incomes. In a similar fashion, Steinberg (2018) generalizes the approach to work for any portfolio choice problem within a many-country, many-asset environment. Yu

[1] In an alternative interpretation, agents (banks) take the possibility that the worst-case scenario with regard to asset returns is realized into consideration. Consequently, the risks of holding an asset affect agents' portfolio in the steady state (Aoki and Sudo, 2012, 2013).

(2015) explores the welfare implications for various countries in a center-periphery framework with endogenous portfolio choice and under several stages of financial integration when the two economies are not equally sized. Finally, Heathcote and Perri (2013) use a more general approach. Specifically, they apply a third-order approximation to the portfolio decision rules and a second-order approximation to the remaining equilibrium conditions as they focus on the portfolio dynamics.

It is also worth to mention that the perturbation-based (local) portfolio solution method by Devereux and Sutherland (2011) is not exempt of limitations since, for instance, there is a difficulty with using the method under the presence of borrowing constraints and idiosyncratic income risk as pointed out by Broer (2017). Also, its performance has been compared to global solution methods by Rabitsch and Stepanchuk (2014) who report that the local method performs well at business cycle frequencies, both in the symmetric and asymmetric settings, while significant differences arise at long horizons in asymmetric settings. Moreover, Rabitsch et al. (2015) document that the method by DS 1) does not capture the direct effect of the presence of risk on portfolio holdings and 2) approximates the policy function around net foreign positions equal to zero, even in presence of cross-country differences. For these reasons, Dlugoszek (2017) proposes an algorithm that combines the bifurcation theory and the nonlinear moving average approximation and whose implementation is based on root-finding algorithms and fixed-point techniques.

There are also alternative portfolio solution methods in the literature. For example, Evans and Hnatkovska (2012) propose a numerical procedure that combines both perturbation methods and continuous-time approximations that allows to capture the conditional heteroskedasticity of the state vector and therefore the endogenous non-stationarity that arises when financial markets are incomplete. Such two-step procedure first relies on log-linearization methods and uses an iterative technique afterwards. Gavilán and Rojas (2009) propose a global (projection) solution method that combines the Parametrized Expectations Algorithms (PEA) with the Samolyak algorithms as the standard PEA is computationally unfeasible. Unlike perturbation (local) methods that focus on the steady-state portfolio, this methods has the advantage of allowing for the study of the effect of permanent shocks. Finally, Tille and van Wincoop (2010) focus on the time-variation in portfolio allocation by computing a third-order expansion of the optimality conditions for portfolio choice that induces first-order changes in portfolio shares. Perhaps, the closest work to mine is given by De Paoli (2009) which focuses on the monetary policy implications of several extents of financial market integration and under nominal rigidities in a New-Keynesian framework.

3 Model

I describe the solution procedure through a highly stylized two-country framework based on Obstfeld and Rogoff (1995). The main purpose of such approach is to explicitly show that agents within the small open economy have the incentive to hedge risks even when both countries' economies coincide in all their characteristics excepting for their relative sizes and their corresponding endowment shocks' distributions. Additionally, and for the sake of clarity, the approximated closed-form solution to be obtained allows to illustrate the required steps and their corresponding implications in a transparent way.

Time is discrete ($t = 0, 1, 2, \ldots$) and the world economy is inhabited by a continuum of individuals indexed in the unit interval $[0, 1]$ and arranged into two countries: Home and Foreign. The mass of identical Home individuals equals n and the mass of identical Foreign individuals equals $1 - n$ with $0 < n < 1$.[2] Since the relative sizes of the Home and Foreign economies are denoted by n and $1 - n$, respectively, the case of a small open economy arises whenever one of those measures tends to zero. For the sake of exposition, and without loss of generality, hereafter I focus on the case in which the Home country constitutes the small open economy ($n \to 0$).

The section A of Table 1 summarizes the decision problem faced by the representative agent of each economy. The corresponding preference relations are defined over streams of units of the unique consumption good (hereafter, referred to as *in real terms*) and summarized by the summations of expected discounted instantaneous utilities 1 and 2. For the representative Home (Foreign) individual's objective in expression 1 (2), the term C_t (C_t^*) denotes her individual consumption level in period t. Moreover, it is assumed that the instantaneous utility function $u : \mathbb{R}_+ \to \mathbb{R}$ is strictly increasing, strictly concave, twice continuously differentiable and satisfies the Inada conditions $\lim_{x \downarrow 0} u'(x) = +\infty$ and $\lim_{x \uparrow +\infty} u'(x) = 0$. The assumptions on the subjective discount factor of Home (Foreign) individuals θ_t (θ_t^*) follow Schmitt-Grohé and Uribe (2003). In this sense, the specification 3 (4) is adopted to guarantee that all the variables are stationary in equilibrium, whereas the term $\bar{C}_t$ ($\bar{C}_t^*$) denotes the "average" consumption per Home (Foreign) individual. The parameters ω, ω^*, η and η^* are all assumed to be strictly positive. Without loss of generality, I further assume

[2] Notice that $n = 0$ is ruled out from the analysis.

$\omega = \omega^*$ and $\eta = \eta^*$ to ensure that Home individuals are as patient as Foreign individuals.

There exist two short-lived one-period assets: Home and Foreign. The gross rate of return of the Home (Foreign) asset is represented by R_t (R_t^*). Let B_t (B_t^*) denote the net real amount of Home assets held by a Home (Foreign) individual at the beginning of period t. The family of budget constraints for each Home (Foreign) individual is displayed in 5 (6) where A_t (A_t^*) denotes the real amount of net assets a Home (Foreign) individual starts with at the beginning of period t.[3] The initial conditions A_0 and B_0 (A_0^* and B_0^*) for the Home (Foreign) individual's problem are taken as given. Everyone makes her choices while taking the sequence of gross rates of return $\{R_t, R_t^*\}$ as given.[4] Finally, the term Y_t (Y_t^*) represents the real endowment of a Home (Foreign) individual per period. This endowment is measured in the same units across countries.

Within the above representation, it is worth to emphasize that the market incompleteness is here reflected by two properties. First, the expression 5 summarizes a collection of budget constraints, one for each combination $(R_t, R_t^*, A_t, B_t, Y_t)$. Second, the lack of Arrow Securities implies that no individual is able to smooth consumption across states of nature. An identical argument applies to the family of restrictions in 6.

The Home (Foreign) asset is assumed to be a one-period-lived equity claim on a fraction $0 < \alpha < 1$ ($0 < \alpha^* < 1$) of the Home (Foreign) endowment. The source of uncertainty for the Home (Foreign) economy is summarized by the endowment process $Y_t = Ye^{u_t}$ ($Y_t^* = Y^* e^{u_t^*}$) where Y (Y^*) is a positive constant. For the sake of exposition, I assume that $Y = Y^*$. Let $\{u_t\}$ ($\{u_t^*\}$) denote a sequence of independent and identically distributed random variables with zero mean and positive variance σ^2 (σ^{*2}). In period t, once the uncertainty has been resolved, the real payoff to a claim on Home (Foreign) equity bought in period $t-1$ is given by αY_t ($\alpha^* Y_t^*$) whereas its real price is denoted by Z_{t-1} (Z_{t-1}^*). Therefore, the gross rate of return on Home (Foreign) assets is given by $R_t = \alpha Y_t / Z_{t-1}$ ($R_t^* = \alpha^* Y_t^* / Z_{t-1}^*$).[5] I also

[3]Let $\tilde{B}_t$ denote the net real amount of Foreign assets held by a Home individual when the period t starts. The corresponding budget constraint is then given by $C_t + B_{t+1} + \tilde{B}_{t+1} \leq R_t B_t + R_t^* \tilde{B}_t + Y_t$. Since $A_t \equiv B_t + \tilde{B}_t$, some algebraic manipulations allow to obtain 5. An analogous procedure is employed to obtain 6.

[4]For all t and $j \geq 0$, let the discount factor D_{t+j}^* be equal to 1 if $j = 0$ and equal to $\prod_{k=1}^{j} R_{t+k}^{*-1}$ otherwise. For both problems to be well defined, the no-Ponzi game conditions $\lim_{j \uparrow +\infty} E_t \left[D_{t+j}^* A_{t+j+1} \right] \geq 0$ and $\lim_{j \uparrow +\infty} E_t \left[D_{t+j}^* A_{t+j+1}^* \right] \geq 0$ are imposed.

[5]This is implied by the one-period nature of assets whereas the case of a Lucas tree leads to $R_t = (Z_t + \alpha Y_t) / Z_{t-1}$. Also, the parameters α and α^* reflect the fact that the real return on assets is linked to the amount of (consumption) goods within each economy. In a more general setting this parameter may

assume that there is no default risk in either economy.[6]

4 Competitive equilibrium

The economic environment described above allows me to consistently define its corresponding competitive equilibrium as follows and where prices and allocations are expressed in real terms.

Definition 1. *A competitive equilibrium is given by sequences of asset prices* $\{Z_t, Z_t^*\}$*, gross returns* $\{R_t, R_t^*\}$*, individual allocations* $\{C_t, A_{t+1}, B_{t+1}\}$ *and* $\{C_t^*, A_{t+1}^*, B_{t+1}^*\}$*, and average allocations* $\{\bar{C}_t, \bar{C}_t^*\}$ *such that for all t:*

a) Given $\{R_t, R_t^*\}$ *and* $\{\bar{C}_t\}$*, the Home individual allocations* $\{C_t, A_{t+1}, B_{t+1}\}$ *solve the utility-maximization problem 1 subject to the definition of the discount factor 3 and the budget constraints in 5,*

b) Given $\{R_t, R_t^*\}$ *and* $\{\bar{C}_t^*\}$*, the Foreign individual allocations* $\{C_t^*, A_{t+1}^*, B_{t+1}^*\}$ *solve the utility-maximization problem 2 subject to the definition of the discount factor 4 and the budget constraints in 6,*

c) The gross returns satisfy $R_t = \alpha Y_t / Z_{t-1}$ *and* $R_t^* = \alpha^* Y_t^* / Z_{t-1}^*$*,*

d) World net assets equal zero: $nA_t + (1-n)\,A_t^* = 0$ *and* $nB_t + (1-n)\,B_t^* = 0$*, and*

e) For each economy, the average and individual consumption levels are consistent with each other: $\bar{C}_t = C_t$ *and* $\bar{C}_t^* = C_t^*$*.*

The equations 7-18 in section B of Table 1 characterize the competitive equilibrium.[7] The expressions 7 and 8 are no-arbitrage conditions that require the corresponding marginal

represent, for instance, the capital's share of output.

[6]Specifically, the conditions $R_t\left\{nB_{t-1} + (1-n)B_{t-1}^*\right\} = n\alpha Y_t$ and $R_t^*\left\{n\tilde{B}_{t-1} + (1-n)\tilde{B}_{t-1}^*\right\} = (1-n)\,\alpha^* Y_t^*$ hold for the Home and Foreign economies, respectively.

[7]The complete characterization also requires the following transversality conditions under incomplete financial markets (see Magill and Quinzii, 1994) to hold:

$$\lim_{j\uparrow+\infty} E_t\left[\omega C_t^{-\eta}\frac{u'(C_{t+1})}{u'(C_t)}A_{t+j+1}\right] = 0 \text{ and } \lim_{j\uparrow+\infty} E_t\left[\omega^* C_t^{*-\eta^*}\frac{u'(C_{t+1}^*)}{u'(C_t^*)}A_{t+j+1}^*\right] = 0.$$

utility of consumption to be uncorrelated with the exceeding return.[8] The conditions 9 and 10 are Euler equations where the marginal utility of current consumption equals the discounted expected marginal utility of next period's consumption. It is worth to notice that in equilibrium there is no distinction between individual and average consumption and therefore, by construction, there is also an impatience effect of consumption.[9] The equations 11 and 12 are the (binding) budget constraints that in equilibrium describe, given the equilibrium consumption and portfolio decisions, the evolution of the net assets position for each economy. The expressions 13 and 14 describe the exogenous endowment processes. The link between the (gross) rates of return on assets and their corresponding prices is reflected in 15 and 16. The condition 17 requires the world net assets to equal zero. Namely, any deficit in one economy must be financed by a surplus in the other economy and vice versa.[10] Without loss of generality, in 18 I further assume that the total home net assets are equal to zero.[11] The reader must remember that, in general, a closed-form solution of these class of models is not possible to be obtained. For this reason, the use of approximation methods has become customary.[12] Also, and since the distinctive feature of this stripped-down model is the presence of two assets, hereafter I focus on the equilibrium properties of the sequence of net (real) amounts of Home assets held by Home individuals $\{B_t\}$ and specially on its *unconditional mean* or *non-stochastic steady-state* value B which has an effect on the dynamics of the current account. For such purpose, a special emphasis is placed on the implications of the Home and Foreign no-arbitrage conditions 7 and 8 which imply

$$E_t \left\{ \left[u' \left(C_{t+1} \right) - u' \left(C^*_{t+1} \right) \right] \left(R_{t+1} - R^*_{t+1} \right) \right\} = 0 \tag{31}$$

In 31, since (in equilibrium) the marginal utilities of consumption within each economy are uncorrelated with the differential of returns (7 and 8), it must be the case that the differential

[8] There is an alternative interpretation of these conditions: since the Home and Foreign assets constitute competing ways of achieving next period's consumption, they must provide the same discounted expected marginal utility of consumption. Otherwise, there exist an incentive for re-allocating portfolio composition towards the asset that provides higher benefits in terms of utility.

[9] In each case, the condition e of Definition 1 and the definitions of θ_t and θ^*_t are already embedded.

[10] That is, the entire world behaves as a closed economy for any $0 < n < 1$.

[11] This assumption is made because it allows, along with the no-default assumption, the budget constraints and the zero-world-net-assets identity, to derive the resource constraint of the world economy $nC_t + (1-n) C^*_t = nY_t + (1-n) Y^*_t$.

[12] King et al. (1988) and Campbell (1994) adopt this approach within the business cycle literature. For the international macroeconomics literature, the study by Obstfeld and Rogoff (1995) constitutes a pioneer work.

Table 1: Model summary

A. Household problem			
$\max_{\{C_t, A_{t+1}, B_{t+1}\}_{t=0}^{\infty}} E_0 \sum_{t=0}^{\infty} \theta_t u(C_t)$ subject to	(1)	$\max_{\{C_t^*, A_{t+1}^*, B_{t+1}^*\}_{t=0}^{\infty}} E_0 \sum_{t=0}^{\infty} \theta_t^* u(C_t^*)$ subject to	(2)
$\theta_{t+1} = \theta_t \omega \bar{C}_t^{-\eta}$, $\theta_0 = 1$	(3)	$\theta_{t+1}^* = \theta_t^* \omega^* \bar{C}_t^{*-\eta^*}$, $\theta_0^* = 1$	(4)
$C_t + A_{t+1} \leq R_t^* A_t + (R_t - R_t^*) B_t + Y_t$	(5)	$C_t^* + A_{t+1}^* \leq R_t^* A_t^* + (R_t - R_t^*) B_t^* + Y_t^*$	(6)
B. Equilibrium			
Financial sector:			
$E_t \left[u'(C_{t+1})(R_{t+1} - R_{t+1}^*)\right] = 0$	(7)	$E_t \left[u'(C_{t+1}^*)(R_{t+1} - R_{t+1}^*)\right] = 0$	(8)
Non-financial sector:			
$u'(C_t) = E_t \left[\omega C_t^{-\eta} u'(C_{t+1}) R_{t+1}^*\right]$	(9)	$u'(C_t^*) = E_t \left[\omega^* C_t^{*-\eta^*} u'(C_{t+1}^*) R_{t+1}^*\right]$	(10)
$C_t + A_{t+1} = R_t^* A_t + (R_t - R_t^*) B_t + Y_t$	(11)	$C_t^* + A_{t+1}^* = R_t^* A_t^* + (R_t - R_t^*) B_t^* + Y_t^*$	(12)
$Y_t = Y e^{u_t}$	(13)	$Y_t^* = Y^* e^{u_t^*}$	(14)
$R_t = \alpha Y_t / Z_{t-1}$	(15)	$R_t^* = \alpha^* Y_t^* / Z_{t-1}^*$	(16)
$nA_t + (1-n) A_t^* = 0$	(17)	$nB_t + (1-n) B_t^* = 0$	(18)
C. Equilibrium (approximation)			
Financial sector:			
$E_t[(r_{t+1} - r_{t+1}^*) - \rho c_{t+1}(r_{t+1} - r_{t+1}^*)] = 0 + \mathcal{O}(\epsilon^3)$	(19)	$E_t[(r_{t+1} - r_{t+1}^*) - \rho c_{t+1}^*(r_{t+1} - r_{t+1}^*)] = 0 + \mathcal{O}(\epsilon^3)$	(20)
Non-financial sector:			
$-\rho c_t = E_t \left[-\eta c_t - \rho c_{t+1} + r_{t+1}^*\right] + \mathcal{O}(\epsilon^2)$	(21)	$-\rho c_t^* = E_t \left[-\eta c_t^* - \rho c_{t+1}^* + r_{t+1}^*\right] + \mathcal{O}(\epsilon^2)$	(22)
$\frac{C}{Y} c_t + a_{t+1} = \frac{1}{\beta} a_t + \frac{B}{\beta Y}(r_t - r_t^*) + y_t + \mathcal{O}(\epsilon^2)$	(23)	$\frac{C^*}{Y^*} c_t^* + a_{t+1}^* = \frac{1}{\beta} a_t^* + \frac{B^*}{\beta Y^*}(r_t - r_t^*) + y_t^* + \mathcal{O}(\epsilon^2)$	(24)
$y_t = u_t + \mathcal{O}(\epsilon^2)$	(25)	$y_t^* = u_t^* + \mathcal{O}(\epsilon^2)$	(26)
$r_t = y_t - z_{t-1} + \mathcal{O}(\epsilon^2)$	(27)	$r_t^* = y_t^* - z_{t-1}^* + \mathcal{O}(\epsilon^2)$	(28)
$nYa_t + (1-n)Y^* a_t^* = 0 + \mathcal{O}(\epsilon^2)$	(29)	$nB + (1-n)B^* = 0$	(30)

of marginal utilities of consumption is in turn uncorrelated with the differential of returns. It is worth to emphasize that the condition 31 constitutes the key expression for computing the approximated equilibrium portfolio since it provides a necessary condition that filters out potential candidates.

5 Approximation

The previous characterization is isomorphic to the class of Dynamic Stochastic General Equilibrium (hereafter, DSGE) models as it comprises utility-maximizing agents and market-clearing conditions. Also, it is well known that the solution of the previous class of models is computed up to an approximation order chosen by the researcher. For this purpose, I employ Taylor expansions around the *non-stochastic steady state* of the model. Firstly, let

$$x_t \equiv \frac{X_t - X}{X}$$

denote, otherwise mentioned, the percent deviation of X_t from its non-stochastic steady state value $X > 0$. Also, let $\mathcal{O}(\epsilon^j)$ denote "terms of j-th order and higher." The expressions

19-29 in section C of Table 1 summarize the approximation of the conditions 7-17 that characterize the competitive equilibrium, respectively, along with 30 which constitutes the steady state version of 18. Within such representation, $\rho \equiv -Yu''(Y)/u'(Y) > 0$ denotes the relative risk aversion coefficient evaluated at the steady state value of the endowment process and $\beta \equiv \omega Y^{-\eta} \in (0,1)$ denotes the steady state discount factor. Following Devereux and Sutherland (2011), a second-order approximation is taken for only the so-called "Financial sector" conditions 7 and 8 and this leads to 19 and 20. The main argument is that a first-order approximation implicitly reduces agents to be risk-neutral and care only about expected future returns. Nonetheless, risk-aversion arguments are needed to pin down the equilibrium portfolio. Standard first-order approximations are taken for the remaining conditions 9-17 and lead to 21-29 where the net asset position deviations are written as $a_t \equiv (A_t - A)/Y$ and $a_t^* \equiv (A_t^* - A^*)/Y^*$.

At this point, the reader should notice that the approximated characterization in section C of Table 1 differs in several ways from the standard approach used in the literature. However, the minimalistic representation therein allows me to elaborate on the minimum structure required for computing the small open economy's equilibrium portfolio.

6 Approximated closed-form solution

In addition to the details of the previous section, I introduce a new insight into the discussion. For such purpose, notice that most of the literature has typically worked out models by directly imposing the small open economy assumption (that is, $n = 0$ is further assumed) along with exogenous processes for the rest-of-the-world variables (variables with a star superscript). Proceeding in that fashion is not innocuous as there is relevant structure that is lost and key to consistently solve the model for the variables of interest.

In this regard, the structure imposed on external variables becomes extremely relevant.[13] To fully understand this, I compare it to other approaches. First, consider a situation in which we approximate the equilibrium such that it implicitly assumes risk-neutral agents and exogenous rest-of-the-world variables. Such case arises if we only used the *Uncovered Interest Parity* $E_t[r_{t+1} - r_{t+1}^*] = 0 + \mathcal{O}(\epsilon^2)$ instead of 19 and imposed exogenous autoregressive processes describing the external variables. It is easy to show that in this case the Home

[13] This has been previously done in the literature (for example, Faia and Monacelli (2008)), although with different purposes.

portfolio is undetermined (i.e. there is one degree of freedom) since it is implicitly assumed that the decision makers are risk neutral at the margin. Second, consider an approximation that considers risk-averse Home agents (condition 19) and assumes an exogenous autoregressive process for the rest-of-the-world variables. Once again, it is easy to show that computing the solution requires the use of numerical methods as a non-linearity arises.

6.1 Solving the large closed economy's non-financial sector

Finally, let's focus back on the structural framework with risk-averse Home and Foreign agents originally considered. The Foreign economy behaves like a closed economy as $n \to 0$ and therefore a closed form solution can be obtained for the non-stochastic steady state ratio $B/(\beta Y)$. Specifically, for any $0 < n < 1$, substitute the Foreign holdings of Home assets $B^* = -[n/(1-n)]B$ from 30 into the Foreign economy's future net assets a^*_{t+1} in 24 to obtain

$$a^*_{t+1} = \frac{1}{\beta} a^*_t - \left(\frac{n}{1-n}\right) \frac{B}{\beta Y} (r_t - r^*_t) + y^*_t - \frac{C^*}{Y^*} c^*_t + \mathcal{O}(\epsilon^2), \tag{32}$$

and notice that, by construction, the portfolio effect $B/(\beta Y)$ of Home variables on a^*_{t+1} vanishes as $n \to 0$ which departs from the common practice of setting $n = 1/2$ based on previous studies (Trani, 2012). On the other hand, the market-clearing condition 29, after taking $n \to 0$, collapses to $a^*_t = 0 + \mathcal{O}(\epsilon^2)$ (world net assets are zero) which in turn reduces 32 to $c^*_t = y^*_t + \mathcal{O}(\epsilon^2)$ (the rest of the world only consumes its own endowment). To compute the equilibrium assets' returns and prices, substitute the former result into 22 to obtain $E_t r^*_{t+1} = -(\rho - \eta) u^*_t + \mathcal{O}(\epsilon^2)$ which in turn implies $z^*_t = (\rho - \eta) u^*_t + \mathcal{O}(\epsilon^2)$.

Once a partial solution is computed for the rest of the world (a large closed economy), the solution procedure for the small open economy is summarized as follows:

> *Step 1.* Provided with the solution for the rest of the world, solve the small open economy's "Non-financial sector" conditions 21, 23, 25, 27 and 29 which are based on a first-order approximation. As expected, the results will depend on the still undetermined portfolio ratio $B/(\beta Y)$.
>
> *Step 2.* Use the results from *Step 1* to solve for the portfolio ratio that satisfies the approximated version of 31 that is implied by the "Financial sector" conditions 19-20.

6.2 Solving the small open economy's non-financial sector

For the small open economy, the budget constraint 23 (after substituting the endowment process 25) and the Euler equation 21 can be represented in compact form by

$$\begin{bmatrix} a_{t+1} \\ E_t c_{t+1} \end{bmatrix} = \underbrace{\begin{bmatrix} 1/\beta & -1 \\ 0 & 1-\eta/\rho \end{bmatrix}}_{\mathbf{A}} \begin{bmatrix} a_t \\ c_t \end{bmatrix} + \underbrace{\begin{bmatrix} 1 & 0 & 1 \\ 0 & -(1-\eta/\rho) & 0 \end{bmatrix}}_{\boldsymbol{\gamma}} \begin{bmatrix} u_t \\ u_t^* \\ \frac{B}{\beta Y}\xi_t \end{bmatrix} + \mathcal{O}(\epsilon^2) \quad (33)$$

where $\xi_t \equiv r_t - r_t^*$ denotes the exceeding return. Moreover, up to a first-order approximation the *Uncovered Interest Parity* $E_t[r_{t+1} - r_{t+1}^*] = 0 + \mathcal{O}(\epsilon^2)$ holds and in the particular case of this model the exceeding return is expressed as the differential of endowment shocks:

$$r_t - r_t^* = u_t - u_t^* + \mathcal{O}(\epsilon^2) \quad (34)$$

for all t (see Appendix A). In 33, since the entries of the principal diagonal of $\mathbf{A}$ satisfy $|1/\beta| > 1$ and $|1-\eta/\rho| < 1$, the conditions of Blanchard and Kahn (1980, Proposition 1) hold and the unique forward-looking solution for c_t is given by (see Appendix B)

$$\begin{aligned} c_t \;=\; & \left[\frac{1}{\beta} - \left(1 - \frac{\eta}{\rho}\right)\right] a_t + \left[1 - \beta\left(1 - \frac{\eta}{\rho}\right)\right] u_t \\ & + \beta\left(1 - \frac{\eta}{\rho}\right) u_t^* + \left[1 - \beta\left(1 - \frac{\eta}{\rho}\right)\right] \frac{B}{\beta Y}\xi_t + \mathcal{O}(\epsilon^2). \end{aligned} \quad (35)$$

As previously mentioned, the *partial* solution for c_t depends on a particular steady state value $B/(\beta Y)$ (i.e. there is one degree of freedom).

6.3 Non-stochastic steady state portfolio

It is worth to notice that the expressions 19 and 20 imply

$$E_t\left[-\rho\left(c_{t+1} - c_{t+1}^*\right)\left(r_{t+1} - r_{t+1}^*\right)\right] = 0 + \mathcal{O}(\epsilon^3). \quad (36)$$

which approximates the condition 31 that, once again, states that if each marginal utility of consumption is uncorrelated with the exceeding return, then it has to be the case that the differential of marginal utilities of consumption across countries must be uncorrelated with the differential of returns as well. Notice that the left hand side of 36 is a second moment expressed as the product of first order terms which in turn can be computed separately:

$c_{t+1} - c^*_{t+1}$ and $r_{t+1} - r^*_{t+1}$. Given this, substituting 34, 35 and the already obtained result for the large closed economy $c^*_t = u^*_t + \mathcal{O}(\epsilon^2)$ into 36 and solving for $B/(\beta Y)$ leads to (see Appendix C)

$$\frac{B}{\beta Y} = -\frac{1}{1 + \sigma^2/\sigma^{*2}} + \mathcal{O}(\epsilon^3) \tag{37}$$

which is an expression that resembles the one obtained by Merton (1969, equation 25) under the case of instantaneous utility functions exhibiting a constant relative risk aversion coefficient.

Finally, the prior results can be used to re-write the small open economy's equilibrium budget constraint in terms of the evolution of its net foreign assets

$$\begin{aligned} c_t &= \left[\frac{1}{\beta} - \left(1 - \frac{\eta}{\rho}\right)\right] a_t + \left[1 - \beta\left(1 - \frac{\eta}{\rho}\right)\right] u_t \\ &\quad + \beta\left(1 - \frac{\eta}{\rho}\right) u^*_t + \left[1 - \beta\left(1 - \frac{\eta}{\rho}\right)\right] \frac{B}{\beta Y}\left(u_t - u^*_t\right) + \mathcal{O}(\epsilon^2) \end{aligned} \tag{38}$$

$$a_{t+1} = \underbrace{\frac{1}{\beta} a_t}_{\text{volume effect}} + \underbrace{\frac{B}{\beta Y}\left(u_t - u^*_t\right)}_{\text{composition effect}} + \underbrace{u_t}_{\text{endowment}} - \underbrace{c_t}_{\text{consumption}} + \mathcal{O}(\epsilon^2) \tag{39}$$

which provides a structural analysis of the current account. In particular, it can be seen that the relative volatility of shocks has not only a direct effect in 39 but also an indirect effect through the response of consumption in 38.

7 Conclusions

Although highly stylized, the two-country framework described here contains two elements present in the class of Dynamic Stochastic General Equilibrium (DSGE) models. Namely, an Euler equation and a law of motion for each economy. The main difference relies on the introduction of foreign marginal conditions that help pin down the equilibrium portfolio for the small open economy. Within these margins, the risk component (contained in the second moments of real returns) is not neglected anymore. This so happens because the approximation order employed does not preclude that decisions are taken by risk-neutral agents. Therefore, the indeterminacy of the equilibrium portfolio no longer holds.

Appendix A Exceeding returns

The conditions 19 and 20 lead to

$$E_t\left[r_{t+1}-r_{t+1}^*\right]=0+\underbrace{E_t\left[\rho c_{t+1}\left(r_{t+1}-r_{t+1}^*\right)\right]+\mathcal{O}(\epsilon^3)}_{\mathcal{O}(\epsilon^2)} \tag{40}$$

and

$$E_t\left[r_{t+1}-r_{t+1}^*\right]=0+\underbrace{E_t\left[\rho c_{t+1}^*\left(r_{t+1}-r_{t+1}^*\right)\right]+\mathcal{O}(\epsilon^3)}_{\mathcal{O}(\epsilon^2)} \tag{41}$$

which imply that, up to a first-order approximation, the sequence of future exceeding returns $\left\{r_{t+1}-r_{t+1}^*\right\}$ behaves as a sequence of zero-mean random variables.

For the specific model under consideration, the conditions 25, 26, 27 and 28 imply

$$r_{t+1} = u_{t+1}-z_t+\mathcal{O}(\epsilon^2) \tag{42}$$

$$r_{t+1}^* = u_{t+1}^*-z_t^*+\mathcal{O}(\epsilon^2). \tag{43}$$

Conditions 41, 42 and 43 imply that $z_t=z_t^*+\mathcal{O}(\epsilon^2)$ for all t which in turn implies

$$r_t-r_t^*=u_t-u_t^*+\mathcal{O}(\epsilon^2),\text{ for all } t. \tag{44}$$

Appendix B Equilibrium consumption

In equation 33, define

$$\mathbf{A}=\begin{bmatrix}1/\beta & -1\\ 0 & 1-\eta/\rho\end{bmatrix}\text{ and }\boldsymbol{\gamma}=\begin{bmatrix}1 & 0 & 1\\ 0 & -(1-\eta/\rho) & 0\end{bmatrix}.$$

Then, it is easy to verify that $\mathbf{A}=\mathbf{BJC}$ where

$$\begin{aligned}\mathbf{B} &= \begin{bmatrix}\mathbf{B}_{11} & \mathbf{B}_{12}\\ \mathbf{B}_{21} & \mathbf{B}_{22}\end{bmatrix}=\begin{bmatrix}\frac{1}{1/\beta-(1-\eta/\rho)} & 1\\ 1 & 0\end{bmatrix},\\ \mathbf{J} &= \begin{bmatrix}\mathbf{J}_1 & 0\\ 0 & \mathbf{J}_2\end{bmatrix}=\begin{bmatrix}1-\eta/\rho & 0\\ 0 & 1/\beta\end{bmatrix}\text{ and}\\ \mathbf{C} &= \begin{bmatrix}\mathbf{C}_{11} & \mathbf{C}_{12}\\ \mathbf{C}_{21} & \mathbf{C}_{22}\end{bmatrix}\begin{bmatrix}0 & 1\\ 1 & -\frac{1}{1/\beta-(1-\eta/\rho)}\end{bmatrix}.\end{aligned}$$

Also, define

$$\boldsymbol{\gamma} \equiv \begin{bmatrix} \gamma_1 \\ \hdashline \gamma_2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 1 \\ \hdashline 0 & -(1-\eta/\rho) & 0 \end{bmatrix}. \tag{45}$$

The forward-looking solution for c_t is implied by the equation 3 in Blanchard and Kahn (1980), which leads to equation 35 in text.

Appendix C Equilibrium portfolio

The solution in 35, along with $c_t^* = u_t^* + \mathcal{O}(\epsilon^2)$, implies that

$$\begin{aligned} c_{t+1} - c_{t+1}^* &= \left[\frac{1}{\beta} - \left(1 - \frac{\eta}{\rho}\right)\right] a_{t+1} \\ &+ \left[1 - \beta\left(1 - \frac{\eta}{\rho}\right)\right] u_{t+1} - \left[1 - \beta\left(1 - \frac{\eta}{\rho}\right)\right] u_{t+1}^* \\ &+ \left[1 - \beta\left(1 - \frac{\eta}{\rho}\right)\right]\left(\frac{B}{\beta Y}\right)\left(u_{t+1} - u_{t+1}^*\right). \end{aligned} \tag{46}$$

Plugging 46 into 36 and solving for $B/(\beta Y)$ leads to the expression 6 in text.

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