Universidad de
SanAndrés

Escuela de Administración y Negocios

Maestría en Finanzas

CROSS SECTIONAL DISPERSION IN ACTIVE PORTFOLIO MANAGEMENT OF THE CAC40 INDEX

- Tesista: Guillaume MANCA guillaumemanca@gmail.com
- Legajo: Pas0000381037
- Mentor: Ignacio WARNES iwarnes@udesa.edu.ar

Buenos Aires, agosto 2015

Abstract:

When active managers try to outperform the benchmark they predict which stocks will perform better than others, forecasting the cross-sectional dispersion (or standard deviation) of returns. This measure is central to understand and enhance performances of the active managers.

Based on the hypothesis set forth by Larry R. Gorman (2010) we will verify whether this measure of the risk impacts systematic and idiosyncratic risks, and impacts the results of the active manager looking for outperforming the benchmark. Moreover we will verify whether active portfolio returns are function of managers' skill and cross-sectional dispersion, on the CAC40, in order to prove that Gorman's work conclusions are verified on the French market.

Introduction:

The goal of active portfolio management is to generate a portfolio with return performing better than the return of the index taken as benchmarking. This new portfolio is built assigning bigger weight on the "winners" and smaller weight on the "losers" in the portfolio considered.

Beyond the ability to rank winners and losers, however, active portfolio management also requires a reasonable degree of return dispersion in order to provide an adequate opportunity set for ranking stocks' relative expected returns. In fact, when active managers predict which stocks will perform better than others, they are essentially forecasting the cross-sectional dispersion (or standard deviation) of returns, which is simply a more formal term for the future distribution of relative winners and losers. It follows intuitively that dispersion, the extent to which stock prices will move in different directions, represents a key consideration in any forecast of relative security returns.

Can characteristics of the cross-sectional dispersion as a risk measure for active portfolio managers be verified? What is its role in manager's performances?

Cross-sectional variability in returns is an important feature of equity markets, particularly from a portfolio management and performance measurement perspective. DeSilva, Sapra, and Thorley (2001) find that cross-sectional variability in individual security returns within a market (which they term "dispersion") is an important determinant of the return dispersion across managed portfolios. In a high-dispersion environment, portfolio returns will tend to differ greatly from each other and from the aggregate market return. Despite the differentiating effect of high dispersion, DeSilva, et al. question whether the diversity in portfolio "alpha" during such periods is an accurate reflection of the diversity of investment management talent.

Ankrim and Ding (2002) argue that when cross-sectional return dispersion is high such as at the end of the 1990s, differences in skill across managers are magnified. Thus, managers with slightly below-average stock picks are penalized disproportionately for those suboptimal weightings. Indeed, DeSilva et al propose making an adjustment to portfolio results for periods of unusually high and low security-return dispersion (their proposal is to weight alphas by the inverse of dispersion).

Yu and Sharaiha (2007) interpret cross- sectional stock return dispersion to reflect the current alpha potential in the equity market. Bouchey et al point out that cross-sectional dispersion may bear a relation to the Fundamental Law of Active Management proposed by Grinold (1989), in which the breadth of investment opportunities is a key input to the information-ratio performance metric. They suggest that cross- sectional dispersion may be thought of as a proxy for Grinold's concept of "market breadth."

Yu and Sharaiha note that the worst type of environment for stock pickers is low time-series return dispersion in the presence of low cross-sectional return dispersion. In such environments, active portfolio managers have few opportunities to distinguish themselves. Yu and Sharaiha suggest that superior portfolio managers could employ techniques such as variance swaps to hedge against low dispersion, thus magnifying returns and achieving more separation from peers.

Smith (2014) is the first to study hedge funds in the context of cross-sectional dispersion. It uses four models to estimate alpha for different equity hedge fund: the capital asset pricing model, the Fama-French 3-factor model, the Carhart 4-factor model, and the Fung and Hsieh 7- factor model. The results across the four models are highly consistent. On average, equity hedge funds have generated positive alpha during the sample period between 1996-2013. Consistent with past findings on mutual funds, they find that hedge fund performance is strongly related to the return-dispersion environment. High-dispersion periods are associated with significantly higher alpha estimated using all four pricing models, which confirms their hypothesis that equity hedge fund managers tend to exploit opportunities to use active management and distinguish their performance from that of peers.

We decided to study the index CAC40 in the context of cross sectional dispersion. Larry R. Gorman, Steven G. Sapra, Robert A. Weigand (2010) worked on the cross sectional-dispersion of returns as a measure of the risk. They show that all key measures of portfolio risk – total, systematic and idiosyncratic – are positively related to return dispersion, with dispersion primarily affecting idiosyncratic risk. Moreover they introduce this measure as a strong parameter of information ratio, active portfolio returns and realized tracking errors, and demonstrate how developing a better understanding of the role of cross sectional dispersion in active management can enhance managers' performance.

From this perspective, they show how the cross sectional-dispersion of returns is more relevant as a measure of risk than time series volatility.

Gorman assumes in his work that the VCV matrix is restricted. Indeed, when a VCV matrix is unrestricted in its structural form, analytical results are often complex and difficult to interpret. The VCV matrix is the covariance matrix (also known as dispersion matrix or variance–covariance matrix) whose element in the i, j position is the covariance between the i th and j th elements of a random vector (that is in our case, of a vector of CAC 40 stocks relative returns).

In Gorman's work, the restricted VCV matrix is symmetrical with stock returns variance all equal to σ^2 and stock by stock covariance all equal to ρσ^2. σ^2 is the average time series volatilities of individual stocks, ρ is the average stock-by-stock correlation.

With the cross sectional dispersion, Gorman measures the risk as an overall reaction of the market stocks against financial events. This restriction of the VCV matrix implies that, in a matter of risk, all stock returns follow a normal law with mean 0 and the same volatility σ^2. The standard deviation of returns is therefore measured *across* all stocks on a particular day or month, and not stock-by-stock. This is the cross sectional dispersion which measures the risk through the whole market and not from individual stocks.

So, this paper is not the first to study cross-sectional dispersion in the context of active portfolio management, but we focus here in the validation of characteristics and conclusions from last paper "The role of cross-sectional dispersion in active portfolio management » for a standard market as the CAC40.

Though, we will test and validate numerically on the French market the Cross Sectional Dispersion's characteristics and role in manager's performance.

Conclusions of this study being strong and relevant, they could then be interpreted and used by many portfolio managers working on the CAC40.

Hypothesis:

To validate the results on the characteristics of the cross sectional dispersion, we define a set of hypothesis that we will verify for a relative return portfolio strategy, based on the index CAC40 as benchmark, from the 22/06/2007 to 20/11/2014, on daily bases.

The 1st hypothesis of this study are set to show how the cross sectional dispersion can be a fair measure of the risk. It has been compared to the time series volatility, which is the most famous indicator of the risk:

- Trend of the cross sectional dispersion from Gorman is the same as the trend of of time series volatility for the CAC40
- Mean and trend are the same between Sigma CS (cross sectional dispersion) from Gorman and numerical instantaneous Sigma CS (from the definition) for the CAC40

The 2nd set of hypothesis aims to identify how the drivers of the cross sectional diversification affect total and idiosyncratic risk.

- Idiosyncratic risk from Gorman formula tends to 0 when n (total number of stocks on the portfolio) tends to N (total number of stocks on the benchmark) for the CAC40

- Systematic risk resulted from Sigma CS in Gorman formula is negligible for the CAC40

The 3rd set of hypothesis will point out the role of the cross sectional in active portfolio management. We will apply the Gorman's optimized portfolio weights to our portfolio against the index CAC40 and verify that the new Information Ratio defined by Gorman is in line with IR defined in the Fundamental Law of Active Management:

- Relative returns of the portfolio optimized by Gorman formula are always positives (return of the portfolio better than the benchmark, index CAC40)

- Information Ratio as known from the Fundamental Law of Active Management is numerically similar as Information Ratio with tracking error as defined by Gorman, for the portfolio optimized

Methodology:

To validate the conclusions made by Larry R. Gorman et al. on the CAC40, we are going to follow the methodology and the plan developed in their paper.

We will translate the conclusions in a set of mathematical relations that we will verify or not by empirical tests, these relations are set as hypothesis in the previous paragraph. Indeed, Gorman structured his work about cross sectional diversion in 3 particular topics:

1) Cross sectional dispersion as a measure of the risk

2) Portfolio diversification and systematic and idiosyncratic risk

3) Active portfolio management

Each set of properties will be numerically validated through empirical tests with Matlab.

We will extract series of daily prices for each stock of the index CAC40 between the 22/06/2007 to 20/11/2014.

1) Cross sectional dispersion as a measure of the risk

- Trend of the cross sectional dispersion from Gorman is the same as the trend of time series volatility for the CAC40

On this test the explicative variables are the set of variables defining the cross sectional diversion by Gorman:

$$\sigma_{CS\ article,t} = \sigma_{TS,t} * \sqrt{1 - \rho_{TS,t}}$$

It will be compared to the definition of times series volatility (in term of relative returns):

$$\sigma_{TS,t} = \left(\frac{1}{252} \sum_{j=0}^{252} \left(\left(R_{i,t-j} - R_{CAC40,t-j} \right) - \overline{\left(R_{i,t} - R_{CAC40,t} \right)} \right)^2 \right)^{\frac{1}{2}}$$

To validate the hypothesis we will compare standard deviation of sigma CS (from Gorman) and sigma TS.

- Mean and trend are the same between Sigma CS (cross sectional dispersion) from Gorman and numerical instantaneous Sigma CS for the CAC40

The same Sigma CS Gorman will be compared to the definition formula of cross sectional diversity at an instant t:

$$\sigma_{CS,t} = \left(\frac{1}{40} \sum_{i=0}^{40} \left(\left(R_{i,t} - R_{CAC40,t} \right) - \overline{\left(R_{i,t} - R_{CAC40,t} \right)} \right)^2 \right)^{\frac{1}{2}}$$

To validate the hypothesis we will compare standard deviation and mean of the two sigmas CS.

2) Portfolio diversification and systematic and idiosyncratic risk

- Idiosyncratic risk from Gorman formula tends to 0 when n (total number of stocks on the portfolio) tends to N (total number of stocks on the benchmark) for the CAC40

The total risk of the portfolio can be decomposed in 2 terms, the systematic risk and the idiosyncratic risk:

$$\sigma_{p,t}^2 = \left(\rho_{TS,t} * \sigma_{TS,t}^2 + \frac{\sigma_{CS,t}^2}{N} \right) + \left(\sigma_{CS,t}^2 * \left(\frac{1}{n} - \frac{1}{N} \right) \right)$$

The 2nd term formula is the idiosyncratic risk, representing the dependant variable for this test.

To validate the hypothesis, we will validate then that this risk numerically tends to 0 when one of his explicative variable n tends to N.

- Systematic risk resulted from Sigma CS in Gorman formula is negligible for the CAC40

To validate the hypothesis we will compare standard deviation and mean of idiosyncratic risk, systematic risk and total risk over the period studied.

3) Active portfolio management

- Relative returns of the portfolio optimized by Gorman formula are always positive (return of the portfolio better than the benchmark, index CAC40)

Optimized portfolio by Gorman is given by following equations:

$$w^*_{a,t} = \frac{E(r)_t}{a_0 \sigma^2_{cs,t}}$$

$$E(r)_t = IC_t \sigma_{cs,t} z_t$$

We will assign the optimal weights to the stocks of our portfolio and will calculate the relative return of the portfolio over the period.

The hypothesis will be validated if the relative returns over the period are always higher than 0.

- Information Ratio as known from the Fundamental Law of Active Management is numerically similar as Information Ratio with tracking error as defined by Gorman, for the portfolio optimized

The information ratio IR is given by formula:

$$IR_t = \frac{E(r)_t}{TE_t} = IC_t \sqrt{n}$$

The right hand part of the formula is the definition of the Information Ratio in the Fundamental Low of Asset Management and the left hand part is the IR defined by Gorman in his work, under the constraint on VCV matrix.

The Tracking Error TE*t* can be calculated with the formula:

$$TE_t = \sigma_{cs,t}{}^2 w^{*\prime}_{at} w^*_{at}$$

To validate our hypothesis, we will calculate mean and standard deviation of the difference between the Information Ratio calculated by one formula and the other.

Data:

Data from the CAC40 stock market are public and daily available, this market being liquid enough. We make an extraction of daily quotations of the CAC40 stocks from Bloomberg.

The time period analysed will be from the 22/06/2007 to 20/11/2014.

Results:

<u>1) Cross sectional dispersion as a measure of the risk</u>

a) Cross sectional dispersion from Gorman VS time series volatility

The hypothesis we want to validate is that the trend of the cross sectional dispersion from Gorman is the same as the trend of time series volatility for the CAC40. If we validate this hypothesis, it then makes sense to use Sigma CS (from Gorman) as a measure of the risk.

On this test the explicative variables are the set of variables defining the cross sectional diversion by Gorman:

$$\sigma_{CS\ article,t} = \sigma_{TS,t} * \sqrt{1-\rho_{TS,t}}$$

This measure is compared to the definition of times series volatility (in term of relative returns):

$$\sigma_{TS,t} = \left(\frac{1}{252}\sum_{j=0}^{252}\left(\left(R_{i,t-j} - R_{CAC40,t-j}\right) - \overline{\left(R_{i,t} - R_{CAC40,t}\right)}\right)^2\right)^{\frac{1}{2}}$$

After calculating the two standard deviations with our data, we found the results under here:

Standard deviation of the cross sectional diversion by Gorman: 0,0057

Standard deviation of the times series volatility: 0,0041

Difference of the two standard deviations: 0,0017 or 29,82%

To get a better understanding of the two measures we plotted for comparison the two measures of the risk:

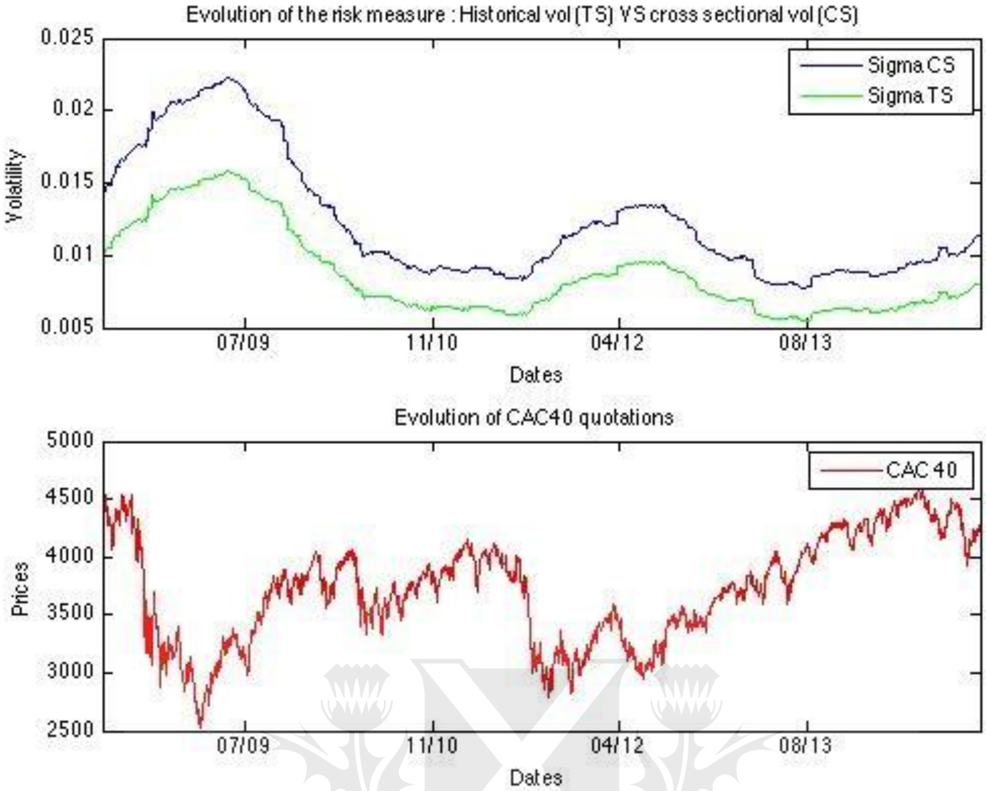


Figure 1: Evolution of the risk measure

We can notice the two similar trends with a bigger jump of Sigma CS in 2008 at the fall of the market for subprime crisis.

CS volatility shows a same trend and evolution as historical volatility, validating our hypothesis.

Moreover, a higher CS volatility standard deviation than historical volatility standard deviation shows us that CS volatility is more sensitive to variations of the market, as the jump in 2008.

In conclusion CS volatility is a good measure of the risk and is even more sensitive than the historical volatility to variations of the market.

b) Cross sectional dispersion from Gorman VS numerical instantaneous Sigma CS

We compare now the cross sectional volatility from Gorman VS the definition of the numerical instantaneous Sigma CS. Our goal is to make sure that Gorman's definition of Sigma CS is in line with the theory.

Our hypothesis is that the mean and trend are the same between Sigma CS (cross sectional dispersion) from Gorman and numerical instantaneous Sigma CS for the CAC40

The same Sigma CS Gorman is compared to the definition formula of cross sectional diversity at an instant t:

$$\sigma_{CS,t} = \left(\frac{1}{40}\sum_{i=0}^{40}\left(\left(R_{i,t} - R_{CAC40,t}\right) - \overline{\left(R_{i,t} - R_{CAC40,t}\right)}\right)^2\right)^{\frac{1}{2}}$$

To validate the hypothesis we compare standard deviation and mean of the two sigmas CS.

Mean of the cross sectional diversion by Gorman: 0,0107

Mean of the numerical instantaneous cross sectional diversion: 0,0153

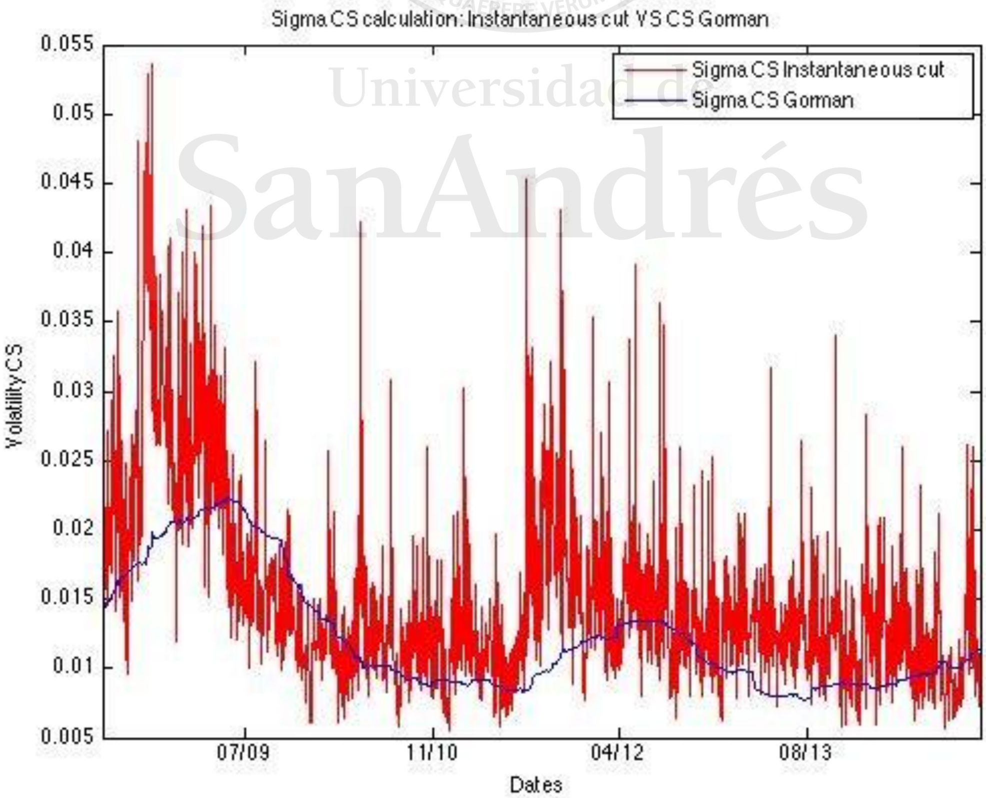
Difference of the two means: 0,0047 or 30,07%

Standard deviation of the cross sectional diversion by Gorman: 0,0057

Standard deviation of the numerical instantaneous cross sectional diversion: 0,0068

Difference of the two standard deviations: 0,0010 or 14,71%

The difference of the two means is too high to validate the hypothesis. To get a better understanding of the two measures we plotted the two cross sectional diversions:



The trend is clearly the same with higher mean value for the numerical instantaneous cross sectional diversion.

In fact, Sigma CS Gorman smooths the numerical instantaneous Sigma CS curve, especially the peak levels, which explains why Sigma CS Gorman mean is 30% smaller than numerical instantaneous Sigma CS mean.

With similar trend and a mean difference explained, we can consider that Gorman's definition of Sigma CS is in line with the theory, and can be exploit.

2) Portfolio diversification and systematic and idiosyncratic risk

a) The impact of portfolio diversification

Our goal here is to identify which part of the total risk is impacted by portfolio diversification, and more specifically, to verify that the idiosyncratic risk is diversifiable.

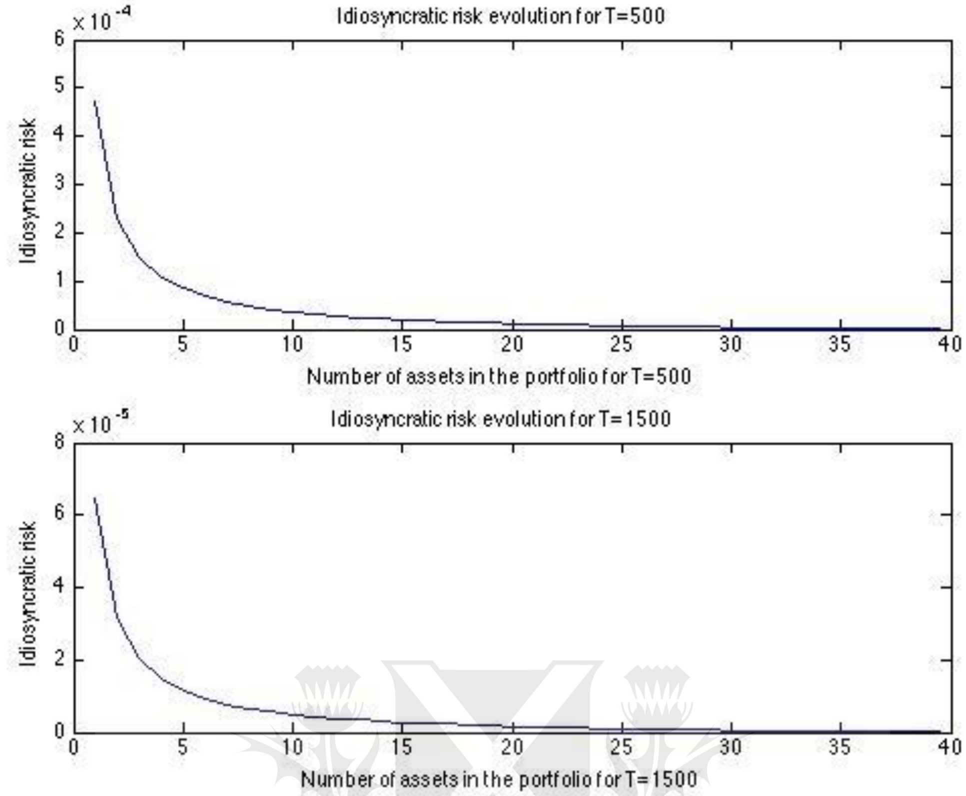
The hypothesis set is that idiosyncratic risk from Gorman formula tends to 0 when n (total number of stocks on the portfolio) tends to N (total number of stocks on the benchmark) for the CAC40

Gorman work states that the total risk of the portfolio can be decomposed in 2 terms, the systematic risk and the idiosyncratic risk:

$$\sigma_{p,t}^2 = \left(\rho_{TS,t} * \sigma_{TS,t}^2 + \frac{\sigma_{CS,t}^2}{N}\right) + \left(\sigma_{CS,t}^2 * \left(\frac{1}{n} - \frac{1}{N}\right)\right)$$

The 2nd term formula is the idiosyncratic risk, specific to each asset, and the 1st term formula is the systematic risk, which is generated by the market.

We calculated and plug the value of the idiosyncratic risk, with n tending to high numerical values, T being the number of time values taken for the calculation.



It appears clearly that the idiosyncratic risk tends quickly to 0 with the number of assets in the portfolio growing.

The hypothesis is validated; the idiosyncratic risk can be highly reduced by diversification of the portfolio.

b) Systematic risk resulted from Sigma CS

In time series based Modern Portfolio Strategy, once the risk diversified, the risk known as diversifiable (resulted from time series volatility) doesn't impact anymore the total risk, as it only remains the systematic risk.

The objective here is to make a parallel between Sigma CS and this time series volatility property. The hypothesis taken is that the systematic risk resulted from Sigma CS in Gorman formula is negligible for the CAC40

In his study, Gorman demonstrated that the systematic risk (the 1st term formula) is composed of one component dependant from sigma CS and another one independent from sigma CS. Besides, he decomposed the systematic risk of the portfolio as:

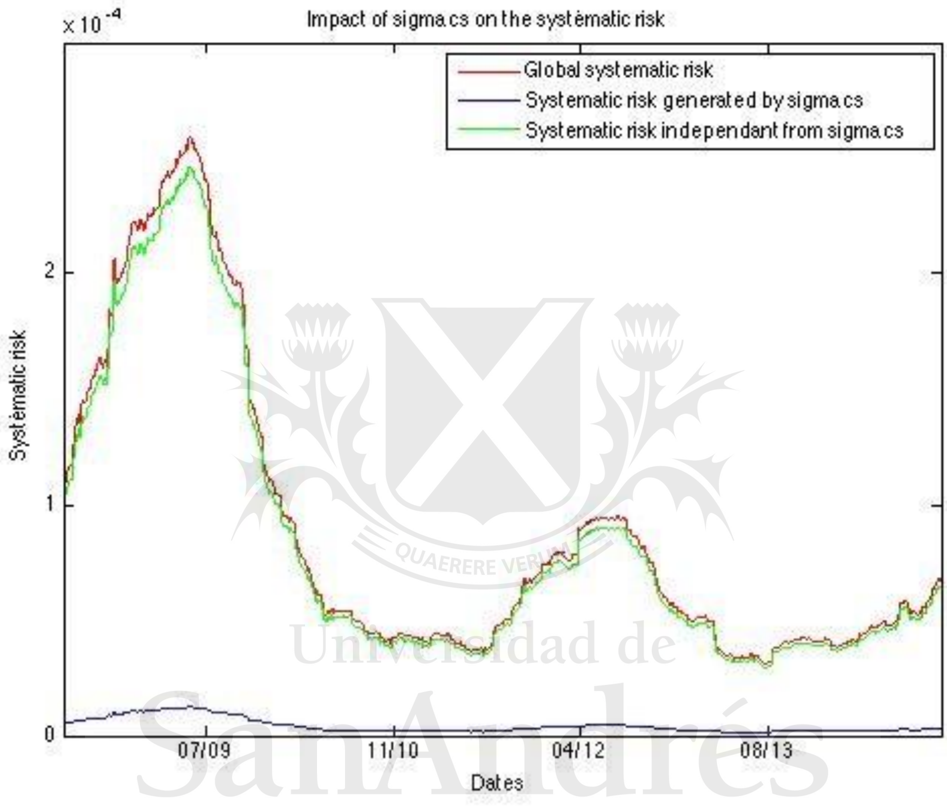
$$\sigma_P^2 = \left(\rho\sigma^2 + \frac{\sigma_{CS}^2}{N} \right) + \sigma_e^2 .$$

N is the total number of assets available for investment in the market, which is usually different than the number of assets in the portfolio n.

It appears then that the global systematic risk is unaffected by diversification.

We compare now numerical values of systematic risks (dependant and independent for sigma cs) and global systematic risk over the period studied.

We plotted our results over the period:



It clearly appears that systematic risk generated by sigma CS is negligible against the systematic risk independent from sigma CS; the hypothesis is validated.

As in time series based Modern Portfolio Strategy, once the non-systematic risk diversified, there is no more risk generated by volatility.

3) Active portfolio management

a) Portfolio optimized by Gorman formula

Gorman solves in his work the equation maximizing the expected return of a portfolio, with the formula of the vector of optimal weights.

We want to validate here, that this formula can apply on the CAC40 index based portfolio. Though our hypothesis is that relative returns of the portfolio optimized by Gorman formula are always positive (return of the portfolio better than the benchmark, index CAC40).

Optimized portfolio by Gorman is given by following equations:

$$w^*_{a,t} = \frac{E(r)_t}{a_0 \sigma^2_{cs,t}}$$

$$E(r)_t = IC_t \sigma_{cs,t} z_t$$

$$w^*_{a,t} = \frac{E(r)_t}{a_0 \sigma^2_{cs,t}}$$

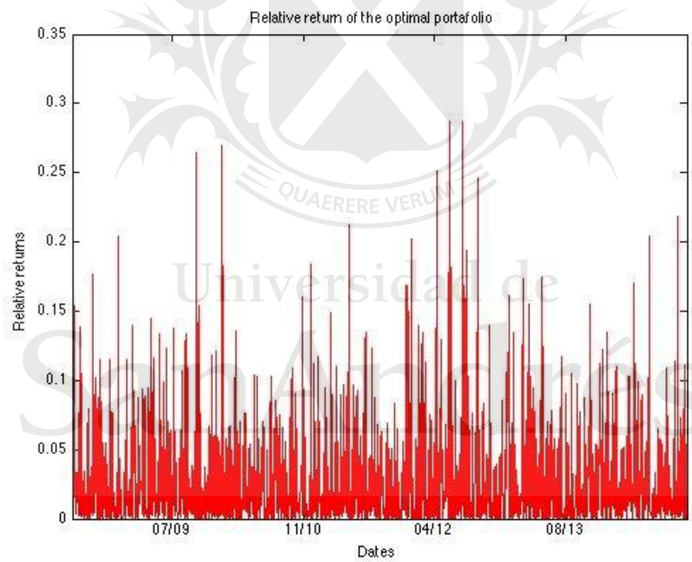
$$E(r)_t = IC_t \sigma_{cs,t} z_t$$

timal weights and:

$$\sigma_{CS,t} = \left(\frac{1}{40}\sum_{i=0}\left(\left(R_{i,t} - R_{CAC40,t}\right) - \overline{\left(R_{i,t} - R_{CAC40,t}\right)}\right)^2\right)^{\frac{1}{2}}$$

$$\sigma^2_{p,t} = \left(\rho_{TS,t} * \sigma^2_{TS,t} + \frac{\sigma^2_{CS,t}}{N}\right) + \left(\sigma^2_{CS,t} * \left(\frac{1}{n} - \frac{1}{N}\right)\right)$$

To validate that this optimal portfolio always outperform the benchmark, index CAC 40, we calculated the relative return of the portfolio over the period:



The relative returns over the period are always higher than 0, the minimum being 2,93.10^(-10). This vector of optimal portfolio underweights "looser" assets and overweights "winner" assets, the hypothesis is validated.

b) Information ratio defined by Gorman VS the Fundamental Law of Active Management

The information ratio (IR) measures a portfolio manager's ability to generate excess returns relative to a benchmark, but also attempts to identify the consistency of the investor. This ratio will identify if a manager has beaten the benchmark by a lot in a few months or a little every month. The higher the IR the more consistent a manager is and consistency is an ideal trait.

This Information Ratio (IR) has been defined by a new formula in Gorman's work, enabling to conclude strong results on the impact of Sigma CS over manager's performances.

To validate that IR by Gorman is in line with IR as known from the Fundamental Law of Active Management (FLAM) we set the hypothesis that IR as known from the FLAM is numerically similar as IR with tracking error as defined by Gorman, for the portfolio optimized.

Indeed, the information ratio IR is given by the formula:

$$IR_t = \frac{E(r)_t}{TE_t} = IC_t\sqrt{n}$$

The right hand part of the formula is the definition of the IR in the FLAM and the left hand part is the IR defined by Gorman in his work, still under the constraint on VCV matrix.

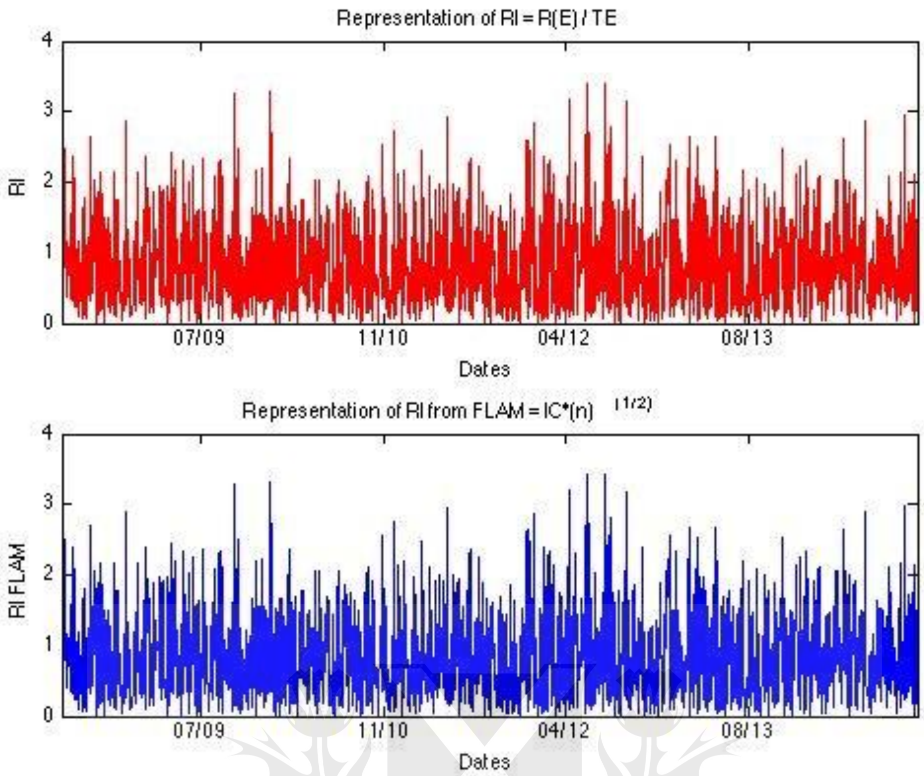
The Tracking Error TE*t*, the divergence between the price behavior of the portfolio and the price behavior of a benchmark, can be calculated with the formula:

$$TE_t = {\sigma_{cs,t}}^2 w_{at}^{*\prime} w_{at}^*$$

This tracking error $TE_t = {\sigma_{cs,t}}^2 w_{at}^{*\prime} w_{at}^*$ increase with a shock of dispersion. On the contrary, the skill of the manager would not be impacted and valuated by an IR (or IC from the FLAM) constant.

That is why this result is strong, holding IR (or IC) fixed, activereturns $E(r)_t$ are linear in cross-sectional dispersion: when dispersion is high (low), active returns will similarly be high (low).

To validate our hypothesis, we first calculate the Information Ratio by one formula and the other, and compare the numerical results over the period.



The results look similar but to get a better precision on our test, we calculate the difference RI – RI FLAM over the period.

The results are satisfying and gave us a mean of 0,0107 (or 1,3% of RI mean) and a standard deviation of 0,0080 or 9,9%.

We can consider with confidence that the test is satisfying, and that the theory of the FLAM is verified.

Conclusion:

Thanks to these empirical tests, we proved that main conclusions of Gorman in his paper are validated on the index CAC 40.

The Cross sectional volatility appears to be a better measure of the risk because it is more conservative in case of crisis. Indeed, Cross sectional volatility includes Time Series volatility effect combined with the correlation between assets of the portfolio. The less there is correlation between the assets, the higher is the cross sectional volatility.

All the risks are positively correlated to this volatility, the systematic risk is lowly correlated and the idiosyncratic risk is highly correlated but it can be reduced by diversification.

Formula of the Information Ratio of an investor deduced by Gorman accords to the theory of the FLAM, and validates a strong result: holding IR fixed, active returns are linear in cross-sectional dispersion: when dispersion is high (low), active returns will similarly be high (low). Indeed, shocks to cross-sectional dispersion do not affect manager's information ratio, because dispersion shocks result in a change in tracking error that is proportional to the change in active return.

This work would enable to use Gorman conclusions on the French market.

Bibliography:

Ernest M. Ankrim and Zhuanxin Ding, Cross-Sectional Volatility and Return Dispersion, *Financial Analyst Journal* Vol. 58 No. 5 *(*September/October 2002) 67-73

Paul Bouchey, Mary Fjelstad and Hemambara Vadlamudi – Measuring alpha potential in the market, Russell Research, (September 2010)

Gregory Connor Sheng Li, Market Dispersion and the Profitability of Hedge Funds (National University Of Ireland, January 2009)

Harindra De Silva, Steven G. Sapra, and Steven Thorley, Return Dispersion and Active Management, *Financial Analysts*, Vol. 57 No. 5 (September/October 2001) 29-42

Larry R. Gorman, Steven G. Sapra and Robert A. Weigand, The Cross-Sectional Dispersion of Stock Returns, Alpha and the Information Ratio, *Journal of Investing* (January 2010)

Larry R. Gorman, Steven G. Sapra and Robert A. Weigand, The Role of Cross-Sectional Dispersion in Active Portfolio Management, *Investment Management and Financial Innovations* (July 2010)

Richard C. Grinold, The fundamental law of active management, *The Journal of Portfolio Management*, Vol.15 N°3 (Spring 1989) 30-37

Antti Petajisto, Active Share and Mutual Fund Performance (Yale School of Management publication, January 2013)

Edward Qian and Ronald Hua, Analysis of Cross Sectional Equity Models (Putnam Investment Publication, February 2002)

Steve Sapra and Manny Hunjan – Active Share, Tracking Error and Manager Style, PIMCO Quantitative Research (October 2013)

David M. Smith, Equity Hedge Fund Performance, Cross-Sectional Return Dispersion, and Active Share, *Research in Finance*, Vol.30 (January 2014) 1-22

Bruno Solnik and Jacques Roulet, Dispersion as Cross-Sectional Correlation, *Financial Analysts Journal*, Vol. 56 No. 1 (January/February 2000) 54-61

H. Raubenheimer – Varying cross-sectional volatility in the South African equity market and the implications for the management of fund managers (University of Cap Town, February 2010)

Yu Wallace and Yazid M. Sharaiha, Alpha budgeting - Cross-sectional dispersion decomposed, *Journal of Asset Management*, Vol.8 (February 2007) 58-72